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MODELS FOR HOUSEHOLD PORTFOLIOS AND LIFE-CYCLE ALLOCATIONS IN THE PRESENCE OF LABOUR INCOME AND LONGEVITY RISK

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Abstract

Campbell (2006) stressed the two main challenges of household finance: empirical analyses that highlight how households do invest (i.e. positive household finance) have important measurement problems, while suggestions about how investments should be made (i.e. normative household finance) encounter modelling problems. In this latter connection the final aim of this paper is to highlight the modelling requirements necessary to analyse the impact of household-specific risks on their portfolios, with special focus on the financial segment. To this end, after an overview of household portfolio theory, special emphasis is given to models incorporating two increasingly more important risks for the household: labour income and longevity risks. The paper concludes with some research directions.

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Jel: D1, D91, G11, J11, J31

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1. Introduction

Recent demographic trends have given impetus to a literature analysing the impact of age on asset allocations. The possible impacts of age on household portfolios rest on the observed heterogeneity in portfolio allocations, which, beside other factors (e.g. income, wealth, education, family size etc.), is determined by the age of the household taking the financial decisions. Such portfolio heterogeneity has been studied in the empirical literature, providing detailed evidence on the issue, and in the theoretical one, aiming to find an explanation for it. More specifically, survey data provide the base for very insightful empirical contributions and can be used to analyse the decision to invest in risky asset (participation decision) and/or the portfolio patterns (allocation decisions). Both descriptive and econometric analyses support an apparent heterogeneity in terms of stock market participation and asset allocations as shown by studies for different countries (see e.g. Guiso, Haliassos and Jappelli (2002) for US, UK, Italy, Germany and Netherlands) . Three main results emerge from the empirical literature: low stock market participation, scarce diversification and a life-cycle of household portfolios that display a hump shape, whereby the investment in stocks peaks at middle ages. This evidence contradicts most portfolio models and popular financial advice suggesting investment in stock decreasing with age (e.g. Malkiel, 1996). However, most recent theoretical models have been progressively extending the seminal Merton-Samuelson model with the aim to explain empirical regularities which are at odds with the predictions of earlier models as stressed by many (e.g. Curcuru et al. (2007) , Gomes and Michaelides, 2005).

We can sum up with Campbell (2006) the two main challenges of household finance as follows: empirical analyses that highlight how households do invest (i.e. positive household finance) have important measurement problems, while suggestions about how investments should be made (i.e. normative household finance) encounter modelling problems.

In this latter connection the final aim of this paper is to highlight the modelling requirements necessary to analyse the impact of new risks for the households on their portfolios, with special focus on the financial segment. Portfolio selection models for the household are based on classical portfolio theory and have closely followed its evolution which, as for the application to the household, has also been fostered by the empirical evidence (see Brunetti (2007) for an overview). The present paper cannot - and by no means aspire to - provide an exhaustive survey of a theory which is vast, manifold and still growing. Rather, by concentrating on models that analyse both consumption/saving and asset allocation decisions, the purpose here is twofold:

i. to recall the evolution of household portfolio theory from early models to more realistic ones highlighting how the basic assumptions have been progressively, and often alternatively, released in order to capture empirical facts and life-time effects (see Sections 2 and 3);

ii. to focus on life-time asset allocation models that account for the effect of age and attain consistency with the empirical findings; in this connection preference is accorded to models which rest on assumptions that better depict two relevant sources of risk for households: labour income and the length of life. To illustrate the state of the art, two representative models will be illustrated in more detail (see Sections 4 and 5).

In other words, preference is here given to models where the causes of heterogeneity are not so much related to external factors (e.g. returns distribution, tax frictions) as rather to household-specific features (e.g. objectives and/or human capital) [1]. Finally, in the last Section we points out some open research questions.

2. The Seminal Models

The basic model for the optimal portfolio selection is the static one-period model based on the maximization of a concave Von Neumann-Morgenstern utility function of end-of-period wealth. Under specific assumptions on absolute risk tolerance or risk aversion (being one measure the reciprocal of the other) and/or the asset return distribution, this model produces a fundamental result in the theory of finance known as the two-fund separation theorem. The theorem, essentially states that all agents choose a portfolio made of two funds: the risk-free asset and a fund comprising all other risky assets. The Markowitz (1952) mean-variance formulation is by far the most well-known one. Its main implications, i.e. the risk-return trade off and portfolio diversification, hinge on either a quadratic expected utility or normally distributed returns.

More generally, the basic static model mainly suffers of two major drawbacks that are particularly apparent in connection with household portfolios: empirically is not supported by the data since real portfolios do not appear to be as diversified as predicted by the model and, theoretically, being a static model, it is not appropriate for the analysis of the life-cycle asset allocations. If the first limitation could be overcome also within the static framework by making the model setup more

[1] Throughout the paper, the viewpoint assumed is not that of an institution such as a pension fund, but that of an individual household decision maker who has to take consumption and portfolio decision over her life.

realistic (e.g. including a form of background risk such as uninsurable labour income), the second one calls for an extension of the model to a multiperiod setting[2].

In the late 1960s multiperiod versions of the basic model were proposed in the pioneering papers by Mossin (1968), Merton (1969, 1971) and Samuelson (1969). As for the modelling setup, the common features of these papers is to solve a dynamic optimization problem for a risk-averse household which maximizes expected utility subject to a budget constraint. The models can differ for the setup (discrete vs. continuous), for the specification of the utility function and the assumption on the asset return process, whereby explicit solutions can be in some cases derived depending on the utility function and the asset return process assumed.

Additionally, the models differ according to their main focus. In fact, the multiperiod setup immediately calls for consideration of optimal decision rules not only for portfolio selection, but also for consumption. However some models (e.g. Mossin, 1968) abstract from the consumption/saving problem and aim to find the optimal portfolio which is specific to retirement (i.e. there is no consumption from portfolio before retirement). It can be shown that in general the optimal multiperiod portfolio consists of two components: the so called myopic portfolio which is equal to the sequence of one-period optimal portfolios and the hedging portfolio, which serves to hedge against changes in the investment opportunities. It follows that the optimality of the myopic strategy depends on assumed features for the household objectives (i.e. the utility function) and returns process (i.e. the investment opportunities). Specifically, the myopic strategy is optimal with Constant Relative Risk Aversion (CRRA) utility - under which the optimization problem is non recursive and utility is homothetic in wealth - if excess returns are independent of the innovations in the state variables[3].

However, by considering intermediate consumption results change since the decision maker can attain smoothing in wealth shocks by means of variations in consumption. In this connection, it is useful to follow Gollier (2002) and stress three effects that play a role in determining the optimal consumption and portfolio rules. The first is the *time diversification effect*: it means that, ceteris paribus, longer horizons allow to better smooth shocks and hence younger household should take up more risk. The second one is the *wealth effect* and accounts for the role of wealth in connection with the time horizon: if the wealth level in each period changes with the horizon, so does consumption

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[2] Gollier (2002) provides an excellent survey on the classical theory of household portfolio, where also the static problem is discussed in detail. In particular, the author highlights the dual interpretation of the very same as either a static portfolio selection problem under uncertainty (Arrow-Debreu portfolio problem) or a lifetime consumption-saving problem under certainty and clearly illustrates the condition for the validity of the two-fund separation theorem (i.e. all agents must have linear absolute risk tolerance with the same slope).

[3] Hakansson (1971) discussed Mossin (1968) results and provided more stringent conditions for optimality of myopic strategies. Brandt (2009) summarizes and argues three cases where the myopic investment strategy is optimal, i.e.: constant investment opportunity set, stochastic but unhedgable investment opportunities and logarithmic utility.

per period (e.g. lower for younger households that face longer horizons) and the overall effect on risk taking depends on the relationship between risk tolerance and consumption[4]. Finally, the *repeated risk effect* captures the idea that, in a multiperiod setting, taking risk today can affect risk – taking in the future: this depends again on risk tolerance.

In sum, in a multiperiod set up, while the time-diversification effect is quite clear-cut, the latter two effects depend on the utility function taken to represent household preferences. An interesting benchmark to illustrate the joint working of these three effects is that of CRRA utility, which is a special case of Hyperbolic Absolute Risk Aversion (HARA) utility displaying linear risk tolerance: in this case, the time diversification effect offsets the wealth effect, while taking risk today does not influence risk taking in subsequent period. It follows that the myopic strategy is the optimal one[5], a result that prevails in the early models. In fact, Merton (1969) paper confirms in continuous time an important result proved by Samuelson (1969) in discrete time: for isoelastic marginal utility (i.e.CRRA) and a Wiener process for the asset price changes, the portfolio-selection decision is independent of the consumption decision and, for Bernoulli logarithmic utility, the separation goes both ways. Moreover, the classic two-fund separation result holds and Merton (1971) extends these results to more general utility functions and asset price assumptions thus showing that the classical Markowitz mean-variance rules hold without the hypothesis of quadratic utility and normal prices.

Therefore, overall seminal portfolio models provide portfolio rules that are independent of wealth, but more importantly of age so that life-cycle implications cannot be analysed. It is thus not surprising that the predictions of the classical models are markedly at odds with the empirical evidence presented in Section 2.

To make a step towards more suitable models for household portfolios, it is necessary to consider a setup which is more household specific.

3. More Realistic Portfolio Models

The review in Section 2 has highlighted some empirical regularities in household portfolios, which although different in magnitude, characterize household portfolios in most countries over the world. Specifically, three main stylized facts are apparent: low stock market participation, scarce diversification and a life-cycle of household portfolios that display a hump shape, whereby the investment in stocks peaks at middle ages .

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[4] Specifically, it is the risk-tolerance degree of homogenity with respect to consumption that determines whether the time diversification effect prevails on the wealth effect (see Gollier, 2002).

[5] Watcher (2002) shows that, assuming CRRA preferences, the possibility of intermediate consumption affects the hedging component of a portfolio only and not the myopic one, with the overall effect of shortening the time horizon of the investor.

The predictions of the seminal models recalled in the previous Section are not consistent with these empirical facts and, as a consequence, a body of literature has been growing since the 90s to reconcile theory with evidence. In doing so the more recent models, while departing from some classical assumptions, have extended and added features of realism to the model setup.

To summarize, three are the main routes the literature has taken to attain life-cycle patterns in households portfolios:

- the modelling of household preferences different from CRRA,
- the predictability in asset returns
- the consideration of trading frictions and market incompleteness.

It should be stressed that, by contrast to the classical papers where the primary assumptions (i.e. stationarity of returns and CRRA preferences) allowed for closed form solutions, most realistic models are not so easily tractable, requiring numerical and approximate solution methods.

A fundamental issue in portfolio models is modelling of household risk preferences via the utility function. In the previous Section, the role of the CRRA utility assumption clearly emerged in determining both desirable results (e.g. analytic tractability, asset demands independent of wealth) and undesirable ones, such as the independence of asset allocations from time horizon. If follows that an important generalization has concerned preferences: some authors (e.g. Campbell and Viceira, 1999) take Epstein-Zin-Weil preferences (Epstein and Zin, 1989) which are a generalization of CRRA preferences based on recursive utility. They retain the wealth scale independence of power utility, but in contrast to CRRA, allow to distinguish risk aversion from the elasticity of intertemporal substitution in consumption[6]: an important feature given that these parameters have different effects on optimal consumption and portfolio choice.

A different departure from the CRRA preference structure consists in assuming that past consumption choices affect current consumption, as in the so called habit formation (or habit persistence) proposed by Costantinides (1990), who shows that this assumption helps in explaining the equity premium puzzle (Mehra and Prescott, 1985) and variation in returns.

Based on some empirical evidence, a strand of literature (e.g. Barberis (2000), Campbell and Viceira, 1999) has assumed the predictability of asset returns. Brandt et al. (2005) also consider the case of an investor that is uncertain about the parameters of the data generating process and learns

[6] It has to be recalled that for CRRA utility functions the elasticity of intertemporal substitution is the inverse of the coefficient of absolute risk aversion. This implies an unrealistic connection between two distinct feature of household preferences: the willingness to substitute consumption intertemporally and the willingness to take up risk.

from realized returns and dividend yields: in this setting they obtain that learning reduces the allocation to stocks due to parameter uncertainty[7].

A feature neglected by the early models were trading frictions, whose consideration in the dynamic framework makes the analysis quite complex. In particular the absence of transaction costs, and specifically of fixed costs, has been demonstrated to be a possible explanation for the low levels of stock market participation (e.g. Basak and Cuoco, 1998). By contrast, the effect of proportional transaction costs is less clear cut: e.g. Costantinides (1986) concludes they do not discourage stock holding, while Heaton and Lucas (1997) find that they shift portfolios towards assets with lower transaction costs. Some kind of taxes play a role similar to proportional transaction costs in that they may prevent from portfolio rebalancing (e.g. Dammon et al., 2004).

The consideration of market incompleteness is often based on liquidity constraints or short selling restrictions, which in fact impede intertemporal smoothing of consumption and portfolio return and can provide an alternative explanation for non participation. Frictions are also behind models that consider the role of uninsurable labour income in explaining life-cycle asset allocations.

In sum, three are the main issues in long-term portfolio choices: the stochastic opportunity set, the consideration of illiquid assets (and particularly labour) and the uncertainty in the length of life. As for the former issue, many papers have contributed especially from the late 90s by considering interest rate or inflation risk and time-varying risk premia, which are more tractable and hence relevance for portfolio choices of institutions[8]. Given the focus of this paper on individuals' decisions, the attention is restricted to the inclusion into life-cycle models of the latter two issues, which are more household specific and increasingly necessary to attain features of realism in the decision scenario of most households. For these reasons in the next Sections we will concentrate on the role of labour income risk and uncertainty in the length of life.

4. Life-cycle Asset Allocation Models in the presence of uninsurable labour risk

A crucial element in setting up a model for life-cycle asset allocation is to account for the main reasons that motivate wealth accumulation, i.e. the precautionary savings motive connected with background risks and the bequest motive.

The most important background risk for household is possibly labour risk, which has been in fact considered in connection with portfolio choices by a strand of literature starting with the Merton

[7] Among others, see also a recent issue of Review of Financial Studies (Vl.21, 4, 2008) where a few articles are centred on the return predictability debate, which is far from being resolved.

[8] Examples are: Campbell and Viceira (1999, 2001), Watcher (2002). More references in the survey by Brandt (2009).

(1971). The author analyses the issue in a framework where labour income can be capitalized and hence the risk insured.

By contrast, in order to see the effect of labour risk on portfolio choices it is fundamental to assume incompleteness so that labour income risk has to be considered, more realistically, uninsurable.

In this connection Cocco et al. (2005) provide a model for consumption and portfolio choices in the presence of uninsurable labour income risk, which is becoming a milestone in the analysis of this issue.[9] The article provides life-cycle consumption and portfolio rules for a realistically calibrated model with non-tradable labour income and borrowing constraints. Although in other and subsequent papers, the model has been extended, the main feature and implications are already present in Cocco et al. (2005).[10] For this reason, we believe it is worth to illustrate the major model assumptions and their implications in order to highlight what are the modelling characteristics that allow to capture features of realism of portfolio rules such as the dependence of asset allocations on age.

Intuitively, the main departure of Cocco et al.(2005) from classical models is the inclusion of uninsurable labour risk (i.e. markets are incomplete). Specifically, the authors maintain that moral hazard problems, via borrowing constraints, prevent household to capitalize future labour income and thus labour income is a risky asset in household portfolios. However, labour income risk has no or very low correlation with the other financial assets: it is thus a substitute of the risk-free asset and plays a role in terms of portfolio diversification: e.g. young households, who already own a sort of risk-free asset in the form of labour income, tend to hold more risky asset than older households, who by contrast have lower "risk-free" labour holdings. Overall, as the authors conclude, "*the share invested in equities is roughly decreasing with age. This is driven by the fact that the labour income profile itself is downward sloping*", a result that is attained with no need to rest on the predictability of asset returns.

To get more insight into the model, its limitations and extensions it is worth to illustrate the benchmark model proposed in Cocco et al. (2005) where, beside the standard ingredients of an optimal portfolio choice problem (i.e. preferences, asset returns processes and various constraints), the modelling labour income risk plays an important role.

[9] Also Heaton and Lucas (1997), Koo (1998) and Viceira (2001) analyse the effect of uninsurable income risk on portfolio composition, but they do it in a infinite-horizon setting and hence in a stationary setup which is less appropriate for the analysis of life-cycle pattern. More related to Cocco et al. (2005) is Bertaut and Haliassos (1997) who also assume a finite horizon.

[10] In Cocco et al.(2008) the authors allow for flexible labour supply but results remain overall qualitatively similar to Gomes et al.(2005) although the ability to increase labour supply represents an alternative to an increase in savings against future income uncertainty so that the portfolio pattern can be less conservative with respect to the case of fixed labour supply.

Let K be a deterministic and exogenous working age, T the uncertain length of life and p_t the probability that the household-investor is alive at date $t+1$. The household i is assumed to have time-separable CRRA *preferences* of the following type:

$$E_t \sum_{t=1}^{T} \delta^{t-1} \left(\prod_{j=0}^{t-2} p_j \right) \left\{ p_{t-1} \frac{C_{it}^{1-\gamma}}{1-\gamma} + b(1-p_{t-1}) \frac{D_{it}^{1-\gamma}}{1-\gamma} \right\} \quad (1)$$

where: $\delta<1$ is the discount factor, $\gamma>0$ is the coefficient of relative risk aversion, C_t and D_t are, respectively, the consumption level and the amount of bequest at time t.

The exogenous *labour income* process, for $t<K$, is assumed to be the sum of a deterministic component that can capture the hump shape of earnings over the life-cycle and a stochastic one, which is made of a persistent part (v_{it}) and a transitory shock (ε_{it}):

$$\log Y_{it} = f(t, Z_{it}) + v_{it} + \varepsilon_{it} \quad (2)$$

with

Z_{it} = vector of individual characteristics

ε_{it} distributed as $N(0,\sigma_\varepsilon^2)$

$$v_{it} = v_{it-1} + u_{it} \quad (3)$$

u_{it} is uncorrelated with ε_{it} and distributed as $N(0,\sigma_u^2)$.

While the transitory component is uncorrelated across households, the permanent shock u_{it} can be decomposed in an aggregate component and a transitory one, both normally distributed with zero mean and constant variance:

$$u_{it} = \xi_t + \omega_{it} \quad (4)$$

The *retirement income*, for $t>K$, is assumed to be a constant fraction λ in the last working year:

$$\log Y_{it} = \log(\lambda) + f(t, Z_{ik}) + v_{ik} \quad (5)$$

In the financial markets, two assets exist: a risk-free asset and a risky one whose gross real *excess return* over the risk-free is modelled as:

$$\boldsymbol{R}_{t+1} - \overline{\boldsymbol{R}}_f = \mu + \eta_{t+1} \tag{6}$$

where $\overline{\boldsymbol{R}}_f$ is the risk-free return and the innovation η_{t+1} is assumed to be normally i.i.d. with constant variance, but correlated with the aggregate component of labour income with a coefficient ρ.

A crucial assumption is the *borrowing constraint* since it impedes the household to capitalize or borrow against future labour income or retirement wealth:

$$\boldsymbol{B}_{it} \geq 0 \tag{7}$$

It is justified by moral hazard/adverse selection arguments, which as the authors stress are particularly stringent in the early years of the household adult life.

The *short-selling constraint*:

$$\boldsymbol{S}_{it} \geq 0 \tag{8}$$

implies non negative allocation in equities at all dates. Borrowing and short-selling constraints imply that the proportion invested in equities is $\alpha_{it} \in [0,1]$ and wealth is non-negative.

Against this setup, the household *i* in period t start with a wealth W_{it} and maximizes (1) subject to constraint (2)-(8) in order to obtain optimal consumption and portfolio rules:

$C_{it}(X_{it}, v_{it})$ and $\alpha_{it}(X_{it}, v_{it})$

which are a function of the state variables: time *t*, cash-on-hand ($X_{it} = W_{it} + Y_{it}$) and the stochastic persistent component of labour income, v_{it}.

Although the dimensionality of the problem can be reduced[11], the model cannot be solved analytically and the authors obtain numerical solutions by backward induction, after appropriate calibration of the model to real data. Given the role of labour income in the model, particular attention is devoted to the calibration of the corresponding process which is done on PSID data (a longitudinal US Panel Study on Income Dynamics) and in line with the literature on the subject (e.g. Attanasio (1995), Hubbard et al., 1995).

Simulations results are presented for the benchmark case of the second education group (i.e. high school) but are shown to remain qualitatively unaltered for the other income groups: this is due to the fact that in this model the different groups are solely characterized by the age at which working age begins. A stronger characterization of the benchmark case lies in the correlation between labour income risk and the stock market, which is assumed to be absent (i.e. ρ=0).

As for the life-cycle pattern of portfolio choices, the result is quite clear: the investment in stocks is roughly decreasing with age. To understand this result, recall that in the paper labour is essentially

[11] The value function is in fact homogeneous with respect to v_{it} which can be normalized to one.

characterized as a bond-like asset and portfolio decisions are determined by the household labour income profile also in relation to wealth. Thus young household, who have a very steep labour income profile, display a rapidly increasing implicit riskless holding (represented by labour income) and diversify by investing in stocks. Later in life the labour income profile is not so steep and the portfolio rule is evaluated at higher wealth levels so that the portfolio moves away from stocks. This overall result rationalizes and supports professional advice suggesting to shift portfolio towards relatively riskless assets as the household ages[12], but, as discussed below, this is in contrast to most empirical evidence on the issue (see Section 2).

However, some extensions considered in the paper attain results which are closer to the empirical evidence. For example, an empirically calibrated small probability of disastrous labour income draw lowers stock holding and produces heterogeneity in young household portfolio choices, while endogenous borrowing can explain non participation decisions of young household. Moreover, the sensitivity analyses performed in many directions pave the way to many subsequent literature contributions. Worth mentioning is the sensitivity analysis with respect to labour income risk and in particular to its correlation with stock returns: a positive correlation has significant portfolio effects indicating a lower level of stock for young and a higher level for middle-aged households. This line of research is taken up by Benzoni et al. (2007) as described later.

A related paper is Gomes and Michaelides (2005) where the labour income process is calibrated as in Cocco et al. (2005), but preferences are Epstein-Zin and a fixed entry cost is assumed. The objective is to provide theoretical support for empirical findings on participation rates and asset allocations conditional on participation: heterogeneity in risk aversion seems explain both. In fact, on one hand households with small risk aversion and small elasticity of intertemporal substitution smooth earning shocks with little buffer wealth and this explain low participation, on the other more risk-averse households accumulate more wealth and hence participate in the stock market since young, but do not invest the whole portfolio in it.

As for the role of labour income risk, Cocco et al. (2005) and related extensions highlight that the assumption of null correlation with stock market risk impedes to obtain realistic portfolio rules. Based on previous evidence on the correlation between human capital and market returns, Benzoni et al. (2007) study, in a continuous time model, the optimal portfolio choice over life-cycle in a setup that is essentially the same[13] as the one just described for Cocco et al. (2005), but for the correlation assumption. Specifically, the author assume that the aggregate component of labour

[12] The typical reference here is the rule suggesting to place (100-age)% of wealth in a well-diversified stock portfolio (see Malkiel, 1996).

[13] Benzoni et al. (2007) do not explicitly model retirement income, but calibrate the bequest function so as to capture the saving necessary for consumption in the retirement years. This is equivalent to assume that the household receive an annuity in retirement years.

income (the equivalent of ξ_t in eq. (4)) is cointegrated with aggregate dividends and hence with the stock returns. The cointegration assumption makes labour income more of a stock-like asset (than a bond-like one as in Cocco et al., 2005). More precisely, the cointegration is modelled as a mean reverting process with *k* being the coefficient of mean reversion, whereby if *k=0* there is no cointegration. Its inverse *1/k* provides the time necessary for cointegration to act. It follows that if the residual working life is long (i.e. young households), the return on the household's human capital is highly exposed to stock market returns and labour income resembles more a risky asset than a risk-free one so that young households, who are already overexposed to stock market risk, find it optimal to go short in stocks or, in the presence of borrowing constraint, to invest the entire wealth in the risk-free bond. For middle-aged the cointegration has no time to act, labour income has bond-like features and the opposite is true. This is still true in the years before retirement, but for sufficiently short time before retirement a second effect prevails: due to lower future labour income, the value of the bond position implicit in human capital decreases and the household start reducing stocks in favour or risk-free assets. This is the hump-shape profile that Benzoni et al.(2007) obtain in the simulations of their model, which, in line with the literature is solved numerically by backward induction using standard difference method. Beyond robustness exercises, the authors also prove that the result still holds in the presence of return predictability.

In sum, by contrast to similar models (even micro-data calibrated ones) and conventional prefessional wisdom, Benzoni et al. (2007) attain a hump-shaped pattern for life-time portfolio that is consistent with most empirical evidence on the topic. The result provides an explanation to limited stock market participation that the authors stress to be different but complementary to those typically put forward in the literature: *"In particular, our paper emphasizes that long-run cointegration between aggregate labor income and aggregate dividends has a first-order effect on the optimal portfolio decisions of an agent over the life cycle."*.

However, it has to be stressed that the fundamental assumption for this result is questionable in that difficult to test. Evidence on the cointegration between human capital and market return is disparate if not weak, due to the well-known lack of power of cointegration tests. Moreover results are very sensitive not only to the very same existence of a cointregation but also to the specific level[14].

[14] In fact, as the author stress, it is econometrically very difficult even to distinguish between k=0 and k=0.05 and the peak of the hump-shape of the portfolio pattern depends on the value of the coefficient governing the cointegration k.

5. Life-cycle Asset Allocation Models in the presence of annuities

The stochastic nature of an household investment horizon calls for the consideration of a further source of risk, which is known as longevity risk, i.e. the risk for the investor of living longer that predicted and hence run out of saving.

To account for this issue, household portfolio models have to consider uncertainty in the length of life in connection with the role played by a particular type of assets, annuities, which in fact allow to transfer longevity risk to the insurer but are an illiquid instrument[15].

The question in a portfolio framework becomes the optimal investment in annuities. The seminal paper to answer this question is Yaari (1965), which ignores other sources of risks other than mortality and provides conditions for full annuitization, i.e. market completeness and no bequest motive, otherwise partial annuitization becomes optimal. Later on, portfolio models have analysed the case of constant life annuities (i.e. annuities providing a fixed payout) but they have often simplified the problem by either imposing full annuitization (e.g. Cairns et al., 2006) or disregarding other important features of the decision problem, such as the irreversibility of the annuities purchase, other sources of risks (e.g. Richard, 1975) or the impact of annuity markets during working life (e.g. Milevsky and Young (2007) consider the case of a retiree).

However, in their recent work Horneff et al. (2008)[16] overcome some limitations of the previous literature and study the optimal consumption and saving strategy in the presence of constant life annuities, bonds and stocks in an incomplete market setting. The model allows for gradual purchase of annuities and considers the three main sources of risks faced by an household: risky stocks, untradable labour income during working life and stochastic time to death. In connection with the latter two features their model can be seen as extending those presented in the previous Section. In particular, the model shares many features of Cocco et al.(2005) and some of the parameters taken for the numerical solution of the optimization problem are borrowed from the former.

More precisely, the problem differs from the one represented by equation (1)-(8) in the following:

i. Since Epstein-Zin preferences are assumed, equation (1) is replaced by the recursive formulation of intertemporal utility that allows to disentangle risk aversion γ from the elasticity of intertemporal substitution ψ.

[15] An overview on the treatment of longevity risk from the viewpoint of annuity providers and pension plans is provided by Biffis and Blake (2009).

[16] Cocco and Gomes (2008) also consider longevity risk and the role of longevity bond in the optimal consumption/saving problem, but they do not study the optimal asset allocation.

ii. An incomplete annuity market is considered. Against the payment of an actuarial premium A_t, the annuitant receives a constant payment L until death.

$$\boldsymbol{A_t} = \boldsymbol{Lh_t} \text{ with } \boldsymbol{h_t} = (1+\delta)\sum_{s=1}^{T-t}\left(\prod_{u=t}^{t+s}\boldsymbol{p_u^a}\right)\boldsymbol{R_f^{-s}} \tag{9}$$

where δ= loading factor and $\boldsymbol{p_u^a}$= survival probability used by the annuity provider, which is higher that the average survival probabilties $\boldsymbol{p_u^s}$.

The annuity provider hedge the guaranteed annuity payments by pooling mortality risks of annuitants: the funds of those who die are allocated among the living member of a cohort and this represent the source of the so called mortality credit, i.e. the excess annuity return over a bond. The market is incomplete because only life-long payouts are available and funds from annuity are invested in bonds only.

iii. Mortality is modelled by means of the Gompertz law. The force of mortality used by the provider and the subjective one are specified as the following function of the parameters m^i and b^i:

$$\lambda_i^t = \frac{1}{\boldsymbol{b^i}}\exp\left(\frac{\boldsymbol{t}-\boldsymbol{m^i}}{\boldsymbol{b^i}}\right) \quad \text{with } i=a,s \tag{10}$$

and the survival probability is then given by

$$\boldsymbol{p_t^i} = \exp\left(-\int_0^1 \lambda_{t+s}^i \boldsymbol{ds}\right) \tag{11}$$

Additionally the subjective force of mortality is taken to be a linear transform of the one derived from average mortality tables.

It follows that wealth accumulation is determined by annuities as well and a borrowing constraint is placed on the annuities too ($A_t \geq 0$). In this setup, beyond demand for stocks, bonds and consumption, there is one more choice variable that has to be determined: the optimal level of annuities in each period. Moreover all policy rules are a function of the same state variables as in Cocco et al. (2005) plus the annuity payouts form previously purchased annuities.

There is a trade-off between liquid financial savings and illiquid annuities which provide the mortality credit. The introduction of annuities thus poses a central question: is the mortality credit high enough to compensate for the illiquidity of the annuity? The irreversibility of the annuity purchase makes the consideration of labour income risk is even more important, given the need for liquidity that income shocks normally bring by. Moreover extreme income shocks can also be interpreted as reflecting healthcare and nursing costs and hence the analysis in relation to annuities becomes even more important.

The model is first solved for the baseline case (no loads, no asymmetries between insurer's and anunuitant's beliefs, no bequest) so as to isolate the role of the annuities in the portfolio problem. As for investment in stocks and bonds results are qualitatively very similar to Cocco et al. (2005) given that the setup implies the same interpretation of labour income, which is more a bond-like asset than a stock-like one (as in Benzoni et al., 2007). However, the presence of annuities in most cases crowds out bonds thus indicating that mortality credit compensate for illiquidity of annuities. Moreover the optimal annuity holding increases over time in contrast to stock holdings and the explanation, as in Cocco et al. (2005) rests on the characterization of labour as a bond-like asset, whose holding decreases with age.

Sentitivity analyses allow to highlight important determinants of optimal annuities holdings. As expected the correlation between human capital and market risk increases the purchase of annuities at least until the retirement period (since retirement income is assumed uncorrelated with the market), while during retirement the purchase of annuities depends positively on the retirement income replacement ratio but does not vanishes even if the latter were set to one, a result that can help in the debate over the pension system.

6. Conclusions and research directions

Since the mid of 1990s, the nexus between age and finance has inspired a lively debate, which has given rise to an increasing number of empirical studies highly diversified in terms of approach, methodology and data. Overall, however, three facts most strikingly emerge from the empirical literature: low stock market participation, scarce diversification and a life-cycle of household portfolios that display a hump shape, whereby the investment in stocks peaks at middle ages[17]. This evidence contradicts popular financial advice suggesting investment in stock decreasing with age and most portfolio models. The overview of this paper highlights that, in order to capture real portfolio patterns, the dynamic nature of most recent models has to be coupled with an appropriate modelling of household specific assets, which share a common feature: illiquidity. The

[17] A related literature investigates the effects that demographic patterns may ultimately have on financial returns (e.g. Poterba (2001, 2004) for US and Brunetti and Torricelli (2009) for Italy).

most important among these is surely human capital, which materializes in the portfolio framework via labour income. This is why particular emphasis was devoted to Cocco et al. (2005) where the inclusion of labour income risk is made in an incomplete market setting so that labour income risk is uninsurable, an assumption that well represents real market conditions. However, an important source of background risk such as labour income risk is not sufficient to explain the hump-shaped pattern of observed portfolio choices if it is modelled as independent of market risk. This is the line taken in Benzoni at al. (2007), where a long-run cointegration between human capital and the stock market is assumed and portfolio rules are concave with respect to age.

We believe this is the direction that future research should take. More specifically, models for household portfolio decisions should account for three main features affecting household decisions in a specific way: finite horizon together with longevity risk, borrowing constraints and non financial assets. As for the latter, it has to be highlighted that non financial wealth represents the major part of an household portfolio so that its features in terms of tradability, insurability and correlation with the stock market are of uttermost importance in determining financial choices.

The two most important component of non financial wealth are by large labour income and housing. A literature that addresses housing choices in connection with portfolio choices has been growing (e.g. Cocco (2004), Flavin and Yamashita (2002), Yao and Zhang, 2005). However, the different nature of housing with respect to other assets (consumption and investment) and its often leveraged status requires a specific focus and deserves a separate study, but its consideration is in some cases essential to explain some portfolio puzzles (e.g. in Cocco (2004) housing crowds out stock holding). As for the link between labour income and financial markets, Benzoni et al. (2007) shows how the correlation between human capital and market returns can change the optimal household portfolio and suggests the inclusion of housing in their setup. This is interesting especially if evidence on the cointegration between the housing and the stock market is strong. However, since results under the cointegration assumption are very sensitive not only to the very same existence of a cointegration (which is also hard to test) but also to the specific level, very high on the research agenda is still the modelling of the link between labour income and financial markets. In particular, the long-run cointegration assumption essentially focus on the time dimension of the link between human capital and financial markets, whereby the idea is that in the long-run only young people will be alive and thus hit by the long-run cointegration. However, we believe that another important dimension to be considered in this connection is the type of human capital as captured for instance by the different education groups, which might deserve more differentiation in term of labour income risk (e.g. recall that in Cocco et al. (2005) education groups are solely characterized by the age at which working age begins). Also the modelling of the post-retirement period leaves room for further investigation: if in Cocco et al. (2005) retirement income is simply a constant fraction of labour income, in Benzoni et al. (2007) the post-retirement consumption and investment decisions are not explicitly modelled. Moreover there is a further element that differentiates the household in terms of income risk beside age: it is the connection between gender and marital status, which has been rarely considered in the portfolio setting (e.g. Bertocchi et al., 2009) and yet not in relation to life-cycle choices.

In the latter two sections of the paper we have illustrated and recalled the numerical solution problems and calibration issues implied by more realistic models. In this connection, we believe that household finance can benefit of a large and useful literature developed in connection with institutional portfolio choices (see e.g. Geyer and Ziemba (2008), Rudolf and Ziemba (2004), Zenios and Ziemba (2006)). In addition, more research on modelling is also needed to provide some support to financial advice and for economic policy considerations in view of current and perspective socio-economic scenarios increasingly characterized by more volatile labour income, important changes in the family structure, population ageing and less generous public pension systems. A further source of worry comes from recent financial market downturns (two in a decade) that cast doubts over the possibility of pension funds and annuity providers to cover, by relying on the market only, demographic risks typical of an ageing society (e.g. Visco, 2008).

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