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Asymmetric semi-volatility spillover in a nonlinear model of interacting markets

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Abstract

This paper develops an heterogeneous agents model with fundamentalists and chartists trading in two different speculative markets. It examines whether investors' behaviour is related to the volatility and its dynamics. We find that investors' heterogeneity in price trends and trading strategies can significantly explain asymmetry in semi-volatility transmission.

Keywords: spillover effects; market risk; asymmetric semi-volatility; numerical simulations

1 Introduction

There is a rich literature contributing to the identifications and duplication of stylized facts of stock returns in financial markets. Some of these regularities can be summarized as asymmetry, excess of kurtosis, volatility clustering, leverage effect and long-range dependence. In particular, the line of research focusing on the role of agent-based financial market models have been quite successful at replicating these facts (see e.g. Brock and Hommes (1998) Chiarella *et al.* (2002), Tramontana *et al.* (2009)). Although, these models are able to match quite well the stylized facts of stock returns, their majority focus on a single market or one risky asset. There are few exceptions that extend the heterogeneous agents models (HAM) framework to price dynamics of multi-asset or multi-markets dynamics. Westerhoff (2004) proposes a multi-asset market model where technical traders can switch between several financial markets. He demonstrates that the stability of the markets can be affected if the composition between chartists and fundamentalists varies. Westerhoff and Dieci (2006) present a model with fundamentalists and chartists that are free to trade in two different markets. The authors show that a uniform transaction tax may stabilize all markets. Huang and Chen (2014) analyse the propagation of financial crisis considering a two-market heterogeneous agents model where each trader can invest in each market.

In these models, the link between markets can occur in different ways. A common approach consists of fitness measures affecting investor composition, as in the seminal paper of Brock and Hommes (1998). The

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works of Westerhoff and Dieci (2006), Anufriev *et al.* (2012) and Huang and Chen (2014), for example, belong to this group. Another proposal is to develop HAM of multi-assets in continuous time as in He *et al.* (2018). In this case, two markets are integrated into one market via the introduction of cross-sectional momentum traders. In addition, Dieci and Westerhoff (2010) consider a three-market model in which two stock markets are linked via foreign exchange market. Finally, Day and Huang (1990) show that nonlinear trading rules may cause endogenous price dynamics with random switching between bull and bear markets.

Our paper combines two literature strands: (i) the first focuses on multi-markets HAM in economics and finance, while (ii) the second concentrates on the impact of asymmetric behaviour of investors in relation to downside and upside risk. In financial markets literature, this phenomenon is known as asymmetric volatility. It is related to the fact that positive changes in volatility are associated with negative returns more often than negative changes in volatility are associated with positive returns. In this connection, in this paper we rely on the total spillover index (TSI) of Caloia *et al.* (2018) as a proxy for the system overall connectedness of markets showing that the volatility transmission exhibits a certain degree of asymmetry.

Our contributions to the existing literature are as follows. First, we follow the framework of Westerhoff and Dieci (2006) and Campisi and Muzzioli (2020). We consider two types of traders, fundamentalists and chartists, who place orders in two speculative markets with the same transaction currency and on the same asset. It is assumed that, in each market, traders switch to the strategy that they believe to perform better. Moreover, the switching mechanism is based on the TSI index of Caloia *et al.* (2018) which is intended to capture the overall connectedness of markets. It considers the ratio of total cross-variance shares over the total (own and cross) variance shares.

Second, we demonstrate that the model is able to replicate the most important stylized facts of financial markets. Indeed, we consider the empirical facts that simultaneously involve multiple markets. To this purpose, following Caloia *et al.* (2018) we use downside and upside volatility as proxies of downside risk and upside opportunities in order to analyse the asymmetries in volatility transmission.

The paper is structured as follows. In Section 2 we present the model where two types of speculators, fundamentalists and chartists, trade in two stock markets. In Section 3 we conduct different numerical simulations of the model to verify its capability to match typical stylized facts documented in the literature, especially asymmetry in semi-volatility transmission. Section 4 concludes.

2 The model

In this section we develop a simple behavioural financial model which allows us to analyse interactions between two speculative markets. We study a two-dimensional discrete-time dynamic model which describes the dynamics of two markets where the same asset is traded. As stressed by Anufriev *et al.* (2012), in financial literature has been shown that the same asset traded in different markets tend to behave differently. In line with this evidence, following Westerhoff and Dieci (2006), Schmitt and Westerhoff (2014) and Huang and Chen (2014), we consider two markets, 1 and 2, each populated by three types of traders (fundamentalists, chartists and a market maker). Fundamentalists believe the stock price returns towards its fundamental value, in this regard, they buy stocks if the current price is below the fundamental value, while they sell stocks if the current price is above the fundamental value. The orders placed by fundamentalists in market i, with $i = 1, 2$ are given by:

$$D^{f_i}_{i,t} = a^{f_i}(F_{i,t} - P_{i,t}) + \epsilon^{f_i}_t \tag{1}$$

where parameter $a^{f_i} > 0$ captures the convergence of the market price i to the expected fundamental value. $\epsilon^{f_i}_t \sim N(0, (\sigma)^{f,i})$ is an independent normally distributed random variable with zero means and constant variance.

Chartists behave in exactly the opposite way to fundamentalists. Indeed, they follow the trend of the market, in this sense, they buy stocks if the current price is above the price of the previous period, otherwise they sell stocks. Their orders in market i, with $i = 1, 2$ can be written as:

$$D^{c_i}_{i,t} = a^{c_i}(P_{i,t} - P_{i,t-1}) + \epsilon^{c_i}_t \tag{2}$$

with a^{c_i} is a positive reaction parameter. $\epsilon^{c_i}_t \sim N(0, (\sigma)^{c,i})$ is an independent normally distributed random variable with zero means and constant variance.

Regarding the market fractions of traders, we follow Campisi and Muzzioli (2020) assuming that the fraction of chartists evolves according to the TSI index, that is:

$$W^{c_i}_{i,t} = \frac{D^{c_j}_{j,t} + D^{f_j}_{j,t}}{D^{c_j}_{j,t} + D^{f_j}_{j,t} + D^{c_i}_{i,t} + D^{f_i}_{i,t}} \tag{3}$$

while the fraction of fundamentalists is determined as follows:

$$W^{f_i}_{i,t} = 1 - W^{c_i}_{i,t} \tag{4}$$

with $i, j = 1, 2$ and $i \neq j$. From Eqs. (3)-(4), we can infer some important facts. First, the fractions of traders are relative to the total demand of all the markets. This represents the first element that connects each market to the other. Second, fundamentalists fraction takes into consideration only the total demand of the market where they trade relative to the total demand. Indeed, expanding Eq (4), the fraction of fundamentalists is equal to:

$$W^{f_i}_{i,t} = \frac{D^{c_i}_{i,t} + D^{f_i}_{i,t}}{D^{c_j}_{j,t} + D^{f_j}_{j,t} + D^{c_i}_{i,t} + D^{f_i}_{i,t}} \tag{5}$$

While the chartists fraction is based on the total demand of the other market relative to the total demand (see Eq (3)). In line with (Caloia *et al.*, 2018), the two agents act as spillovers of volatility. In fact, the

TSI index can be decomposed into two parts:[1]

$$TSI^i = DS^i_{*\to} + DS^i_{\to *} \tag{6}$$

with $i, j = 1, 2$ and $i \neq j$. $DS^i_{*\to}$ is the directional spillover received by market i from the other market j, while $DS^i_{\to *}$ is the directional spillover transmitted by market i to the other market j. In our model $DS^i_{*\to}$ corresponds to the fraction of chartists (Eq. (3)) , while $DS^i_{\to *}$ is the equivalent of the fraction of the fundamentalists (Eq. (5)). Given that, in our model, $W_i^{f_i}$, $W_i^{c_i}$ are the fractions of fundamentalists and chartists of type i in market i, with $i = 1, 2$, respectively, and $W_i^{f_i} + W_i^{c_i} = 1$ for every $i = 1, 2$, we conclude that the total number of agents in each market give us a proxy for the system overall connectedness(i.e. the Total Spillover Index).

The stock market is characterized by the presence of a market maker that sets the stock (log) price $P_{i,t+1}$ with $i = 1, 2$ according to total excess demand:

$$T : \begin{cases} P_{1,t+1} = P_{1,t} + a^m \Big(W^{f_1}_{1,t} D^{f_1}_{1,t} + W^{c_1}_{1,t} D^{c_1}_{1,t} \Big) \\ P_{2,t+1} = P_{2,t} + a^m \Big(W^{f_2}_{2,t} D^{f_2}_{2,t} + W^{c_2}_{2,t} D^{c_2}_{2,t} \Big) \end{cases} \tag{7}$$

where a^m is a positive price adjustment parameter that measures the market power of traders. Taking into account the trading strategies and the corresponding fractions in (1)-(4), System (7) takes the following form:

$$T : \begin{cases} P_{1,t+1} = P_{1,t} + a^m \Big[\Big(\dfrac{D^{c_1}_{1,t} + D^{f_1}_{1,t}}{D^{c_2}_{2,t} + D^{f_2}_{2,t} + D^{c_1}_{1,t} + D^{f_1}_{1,t}} \Big) * D^{f_1}_{1,t} + \Big(\dfrac{D^{c_2}_{2,t} + D^{f_2}_{2,t}}{D^{c_2}_{2,t} + D^{f_2}_{2,t} + D^{c_1}_{1,t} + D^{f_1}_{1,t}} \Big) * D^{c_1}_{1,t} \Big] \\ P_{2,t+1} = P_{2,t} + a^m \Big[\Big(\dfrac{D^{c_2}_{2,t} + D^{f_2}_{2,t}}{D^{c_2}_{2,t} + D^{f_2}_{2,t} + D^{c_1}_{1,t} + D^{f_1}_{1,t}} \Big) * D^{f_2}_{2,t} + \Big(\dfrac{D^{c_1}_{1,t} + D^{f_1}_{1,t}}{D^{c_2}_{2,t} + D^{f_2}_{2,t} + D^{c_1}_{1,t} + D^{f_1}_{1,t}} \Big) * D^{c_2}_{2,t} \Big] \end{cases} \tag{8}$$

Based on the above considerations, introducing the auxiliary variables $X_{1,t}$ and $X_{2,t}$, we obtain a first order four-dimensional dynamical system, which describes the prices evolution over time in each market, driven by the following 4-D map $S : (X_{1,t}, X_{2,t}, P_{1,t}, P_{2,t}) \mapsto (X_{1,t+1}, X_{2,t+1}, P_{1,t+1}, P_{2,t+1})$

$$S : \begin{cases} X_{1,t+1} = P_{1,t} \\ X_{2,t+1} = P_{2,t} \\ P_{1,t+1} = P_{1,t} + a^m \Big[\Big(\dfrac{D^{c_1}_{1,t} + D^{f_1}_{1,t}}{D^{c_2}_{2,t} + D^{f_2}_{2,t} + D^{c_1}_{1,t} + D^{f_1}_{1,t}} \Big) * D^{f_1}_{1,t} + \Big(\dfrac{D^{c_2}_{2,t} + D^{f_2}_{2,t}}{D^{c_2}_{2,t} + D^{f_2}_{2,t} + D^{c_1}_{1,t} + D^{f_1}_{1,t}} \Big) * D^{c_1}_{1,t} \Big] \\ P_{2,t+1} = P_{2,t} + a^m \Big[\Big(\dfrac{D^{c_2}_{2,t} + D^{f_2}_{2,t}}{D^{c_2}_{2,t} + D^{f_2}_{2,t} + D^{c_1}_{1,t} + D^{f_1}_{1,t}} \Big) * D^{f_2}_{2,t} + \Big(\dfrac{D^{c_1}_{1,t} + D^{f_1}_{1,t}}{D^{c_2}_{2,t} + D^{f_2}_{2,t} + D^{c_1}_{1,t} + D^{f_1}_{1,t}} \Big) * D^{c_2}_{2,t} \Big] \end{cases} \tag{9}$$

where $D^{c_i}_{i,t} = a^{c_i}(P_{i,t} - X_{i,t}) + \epsilon^{c_i}_t$ with $i = 1, 2$.

[1] In order to make comparison with the paper of Caloia *et al.* (2018) easier, we maintain the same terminology adopted by the authors.

3 Semi-volatilities spillovers and stylized facts

The model presented in the previous section, and summarized by Map (9), describes our HAM aimed at studying how volatility is transmitted from one market to the other and, its consequences to the price dynamics. The channel through which volatility spreads among markets is the sentiment index, that relies on the TSI of Caloia *et al.* (2018). In the building of our model, we have seen that the sentiment index is responsible for the switching mechanism of traders. Overall, our paper is in line with the literature on sentiment traders that are responsible to the emergence of complex dynamics in financial markets (Lux (1995), Brock and Hommes (1998), Dieci and He (2018)). For example, Gardini *et al.* (2022) find that when sentiment traders enter the market, the fundamental fixed point is never reached. Moreover, the presence of sentiment traders is responsible to the excess volatility and mispricing.

This model is inspired by the work of Westerhoff and Dieci (2006) where the introduction of a transaction tax is evaluated on the dynamics of the price variability in two different speculative markets with technical and fundamental analysts. Unlike Westerhoff and Dieci (2006), the focus of the model is on the transmission of volatility spillovers between two speculative markets where two types of traders invest on the same asset. Moreover, from the assumption of the model, the switching mechanism is limited on the same market considering the effects of information coming from the other market. Finally, we stress that the proportion of switching agents choosing a certain trading rule is governed by the sentiment index according to Eqs. (3) and (5) following Day and Huang (1990), Lux (1995), Chiarella *et al.* (2002) unlike other authors using a discrete-choice model (see Brock and Hommes (1998), Dieci *et al.* (2006), Westerhoff and Dieci (2006), among others).

In what follows we provide evidence of the capability of our model to replicate typical stylized facts of financial markets. We concentrate on the stylized facts observed between markets. In what follows, the simulations are performed using Map (9), where a stochastic component is added to the demand of each trader. However, instead of prices, numerical simulations focus on daily log-returns, specified as:

$$r_t = \ln(P_t) - \ln(P_{t-1}) \tag{10}$$

We analyze the stock market semi-volatility dynamics demonstrating that our model exhibits asymmetry in semi-volatility transmission. This fact implies that, the correlation of large and negative returns is larger than the correlation of large positive returns.

Figure (1) shows the dynamics of upside semi-volatility (panels (a), (d) and (g)), downside semi-volatility (panels (b), (e) and (h)) and volatility (panels (c), (f) and (i)) for different frequencies (daily, weekly and monthly from the top to the bottom respectively). It is worth noting that volatility shows the two asymmetries discussed above. In particular, for all frequencies, downside semi-volatility is larger than upside volatility. Moreover, we also observe that long-term volatility has a stronger influence on those at shorter interval. This confirms that our model is able to replicate typical empirical facts concerned with volatility and its dynamic as found by many authors (Caloia *et al.* (2018), Müller *et al.* (1997), among others).

Following Schmitt and Westerhoff (2014), we consider cross-correlation between markets. Table (1) presents estimates of the cross-correlation function of raw and absolute returns for between the two stock

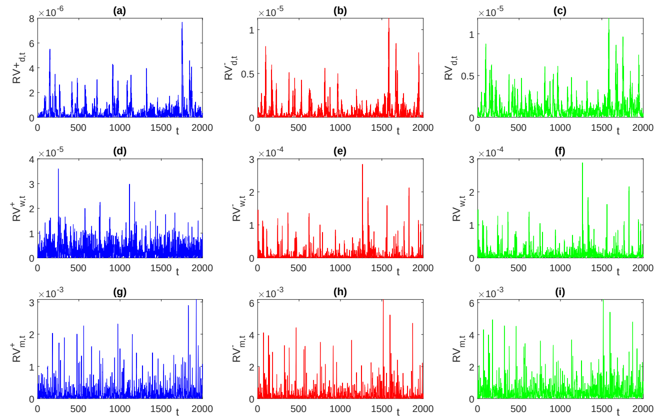

Figure 1: Asymmetric semi-volatilities. Upside semi-volatility (left), downside semi-volatility (middle) and volatility (rigth) of market 1 for different frequencies. In panels (a)-(c) daily semi-volatilities and volatility, in panels (d)-(f) weekly semi-volatilities and volatility, and panels (g)-(i) monthly semi-volatilities and volatility.

Table 1: Statistical properties of the two markets. The table reports the cross-correlation function of raw returns $cc^i_{r_t}$ for lags $i \in \{-1, 0, 1\}$ and the cross-correlation function of absolute returns $cc^i_{|r_t|}$ for lags $i \in \{-50, -2, 0, 2, 50\}$.

	$cc^{-1}_{r_t}$	$cc^{0}_{r_t}$	$cc^{1}_{r_t}$	$cc^{-50}_{\|r_t\|}$	$cc^{-2}_{\|r_t\|}$	$cc^{0}_{\|r_t\|}$	$cc^{2}_{\|r_t\|}$	$cc^{50}_{\|r_t\|}$
Mean	-0.038	0.99	0.004	0.124	0.347	0.989	0.304	0.095

markets. It shows the contemporaneous correlation in raw returns with a coefficient of 0.99 but after the lags of ± 1 the coefficient is insignificant. On the other hand, absolute returns exhibit long-range dependence of correlations, indeed, the cross-correlation functions decay slowly also for time difference of ± 50 lags where their coefficients reach values of 0.124 and 0.095.

4 Conclusions

The model proposed in this paper introduces a sentiment indicator in a simple asset-pricing framework. The sentiment index relies on a proxy of the cross-variance between two different markets where the same asset is traded, and is responsible of the switching mechanism generated in the model.

In particular, we have shown that the traders' demands could be compared with the two component of the TSI index, i.e., the directional spillover received and transmitted from each market. To this purpose, fundamentalists played the role of directional spillovers transmitted by one market to the other, while chartists represented directional spillovers received by one market from the other. Although transactions only take place within the same market and not between markets, through the number of agents it is possible to go back to a proxy of the system overall connectedness thanks to the sentiment index

introduced.
Finally, numerical simulations have exhibited the capability of our model to replicate some of the stylized facts of financial markets, i.e., comovements of prices and cross-correlations of volatilities. Only few works have jointly explained both statistical regularities (for example Schmitt and Westerhoff (2014)), and we have shown that our model pass this test with success.
Further extensions of our model could be considered. From a theoretical point of view, we can make our model discontinuous extending the analysis on the border collision bifurcations.

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