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The Term Structure of Capital Values: An accounting-based framework for measuring economic profitability

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The Term Structure of Capital Values: An accounting-based framework for measuring economic profitability

Carlo Alberto Magni¹ and Ken V. Peasnell²

Abstract. This paper shows how the outputs of the accounting measurement process can be translated into terms that can be used in economic decisions. We introduce the notion of Term Structure of Capital Values (*TSCV*), uniquely associated to a Term Structure of Interest Rates (*TSIR*). We show that the state of temporary disequilibrium created by an asset (project, firm, etc.) introduced in a market can be described in terms of a pair of *internal TSCV* and *TSIR*. Any internal *TSCV* determines a specific depreciation schedule for the asset and the corresponding *TSIR* determines an economic rate of return obtained as a capital-weighted average of the Return On Assets collected in the *TSIR*. The difference between this economic rate of return and the overall equilibrium rate (itself an average of the equilibrium forward rates) correctly captures value creation. The determination of a specific (internal) *TSCV* depends on the way the market sweeps away arbitrage opportunities and restores the equilibrium. As each possible accounting system can be viewed as being associated with a corresponding disequilibrium-to-equilibrium process and therefore with a corresponding economic rate of return, the paper shows that the determination of an economic rate of return is both a matter of accounting (in terms of a specific internal *TSCV*) and a matter of finance (in terms of a specific disequilibrium-to-equilibrium process). This evidently calls for a theory of capital valuation capable of associating the proper *TSCV* with each asset so that the intrinsic *underdetermination* of the economic rate of return can be solved.

Keywords. Average ROA, value creation, disequilibrium, term structure.

JEL codes. M41, G31, G11, G12.

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Introduction

The quest for profit plays a pivotal role in the workings of capitalist economies. A well-functioning market economy will direct resources into more profitable activities and withdraw them from less profitable ones. The determination of economic profitability is therefore a matter of central importance; this is reflected in the fact that the process governing the production of earnings numbers is subject to considerable oversight by auditors and regulators. The rules governing accounting and financial reporting now run to thousands of pages, and considerable time, energy and resources are devoted to assessing the results. The critical issue addressed in this paper is how the outputs of the accounting measurement process can be translated into terms that can be used in economic decisions.

There is a general agreement, in the financial and accounting literatures, as to the strengths of Net Present Value (*NPV*) as a measure of economic profitability, for it quantifies the monetary increase of the investors' wealth. However, in real-life applications, there is a demand to express economic profitability as a relative measure (e.g. a rate) rather than an absolute one. The reason seems to lie in the greater intuitive appeal of a relative metric such as a percentage return as opposed to an absolute amount of money (see Evans and Forbes, 1993). Value creation depends on the cost of capital, a relative measure itself; so, comparing two relative measures of worth is more natural than checking the sign of an absolute measure which depends on a relative one. To such an extent is a relative metric preferred by practitioners that scholars are sometimes criticized for endorsing the use of an absolute measure: "For abstruse reasons ... college professors prefer *NPV* analysis over *IRR* analysis. But all surveys indicate that lenders find it more appealing to analyze potential investments in terms of percentage rates of return rather than by comparing dollars of *NPV*" (Yung and Sherman, 1995, p. 18).

Unfortunately, there is no consensus on what "profitability" means in relative terms. A variety of relative metrics are used by firms in investment decision-making (Gitman and Forrester, 1977; Stanley and Block, 1984; Remer, Stokdyk and Van Driel, 1993; Burns and Walker, 1997; Graham and Harvey, 2001; Ryan and Ryan, 2002; Brounen, de Jong and Koedijk, 2004). Most corporate finance and

engineering economy textbooks (and some actuarial mathematics ones) therefore devote several pages to a variety of metrics the connections between which have received relatively little attention (see Remer and Nieto, 1995a,b; Hartman, 2007; Broverman, 2008; Kellison, 2009; Ross, Westerfiel and Jordan, 2011; Brealey, Myers and Allen, 2011; Blank and Tarquin, 2014, Park, 2013). The most commonly mentioned of these metrics are the Internal Rate of Return (*IRR*) and the Average Accounting Return (*AAR*).

Traditionally, the *IRR* has been viewed as the “gold standard” measure among these alternatives to *NPV*, as reflected in the fact that it is often called the “economic rate of return” against which the *AAR* (and related variants) are to be judged (Harcourt, 1965; Solomon, 1966; Livingstone and Salamon, 1970; Kay, 1976; Peasnell, 1982a,b; Fisher and McGowan, 1983; Luckett, 1984; Salamon, 1985; Whittington, 1988; Peasnell, 1996; Stark, 2004. See also Feenstra and Wang’s, 2000, review and references therein). However, Magni (2010, 2013, 2015) show that the *IRR* is itself just a special case of a more general class of metrics, highlighting the critical role of a project’s capital values in the determination of *any* rate of return measure, and thereby undermining the case for according the *IRR* special status as an economic concept.³ The present paper builds on these foundations. Starting from the well-known notion of the “term structure of interest rates” (*TSIR*) of an equilibrium market, we introduce the notion of the “Term Structure of Capital Values” (*TSCV*). We show that any asset introduced in the market creates a state of temporary disequilibrium. The process according to which equilibrium is restored by the market is not unique and each feasible disequilibrium-to-equilibrium process generates a corresponding *internal TSIR*, which represents a vector of internal holding period rates. Each such *TSIR* is uniquely related to an internal *TSCV* and the project rate of return is shown to be an average of the *TSIR*’s holding period rates weighted by the (present values of the) components of the *TSCV*.

³ As far as we can ascertain, Bailey (1959) was first to recognize that the *IRR* is a single, long-term rate having no inherent economic superiority to the whole sequence of time-variant holding-period rates associated with a multi-period project: “Recognition of the correct general solution of the investment problem has been hindered by the habit of thinking in terms of a single, long-term rate of interest” (p. 477); “This is an example of the “paradox” that have attracted so much attention in connection with investment decision criteria. It should be evident, however, that this paradox is merely an accident of the simplifying device of dealing with a single long-term rate of interest, and that it has no special importance in the more general framework used here” (pp. 478-479).

We study the implications of this framework for accounting. The choice of the appropriate *TSCV* for describing a project represents a choice as to how the capital values (i.e., the asset) is to be represented over the life of the project. This is strictly related to the way the market resolves the disequilibrium generated by the announcement of a project's acceptance by a firm's management team. Different choices of *TSCV* lead to different internal *TSIRs*, that is, to different sequences of Return on Assets (*ROA*), which then correspond to different ways in which the market reacts to restore the equilibrium. As a result, different Average *ROAs* capable of signalling wealth creation are available to the analyst. The choice as to which Average *ROA* is most appropriate is a domain-specific issue and a matter of value judgment.

The remainder of the paper is structured as follows. Section 1 introduces the notion of *TSCV* and shows that it is biunivocally associated with a given *TSIR*. Section 2 focuses on the state of temporary disequilibrium generated in the market by the announcement of a project's acceptance and describes how an efficient market rapidly restores the equilibrium. Section 3 introduces the notions of *internal* (i.e., disequilibrium) *TSCV* and *TSIR*: Each internal *TSCV* (and corresponding *TSIR*) is an expression of a conceptually well-determined pricing mechanism that tends to restore, more or less rapidly, the equilibrium in the market. This section also shows the relations between said pricing mechanism and the capital-weighted average of the internal interest rates, which validly captures the asset's economic profitability in an *NPV*-consistent way. The relations with the shareholders' rate of return are illustrated and the well-known profitability index is shown to be an instantaneous rate of return linked to the windfall gain earned by shareholders in an efficient market. Section 4 applies the results of the previous sections to accounting rates of return and shows that the average *ROA* represents an economically significant rate of return related to a gradual disequilibrium-to-equilibrium process associated with the internal *TSCV*. Section 5 explains that the selection of the disequilibrium-to-equilibrium process (and, therefore, the choice of the *TSCV*) cannot be made on a mathematical basis: It is domain-specific and a matter of accounting. The domain-specificity of the rate of return calls for a theory of capital valuation,

on the understanding that a project is not a mere sequence of cash flows but a sequence of capital values as well, whose determination affects the notion of rate of return. Some concluding remarks end the paper.

1. The term structure of interest rates and the term structure of capital values

To fix ideas, we adopt the usual convention of the flows in period t (i.e., the period between time $t - 1$ and time t) occurring at the end of the period and assume the existence of a security market which is efficient and in equilibrium. This means that arbitrage opportunities are not available and, when disequilibrium arises, the market quickly re-establishes a new equilibrium. The term structure of interest rates (*TSIR*), graphically represented by the yield curve, is described by a sequence $\mathbf{s} = (s_1, \dots, s_n) \in \mathbb{R}_+^n$ of spot rates, which is uniquely related to the sequence of spot prices $\mathbf{v} = (v_{1,0}, \dots, v_{n,0}) \in \mathbb{R}_+^n$, where $v_{t,0} = (1 + s_t)^{-t}$ is the unit price of a zero-coupon bond (zcb). The *TSIR* can be alternatively represented by a sequence of implicit forward rates $\mathbf{r} = (r_1, \dots, r_n) \in \mathbb{R}_+^n$, recursively defined as $(1 + s_t)^t = (1 + s_{t-1})^{t-1}(1 + r_t)$. The interest rate r_t then represents the one-period return of a traded asset per unit of capital invested at the beginning of the t -th period. The unit price of a zcb of length t can be written as $v_{t,0} = (1 + r_1)^{-1} \cdot (1 + r_2)^{-1} \cdot \dots \cdot (1 + r_t)^{-1}$. We allow for the possibility that the yield curve need not be flat, that is, that spot and forward rates are time-variant.

Let $\mathbf{F} = (F_1, \dots, F_n) \in \mathbb{R}^n$ be any sequence of cash flows. The equilibrium assumption imposes that the price of any such cash-flow stream must be given by the present value of the prospective cash flows, discounted at the spot (or forward) rates. Denoting as V_0 the price of the asset,

$$V_0 = \sum_{h=1}^n \frac{F_h}{(1 + r_1) \cdot (1 + r_2) \cdot \dots \cdot (1 + r_h)}. \quad (1a)$$

Equation (1a) can be written as

$$-V_0 + \sum_{h=1}^n \frac{F_t}{(1 + r_1) \cdot (1 + r_2) \cdot \dots \cdot (1 + r_h)} = 0 \quad (1b)$$

which means that such an asset has a net present value (NPV) equal to zero. In general, the economic (i.e., market) value implied by the $TSIR$ is such that $V_t = V_{t-1}(1 + r_t) - F_t$, which describes the evolution of the capital invested. The amount $r_t V_{t-1}$ is the investor's return. Also, by backward induction,

$$V_t = \frac{V_{t+1} + F_{t+1}}{1 + r_{t+1}} = \sum_{h=t+1}^n \frac{F_h}{(1 + r_{t+1}) \cdot (1 + r_{t+2}) \cdot \dots \cdot (1 + r_h)} \quad (2)$$

for $t = 0, 1, \dots, n - 1$. Equation (1) is, of course, a special case of (2), when $t = 0$.

Given any asset \mathbf{F} , the sequence $\mathbf{V} = (V_0, V_1, \dots, V_{n-1}) \in \mathbb{R}^n$ of economic values is uniquely related to \mathbf{r} via (2). In other words an implicit “Term Structure of Capital Values” ($TSCV$) exists, for any given \mathbf{F} , which is biunivocally associated with the $TSIR$. The triplet $(\mathbf{F}, \mathbf{r}, \mathbf{V})$ completely identifies an asset in the market in which the choice of any two of the elements is sufficient to completely describe the asset. The elements of \mathbf{r} are the period return rates, while the elements of \mathbf{V} represent the invested capital amounts period by period.

2. From disequilibrium to equilibrium: Net Present Value and wealth creation

Suppose a firm has the opportunity of undertaking a project, whose cash flows are $F_t, t = 1, 2, \dots, n$. Henceforth, we will assume for analytical purposes that flows are certain or, equivalently, that they are uncertain and that risk is dealt with by representing the flows as certainty equivalents.⁴ Let C_0 be the project cost. The acceptance of the project by the firm's managers creates a temporary state of disequilibrium (Rubinstein, 1973). If the market is efficient, the disequilibrium is arbitrated away rapidly by the market forces in the following way. Before acceptance of the project, the equilibrium value \mathcal{V}_0 of the firm is

$$\mathcal{V}_0 = \sum_{t=1}^n \frac{\mathcal{F}_t}{\prod_{h=1}^t (1 + r_h)} \quad (3a)$$

$$\mathcal{V}_0 = N \cdot P \quad (3b)$$

⁴ In practice, risk is usually dealt with by expressing cash flows as expected values and making risk adjustments via the discount rates(s). The results reported in this paper are unchanged if risk is dealt with in this manner.

where \mathcal{F}_t = firm's free cash flows at time t , N = number of shares outstanding at time 0, and P = current share price. With no loss of generality, we assume the project is equity-financed. When the acceptance of the project is announced, the market reacts and the new equilibrium firm's value \mathcal{V}'_0 is set as

$$\mathcal{V}'_0 = \sum_{t=1}^n \frac{\mathcal{F}_t + F_t}{\prod_{h=1}^t (1 + r_h)} \quad (4a)$$

$$\mathcal{V}'_0 = (N + N') \cdot P' \quad (4b)$$

where P' is the new equilibrium price and $N' = C_0/P'$ is the number of new shares issued at the price P' to finance the project.⁵ Subtracting (3b) from (4b),

$$\mathcal{V}'_0 - \mathcal{V}_0 = N \cdot (P' - P) + N'P'. \quad (5a)$$

Also, subtracting (3a) from (4a),

$$\mathcal{V}'_0 - \mathcal{V}_0 = \sum_{t=1}^n \frac{F_t}{\prod_{h=1}^t (1 + r_h)} = V_0 \quad (5b)$$

Using (5a), (5b), and the financing requirement that $N'P' = C_0$,

$$V_0 - C_0 = N(P' - P). \quad (6)$$

The left-hand side of (6) is the project's *NPV*, the right-hand side is shareholders' wealth increase/decrease.⁶ Wealth is created for shareholders if and only if $P' > P$, which occurs if and only if $NPV > 0$. The *NPV* is a 'windfall gain' that benefits the firm's existing shareholders.⁷

In general, whenever any asset with cash-flow stream \mathbf{F} is introduced in the market at a price of C_0 that is inconsistent with the *TSIR* (i.e., $C_0 \neq V_0$), the one-price law is infringed and an arbitrage opportunity arises. An efficient market reacts (i.e., arbitrageurs intervene) and the price C_0 of the new asset increases until it reaches V_0 ,⁸ so that equilibrium is re-established.⁹

3. The equilibrium condition and the *internal* market

⁵ This analysis assumes that the market is not so efficient as to be able to anticipate the project's acceptance and yet it is sufficiently efficient that when it learns of the project it forms expectations that accord with those of the managers. If either (or both) of these assumptions are false, the equilibrating process will not be instantaneous. We address the issue of the equilibrating process in section 3 below.

⁶ $N(P' - P)$ is the wealth increase/decrease for the "old" shareholders, not the "new" shareholders, who purchase the new N' shares at the new equilibrium price P' (i.e., the latter make a zero-*NPV* investment).

⁷ *NPV* is "a 'windfall gain', which accrues to the owners of the firm as a result of their being able to invest in a project that is more profitable than the standard market rate" (Robichek and Myers, 1965, p. 11).

⁸ "Given the market strives for equilibrium, the market value of the stock will increase by more than the investment outlay" (Bierman and Hass, 1973, p. 122).

⁹ As usual in this kind of analysis, we assume that the number of assets in the security market is so large that the introduction of the new asset does not affect the *TSIR*.

The disequilibrium-to-equilibrium process that a project undertaking generates can be equivalently illustrated in terms of divergence between two alternative markets. To this end, consider a disequilibrium *TSIR*, $\mathbf{i} = (i_1, \dots, i_n)$ such that

$$C_0 = \sum_{h=1}^n \frac{F_h}{(1 + i_1) \cdot (1 + i_2) \cdot \dots \cdot (1 + i_h)}. \quad (7)$$

Equation (7) can be interpreted as the arbitrage-free condition of a hypothetical market where the equilibrium *TSIR* is \mathbf{i} . We call such a hypothetical market the ‘internal market’ and \mathbf{i} is the ‘internal *TSIR*’); the price C_0 is then the equilibrium price of the project in the internal market. The difference between the two equilibrium prices in (2) and (7) measures the magnitude of the wealth created by the project undertaken by the firm. In this way, we formalize the disequilibrium as a difference between two alternative equilibrium prices, holding in alternative markets. Such a deviation just expresses the project’s economic profitability: $V_0 - C_0$.

There are many ways in which (7) can deviate from (2) in terms of interest rates; equivalently stated, there are many ways in which disequilibrium is resolved and equilibrium is restored. Equation (7) is solved by infinitely many internal *TSIRs*, $\mathbf{i} = (i_1, i_2, \dots, i_n)$. Let γ_t be such that $(1 + i_t) = (1 + r_t)(1 + \gamma_t)$ so that the γ_t ’s represent the relative deviations of the internal *TSIR* \mathbf{i} from the yield curve \mathbf{r} . Therefore, (7) can be written as

$$C_0 = \sum_{h=1}^n \frac{F_h}{\prod_{j=1}^h (1 + r_j)(1 + \gamma_j)} \quad (8)$$

There are an infinite number of solutions $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, \dots, \gamma_n)$ to (8). Different solutions describe (different *TSIRs* \mathbf{i} and therefore) different equilibrium conditions in the internal market resulting in the same price, C_0 . To any possible deviating vector $\boldsymbol{\gamma}$ (and to any possible \mathbf{i}) there corresponds a well-defined internal *TSCV*, $\mathbf{C} = (C_0, C_1, \dots, C_{n-1})$, such that $C_t = C_{t-1}(1 + i_t) - F_t$ or, alternatively,

$$C_t = \frac{C_{t+1} + F_{t+1}}{1 + i_{t+1}} = \sum_{h=t+1}^n \frac{F_h}{\prod_{j=t+1}^h (1 + r_j)(1 + \gamma_j)} \quad t = 0, 1, \dots, n-1 \quad (9)$$

(note that (8) is a particular case of (9) for $t = 0$). One possible solution of (8) is $\boldsymbol{\gamma} = (\gamma_1, 0, 0, \dots, 0)$, which means that $\mathbf{i} = (j, r_2, \dots, r_n)$ such that $(1 + j) = (1 + r_1)(1 + \gamma_1)$. Therefore, the internal *TSIR* is equal to \mathbf{r} except in the first period. Thus, (8) becomes

$$C_0 = \sum_{t=1}^n \frac{F_t}{(1 + r_1)(1 + \gamma_1) \prod_{h=2}^t (1 + r_h)} \quad (10)$$

It is worth noting that (10) is a formal description of the pricing mechanism illustrated in the previous section, when the market wipes out disequilibrium rapidly, within one period. Indeed, solving (10) for γ_1 ,

one finds $C_0(1 + \gamma_1) = V_0$, whence $\gamma_1 = PI$ where $PI = NPV/C_0$ is the well-known profitability index. PI represents an interest rate that marks up the first-period cost of capital r_1 to the first-period rate of return j . From the definition of γ_1 one finds

$$PI = \frac{j - r_1}{1 + r_1}. \quad (11)$$

Multiplying (11) by C_0 ,

$$NPV = \frac{C_0 \cdot (j - r_1)}{1 + r_1} \quad (12)$$

The numerator on the right-hand side of Equation (12) is the excess return generated in the first-period . Thus, NPV is equal to the period 1 excess return discounted at the required rate for that period. In the subsequent periods the excess return is zero: $\gamma_t = 0$ implies $i_t = r_t$ for $t > 1$ which implies $C_{t-1}(i_t - r_t) = 0$ in those periods. Therefore, the first-period rate j correctly captures wealth creation when compared to r_1 : from (12),

$$NPV > 0 \quad \text{iff} \quad j > r_1 \quad (13)$$

The first-period rate j properly captures, in relative terms, such a pricing mechanism. The intuition behind it is very simple: Investors invest C_0 at time 0 and receive an asset which is worth V_1 at the end of the first period, alongside the cash flow F_1 . Therefore, the rate of return in the first period is $\frac{V_1 + F_1}{c} - 1$. The latter is equal to j , since

$$\frac{V_1 + F_1}{c} - 1 = \frac{V_0(1 + r_1) - C_0(1 + r_1) + r_1 C_0}{C_0} = r_1 + PI(1 + r_1) = j. \quad (14)$$

To sum up, in the case of a mispriced asset, the market is assumed to react at time 0 so as to boost the misaligned price up by PI in order to reestablish the equilibrium price V_0 (i.e., $PI = \frac{V_0 - C_0}{C_0}$). At the beginning of period $t > 1$, the invested capital is the economic value V_{t-1} and the return is $r_t V_{t-1}$, $t = 2, \dots, n$, so no more value is created. This process is typical of an efficient market: “after equilibrium is restored, nothing else occurs, for there is no tendency for subsequent increases and decreases” (Ross, Westerfield and Jordan, p. 327). By (9), this also implies that the TSCV corresponding to $\mathbf{i} = (j, r_1, \dots, r_n)$ is $\mathbf{C} = (C_0, V_1, \dots, V_n)$. The elements of $\mathbf{i} = (j, r_1, \dots, r_n)$ describe the project’s relative performance period by period, while the elements of \mathbf{C} represent, period by period, the invested capital to which the period rates are applied.

We are now able to express the project’s economic profitability in relative terms. Taking into account the time-value of money, the project’s overall return x is the sum of the period returns; the latter can be expressed as sum of products of period rate and invested capital:

$$x = jC_0 \cdot v_{1,0} + r_2 V_1 \cdot v_{2,0} + \dots + r_n V_{n-1} \cdot v_{n,0}, \quad (15)$$

where, recall, $v_{t,0} = v = \prod_{h=1}^t (1 + r_h)^{-1}$ is the spot price of cash in period t . The return can be written as $x = \bar{J} \cdot C$, where $C := C_0 + V_1 \cdot v_{1,0} + \dots + V_{n-1} \cdot v_{n-1,0}$ so that

$$\bar{J} = \frac{x}{C}. \quad (16)$$

Equation (16) can also be framed as a linear combination of the project's holding period rates, weighted by the project's values collected in the internal TSCV: $\theta_1 j + \theta_2 r_2 + \dots + \theta_n r_n$, with $\theta_1 = \frac{C_0 v_{1,0}}{C}$, $\theta_t = \frac{V_{t-1} v_{t,0}}{C}$, $t = 2, \dots, n$. Also, it is easily seen that

$$NPV = C \cdot (\bar{J} - \bar{r}) \quad (17)$$

where

$$\bar{r} = r_1 \theta_1 + \dots + r_n \theta_n.$$

The project is then interpretable as an investment of C dollars at the rate of return \bar{J} , while \bar{r} represents the opportunity cost of capital, that is, the overall equilibrium rate of return that investors would get if capital C were invested in a portfolio traded in a market where the TSIR is equal to r . Equation (17) implies

$$\bar{J} = \bar{r} + \frac{NPV}{C}. \quad (18)$$

From (17) or (18),

$$NPV > 0 \quad iff \quad \bar{J} > \bar{r}. \quad (19)$$

The case where $C < 0$ represents a situation where the asset is a net borrowing and \bar{J} represents a borrowing rate, so that $NPV > 0$ if and only if $\bar{J} < \bar{r}$. We will call \bar{J} the 'shareholder rate of return' (SRR). Both \bar{J} and \bar{r} depend on $C = (C, V_1, \dots, V_n)$, which is the expression of the pricing mechanism of an efficient market that sweeps away disequilibrium via a windfall gain. The excess returns $(j - r_1)$ and $(\bar{J} - \bar{r})$ are then relative measures that quantify the wealth created by the project per unit of invested capital C_0 and per unit of overall invested capital C , respectively.

The previous case where $\gamma = (PI, 0, 0, \dots, 0)$ presupposes that the interim values of the project are equal to the economic values determined by an efficient market: $C_t = V_t$ for $t \geq 1$. More generally, if $\gamma_t \neq 0$ for $t > 1$ the internal TSCV is such that $C_t \neq V_t$ for $t \geq 1$. This is equivalent to saying that wealth is created gradually, not as a windfall gain (i.e., excess return $C_{t-1} \cdot (i_t - r_t)$ is nonzero for every $t > 1$). This implies that the overall return is $x = \sum_{t=1}^n i_t C_{t-1} \cdot v_{t,0}$. Hence, one gets the project's overall rate of return, \bar{i} :

$$\bar{i} := \frac{x}{C} = i_1 w_1 + i_2 w_2 + \dots + i_n w_n \quad (20)$$

where $C := C_0 + C_1 v_{1,0} + \dots + C_{n-1} v_{n-1,0}$ and $w_t = \frac{C_{t-1} v_{t,0}}{C}, t = 1, 2, \dots, n$. It is easy to see that

$$NPV = C \cdot [\bar{\iota} - \bar{r}] \quad (21)$$

where

$$\bar{r} = r_1 w_1 + r_2 w_2 + \dots + r_n w_n \quad (22)$$

represents the project's cost of capital (i.e., the overall equilibrium rate of return). Equation (21) can also be written as

$$\bar{\iota} = \bar{r} + \frac{NPV}{C}. \quad (23)$$

This implies

$$NPV > 0 \quad \text{iff} \quad \bar{\iota} > \bar{r} \quad (24)$$

(if $C < 0$, then wealth is created if and only $\bar{\iota} < \bar{r}$). Note that $\bar{\iota} \neq \bar{r}$, since both the overall return and the overall capital are different. However, the overall value created, NPV , is equal.

Recall that equation (7) has an infinite number of solutions. Let $S_i = \{\mathbf{i} \in \mathbb{R}^n : \mathbf{i} = (i_1, \dots, i_n)\}$ be the set of the internal *TSIRs* and $S_C = \{\mathbf{C} \in \mathbb{R}^n : \mathbf{C} = (C_0, C_1, \dots, C_{n-1})\}$ be the related set of internal *TSCVs* that fulfill (7). As the elements of S_i and S_C are in a one-to-one correspondence, we can write $\bar{\iota} = \bar{\iota}(\mathbf{C})$ as a function of \mathbf{C} : Different assumptions on the internal *TSCV* (i.e., different assumptions on the vector \mathbf{i} of interest rates) lead to different values of the project's rate of return $\bar{\iota}(\mathbf{C})$.

Among the various solutions, consider the capital sequence such that the capital grows at a constant force of interest: $C_t = C_t(i) = C_{t-1}(i)(1+i) - F_t$, so that $\mathbf{i} = (i, \dots, i)$. Equation (7) becomes

$$C_0 = \sum_{t=1}^n \frac{F_t}{(1+i)^t}.$$

The rate i is therefore the *IRR*. It can be shown that there exists a subset $S'_C \subset S_C$ such that $C_t = \sum_{h=t+1}^n F_h \cdot \prod_{h=t+1}^n (1+i_h)^{-1}$ and $\bar{\iota}(\mathbf{C}) = i$ for any $\mathbf{C} \in S'_C$ (see Magni, 2010, Theorem 3). This implies, from (21), $NPV = C(i - \bar{r})$ for any $\mathbf{C} \in S'_C$ and

$$NPV > 0 \quad \text{iff} \quad i > \bar{r} \quad (25)$$

(again, the sign of the last inequality is reversed if $C < 0$). Equation (25) can therefore be viewed as a generalized internal-rate-of-return rule, which allows for time-variant costs of capital. Note that, in case of multiple *IRRs*, any project's *IRR* is a rate of return associated to a well-defined set S'_C of internal *TSCVs*.¹⁰

¹⁰ Assuming a constant cost of capital, r , Magni (2010) shows that S'_C is an equivalence class of capital streams such that any capital structure $\mathbf{C} \in S'_C$ shares the same discounted sum, equal to $NPV/(i - r)$. This equivalence class is named the *Hotelling class* in Magni 2010. The notion of an equivalence class of capital values (also named depreciation class or investment class) has been introduced in Magni (2009, 2010 2011) and extensively used in subsequent papers.

As seen, the vector \mathbf{C} such that $C_t = C_t(i)$ is the element of S'_C that imply $\mathbf{i} = (i, i, \dots, i)$: This means that a flat yield curve is imposed to the internal market. In other words, it is as if one assumed that disequilibrium fades away gradually, in such a way that the project's value grows at a constant force of return equal to i . (Note that the choice $\mathbf{i} = (i, i, \dots, i)$ is possible only if the *IRR* exists).¹¹

To sum up, to any pair $(\mathbf{i}, \mathbf{C}) \in S_i \times S_C$, a unique pair of economic rate of return and cost of capital (\bar{r}, \bar{r}) is generated, such that

$$NPV = C \cdot [\bar{r}(\mathbf{C}) - \bar{r}(\mathbf{C})] \quad (26)$$

so that value is created if and only if $\bar{r}(\mathbf{C}) > \bar{r}(\mathbf{C})$. The *SRR*, \bar{r} , is equal to the economic rate of return $\bar{r}(\mathbf{C})$ generated by a *TSCV* equal to $\mathbf{C} = (C_0, V_1, \dots, V_{n-1})$ and it is equivalent to the average rate of return obtained when disequilibrium is swept away rapidly by an efficient market. In contrast, if disequilibrium peters out gradually, there are as many solutions as there are ways in which equilibrium is restored. For example, the *IRR*, i , is equal to the economic rate of return $\bar{r}(\mathbf{C})$ associated with a *TSCV* equal to $\mathbf{C} = (C_0, C_1(i), C_2(i), \dots, C_{n-1}(i))$ and is the average return rate that would be obtained if capital grew at a constant pace equal to i . Other solutions are possible, depending on the assumption made on \mathbf{C} (or, which is the same, on \mathbf{i}).

Assume now the yield curve is flat, so that $r_t = s_t = r$ for every t : The equilibrium rate (cost of capital) is constant in every period. In this case, letting $C := \sum_{t=1}^n c_{t-1} v^{t-1}$ and $v^t := (1+r)^{-t}$, one gets

$$NPV = C \cdot [\bar{r}(\mathbf{C}) - r] \quad (27)$$

where $\bar{r}(\mathbf{C}) = \sum_{t=1}^n i_t w_t$, $w_t = \frac{c_{t-1} v^t}{C}$. Hence,

$$\bar{r}(\mathbf{C}) = r + \frac{NPV}{C} \quad (28)$$

which holds for any \mathbf{C} . This implies

$$NPV > 0 \quad \text{iff} \quad \bar{r}(\mathbf{C}) > r \quad (29)$$

(see also Magni 2010, 2013). In equation (28), $\bar{r}(\mathbf{C})$ is a function of the overall capital invested, C , indicating that any project is associated with an indifference curve on the (C, \bar{r}) -plane, where different combinations of invested capitals and rates of return give rise to the same *NPV*. The higher the *NPV*, the

¹¹Note that the yield to maturity y_n such that $V_0 = \sum_{t=1}^n F_t (1+y_n)^{-t}$ may not exist or be multiple, even if the project's *IRR* exists and is unique. For example, consider a two-period investment in which $C_0 = -4$, $F = (12, -9)$ and a term structure of interest rates in the market such that $\mathbf{r} = (15\%, 40\%)$. In this case, the project's unique *IRR* is $i = 50\%$ and its market value is $V_0 = 4.845$, but the yield to maturity y_2 does not exist (the equation $4.845 = \frac{12}{1+y_2} - \frac{9}{(1+y_2)^2}$ has no real-valued solutions). In this case, i could not be compared with y_2 to signal wealth creation, but can be compared with the average cost of capital \bar{r} .

further the arms of the hyperbola (see Figure 1). Following from our earlier analysis, it follows that this “iso-value line” is a geometrical representation of the equilibrium condition, under the assumption of constant cost of capital. In particular, to each solution γ of the equilibrium condition (8) there corresponds a unique pair $(C, i) \in S_C \times S_i$ which in turn induces a unique pair (\bar{C}, \bar{i}) :

$$\gamma \rightarrow (C, i) \rightarrow (\bar{C}, \bar{i}).$$

The choice of a particular solution γ to the equilibrium condition (8) is then the choice of a particular pair (C, i) . To derive an economic rate of return among the infinite other economic rates of return (including IRR) boils down to making implicit or explicit assumptions about the appropriate internal $TSCV$ or, equivalently, to making an assumption on how disequilibrium is removed through time.

4. The Average Accounting Rate

Corporate finance textbooks commonly advise the reader to disregard accounting numbers and focus exclusively on the relevant cash flows. However, in practice, accounting numbers play a significant role in the decision process. Sometimes this is because the decision makers know that subsequent evaluations of the decision outcomes will be expressed in terms of accounting profits and accounting rates of return—in itself sufficient reason for the decision to take account of projected accounting data as well as (or in place of) projected cash flows. There is an additional reason: In many situations, the economic phenomena can only be really understood in terms of accounting numbers.

Consider a capital asset investment. To forecast the cash flows of many projects, financial managers begin by forecasting earnings (Berk and DeMarzo, 2011; Titman and Martin, 2011; Finnerty, 2013). Among the types of investments that invariably require pro forma financial statements are start-up firms, build-up leveraged buyout transactions, and project financing transactions. A standard procedure is to start with forecasts of sales, an accounting construct, then deduct cost of sales and expenses (also accounting constructs). These are needed because the firm has to think about the competition and what they might do in reaction, and for this they need to consider profit margins; this cannot be done with cash flows. It is then necessary to add back to profit depreciation and changes in working capital (more accounting constructs), to arrive at so-called *free cash flow* (see Titman and Martin, 2011; Brealey, Myers and Allen 2011). In other words, accounting values are first-order variables and cash flows are of a second-order magnitude: “accounting variables are the ‘independent’ variables and net dividends the ‘dependent’ variable, not the other way around” (Brief, 1996, p. 28). It is worth noting that, in this circumstance, the estimation of (accounting) capital is not arbitrary. Cash flows are derived from the relation $F_t = I_t - \Delta B_t$ where $\Delta B_t = B_t - B_{t-1}$ is capital depreciation. Obviously, $\Delta B_t = \Delta NFA_t +$

ΔWC_t , where NFA_t = Net Fixed Assets, WC_t = Working Capital. Therefore, both fixed asset depreciation and change in working capital affect the estimation of prospective cash flows.¹²

Needless to say, accounting book values do not always provide good estimates of the economic value of the resources of a firm. The stock market values of the shares of the major pharmaceutical firms greatly exceed the book values of their equity because their balance sheets omit their most valuable assets. However, a cash flow representation would fare even worse: The only asset would be the company's cash balances. So while ΔNFA_t and ΔWC_t might not reflect the "correct" change in value of fixed assets and working capital, they are probably the best that is available to the decision maker. This implies pro forma financial statements are likely to be useful in the sense that they will provide book value forecasts that are the best available estimates of the invested capital in the periods under examination. The results found in the previous sections enable us to interpret these book values as elements of a specific internal *TSCV*:

$$\mathbf{C} = (B_0, B_1, \dots, B_n) \in S_c \quad \text{with } B_0 = -F_0.$$

Likewise, the estimated *ROAs* represent the associated internal *TSIR*:

$$\mathbf{i} = \mathbf{ROA} = (ROA_1, ROA_1, \dots, ROA_n) \in S_i$$

where $ROA_t = I_t/B_{t-1}$ depends on the estimation of book values. The equilibrium condition (7) is fulfilled:

$$B_0 = \sum_{t=1}^n \frac{F_t}{\prod_{h=1}^t (1 + ROA_h)}. \quad (30)$$

However, in this case, F is not exogenously given; as seen above, it is derived from the internal *TSIR* and *TSCV* as $F_t = I_t - \Delta B_t$ so that

$$C_0 = \sum_{t=1}^n \frac{I_t - \Delta NFA_t - \Delta WC_t}{\prod_{h=1}^t (1 + ROA_h)} \quad (31a)$$

or, equivalently,

$$C_0 = \sum_{t=1}^n \frac{I_t - \Delta B_t}{\prod_{h=1}^t (1 + I_h/B_h)}. \quad (31b)$$

¹² In many countries, the tax rules specify that depreciation can only be claimed as a business expense if the rate used to compute taxable income is the same as the rate used for financial reporting. In this case, fixed asset depreciation directly affects cash flow via the tax rate. But the more general point is all changes in the balance sheet are relevant in the determination of cash flows because they come, in an important sense, logically prior to the estimate of cash flows.

From this we can determine a weighted-average rate of return $\bar{r}(\mathbf{C})$. As the internal forward rates are the estimated ROAs, we denote such a rate as \overline{ROA} . Applying (20), we find

$$\overline{ROA} = ROA_1 w_1 + ROA_2 w_2 + \cdots + ROA_n w_n \quad (32)$$

with $w_t = B_{t-1} v_{t-1,0} / B$, where $B = \sum_{t=1}^n B_{t-1} v_{t-1,0}$ is the aggregate book value, which represents the (estimated) overall capital invested in the project. Hence, (26) becomes

$$NPV = B \cdot [\overline{ROA} - \bar{r}] \quad (33)$$

where $B = \sum_{t=1}^n B_{t-1} v_{t-1,0}$, $\bar{r} = \sum_{t=1}^n r_t w_t$, which implies

$$NPV > 0 \quad \text{iff} \quad \overline{ROA} > \bar{r}. \quad (34)$$

If the yield curve is flat, $NPV = B \cdot [\overline{ROA} - r]$ and $\overline{ROA} = r + NPV/B$.

We can restate (32) in a more general way as the ratio of average return to average book value:

$$\overline{ROA} = \frac{\bar{I}}{\bar{B}} \quad \text{where } \bar{I} = \frac{1}{n} \sum_{t=1}^n I_t v^t \quad \text{and} \quad \bar{B} = \frac{1}{n} \sum_{t=1}^n B_{t-1} v^{t-1}. \quad (35)$$

This shows that the estimated average ROA is an accounting measure that correctly captures economic profitability. More precisely, it is the project's overall rate of return associated with the book values of capital. The usefulness of average ROA can thus be seen to directly depend on the extent to which average book value reflects the economic value of the resources at the firm's disposal.

It is also worth noting that the average ROA is unique: The estimation of \mathbf{F} is obtained by the estimation of accounting constructs. To the estimated sequences of book values and $ROAs$ there corresponds a (unique cash flow stream \mathbf{F}) and a unique rate of return \overline{ROA} , as well as a unique cost of capital \bar{r} . So, in this case, the uniqueness of the solution of (7) is intrinsic in the process of estimation of the accounting constructs (and, therefore, of the project's cash flows). To put it equivalently, the way disequilibrium is resolved is explicitly managed in the estimations of the accounting constructs.

Among the various cases, a significant one is cash flow accounting, in which investments are immediately expensed as incurred. In such a system, $B_1 = B_2 = \cdots = B_n = 0$, and $I_1 = F_1 - B_0$, $I_t = F_t$ ($t = 2, \dots, n$). After the first year, ROA_t will be infinitely large (or undefined, given that $B_t = 0$) and $w_t = 0$, dictating that \overline{ROA} be computed by equation (35) rather than (32). Cash flow accounting is a limiting case, where no attempt is made to incorporate any additional information about the non-cash resources available to the firm or to match outlays against the revenues to which they give rise. This

implies a significant relation between cash flow accounting and the way disequilibrium is solved by the market: Denoting as \overline{ROA}^{CF} the cash-flow-accounting average ROA ,

$$\overline{ROA}^{CF} = \frac{(F_1 - B_0)v_{1,0} + F_2v_{2,0} + \dots + F_nv_{n,0}}{B_0} = \frac{V_0 - C_0v_{1,0}}{C_0}. \quad (36a)$$

The first-period rate of return is $j = \frac{(V_1 + F_1) - C_0}{C_0} = \frac{V_0(1+r_1) - C_0}{C_0}$. Hence (36a) can be rewritten as

$$\overline{ROA}^{CF} = j \cdot v_{1,0} \quad (36b)$$

revealing that the average ROA implied by cash flow accounting is equal to the (present value of the) first-period rate of return generated in an efficient market. As a result, cash flow accounting is appropriate for getting information on the way an efficient market wipes out disequilibrium. Therefore, the excess return $\overline{ROA}^{CF} - \frac{r_1}{1+r_1} = \frac{j-r_1}{1+r_1} = PI$ measures the magnitude of the misalignment (i.e., wealth created).

Reminding that the internal $TSCV$ related to SRR is $\mathbf{C} = (C_0, V_1, \dots, V_{n-1})$ the SRR can then be rewritten as

$$\bar{J} = \left(\frac{B_0}{C}\right)\overline{ROA}^{CF} + \left(\frac{V_1v_{2,0}}{C}\right)r_2 + \dots + \left(\frac{V_{n-1}v_{n,0}}{C}\right)r_n \quad (37)$$

where $C := B_0 + \sum_{t=2}^n V_{t-1}v_{t-1,0}$.

The above result establishes a link with the relevant case of fair value accounting. Consider a firm, incorporated at time 0 with $B_0 = C_0$ dollars, and suppose that fair value accounting is followed from time 1 to time n .¹³ This implies that the internal $TSCV$ is $\mathbf{C} = (C_0, V_1, \dots, V_{n-1})$ so that the corresponding average ROA is

$$\overline{ROA}^{FV} = \left(\frac{B_0v_{1,0}}{C}\right)j + \left(\frac{V_1v_{2,0}}{C}\right)r_2 + \dots + \left(\frac{V_{n-1}v_{n,0}}{C}\right)r_n \quad (38)$$

where $C := B_0 + \sum_{t=2}^n V_{t-1}v_{t-1,0}$, so that the fair value accounting average ROA is equal to the SRR :

$$\overline{ROA}^{FV} = \bar{J}. \quad (39)$$

¹³ At time 0, the initial investment of a project is recorded as $B_0 = -F_0$.

Equation (39) sets a relation between cash flow accounting and fair value accounting: The SRR (or the profitability index, PI , of which SRR is a transformation), is the fundamental link between cash flow accounting and fair value accounting.

5. Value judgments and the domain-specificity of the economic rate of return

The results of the previous sections show that the determination of an economic rate of return for a project strictly depends on the choice of a particular equilibrium condition among infinite ones described by (7). This in turn boils down to necessarily choosing, implicitly or explicitly, an internal $TSCV$, $\mathcal{C} \in S_c$, which generates a capital-weighted average rate of return $\bar{r}(\mathcal{C})$. The internal $TSCV$ and its corresponding $TSIR$ give expression to the way wealth increase is apportioned across periods (or, equivalently, the way disequilibrium dissolves); it represents the economic features of the internal market. The choice involves a value judgment, which means that an economic rate of return is a domain-specific measure, obtained as an average ROA from a specific internal $TSCV$. The appropriateness of the choice depends on the purpose of the analysis and on the practical environment where the asset is generated and where the investor operates. A project is not a mere sequence of cash flows, but a set of economic transactions made by the investors in relation with other economic subjects (providers, customers, government etc.). These actions and transactions (as well as the pieces of information required by the evaluator) affect the choice of $\mathcal{C} \in S_c$.

This suggests there is no single way to determine the appropriate $TSCV$ (and therefore the appropriate average ROA). This insight was anticipated by Vatter (1966) half a century ago: “unless capital recovery process is specified, there is no single way to measure the annual productivity of the investment” (Vatter, 1966, p. 687). The problem of deriving the appropriate $TSCV$ is an example of a problem which is well-known in the philosophy of science: The so-called *underdetermination of theory by data* (see Duhem, 1914; Schlick, 1931; Quine, 1951): Given a sequence of empirical data, there are many infinite functions that exactly pass through those points (Schlick, 1931). That is, the scientific law is underdetermined by data. Applied to our topic, this means that we may only observe two data: At time 0,

the capital invested is C_0 and, at time n , after liquidation, the capital is $C_n = 0$; but, given two points $(0, C_0)$ and (n, C_n) on the (t, C_t) -plane, any depreciation schedule is a function that passes through those two points; therefore, the internal *TSCV* is undetermined by the economic data (see Figure 2).¹⁴

As an example, consider a share trader, who typically has a short-term horizon and looks at changes in market values, as he aims at selling the shares of a firm as soon as their price is sufficiently high. In this case, the appropriate *TSCV* is given by fair value accounting, which implies that, to a share trader, the *SRR* is the appropriate economic rate of return. The same holds for an investment fund. In this case, as the setting is one where the decision maker seeks out assets that are liquid, it follows that invested capital should be measured in terms of the price at which the asset could be sold in the security market. However, even when dealing with securities, judgmental evaluations are needed: “the determination of appropriate asset values for the securities is an issue on which universal agreement does not exist ... Valuation methods will often vary depending upon the situation at hand” (Kellison, 2009, p. 234).

Identical securities may have different reported values for different subjects: A firm owned by a share trader likely has a different business model to a firm owned as by a long-term investor like Warren Buffett. The former aims at selling the shares as soon as they have achieved a sufficiently high value, whereas the latter is more interested in long-term value. In the former case, market values are appropriate; in the second case they might not be if the decision maker believes the market prices incorporate less information than is available to the decision maker. Valuation methods will also vary depending on the type of security. For example, an appropriate method for bonds may differ from an appropriate method for common stock. Market values have some advantages: They are objective and easily understood. However, market values are subject to market fluctuations and lack of stability; the market may be itself considered inefficient (e.g., if the asset is thinly traded and transaction costs are high), so that it misprices the appropriate value of the project; also, unrealized gains are taken into account and market values may include speculative bubbles. Moreover, in real life, observed market values can differ from the theoretical

¹⁴ If economic profitability is to be measured for a slice of time $[a, b]$, $a > 0, b < n$, the problem is even more serious, as the capital values in a and b are usually not observable.

market values V_t , so the use of either value depends upon the belief that the market is fairly pricing the asset or mispricing it. Market values may be even less appropriate for certain real assets: What is the market value of the rail tracks owned by a rail company? Historical cost will sometimes be the best information available. Historical costs have the advantage of producing asset values which are more stable, objective, and easily understood and are tied to the financial records of the firm. Also, they avoid speculation and prevent recording an increase in value due to an increase in an input's value. For example, suppose a firm makes use of an input, such as coal, for making steel. If the price of coal goes up, this does not benefit the firm's shareholders, for the firm is not in the business of selling coal, but in the business of making steel. "Indeed, the higher coal price may mean lower profits from steel making... The coal price is timely, but not an indicator of value to a steel maker" (Penman, 2010, pp. 175). In other words, the firms is not "betting on price". Therefore, the use of market values would be inefficient in this case. Historic accounting may be a good basis for a firm's economic profitability. Gross deficiencies of historical accounting (e.g., those can arise with very long-lived assets in highly inflationary settings) can be overcome with by using replacement costs (or variants such as deprival value accounting), although this approach is more subjective (it is not easy to find prices for old plant and equipment). And then there is the problem that numbers might be manipulated by various actors in organisations, giving rise for measurement processes that are less easily manipulable. Penman (2010) endorses the use of "hard" balance sheets: This means "historical cost rather than fair values, but also impairment of carrying values when future losses are forecast" (p. 200) and implies that focus is on transactions, for "transactions are concrete" (p. 201).

The case of a loan is particularly interesting. Under historical cost accounting, these will be shown at face value plus or minus any amortization of premium or discount arising at initiation date. In this case, if the interest rate is constant, then it coincides with the IRR , i , which may be considered the appropriate economic rate of return. However, even the accounting of a loan might be not so obvious and IRR might not be the right choice if there are major doubts regarding its repayment. This has been traditionally dealt with by setting up loan-loss provisions that are then deducted from the amortized book

value, resulting in income being the difference between the gross income computed using the *IRR* and the changes recorded during the period in the loan-loss provision. An alternative approach that has been mooted is to reverse the procedure as follows:

“book the loan at the (transaction) amount lent to the borrower and then, for a few years until the borrower has established his or her good-credit credentials, record interest income on the loan only at the risk-free rate government rate. After creditworthiness has been established ... amortized the cumulated credit spread on the loan (the difference between the lending rate over the government rate) into subsequent earnings.” (Penman, 2010, p. 201)

This involves the use of a well-defined internal *TSCV*. In particular, setting as τ the time where creditworthiness has been established, the *TSCV* proposed is such that $C_t = C_{t-1}(1 + r_t) - F_t$ for $t \leq \tau$ and $C_t = C_{t-1}(1 + \alpha_t) - F_t$ for $t > \tau$ where α_t is the lending rate for the t -the period.¹⁵ In this way, the suggested internal *TSIR* is $\mathbf{i} = (r_1, r_2, \dots, r_\tau, \alpha_{\tau+1}, \alpha_{\tau+2}, \dots, \alpha_n)$. Equation (7) is then used as follows:

$$C_0 = \sum_{t=1}^{\tau} \frac{F_t}{\prod_{h=1}^t (1 + r_h)} + \sum_{t=\tau+1}^n \frac{F_t}{\prod_{h=1}^{\tau} (1 + r_h) \cdot \prod_{h=\tau+1}^n (1 + \alpha_h)}.$$

The loan’s economic rate of return, in this case, is

$$\bar{r}(\mathbf{C}) = w_1 r_1 + \dots + w_\tau r_\tau + w_{\tau+1} \alpha_{\tau+1} + \dots + w_n \alpha_n$$

with obvious definition of w_t , whereas the cost of capital is $\bar{r}(\mathbf{C}) = w_1 r_1 + \dots + w_\tau r_\tau + w_{\tau+1} r_{\tau+1} + \dots + w_n r_n$: If the former is greater than the latter, the loan increases investors’ wealth.

The *IRR* is widely used in Private Financial Initiative (PFI) schemes. However, the advice which the UK Treasury gives to public sector bodies commissioning is not to use it if the project has an uneven profile of cash flows.

The widespread use of *IRRs* in PFI projects reflects the generally even pattern of year-on-year operational cash flows in such projects. However, if a project has an uneven cash flow profile, the Authority should exercise great caution in using *IRR* as the basis of valuing investment in the project.” (Treasury, 2004).

¹⁵ Note that, for $t = n$, the boundary condition $C_n = 0$ implies $\alpha_n = \frac{F_n}{C_{n-1}} - 1$.

(See also Cuthbert and Cuthbert 2012). This means that, either an absolute measure such as the *NPV* is used or a different relative metric based on a different internal *TSCV* should be employed.

The use of accounting rates of return to measure whether a firm is making monopoly profits and, more generally, to regulate the setting of prices in privatized monopolies such as water utility companies has a long history in the USA. The major criticism is that the regulator might not take proper account of the effect of past investments that have long been expensed and therefore are excluded from the capital base when determining whether the prices the company charges are excessive. This can be greatly ameliorated if average *ROA* is used which takes into account past capital expenditures. To see this set $t = 0$ as the current date and let $[-n, 0]$ be the past interval analysed by the regulator. Letting $\mathbf{r} = (r_{-n+1}, r_{-n+2}, \dots, r_0)$ be the structure of normal profit rates and let \overline{ROA} and \bar{r} be, respectively, the ex post average *ROA* and the average normal profit rate:

$$\overline{ROA} = \frac{\bar{I}}{\bar{B}} = \frac{I_{-n+1} \cdot v_{-n+1,-n} + I_{-n+2} \cdot v_{-n+2,-n} + \dots + I_0 \cdot v_{0,-n}}{B_{-n} + B_{-n+1} \cdot v_{-n+1,-n} + \dots + B_{-1} \cdot v_{-1,-n}}$$

$$\bar{r} = \frac{r_{-n+1} \cdot B_{-n} \cdot v_{-n+1,-n} + r_{-n+2} \cdot B_{-n+1} \cdot v_{-n+2,-n} + \dots + r_0 \cdot B_{-1} \cdot v_{0,-n}}{B_{-n} + B_{-n+1} \cdot v_{-n+1,-n} + \dots + B_{-1} \cdot v_{-1,-n}}$$

where B_{-t} represents the book value recorded t periods back and $v_{-t,-n} := 1 / [(1 + r_{-n+1}) \cdot (1 + r_{-n+2}) \dots \cdot (1 + r_{-t})]$, $t = -n + 1, \dots, -1, 0$. The difference between the two rates measures the magnitude of the monopoly position.

Another relevant case is incentive compensation. Either in a corporate setting or in investment portfolio management, compensation is often based on rates of return. A manager is compensated if performance exceeds a certain hurdle rate (or it is tied to a set of increasing hurdle rates). In this case, the choice of depreciation policy changes the economic rate of return, which changes the amount of the reward. So, the determination of a rate of return for this purpose should derive from an appropriate choice of capitals, based on the aims and the purpose of the compensation plan.

These examples are sufficient to highlight the domain-specificity of the rate of return. Any theory of capital valuation needs to recognize that in the real world it is not helpful to view a project simply as a

mere sequence of cash flows that can be represented as a vector \mathbf{F} and then priced and turned into a return metric using the same-size-fits-all calculus. There are infinite equilibrium conditions (7) associated with the same vector of cash flows; and every equilibrium condition is the expression of a well-determined (internal $TSIR$ and) a well-determined internal $TSCV$, which in turn determines a specific economic rate of return $\bar{i}(\mathbf{C})$. Projects (and firms) are entities that are more properly described by

- (i) a set of actions undertaken by the investors,
- (ii) a set of economic transactions involving several different subjects (managers, shareholders, employees, customers, providers, government etc.), and
- (iii) a collection of tools that are necessary to undertake the actions and make the transactions.

However, these can be used as domain-specific elements in a general framework. Formally, all these elements converge into a well-determined triplet $(\mathbf{C}, \mathbf{i}, \mathbf{F})$. We therefore suggest the following formal definition of project:

Definition (project). Formally, a project is represented by a triplet

$$(\mathbf{C}, \mathbf{i}, \mathbf{F}) = (c, \dots, c_{n-1}; i_1, \dots, i_n; F_1, \dots, F_n)$$

of capitals, rates of returns, cash flows.¹⁶

This definition in turn induces a new definition of economic rate of return as a capital-weighted average rate of return. The new definition of a project necessarily involves judgment. Therefore, the choice of an appropriate economic rate of return involves judgment as well. Any reluctance to make value judgments will prevent the achievement of the appropriate economic rate of return.

However, it is also worth noting that different estimates of capital employed in the project affects only the rate of return, not the sign of the excess return $\bar{i} - \bar{r}$, which will always be the same as that of NPV (or opposite if $C < 0$). Therefore, *any TSCV* will correctly capture wealth creation or wealth

¹⁶ Note that the pair (\mathbf{C}, \mathbf{i}) are sufficient to represent a project, as \mathbf{F} is derived from the pair (in an ex ante corporate setting) and *can* be unraveled back from it in an ex post setting.

destruction.¹⁷ This fact gives accounting data a primary role in assessing economic profitability of an investment/firm both in ex ante and in ex post settings. In ex ante decision-making, the equilibrium condition from which the economic rate of return is derived is automatically fulfilled by the accounting data used for estimating cash flows. So, in this case, we have the investment's ex ante average ROA is an appropriate economic rate of return. In ex post assessment of economic profitability, even if the ex post average ROA were not the economic rate of return, wealth creation will nonetheless be captured by the difference $\overline{ROA} - \bar{r}$. This also means that the choice of a particular accounting policy does not impair the appropriateness of the average ROA to measure the overall economic profitability of the investment/firm: Wealth creation is unambiguously signalled. And given that accounting data are available to external users for ex post performance measurement, the average ROA turns out to play a role of paramount importance not only in an ex ante setting (for capital budgeting decisions) but also for ex post analyses, especially considering that the average ROA is unique.¹⁸

Concluding remarks

This paper introduces the notion of the Term Structure of Capital Values (*TSCV*), which is uniquely associated with the term structure of interest rates (*TSIR*). We show that any asset (project, firm etc.) introduced in a market which is in equilibrium creates a state of temporary disequilibrium that can be described in terms of a pair of *internal* (disequilibrium) *TSCV* and *TSIR*. Any internal *TSCV* determines a specific depreciation schedule for the asset and the corresponding *TSIR* determines an economic rate of return as an average of the internal period rates of the *TSIR*. The difference between such an economic rate of return and the overall equilibrium rate (itself an average of the equilibrium forward rates) correctly captures value creation. The determination of a specific (internal) *TSCV* depends on the way the market

¹⁷ In the light of what we have found, to signal wealth creation with a rate of return and to computing the *correct* rate of return is not the same task: The latter always signals wealth creation, but the former is not necessarily the correct rate of return.

¹⁸ Obviously, two different firms may have the same *NPV* (and even the same cash flow stream) but this does not make them the same firm. Therefore, a firm's average ROA does not suffer from any problem of multiplicity. To a given firm, one and only one average ROA is associated. In general, given the new definition of project, to every project there corresponds a unique economic rate of return.

sweeps away arbitrage opportunities and restores the equilibrium. Therefore, our paper shows that an economic rate of return is both a matter of accounting (in terms of a specific internal *TSCV*) and a matter of finance (in terms of a specific disequilibrium-to-equilibrium process). Therefore, an economic rate of return is an average Return On Asset (*ROA*) associated with a specific internal *TSCV*.

An efficient market sweeps disequilibrium away instantaneously, and the *NPV* is a windfall gain for shareholders: We show that the resulting instantaneous rate of return is the well-known profitability index (*PI*), and that the shareholders' first-period rate of return is equal to (the present value of the) average *ROA* derived from a *TSCV* where a cash-flow accounting system is used. We also show that, in such an efficient market, the overall shareholders' rate of return is equal to the average *ROA* that would obtain by employing a fair value accounting system (which implies that the internal period rates after equilibrium has been restored coincide with the equilibrium forward rates). Furthermore, we show that the *IRR* is the average *ROA* generated by an equivalence class of depreciation schedules representing a gradual disequilibrium-to-equilibrium process equivalent to a constant-force-of-interest growth of the capital values.

As each possible accounting system can be viewed as being associated with a corresponding disequilibrium-to-equilibrium process and therefore with a corresponding economic rate of return, the findings of this paper conform to the well-known problem of *underdetermination of a theory by the data* which, in our case, becomes an underdetermination of the *TSCV* (and, therefore, of the economic rate of return) by the available economic data. So, the relevant question is which internal *TSCV* is the most economically appropriate one in the circumstances and why. We argue that the determination of the *TSCV* is domain-specific and a matter of value judgment (and, therefore, a matter of accounting). We present some situations which are economically different and which therefore may trigger different *TSCVs*. This evidently calls for a theory of capital valuation capable of associating the proper *TSCV* with each asset so that the intrinsic underdetermination of the economic rate of return can be solved. This also implies that the notion of project (firm) is not that of a mere sequence of cash flows, but rather as a specific pair of *TSCV* and *TSIR* brought about by (i) a set of actions undertaken by the investors, (ii) a set of economic

transactions involving various economic agents, and (iii) a collections of tools that are necessary to undertake the actions and make the transactions.

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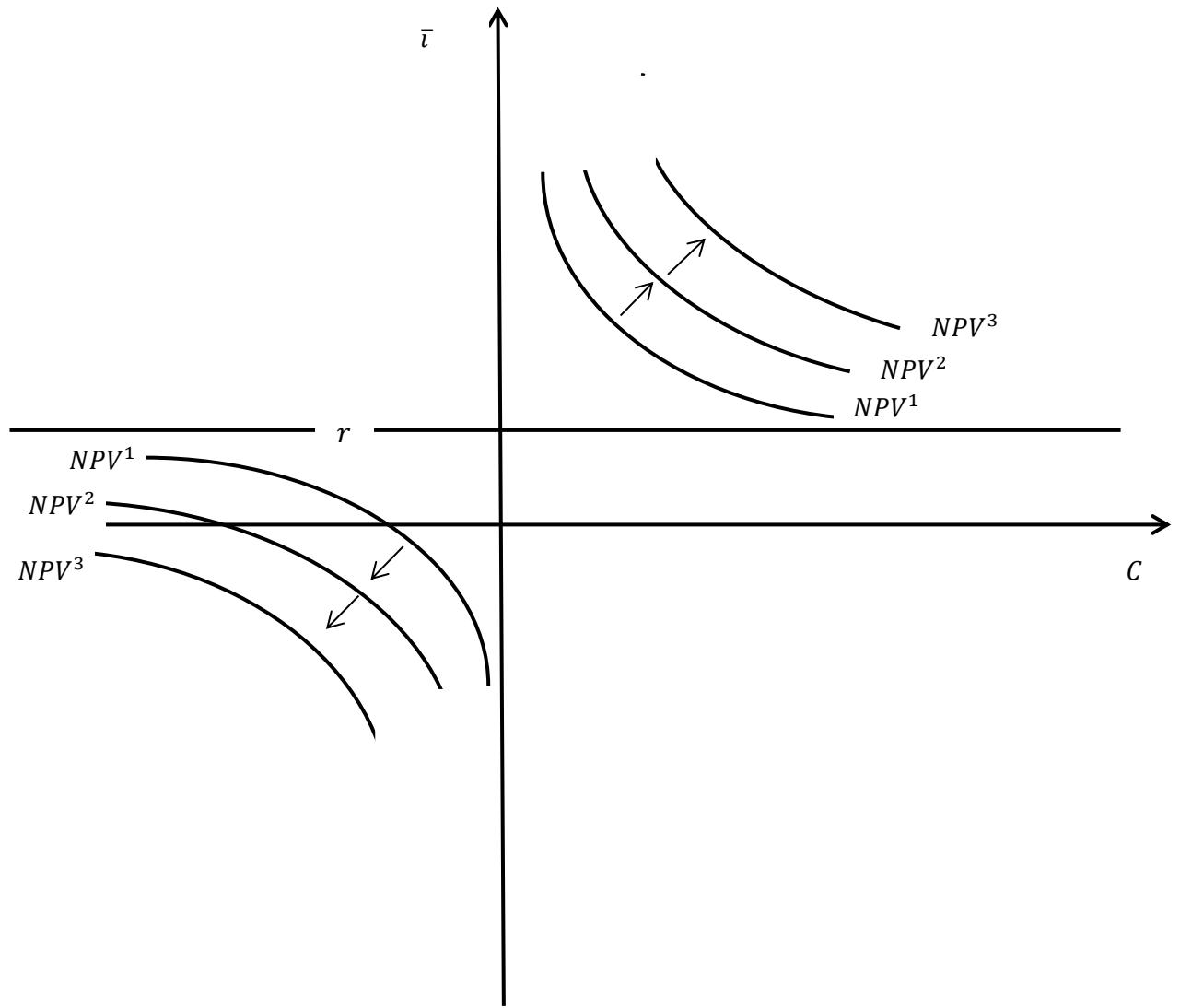


Figure 1. The iso-value line.

The iso-value line represents different combinations of capital and rate of return. The case is represented where the NPV is positive.

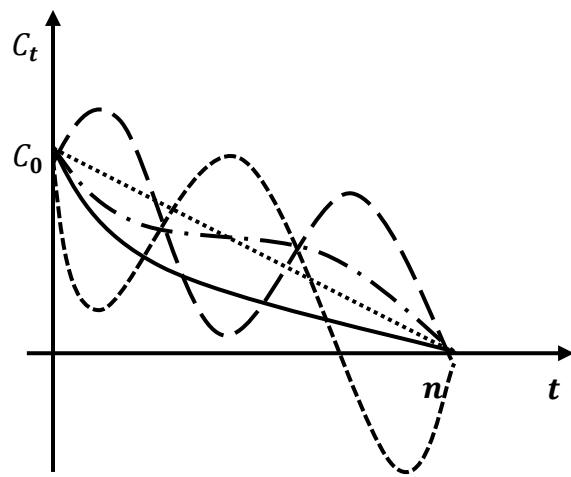


Figure 2. Underdetermination of the TSCV by the economic data. Capital is observed at time 0 and at time n . Each function (TSCV) describes a different disequilibrium-to-equilibrium process and, therefore, different economic of return (i.e., average ROA). (The functions are continuous for illustrative purposes).