Quality cost-based allocation of training hours using learning-forgetting curves

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Abstract

Suppliers and inbound quality inspectors training are common development strategy to increase the supply chain quality performance, under budget constraints these actors compete for a limited amount of training hours. In the proposed model a decision maker allocates these hours minimizing a total quality cost function composed of prevention, appraisal, and failure costs, and sets the inspection rates defining the inspection policies. The relationship between decision variables and costs is expressed through organisational and individual learning-forgetting curves, for suppliers and quality inspectors respectively, and the effect of the training hours on quality improvement is measured in terms of failure rates. To the best of our knowledge, a total quality cost model with such decision variables is new in the literature, as it is a model including both organisational and individual learning-forgetting phenomena.

A nonlinear optimisation approach was adopted to solve such a complex problem. The experimental section includes both a decision trees analysis of simplified scenarios, to interpret the model functioning, and a complex numerical example to extrapolate managerial insights.

Keywords: Supplier development; Training; Quality inspection; Learning-forgetting; Nonlinear optimisation

1. Introduction

The most recently released ISO 9001:2015 enforced the section dedicated to the control of externally provided processes, products and services (clause 8.4), with more rigorous requirements for managing suppliers than in the previous ISO 9001:2008. A step-by-step supplier management approach comprises supplier development in order to improve continuously its capability and performance (Wagner, 2006; Wagner, 2010), which relies on a multitude of manageable activities (Bai & Sarkis, 2011) including training. For a review on supplier development, see Glock, Grosse, & Ries (2017). Quality improvement in products and processes, adaptation to quality standards or reengineering of new components are common examples of goals requiring training activities for suppliers, with the active involvement of the buyer’s management. There are multi-echelon supply chains with suppliers all over the world, often operating under different quality standards and thus requiring periodic monitoring and training. Many firms plan training activities for suppliers as a strategy to improve the suppliers' performance, to such an extent that several consulting companies provide training on behalf of third parties.

This type of supplier development is direct, whereas in indirect approaches the management plans actions that influence the environment in which the suppliers operate in order to create an incentive for them to improve their performance by themselves (Wagner, 2010).

In accordance with the systematic approach to human resources contained within ISO 9001:2015 (e.g. clauses 4.1.2 and 7.1.2), training activities should also be planned for internal employees, with a focus on the quality inspectors operating within inbound inspection sites. After receipt of the items, quality inspectors visually inspect and/or operate different types of equipment, e.g. coordinate-measuring machines, voltmeters, hardness testing tools and so on. These inspection processes require specific skills which inspectors also acquire by training.

However, training suppliers and quality inspectors is costly because it involves trainers, especially when long business trips are needed to reach the suppliers. Moreover, trainers are specialized consultants who charge high prices.

This proposal presented in this paper focuses on how many training hours should be allocated and to whom on a single-period basis. Indeed, very few decision support models have been proposed in the literature to support resources allocation to development programs (Glock, 2016), despite the aforementioned importance of this topic in real settings. To the best of our knowledge, no studies also include the possibility of choosing between different stakeholders, i.e. suppliers and quality inspectors, to be involved in the development programs.

The reasoning for the choice of suppliers and quality inspectors as potential stakeholders of training is strictly quality cost-based. In fact, in accordance with the well-known Prevention, Appraisal, and Failure (PAF) taxonomy of quality costs (Feigenbaum, 1956), the supplier training might be focused, among other supplier attributes, on the replenishment quality, where the development measure used here to monitor quality improvement is the failure rate, which directly affects the failure costs (F). Conversely, the training of inspectors decreases inspection times and thus the appraisal costs (A). At the same time, training suppliers and inspectors increases the prevention costs (P). The allocation of training hours to suppliers and quality inspectors is thus a non-trivial problem

from a continuous improvement viewpoint, and this is the primary goal of our proposal.
To enrich our analytical setting further, the additional decision variables included in the model are the inspection rates to assign to the replenished items on a period by period basis. These variables determine the inspection policies to apply to suppliers and affect both failure and appraisal costs. The effect of the inspection rates on the quality cost function can be revealed by considering that failure costs are higher in downstream than in upstream stages. For instance, an inspection policy with a unitary inspection rate maximises appraisal costs and minimizes failure costs.
The relationship of the training hours as development resources with the quality costs is highlighted by learning-forgetting curves that capture the effects of training on failure and appraisal costs over time. In other words, suppliers and inspectors are subject to organisational and individual learning mechanisms, respectively, which depend on training activities (i.e. induced learning) and repetitions (i.e. autonomous learning) and constitute the foundation of the optimal allocation of the training hours. The need to interconnect continuous improvement with learning curves has been investigated by (Zangwill & Kantor, 1998), and more recently by other authors (e.g. Wang, Plante, & Tang, 2013; Lolli et al., 2016).
Our contribution is two-fold: i) to solve the problem of allocating training hours both to suppliers and to quality inspectors by a single-period decision support system based on learning-forgetting curves, organizational and individual, respectively; ii) to show the relationship that inspection policies have with the optimal allocation solution. Neither issues have been addressed jointly in the related literature by means of mathematical programming approaches. In addition, to the best of our knowledge, the coexistence of organisational and individual learning-forgetting effects in a quality cost-based trade-off problem, i.e. the allocation of training hours, is new in the literature related to learning theory.
The paper is organised as follows. Section 2 provides a review of the literature on supplier development and human learning. Section 3 details the operative environment for which our decision support system has been designed, along with the notation adopted throughout the manuscript. Section 4 focuses on the learning processes underlying the model described in Section 5. Sections 6 and 7 detail the design of the experiment and the results achieved, respectively, while Section 8 provides conclusions and ideas for further research.

2. Literature review

Supplier development is a broad topic in industrial engineering, where three stages can be identified (Glock et al., 2017):

i) Preparation of supplier development. This stage identifies the suppliers that the buyer intends to develop.

ii) Supplier development in the strictest sense, which is aimed at defining the development initiatives that constitute either the direct or the indirect development program.

iii) Monitoring and evaluation of supplier development.

The resource allocation to development programmes represents a step in stage ii), and is also the core of our proposal. Wagner (2011), Glock et al. (2017) and Meisel & Glock (2018) have already underlined that the majority of previous contributions have dealt with supplier development either theoretically or empirically. Regarding stage ii), there are still only a few mathematical models available, especially those dealing with the allocation of resources to development programs; some of them are overviewed in the following.

Kim (2000) investigated a single-buyer single-supplier supply chain where the buyer must evaluate the need for a subsidy to lower the production costs at the supplier via a learning curve modelling. The development attribute under control is thus the production cost at the supplier, and the development program is direct due to the active involvement of the buyer. A lower production cost at the supplier leads in turn to lowering the selling price on the market. In the case of a price-sensitive demand, supplier development shows the greatest benefits. A similar setting was analysed by Proch, Worthmann, & Schlüchtermann (2017), who formulated a continuous time optimal control model for the capital investments in supplier development. They considered both direct and indirect development initiatives through win-to-win perspective, where the buyer can intensify the supplier's participation by subsidizing a share of the investment costs. The efficient level of subsidy over time is therefore the variable to optimise in order to make development profitable. Zhu, Zhang, & Tsung (2007) specifically focused on quality-based development programmes for suppliers undertaken by the buyer in order to reduce the expected number of non-conforming units, whose related failure costs fall both on the buyer and on the suppliers. They analytically derived the optimal order (buyer) and production (supplier) quantities in accordance with the quality-based development actions undertaken.

A quality-based development of suppliers was explored by Lolli et al. (2016) with the rate of non-conforming units as the development attribute to reduce over time periods. They introduced a single-period constrained nonlinear optimization approach for allocating training hours to suppliers. In order to assess the relationship between the improvement in the rate of non-conforming units (dependent variable) and training hours (i.e. induced learning source) and cumulative production volume (i.e. autonomous learning source), a linear learning curve with time-varying learning rates was modelled. Bhattacharyya & Guiffrida (2015) adopted untimely deliveries by suppliers as the development attribute, and introduced an optimization approach, constrained by an upper bound of the available budget, to find the optimal investment to spend in such a development program. Undertaking a development program with suppliers was also investigated by Marchi, Ries, Zanoni, & Glock (2016), where the buyer exploits a lower interest rate than its supplier. This condition enables the buyer to invest in increasing the supplier's productivity with a certain amount of risk. Cui, Deng, Liu, Zhang, & Xu (2017) dealt with the exactness of the suppliers' inventory status, since inventory inaccuracies have severe consequences on the effectiveness of the supply chain. The development program in their work is specifically focused on investments in RFID technology. Capital allocation to suppliers for development programmes has also been studied by Mizgier, Pasia, & Talluri (2017), where a buyer has to select the suppliers to develop while considering the investment risks. The authors proposed a multi-objective optimization approach for such a selection problem. Bai & Sarkis (2016) adopted various game theoretical models to evaluate different development investment strategies, e.g. tangible actions such as capital resources and sharing costs of capital resources as well as intangible actions such as knowledge investments. Different types of supplier development (i.e. learning capabilities, knowledge transfer, and external acquisition of knowledge) have also been taken into account by Glock (2016) and Glock, Jaber, & Guiffrida (2011), focused in particular on delegating workers as a source of supplier training, and proposed a profit-based model to achieve their optimal number, as well as the timing and duration of supplier training. They adopted a Cobb-Douglas type learning curve, based on Wright's pioneering power curve (Wright, 1936), for modelling unitary production costs, and obtained the group learning rates directly by multiplying the individual learning rates for the number of workers employed in the production process (Glock & Jaber, 2014). Meisel & Glock (2018) proposed a profit-based optimisation approach to support the buyer in selecting suppliers to develop and choose the type of development to undertake. The authors referred to self-induced performance improvement as the autonomous learning mechanism arising by allocating additional production quantities to suppliers, as opposed to direct project-based development programmes. A learning curve similar to the Stanford-B model (Carlson, 1987) was adopted in their work.

As shown above, several papers used learning curves to model the relationship between the development attribute (e.g. cost, quality, capacity, service level and so on) and the control variable (e.g. capital investment, workers, training hours and so on). This is not surprising. Learning curves – individual, group, and organizational – provide progress functions with certain independent variables, including the cumulative production volume which is the standard one in univariate models, affecting the dependent variable, which is typically the unitary production time subjected to experience. Consequently, human learning modelling has several applications for supplier development.

From the early definition of the well-known power curve (Wright, 1936), where the unitary production time decreases due to the increasing cumulative number of produced items (i.e. autonomous learning), several papers have focused on specialising and diversifying learning curves (e.g. Jaber & Guiffrida, 2004; Jaber & Guiffrida, 2008; Jaber, Goyal, & Imran, 2008; Jaber & Glock, 2013). The reverse of learning is forgetting, leading to poorer performance. Empirical findings (e.g. Globerson, Levin, & Shtub, 1989) indicate that forgetting depends both on the length of interruption and on the cumulative experience gained prior to the interruption: see Sikström (2002) and Sikström & Jaber (2002, 2012). Jaber & Bonney (1996) composed learning and forgetting into a single power law curve for the first time. Jaber, Kher, & Davis (2003) investigated the effect of cross training and deployment in order to reduce the effects of forgetting. Jaber, Givi, & Neumann (2013) also incorporated fatigue and recovery into the model, and Givi, Jaber, & Neumann (2015) used a learning-forgetting model to estimate the human-related error rate.

It is worth highlighting how numerous contributions have focused both on supplier development and on the definition and application of learning curves. Nevertheless, there is currently no total quality cost-oriented model with learning-forgetting effects that also evaluates the quality inspectors as potential stakeholders of the training programmes. It follows that organizational and individual learning-forgetting curves, for suppliers and quality inspectors respectively, interact for the optimal allocation of resources for training programmes; which has never been addressed in previous works. To highlight the novelty of this proposal further, inspection rates have been considered as additional variables to optimise. In fact, inspection rates have already been dealt with in different optimisation approaches, for instance by game models (e.g. Hsieh & Liu, 2010; Aust, Bräuer, &

Buscher, 2014), however never within an optimisation approach based on learning-forgetting effects for the allocation of development resources.

3. The operative environment

In our method, a single-buyer multi-supplier supply chain is considered (Figure 1). Time is discretized in learning cycles in line with the continuous improvement concept proposed in Zangwill & Kantor (1998). In each cycle, the suppliers replenish the buyer with deterministic quantities of items with a certain rate of non-conforming units, and each supplier is associated with a single item. The multi-item material flow from suppliers passes through the inbound inspection site where quality inspectors operate with an error-free inspection process, i.e. without type-I and type-II errors, on the basis of different inspection rates among suppliers, and each inspector is associated with a single item. Suppliers, items and inspectors are thus in a one-to-one relationship, but this does not limit the applicability of our proposal for allocating training hours to suppliers and inspectors. In fact, it is reasonable to refer to single items both for replenishment and for inspection requiring specific knowledge and equipment. Conforming units after the inspection, along with the units that have not been inspected, pass to the downstream production/assembly stages. A non-conforming unit inspected within the inbound inspection site generates a lower failure cost (rework or scrap) than the cost related to defects in uninspected units that are revealed in subsequent production/assembly stages. Therefore, two levels of failure costs are considered.

The optimisation model for the allocation to training hours is single-period, which means that at the beginning of a cycle, the buyer has to allocate training hours for the incoming cycle both to suppliers and to inspectors, as well as to establish the optimal inspection rate to be adopted for each supplier. The learning processes governing the temporal evolution of the suppliers' and inspectors' performance are described in Section 4. The notation in Table 1 is adopted throughout the manuscript. Although the proposed model is single-period, the subscript t is kept for several costs, total costs and parameters reported in Table 1 in order to model their dynamics due to learning-forgetting effects. In experiment 2 (see Section 6), the model is also launched over consecutive cycles.

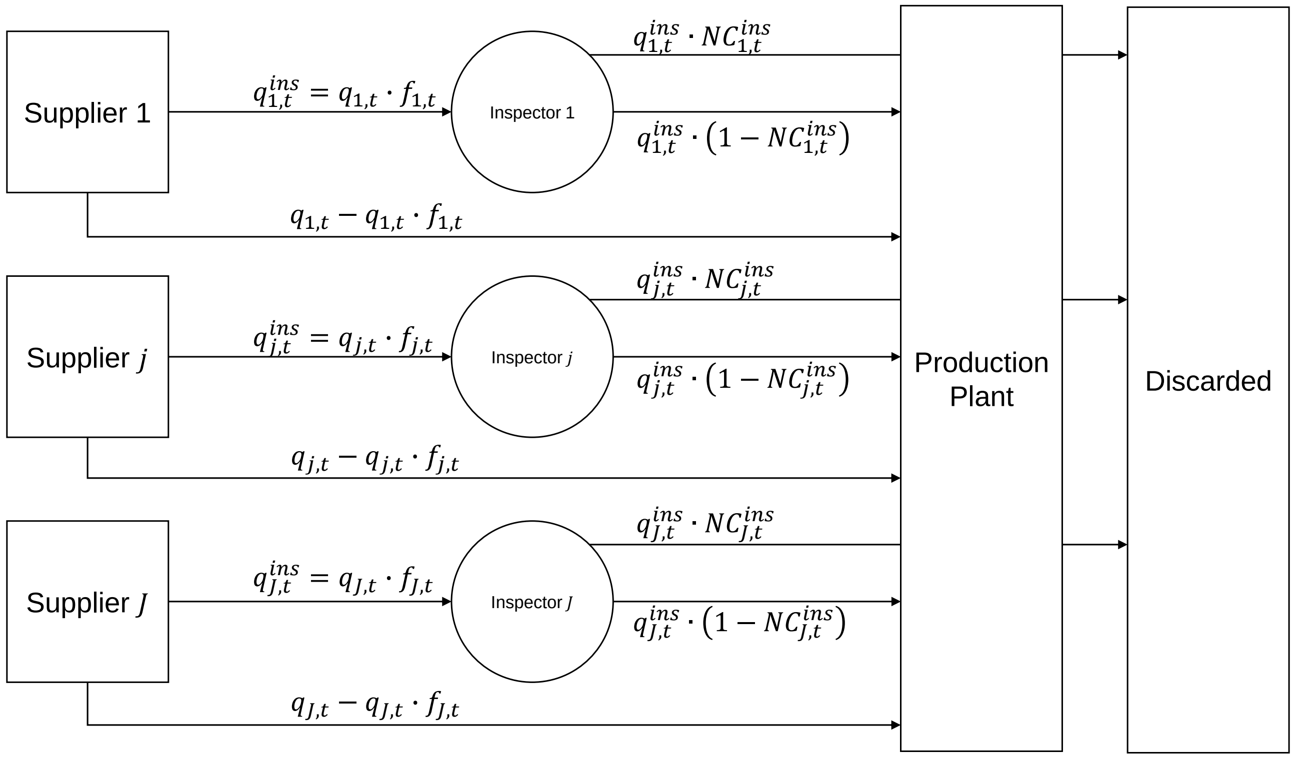

Figure 1. Single-buyer multi-supplier supply chain.

Indexes and number of inputs	
J	Number of suppliers (i.e. types of items) involved in the training program.
j	Supplier, with $j = 1, \dots, J$.
t	Consecutive learning cycles.
m	Consecutive forgetting cycles.
Constraints	
$maxQ^{inb}$	Maximum number of items inspected.
H^{sup}	Maximum number of training hours assignable to the suppliers.
$maxh_j^{sup}$	Maximum number of training hours assignable to supplier j.
H^{ins}	Maximum number of training hours assignable to the inspectors.
$maxh_j^{ins}$	Maximum number of training hours assignable to inspector j.

T_{max}^{ins}	Maximum inspection time.
Decision variables	
$h_{j,t}^{sup}$	Number of training hours assigned to supplier j in learning cycle t.
$h_{j,t}^{ins}$	Number of training hours assigned to inspector j in learning cycle t.
$f_{j,t}$	Inspection sample rate for supplier j in learning cycle t.
Costs	
$c_{j,t}^{app}$	Unitary appraisal cost for inspector j at the end of learning cycle t.
$c_{j,min}^{app}$	Minimum unitary appraisal cost for inspector j.
$\tilde{c}_{j,0}^{app}$	Unitary appraisal cost for inspector j at the beginning of forgetting cycle 1.
$\bar{c}_{j,t}^{app}$	Mean unitary appraisal cost for inspector j at the end of learning cycle t.
c_{j}^{sup}	Unitary training cost for supplier j.
c_{j}^{ins}	Unitary training cost for inspector j.
c_{j}^{inb}	Unitary failure cost for supplier j when a non-conforming unit is detected within the inbound inspection site.
c_{j}^{st}	Unitary failure cost for supplier j when a non-conforming unit is detected in the production/assembly stages.
c_h	Hourly cost of an inspector.
Total costs	
$EPC_{j,t}$	Prevention cost for supplier j in learning cycle t.
$EAC_{j,t}$	Appraisal cost for supplier j in learning cycle t.
$EFC_{j,t}$	Failure cost for supplier j in learning cycle t.

Learning rates	
a_j	Autonomous learning rate of supplier j.
b_j	Induced learning rate of supplier j.
l_j^{ins}	Autonomous learning rate of inspector j.
α_j	Induced learning rate of inspector j.
f_j^{ins}	Autonomous forgetting rate of inspector j.
f_j^{sup}	Autonomous forgetting rate for supplier j.
$leq_{j,t}^{ins}$	Equivalent learning rate of inspector j at the end of learning cycle t.
Parameters	
$NC_{j,t}$	Rate of non-conforming units of supplier j at the end of learning cycle t.
$\widetilde{NC}_{j,0}$	Unitary appraisal cost for supplier j at the beginning of forgetting cycle 1.
$q_{j,t}$	Number of items replenished by supplier j in learning cycle t.
$Q_{j,t}$	Cumulated number of items replenished by supplier j in learning cycle t.
$q_{j,t}^{ins}$	Number of items replenished by supplier j and inspected in learning cycle t.
$Q_{j,t}^{ins}$	Cumulated number of items replenished by supplier j and inspected items at learning cycle t.
$\tilde{q}_{j,m}$	Number of items that would have been replenished by supplier j in forgetting cycle m, had the interruption not occurred.
$\tilde{q}_{j,m}^{ins}$	Number of items that would have been replenished by supplier j and inspected in forgetting cycle m, had the interruption not occurred.
$\tilde{Q}_{j,m}$	Cumulated number of items that would have been replenished by supplier j in forgetting cycle m, had the interruption not occurred.
$\tilde{Q}_{j,m}^{ins}$	Cumulated number of items that would have been replenished by supplier j and inspected in forgetting cycle m, had the interruption not occurred.

$nc_{j,t}$	Number of non-conforming units from supplier j in learning cycle t.

Table 1. Notation.

4. **The underlying learning-forgetting processes**

Autonomous and induced learning-forgetting phenomena affect two dependent variables throughout the cycles, and when the autonomous learning is not operating a forgetting process takes place decreasing the overall efficiency of the system.

The first dependent variable is the failure rate (Section 4.1); suppliers autonomously learn by supplying as well as by receiving training hours (induced learning) and forget while not supplying. The number of supplied items cannot be adjusted, while the training hours for each supplier are controlled by management and act as independent variables.

The second dependent variable is the appraisal cost, which is linearly related to the unitary inspection time of the quality inspectors. The unitary inspection time decreases both autonomously and by means of the training hours allocated to the inspectors (induced learning), while not inspecting leads to forgetting that subsequently increases the appraisal cost.

To sum up, four processes have been identified:

i) External autonomous learning-forgetting, involving suppliers due to repetitions-disruption.
ii) External induced learning-forgetting, involving suppliers due to training-not training.
iii) Internal autonomous learning-forgetting, involving quality inspectors due to repetitions-disruption.
iv) Internal induced learning-forgetting, involving quality inspectors due to training-not training.

4.1 The suppliers' learning-forgetting

In this paper, the rate of non-conforming units $NC_{j,t}$ of supplier j at the end of learning cycle t is adopted as the quality metric to control rather than the more traditional process variance, while the supplied quantity $Q_{j,t}$ and the number of external training hours $h_{j,t}^{sup}$ are adopted as independent variables of autonomous and induced learning, respectively. The choice of the rate of non-conforming units as the quality metric is justified in real settings, where the quality inspections operate as stop-and-go filters.

A cycle is considered here either as a learning t or as a forgetting m cycle. After a set of consecutive learning cycles, the first forgetting cycle starts with $m = 1$ while, after a set of consecutive forgetting cycles, the first learning cycle starts with $t = 1$.

Non-linear models with fixed learning rates have been traditionally used for individual learning from the pioneering contribution of Wright (1936) and validated in several laboratory settings (e.g. Bailey, 1989). We adopt the standard power form with different dependent (i.e. $NC_{j,t}$) and independent (i.e. $Q_{j,t}$ and $h_{j,t}^{sup}$) variables as follows:

$$NC_{j,t} = NC_{j,0}\left(Q_{j,t}\right)^{-a_j-b_jh_{j,t}^{sup}} \tag{1}$$

with $0 < a_j + b_jh_{j,t}^{sup} < 1$, a_j and b_j being the autonomous and induced learning rates of supplier j. $NC_{j,0}$ is the initial rate of non-conforming units, obtained after a single quantity of product is supplied ($Q_{j,0} = 1$), and $Q_{j,t} = Q_{j,t-1} + q_{j,t}$ is the cumulated supplied quantity at the end of learning cycle t.

If $NC_{j,0}$ is unknown, its value can be recovered from Equation 1 as:

$$NC_{j,0} = NC_{j,t}\left(Q_{j,t}\right)^{a_j+b_jh_{j,t}^{sup}} \tag{2}$$

Equation 1 can be rewritten in a one-period-ahead formulation as:

$$NC_{j,t} = NC_{j,t-1}\left(\frac{Q_{j,t-1}}{Q_{j,t}}\right)^{a_j+b_jh_{j,t}^{sup}} \tag{3}$$

Under the assumption that $h_{j,t}^{sup}$ is concentrated at the beginning of the learning cycle, the number of non-conforming units from supplier j during cycle t is given by:

$$nc_{j,t} = \frac{NC_{j,t-1}}{1-a_j-b_jh_{j,t}^{sup}}\left(\left(Q_{j,t}\right)^{1-a_j-b_jh_{j,t}^{sup}}\left(Q_{j,t-1}\right)^{a_j+b_jh_{j,t}^{sup}} - Q_{j,t-1}\right) \tag{4}$$

Proofs for Equations 3 and 4 are provided in Appendix 1.

The forgetting phenomenon arises as a mirror process of autonomous learning and takes place when the supply of item j is interrupted ($q_{j,m} = 0\ \forall m$). The forgetting curve adopted here follows a power form (Carlson & Rowe, 1976):

$$NC_{j,m} = \widetilde{NC}_{j,0}\left(\tilde{Q}_{j,m} + 1\right)^{f_j^{sup}} \tag{5}$$

where $f_j^{sup} > 0$ is the autonomous forgetting rate, $\widetilde{NC}_{j,0}$ is the initial rate of non-conforming units ($\tilde{Q}_{j,0} = 0$), and $\tilde{Q}_{j,m} = \tilde{Q}_{j,m-1} + \tilde{q}_{j,m}$ is the amount of items j that would have been inspected during m forgetting cycles, had the interruption not occurred. $\tilde{Q}_{j,m}$ is generally unknown but can be hypothesized, especially in cases of supplied quantities stationary in mean.

Equation 5 can be rewritten in a one-period-ahead formulation as:

$$NC_{j,m} = \widetilde{NC}_{j,m-1}\left(\frac{\tilde{Q}_{j,m-1}+1}{\tilde{Q}_{j,m}+1}\right)^{-f_j^{sup}} \tag{6}$$

The concept of total forgetting described in Jaber & Bonney (1996) enables us to determine the forgetting rate f_j^{sup} without any other assumption. $NC_{j,m}$ is assumed to increase to $NC_{j,0}$ after a certain $\tilde{Q}_{j,m}$, which is the number of 'uninspected' items annulling the effects of the last learning cycles.

At the end of each learning cycle t, an equivalent learning rate $leq_{j,t}$ is computed as:

$$leq_{j,t} = -\frac{ln\left(\frac{NC_{j,t}}{NC_{j,0}}\right)}{ln(Q_{j,t})} \tag{7}$$

This is the fixed learning rate required to reach $NC_{j,t}$ from $NC_{j,0}$ after $Q_{j,t}$ inspected items. After each forgetting cycle, the number of inspected items $Q_{j,t}$ is recomputed as:

$$Q_{j,t} = \left(\frac{NC_{j,0}}{NC_{j,t}}\right)^{\frac{1}{leq_{j,t}}} \tag{8}$$

The forgetting cycle virtually reduces the number of inspected items, and in the extreme case of total forgetting ($NC_{j,m} = NC_{j,0}$) the learning restarts from scratch with only one unit inspected.

4.2 The inspectors' learning-forgetting

The unitary inspection time for supplier j during learning cycle t is subject to learning, the appraisal costs due to the inbound quality inspections $c_{j,t}^{app}$ are linearly related to it therefore such costs are treated as the dependent variables of the internal learning-forgetting phenomenon.

As for the suppliers' learning a power form is adopted for the appraisal costs, with inspected quantity $q_{j,t}^{ins}$ and the number of external training hours $h_{j,t}^{ins}$. Such a power form is corrected by introducing the plateau $c_{j,min}^{app}$, the lower bound of $c_{j,t}^{app}$ below which there is no further improvement:

$$c_{j,t}^{app} = c_{j,0}^{app}\left(Q_{j,t}^{ins}\right)^{-l_j^{ins}-\alpha_j h_{j,t}^{ins}} + c_{j,min}^{app} \tag{9}$$

with $0 < l_j^{ins} + \alpha_j h_{j,t}^{ins} < 1$, l_j^{ins} and α_j being the autonomous and induced learning rates of supplier j. $c_{j,0}^{app}$ is the appraisal cost achieved after inspecting one item ($Q_{j,0}^{ins} = 1$) and $Q_{j,t}^{ins} = Q_{j,t-1}^{ins} + q_{j,t}^{ins}$ is the cumulated number of inspected items at the end of cycle t. $c_{j,min}^{app}$ is fixed at the theoretical unitary cost due to the lowest unitary appraisal time, which is the time spent by the maximally expert inspector for inspecting one item j.

If $c_{j,0}^{app}$ is unknown its value can be recovered from Equation 9 as:

$$c_{j,0}^{app} = \left(c_{j,t}^{app} - c_{j,min}^{app}\right)\left(Q_{j,t}^{ins}\right)^{l_j^{ins}+\alpha_j h_{j,t}^{ins}} \tag{10}$$

Equation 9 can be rewritten in a one-period-ahead formulation as:

$$c_{j,t}^{app} = \left(c_{j,t-1}^{app} - c_{j,min}^{app}\right)\left(\frac{Q_{j,t-1}^{ins}}{Q_{j,t}^{ins}}\right)^{l_j^{ins}+\alpha_j h_{j,t}^{ins}} + c_{j,min}^{app} \tag{11}$$

The forgetting phenomenon takes place when the supply of item j is interrupted ($q_{j,m} = 0\ \forall m$):

$$c_{j,m}^{app} = \tilde{c}_{j,0}^{app}\left(\tilde{Q}_{j,m}^{ins} + 1\right)^{f_j^{ins}} \tag{12}$$

where $f_j^{ins} > 0$ is the autonomous forgetting rate, $\tilde{c}_{j,0}^{app}$ is the initial appraisal cost ($\tilde{Q}_{j,0}^{ins} = 0$), and $\tilde{Q}_{j,m}^{ins} = \tilde{Q}_{j,m-1}^{ins} + \tilde{q}_{j,m}^{ins}$ is the amount of items j that would have been inspected during m forgetting cycles, had the interruption not occurred.

Equation 12 can be rewritten in a one-period-ahead formulation as:

$$c_{j,m}^{app} = c_{j,m-1}^{app}\left(\frac{\tilde{Q}_{j,m-1}^{ins}+1}{\tilde{Q}_{j,m}^{ins}+1}\right)^{-f_j^{ins}} \tag{13}$$

At the end of each learning cycle t, an equivalent learning rate $leq_{j,t}^{ins}$ is computed as:

$$leq_{j,t}^{ins} = -\frac{ln\left(\frac{c_{j,t}^{app}-c_{j,min}^{app}}{c_{j,0}^{app}}\right)}{ln\left(Q_{j,t}^{ins}\right)} \tag{14}$$

This is the fixed learning rate required to reach $c_{j,t}^{app}$ from $c_{j,0}^{app}$ after $Q_{j,t}^{ins}$ inspected items.

After each forgetting cycle, the number of inspected items $Q_{j,t}^{ins}$ is recomputed as:

$$Q_{j,t}^{ins} = \left(\frac{c_{j,0}^{app}}{c_{j,t}^{app}-c_{j,min}^{app}}\right)^{\frac{1}{leq_{j,t}^{ins}}} \tag{15}$$

The forgetting cycle virtually reduces the number of inspected items, and in the extreme case of total forgetting $c_{j,m}^{app} = c_{j,0}^{app} + c_{j,min}^{app}$ the learning restarts from scratch with only one unit inspected.

Given the single-cycle formulation of total costs, $\bar{c}_{j,t}^{app}$ is the mean value of the unitary appraisal cost within cycle t, i.e. between $Q_{j,t-1}^{ins}$ and $Q_{j,t}^{ins}$ inspected items. Equation 11 is integrated and divided by $Q_{j,t}^{ins} - Q_{j,t-1}^{ins}$ leading to:

$$\bar{c}_{j,t}^{app} = \frac{c_{j,t-1}^{app}-c_{j,min}^{app}}{1-l_j^{ins}-\alpha_j h_{j,t}^{ins}} \cdot \frac{{Q_{j,t}^{ins}}^{1-l_j^{ins}-\alpha_j h_{j,t}^{ins}}\, {Q_{j,t-1}^{ins}}^{l_j^{ins}+\alpha_j h_{j,t}^{ins}} - Q_{j,t-1}^{ins}}{Q_{j,t}^{ins}-Q_{j,t-1}^{ins}} + c_{j,min}^{app} \tag{16}$$

$Q_{j,t}^{ins}$ depends on the inspection policy applied to supplier j in learning cycle t, $Q_{j,t}^{ins} = Q_{j,t-1}^{ins} + q_{j,t}f_{j,t}$, the inspection ratio $f_{j,t}$ being a decision variable in the model:

$$\bar{c}_{j,t}^{app} = \frac{c_{j,t-1}^{app} - c_{j,min}^{app}}{1 - l_j^{ins} - \alpha_j h_{j,t}^{ins}} \cdot \frac{\left(Q_{j,t-1}^{ins} + q_{j,t} f_{j,t}\right)^{1 - l_j^{ins} - h_{j,t}^{ins}} {Q_{j,t-1}^{ins}}^{l_j^{ins} + \alpha_j h_{j,t}^{ins}} - Q_{j,t-1}^{ins}}{q_{j,t} f_{j,t}} + c_{j,min}^{app} \quad (17)$$

5. The minimization cost model

Prevention, appraisal and failure costs are combined into a total cost function to minimise in each learning cycle.

5.1 Prevention cost

To reduce the appraisal and failure costs, instead of generic induced learning variables as in Wang, Plante, & Tang (2013), the training hours are made explicit as the induced learning source, which does not limit the applicability of our proposal to other induced learning sources. The external and internal training activities generate in learning cycle t the expected prevention cost $EPC_{j,t}$ due to item j:

$$EPC_{j,t} = c_j^{sup} h_{j,t}^{sup} + c_j^{ins} h_{j,t}^{ins} \quad (18)$$

The first and the second terms refer to the external and internal training costs, respectively.

5.2 Appraisal cost

The cost incurred in cycle t due to the inspection of item j depends on the number of inspected items $f_{j,t}q_{j,t}$ and therefore on the inspection policy adopted. From Equation 16, the expected appraisal cost for item j in cycle t is as follows:

$$EAC_{j,t} = \bar{c}_{j,t}^{app} \, q_{j,t} f_{j,t} \quad (19)$$

5.3 Failure cost

The expected failure cost depends on when the failure is detected leading to different unitary failure costs for the inbound inspection site and the subsequent assembly/production stage:

$$EFC_{j,t} = \left(c_j^{inb} f_{j,t} + c_j^{st}\left(1 - f_{j,t}\right)\right) nc_{j,t} \quad (20)$$

5.4 Total cost

Combining Equations 18, 19, and 20 for all the items, the total cost function to minimize is:

$$min \sum_{j=1}^{J} \left(EPC_{j,t} + EAC_{j,t} + EFC_{j,t}\right) \tag{21}$$

s. t.

$$h_{j,t}^{sup} \leq maxh_j^{sup} \quad \forall j = 1, \dots, J \tag{22}$$

$$\sum_{j=1}^{J} h_{j,t}^{sup} \leq H^{sup} \tag{23}$$

$$h_{j,t}^{ins} \leq maxh_j^{ins} \quad \forall j = 1, \dots, J \tag{24}$$

$$\sum_{j=1}^{J} h_{j,t}^{ins} \leq H^{ins} \tag{25}$$

$$\sum_{j=1}^{J} q_{j,t}\, f_{j,t} \leq maxQ^{inb} \tag{26}$$

$$\sum_{j=1}^{J} \frac{EAC_{j,t}}{c_h} \leq T_{max}^{ins} \tag{27}$$

$$a_j + b_j h_{j,t}^{sup} < 1 \quad \forall j = 1, \dots, J \tag{28}$$

$$l_j^{ins} + \alpha_j h_{j,t}^{ins} < 1 \quad \forall j = 1, \dots, J \tag{29}$$

$$0 \leq f_{j,t} \leq 1 \quad \forall j = 1, \dots, J \tag{30}$$

$$h_{j,t}^{sup} \in \mathbb{R} \quad \forall j = 1, \dots, J \tag{31}$$

$$h_{j,t}^{ins} \in \mathbb{R} \quad \forall j = 1, \dots, J \tag{32}$$

$$f_{j,t} \in \mathbb{R} \quad \forall j = 1, \dots, J \tag{33}$$

The continuous non-linear problem above contains $3J$ decision variables. Equations 22 and 23 constrain the number of training hours allocated to the suppliers to not exceed their upper bounds, both on the individual suppliers ($maxh_j^{sup}$) and overall (H^{sup}). Similar constraints are imposed in Equations 24 and 25 for the training hours allocated to the quality inspectors. Equations 26 and 27 constrain the capacity of the inbound inspection site during a learning cycle. Equation 26 imposes an upper bound of the total number of inspected items $maxQ^{inb}$, a space capacity constraint of the inbound inspection site, and the Equation 27 guarantees that the total inspection time does not exceed an upper bound T_{max}^{ins}, with c_h equal to the hourly cost of an inspector. The unitary inspection time of item j in period t is a hidden variable dependent on the learning process modelled in Section 4.2 in terms of appraisal costs; which can be obtained as $\bar{c}_{j,t}^{app}/c_h$. Equations 28 and 29 ensure that the learning rates are always less than 1, even when training hours are allocated.

6. Design of experiment

6.1 Data setup

The validation of the model was carried out in two experiments:

i) A test of total cost minimization in a single learning cycle given different inputs.
ii) A test of the model during consecutive cycles to show the learning-forgetting dynamics.

The first experiment captures the optimizer behaviour in different circumstances by validating the single cycle coherence of the model, while the second one captures the multi-cycle behaviour of the model by validating its coherence and ability to describe real life scenarios. Note that the model is again single-period in the second experiment and is simply launched over consecutive cycles.

In the first experiment, 512 scenarios arise from all the combinations of the variables' values reported in Table 2. These fixed variables are designed to loosen the constraints in Equations 22, 23, 24, 25, 26 and 27 and let the optimizer train the supplier and inspector up to their maximum cost effectiveness, outlining in each scenario the trade-off between $f_{1,1}$, $h_{1,1}^{sup}$ and $h_{1,1}^{ins}$. The design choice of implementing a single supplier and a single inspector $(J = 1)$ arises from the need for clarity, while one-dimensional inputs and outputs make the optimizer logic simpler and thus the results easier to analyse.

In the second experiment, 100 consecutive cycles are generated. Each cycle has either a replenishment of 500 items $\left(q_{j,t} = 500\right)$ or no replenishment $\left(q_{j,m} = 0\right)$, and a cycle with replenishment is equivalent in the forgetting process to 250 items not inspected $\left(\tilde{q}_{j,m}^{ins} = 250\right)$. The probability of generating a replenishment cycle is 0.9 and the cycle takes place simultaneously for all three suppliers $(J = 3)$. The values of the variables are outlined in Table 3. If there is a replenishment, the decision variables f_{1t}, $h_{1,t}^{sup}$ and $h_{1,t}^{ins}$ are optimized by minimizing the total cost of the model in Equations 21 to 33. If there is no replenishment, forgetting takes place as in Equations 13, 14 and 15.

In Tables 2 and 3, t_0 refers to the last period before the simulation takes place; it differs from zero since the cumulated number of items replenished and inspected at the beginning of the first period, Q_{1,t_0} and Q_{1,t_0}^{ins} respectively, does not necessarily equal 1. Equations 2 and 10 are used, with $h_{j,t_0}^{sup} = 0$ and $h_{j,t_0}^{ins} = 0$, to evaluate $NC_{1,0}$ and $c_{1,0}^{app}$. Note that the ranges of the autonomous learning rates adopted in both experiments conform, up to a scale parameter, to those reported in the literature from many industries (e.g. Dar-El, 2000). The induced learning, in absence of reference values, represent the fraction of learning hours impacting the learning process. These values are applied in Equations 7, 8, 14 and 15 to find the equivalent learning rates and adjust the amount of supplied and inspected items.

Equations 28 and 29 are implemented in both experiments as non-strict inequalities:

$$a_j + b_j h_{j,t}^{sup} \leq 0.95 \quad \forall j = 1, \dots, J \tag{34}$$

$$l_j^{ins} + \alpha_j h_{j,t}^{ins} \leq 0.95 \quad \forall j = 1, \dots, J \tag{35}$$

This implementation is required to solve the problem using a sequential quadratic programming algorithm.

Parameter	Min value	Max value
J	1	
NC_{1,t_0}	0.1	0.9
$q_{1,1}$	100	
Q_{1,t_0}	100	
Q_{1,t_0}^{ins}	100	
$maxQ^{inb}$	120	
H^{sup}	16	
$maxh_1^{sup}$	16	
H^{ins}	16	
$maxh_1^{ins}$	16	
T_{max}^{ins}	1000	
a_1	0.1	0.9
b_1	0.1	0.9
c_{1,t_0}^{app}	1	10
$c_{1,min}^{app}$	0	

l_1^{ins}	0.1	0.9
α_1	0.1	0.9
c_1^{sup}	1	20
c_1^{ins}	1	20
c_1^{inb}	0	
c_1^{st}	1	20
c_h	10	

Table 2. First experiment data.

Parameter	Value		
J	3		
NC_{j,t_0}	0.9		
Q_{j,t_0}	1		
Q_{j,t_0}^{ins}	1		
$maxQ^{inb}$	750		
H^{sup}	16		
$maxh_j^{sup}$	8		
H^{ins}	1.6		
$maxh_j^{ins}$	0.8		
T_{max}^{ins}	40		
a_j	0.00862	0.00468	0.00074
b_j	0.01	0.01	0.01
c_{j,t_0}^{app}	16		
$c_{j,min}^{app}$	2		

l_j^{ins}	0.862	0.468	0.074
α_j	0.1	0.1	0.1
f_j^{sup}	0.0468	0.0468	0.0468
f_j^{ins}	0.468	0.468	0.468
c_j^{sup}	50		
c_j^{ins}	50		
c_j^{inb}	10		
c_j^{st}	100		
c_h	50		

Table 3. Second experiment data.

6.2 Data analysis

In the first experiment, nine variables (NC_{1,t_0}, a_1, b_1, c_{1,t_0}^{app}, l_1^{ins}, α_1, c_1^{sup}, c_1^{ins}, c_1^{st}) are modified obtaining three results ($f_{1,1}$, $NC_{1,1}$, $c_{1,1}^{app}$). The system described in Equations 21 to 33 is highly non-linear, thus a linear regression analysis is unlikely to identify meaningful relations between inputs and outputs. The aim of the constraint relaxation, embedded in the first experiment, is to obtain results that are easier to analyse: the optimizer is expected to concentrate either on the supplier, $f_{1,1} = 0$, or on the inspector, $f_{1,1} = 1$, training only the most profitable stakeholder and uncovering the implicit trade-offs in the system.

If the results present binary values for $f_{1,1}$, a decision tree (Appendix 2) is implemented to identify under which conditions the optimizer focuses on suppliers or on inspectors, the input variables are predictors for the classification problem, and $f_{1,1} = 0$ and $f_{1,1} = 1$ are two classes. If a linear regression analysis is performed after the classification step, each leaf of the tree can be analysed independently. This is thus a different scenario, and the variables used to split before such leaf can be discarded. The obtained linear regressions are expected to be simpler than the one performed using all the data and more meaningful since they focus on specific scenarios.

7. Results and discussion

The inspection rate $f_{1,1}$ obtained in first experiment is either 0 or 1. In fact the optimizer, which is free of constraints, either inspects all the replenished items or none of them depending on the scenario variables. Figure 2 depicts the decision tree obtained using the impurity gain as the split criterion and not restricting the tree depth, the resulting leaves are pure.

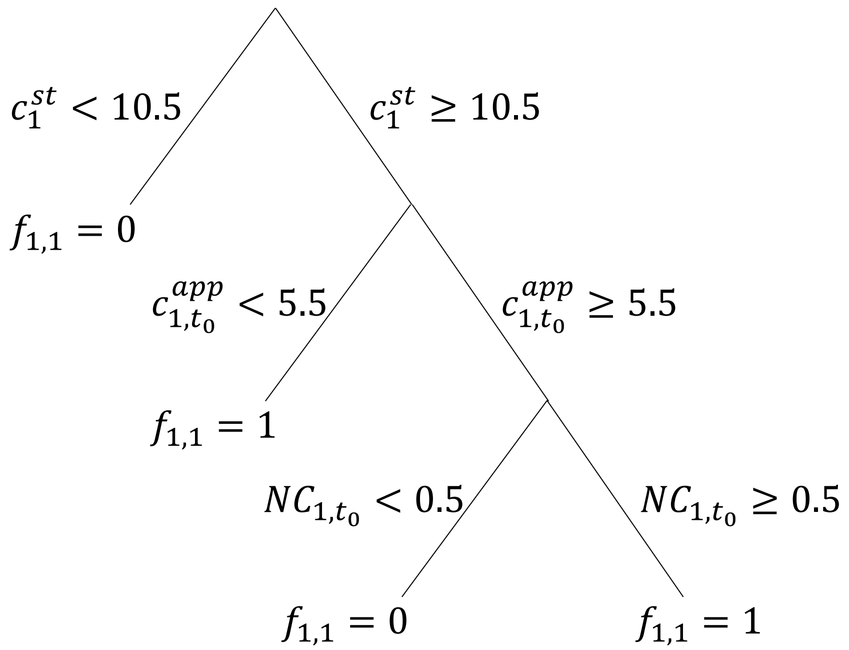

Figure 2. Decision tree classifying the inspection rate in the first experiment.

The optimizer first discriminates between $c_1^{st} = 1$ and $c_1^{st} = 20$; in the first case the unitary outbound failure cost is low, thus inspections are not needed $(f_{1,1} = 0)$, while in the second case inspections might be necessary. If $c_1^{st} = 20$, the optimizer discriminates between $c_{1,t_0}^{app} = 1$ and $c_{1,t_0}^{app} = 10$; in the first case the unitary appraisal cost is low, thus the optimizer leverages it by inspecting all the replenished items $(f_{1,1} = 1)$. If $c_{1,t_0}^{app} = 10$ the optimizer discriminates between $NC_{1,t_0} = 0.1$ and $NC_{1,t_0} = 0.9$; in the first case the initial rate of non-conforming units is low, thus the optimizer does not schedule any inspection $(f_{1,1} = 0)$, while in the second case a significant number of non-conforming units leads to the optimizer inspecting everything $(f_{1,1} = 1)$.

Table 4 outlines the fixed relations between $f_{1,1}$, $h_{1,1}^{sup}$ and $h_{1,1}^{ins}$. If no replenished item is inspected $(f_{1,1} = 0)$, the inspectors are not trained, in fact training the inspectors would only lead to costs with no added benefits. If all the replenishment items are inspected $(f_{1,1}=1)$, the suppliers are not trained, inspections are carried out in any case and the unitary failure cost for units detected inbound c_1^{inb} is zero, thus there is no need to train the suppliers and decrease the rate of non-conforming units $NC_{1,1}$. It should be noted that the converse of these cases is not necessarily true, no replenished items inspected $(f_{1,1}=0)$ does not necessarily lead to supplier's training and full replenished item inspection $(f_{1,1}=1)$ does not necessarily lead to inspector's training. For instance, a case with a high unitary outbound failure cost c_1^{st} and low initial unitary appraisal cost c_{1,t_0}^{app} could lead to full inspection $(f_{1,1}=1)$ without any need for any further improvement in the inspector, as c_{1,t_0}^{app} is already low.

$f_{1,1}=0$	$h_{1,1}^{ins}=0$
$f_{1,1}=1$	$h_{1,1}^{sup}=0$

Table 4. Fixed relations between output variables.

Each leaf in Figure 2 can be individually analysed with a linear regression, whose objective is to obtain either the values of $h_{1,1}^{ins}$ or $h_{1,1}^{sup}$ using the variables as features. If a subset of variables can accurately predict $h_{1,1}^{ins}$ or $h_{1,1}^{sup}$, then those are the variables evaluated by the optimizer in that scenario. Conversely, the subset of variables that are non-significant for the prediction are disregarded by the optimizer's logic.

The analysis is carried out for each leaf as follows:

1. Only the cases categorized in the leaf at hand are used for linear regression.
2. An initial linear regression without interactions is carried out using all the variables with more than one value as features. If there is no inspection $(f_{1,1}=0)$ then $h_{1,1}^{sup}$ is predicted, while if there is an inspection $(f_{1,1}=1)$, the regression predicts $h_{1,1}^{ins}$.
3. A t-test is carried out for each variable to assess its significance, with p-values higher than 0.05 leading to discarding.
4. A new regression is carried out with the remaining variables, including the interactions among variables as features. The new regression is preferred if an F-test between the two results in a p-value lower than 0.05.
5. A new regression is carried out including more complex interactions and executing

an F-test at each stage. The analysis stops if no more interactions can be included or the F-test reveals no significance; this last case leads to the last significant model.

Table 5 contains the significant variables for each leaf, the predicted variable, the mean squared error obtained, and the R-squared statistics. In all cases the most significant model is always the one with all the possible interactions included.

Fixed variables values	Significant variables	Predicted variable	MSE	R
$c_1^{st} = 1$	a_1 b_1 c_1^{sup} NC_{1,t_0}	$h_{1,1}^{sup}$	$6.67 \cdot 10^{-7}$	1
$c_1^{st} = 20$ $c_{1,t_0}^{app} = 1$	l_1^{ins} α_1 c_1^{ins}	$h_{1,1}^{ins}$	$1.38 \cdot 10^{-7}$	1
$c_1^{st} = 20$ $c_{1,t_0}^{app} = 10$ $NC_{1,t_0} = 0.1$	a_1 b_1 c_1^{sup}	$h_{1,1}^{sup}$	$6.38 \cdot 10^{-8}$	1
$c_1^{st} = 20$ $c_{1,t_0}^{app} = 10$ $NC_{1,t_0} = 0.9$	l_1^{ins} α_1	$h_{1,1}^{ins}$	$8.71 \cdot 10^{-8}$	1

Table 5. Results of the regression at each leaf.

Table 5 shows the coherence in the optimizer's decisions. In the leaves with no inspection $(f_{1,1} = 0)$, the number of training hours for the supplier $h_{1,1}^{sup}$ is determined by accounting for the autonomous a_1 and induced b_1 learning rate of the supplier as well as the unitary training cost c_1^{sup} and the initial rate of nonconforming units NC_{1,t_0}. In the leaves with full inspection $(f_{1,1} = 1)$, the number of training hours for the inspectors $h_{1,1}^{ins}$ is determined by accounting for the autonomous l_1^{ins} and the induced α_1 learning rate of the inspector as well as the unitary training cost c_1^{ins}. It should also be noted that the variables used in the nodes are evaluated in the node itself and thus do not appear in the decision process used in the subsequent leaves.

The only regression that can be tri-dimensionally plotted in Figure 3 is in the last row of Table 5, corresponding to the last leaf of the tree in Figure 2. In this case the number of training hours increases in a non-linear fashion as the autonomous l_1^{ins} and the induced α_1 rate of the inspector decreases. The optimizer allocates the maximum number of training hours while satisfying Equation 35.

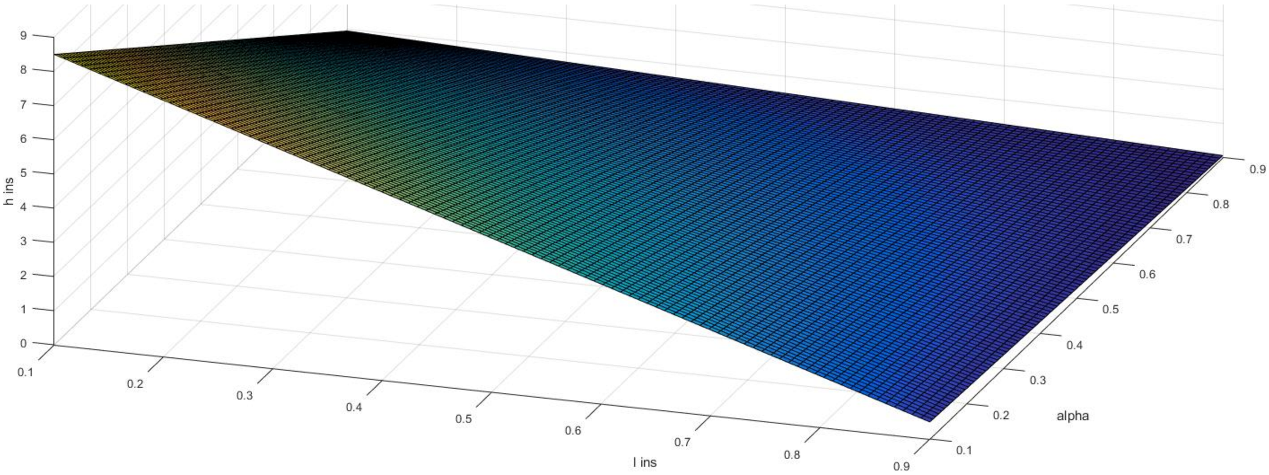

Figure 3. Linear regression predicting $h_{1,1}^{ins}$ as a function of l_1^{ins} and α_1.

The results of the second experiment are shown in Figures 3, 4, 5, where a dotted line represents the supplier or inspector 1, a dashed line represents the supplier or inspector 2, and a solid line, the supplier or inspector 3. Figure 4 presents $c_{j,t}^{app}$ and shows that inspectors 1 and 2 are trained reaching very low $c_{1,t}^{app}$ and $c_{2,t}^{app}$ while inspector 3 is never trained and, as a result, no unit from supplier 3 is ever inspected ($f_{3,t} = 0\ \forall t$). Figure 5 depicts $NC_{j,t}$, and indicates that most of the training is allocated to suppliers 2 and 3, thus supplier 1 presents a much higher $NC_{1,t}$ overall. Figure 6 depicts the total cost for each: Given a set of consecutive learning cycles, the total cost decreases rapidly reaching a plateau and, if a forgetting cycle takes place, it increases sharply undoing most of the past learning. There is no steady-state as learning and forgetting push in turn the total cost in different directions.

From an optimization standpoint, Figures 3, 4 and 5 reveal an overall preference for suppliers and inspectors 1 and 2 over supplier and inspector 3. In fact, the presence of multiple stakeholders generates trade-off scenarios where choices are made not only between $NC_{j,t}$ and $c_{j,t}^{app}$ within the same supplier/inspector couple, but also between different stakeholders competing for a limited amount of resources (e.g. $h_{j,t}^{sup}$and $h_{j,t}^{ins}$). Figures 6 to 10 show these trade-offs by analysing how each decision variable (e.g. $h_{1,t}^{sup}$) in a period t is affected by both the previous state of the system ($c_{j,t-1}^{app}\ \forall j$, $NC_{j,t-1}\ \forall j$, $q_{j,t-1}\ \forall j$, $q_{j,t-1}^{ins}\ \forall j$) and the other decision variables (e.g. $h_{2,t}^{sup}$, $h_{3,t}^{sup}$, $h_{1,t}^{ins}$, $h_{2,t}^{ins}$ and $h_{3,t}^{ins}$). In Figure 7, $h_{1,t}^{sup}$ does not depend on other suppliers or inspectors, and a low $f_{1,t}$ coupled with a high rate of $NC_{1,t-1}$ is what triggers the training. In Figure 8, $h_{2,t}^{sup}$ depends not only on supplier and inspector 2 but also on supplier and inspector 1. For instance, from the second branch of the decision tree, if $f_{1,t}$ is low then the training hours are assigned to supplier 1 instead. To simplify the classification, the single case with $h_{2,t}^{sup} = 3.92$ is not fed to the decision tree. Overall $h_{2,t}^{sup}$ follows a more complex logic than $h_{1,t}^{sup}$, factoring in not only $f_{2,t}$ and $NC_{2,t-1}$ but also $c_{2,t-1}^{app}$: a low $c_{2,t-1}^{app}$ results in $h_{2,t}^{sup} = 0$ as inspector 2 takes care of the non-conforming units.

In Figure 9, $h_{3,t}^{sup}$ depends only on inspector 1 through $c_{1,t-1}^{app}$. If $c_{1,t-1}^{app}$ is low, supplier 3 is not trained. There is no relation between $h_{3,t}^{sup}$ and the other features of inspector or supplier 3. In Figure 10, $h_{1,t}^{ins}$ depends only both on supplier and inspector 1, similarly to $h_{1,t}^{sup}$: training takes place in cases of high $NC_{1,t-1}$ and $h_{1,t}^{sup}$, when it is the most impactful. In Figure 11, $h_{2,t}^{ins}$ behaves similarly to $h_{3,t}^{sup}$ given that $h_{2,t}^{ins}$ does not depend on inspector or supplier 2 features but only on $NC_{3,t-1}$.

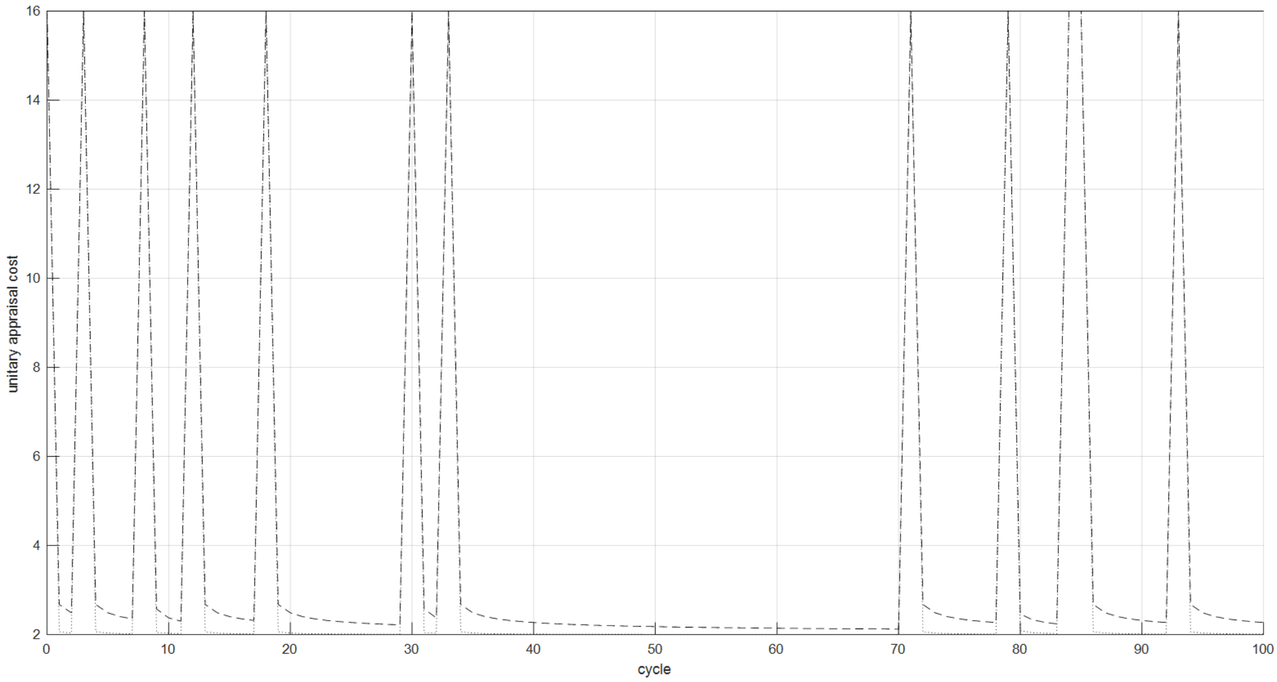

Figure 4. Unitary appraisal cost for each learning cycle.

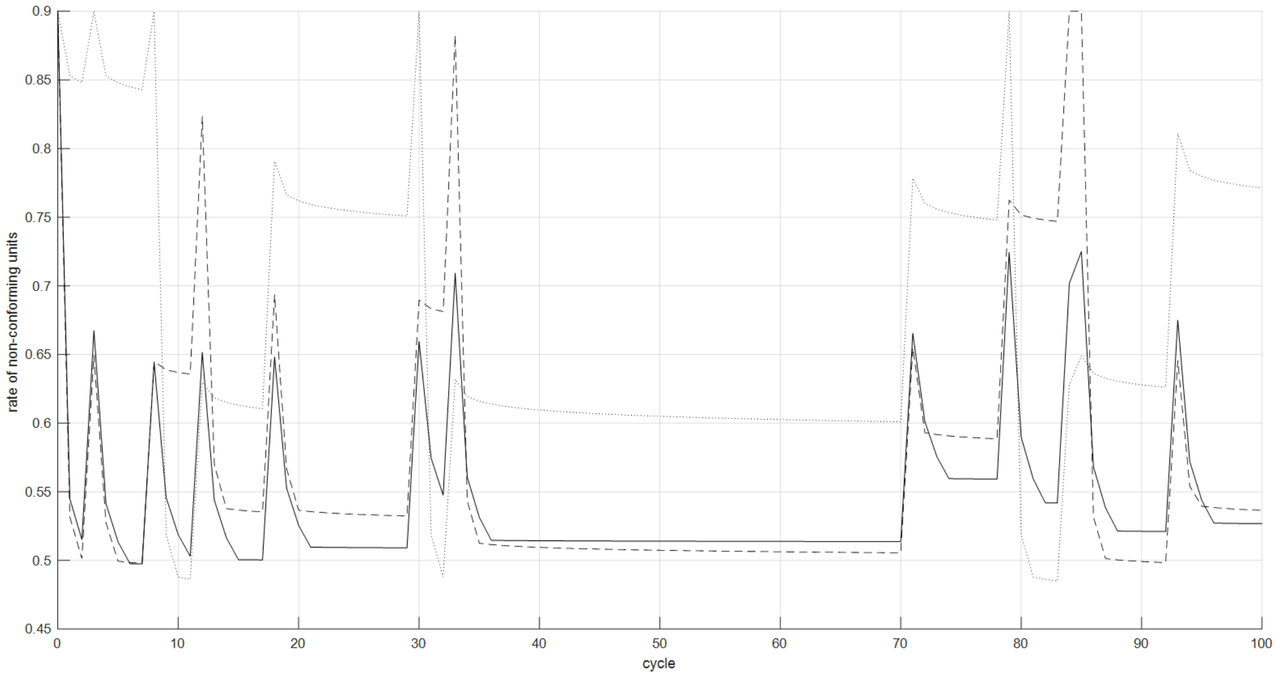

Figure 5. Rate of non-conforming units for each learning cycle.

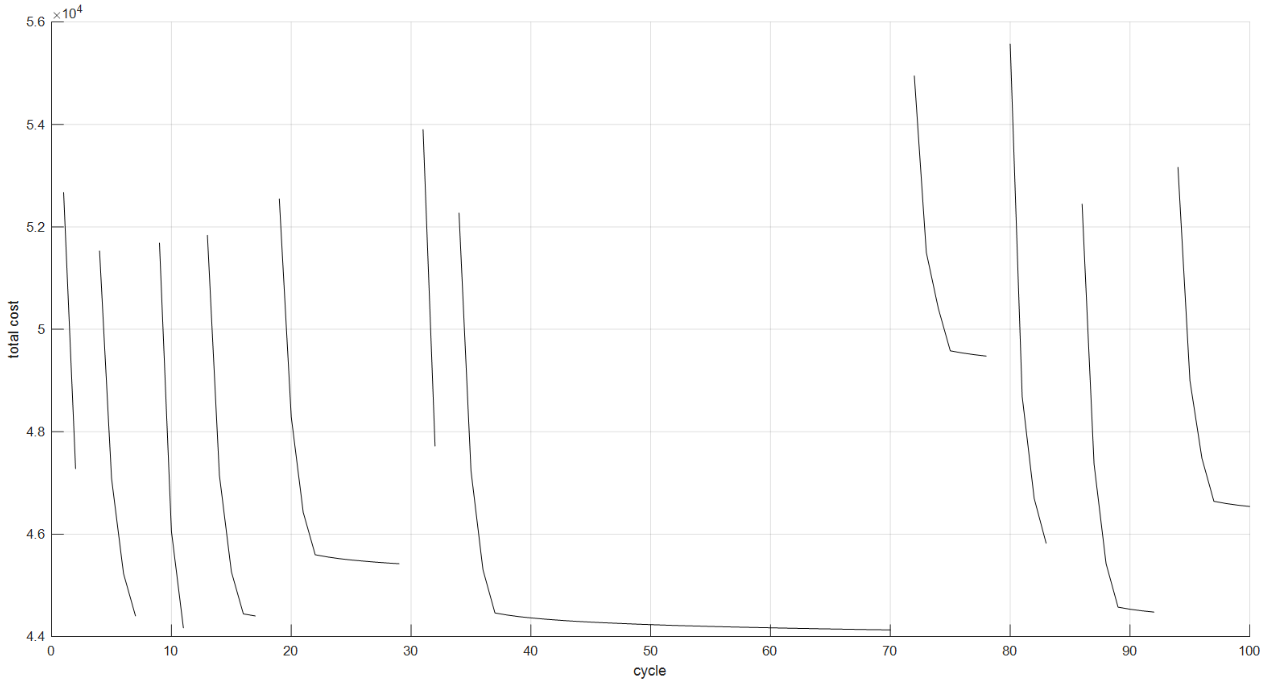

Figure 6. Total cost for each learning cycle.

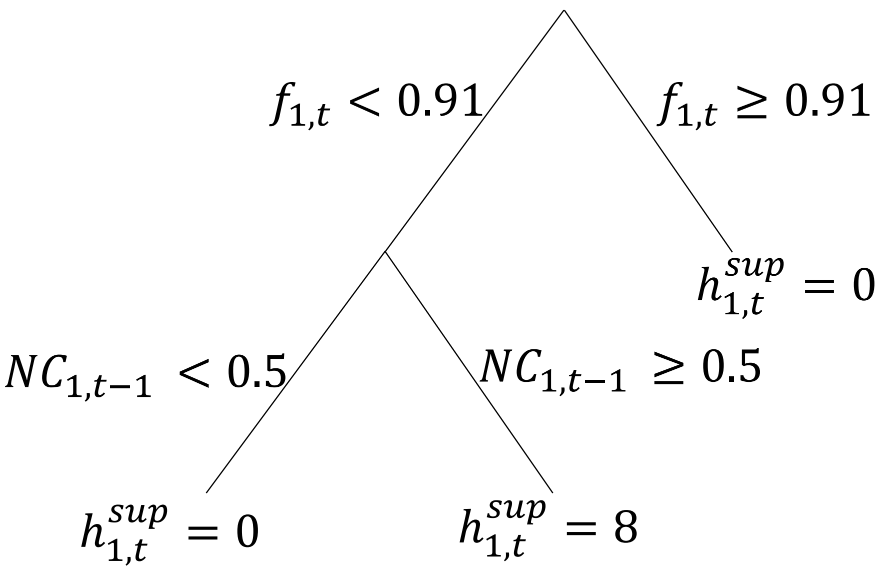

Figure 7. Number of training hours assigned to supplier 1 for each learning cycle.

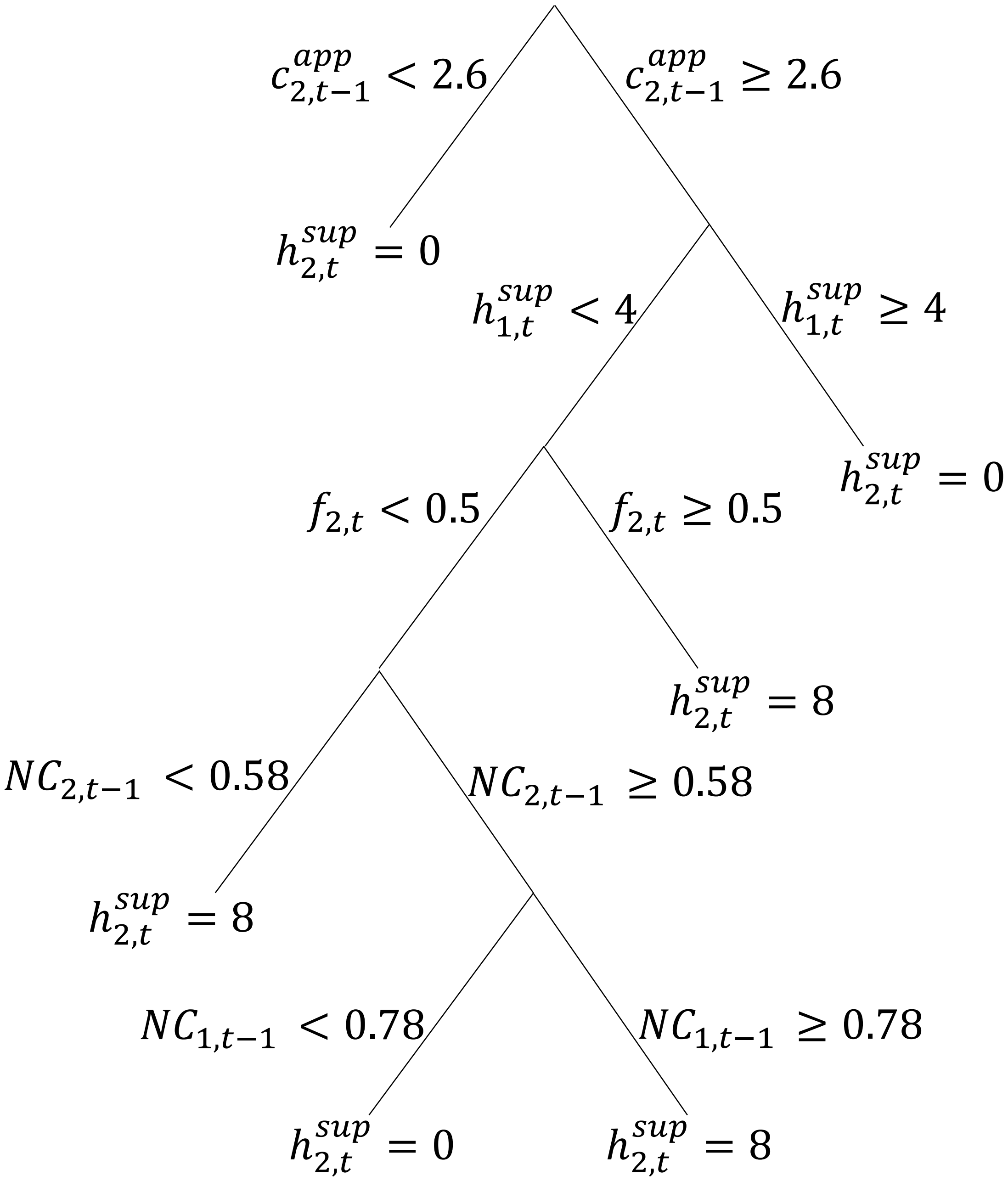

Figure 8. Number of training hours assigned to supplier 2 for each learning cycle.

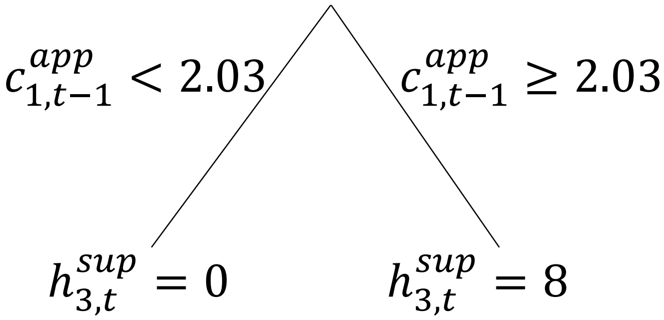

Figure 9. Number of training hours assigned to supplier 3 for each learning cycle.

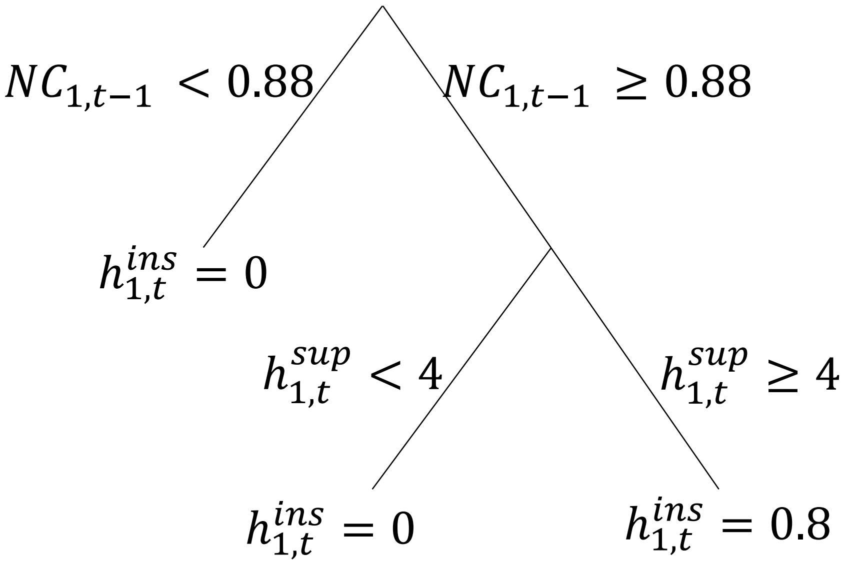

Figure 10. Number of training hours assigned to inspector 1 for each learning cycle.

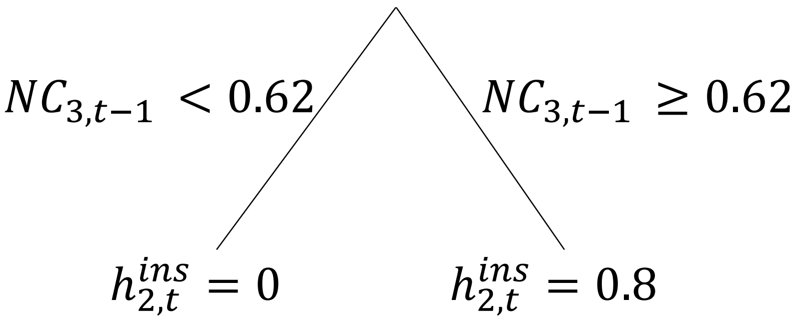

Figure 11. Number of training hours assigned to inspector 2 for each learning cycle.

8. Conclusions and further research agenda

Supplier training is receiving increasing attention as a part of a broad supplier development strategy. However, there are still few mathematical models and decision support systems designed to allocate budget-constrained resources to training activities. In this paper have addressed this allocation problem. Starting with a total quality cost function composed of prevention, appraisal, and failure costs, a further training stakeholder was identified, that is the quality inspectors operating within the inbound inspection site. Finally, the inspection rates applied to suppliers were considered as the third type of decision variables.

The analytical interaction between suppliers, quality inspectors, and inspection policies has never been tackled before but, in our work, it was made possible by a total quality function. In fact, every development activity, such as training, is strictly related to learning-forgetting curves, where the improvement achieved (the decrease in failure rate, in our case) is the effect of manageable control variables. Training hours are control variables leading to induced learning-forgetting, along with the production/inspection volume as autonomous learning-forgetting sources. Suppliers and quality inspectors are subjected to organisational and individual learning-forgetting mechanisms, respectively, which have never been undertaken jointly in a total cost function. In our opinion, this is a further contribution of our proposal.

A single-period non-linear optimisation model is proposed and validated by two experiments: the first considers 512 different single-period mono-supplier scenarios and gauges the optimizer logic; the second computes a multi-period multi-supplier case in order to assess the system capability of managing longer time frames and trade-offs between the control variables.

From a methodological viewpoint with regards to the results analysis, an approach based on decision trees was applied in order to improve the interpretability of the achieved results. This is a new idea with future potential when too many decision variables prevent standard statistics from being applied. In fact, the results obtained are case-sensitive, but the decision trees showed that the presence of multiple stakeholders generates trade-off scenarios where choices are made not only within the same supplier/inspector couple but also between different stakeholders competing for a limited amount of resources.

Our proposal has some limitations however, which we plan to address in future research. First, the failure costs due to non-conforming units increase from the inbound inspection site to the downstream stages, but they are not affected by experience. Hence, learning-forgetting effects in the reworking could be added to the model. Moreover, the inspection process was considered as error-free, thus type-I and type-II errors could be added to the model. Finally, the learning-forgetting process induced by the training was modelled by means of the standard power law curve. A cognitive model of memory decay could thus be adopted and might help solve the problem of allocating massed vs spaced training hours.

Appendix 1

At the end of cycle $t-1$ the parameters $NC_{j,t-1}$ and $Q_{j,t-1}$ are available while, using a one-period-ahead formulation, $NC_{j,0}$ is unknown. A value for $NC_{j,0}$ can be obtained from Equation 1 assuming $NC_{j,t-1}$ and $Q_{j,t-1}$ are generated using the same learning rate as $NC_{j,t}$:

$$NC_{j,0} = NC_{j,t-1}\left(Q_{j,t-1}\right)^{a_j+b_j h_{j,t}^{sup}} \tag{36}$$

Equation 1 and 36 lead to:

$$NC_{j,t} = NC_{j,t-1}\left(Q_{j,t-1}\right)^{a_j+b_jh_{j,t}^{sup}}\left(Q_{j,t}\right)^{-a_j-b_jh_{j,t}^{sup}} \tag{37}$$

This simplifies into Equation 3.

The integral of Equation 3 is the number of non-conforming units from supplier j during cycle t:

$$nc_{j,t} = \int_{Q_{j,t-1}}^{Q_{j,t}} NC_{j,t-1}\left(\frac{Q_{j,t-1}}{Q}\right)^{a_j+b_jh_{j,t}^{sup}} dQ \tag{38}$$

which can be simplified to:

$$nc_{j,t} = NC_{j,t-1}\left(Q_{j,t-1}\right)^{a_j+b_jh_{j,t}^{sup}} \int_{Q_{j,t-1}}^{Q_{j,t}} (Q)^{-a_j-b_jh_{j,t}^{sup}} dQ \tag{39}$$

And is solved as:

$$nc_{j,t} = NC_{j,t-1}\left(Q_{j,t-1}\right)^{a_j+b_jh_{j,t}^{sup}}\left[\frac{(Q)^{1-a_j-b_jh_{j,t}^{sup}}}{1-a_j-b_jh_{j,t}^{sup}}\right]_{Q_{j,t-1}}^{Q_{j,t}} \tag{40}$$

$$nc_{j,t} = \frac{NC_{j,t-1}}{1-a_j-b_jh_{j,t}^{sup}}\left(Q_{j,t-1}\right)^{a_j+b_jh_{j,t}^{sup}}\left(\left(Q_{j,t}\right)^{1-a_j-b_jh_{j,t}^{sup}} - \left(Q_{j,t-1}\right)^{1-a_j-b_jh_{j,t}^{sup}}\right) \tag{41}$$

That simplifies in Equation 4.

Appendix 2

Given a dataset with m datapoints, $\{\vec{x}_i, y_i\}$ $i = 1,..,m$, each input vector being n-dimensional, $\vec{x}_i \in R^n$ $\forall i$, and each output being binary, $y_i \in \{0,1\}$ $\forall i$, the aim of a decision tree is to predict y_i given $\vec{x}_i$.

A decision tree (e.g. Figure 9) is composed of splits, branches and leaves. Each split s_k sends an input vector $\vec{x}_i$ requiring prediction through one of two branches, $b_{k,1}$ and $b_{k,2}$, based on its value along one input variable j. If $x_{i,j} < c_k$ $\vec{x}_i$ is sent to branch $b_{k,1}$otherwise it is sent to $b_{k,2}$ where c_k is a cut point associated with the split s_k. A branch can lead to a split, where $\vec{x}_i$ is sorted again, or to a leaf l_h where a prediction over y_i is made and the process stops.

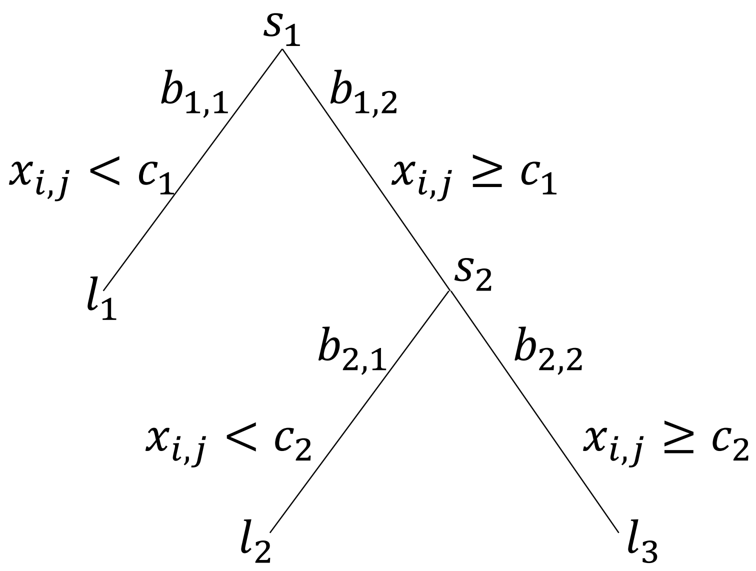

Figure 9. Example of decision tree.

A decision tree is constructed (trained) using a dataset and starting from the first branch, if all the outputs are equal, the branch becomes a leaf predicting such output regardless of the input, otherwise a cut point is created. Following the cut point logic, the dataset is divided into two branches leading to other splits. At this stage each branch contains a subset of the original dataset, if the outputs of a subset are all equal the branch becomes a leaf node predicting such an output, otherwise a cut point is created, and the training continues. Each cut point is generated by selecting the variable and the value maximizing the impurity gain:

$$\Delta I = nI + n_1 I_1 - n_2 I_2 \tag{42}$$

where I is the Gini impurity in the cut point, I_1 is the Gini impurity in the first branch, and I_2 is the Gini impurity in the second branch. n is the number of datapoints in the cut point, n_1 is the number of datapoints that will end in the first branch, and n_2 is the number of datapoints that will end in the second branch.

The Gini impurity in a cut point or in a branch is:

$$I = p_0(1 - p_0) + p_1(1 - p_1) \tag{43}$$

where p_0 is the probability that a datapoint in the cut point or branch has $y_i = 0$, and p_1 is the probability a data point in the cut point or branch has $y_i = 1$. This probability is computed as:

$$p_0 = \frac{1}{|y_i = 0|} \tag{44}$$

$$p_1 = \frac{1}{|y_i = 1|} \tag{45}$$

where $|y_i = 0|$ is the number of data points, in the cut point or branch with $y_i = 0$, and

$|y_i = 1|$ is the number of data points in the cut point or branch with $y_i = 1$.

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