

This is a pre print version of the following article:

Tensile constitutive behavior of high and ultra-high performance fibre-reinforced-concretes / Savino, Vincenzo; Lanzoni, L.; Tarantino, A. M.; Viviani, M.. - In: CONSTRUCTION AND BUILDING MATERIALS. - ISSN 0950-0618. - 186:(2018), pp. 525-536. [10.1016/j.conbuildmat.2018.07.099]

*Terms of use:*

The terms and conditions for the reuse of this version of the manuscript are specified in the publishing policy. For all terms of use and more information see the publisher's website.

09/01/2026 14:08

(Article begins on next page)

# Tensile Constitutive Behaviors of FR mortars and HPFRCs

V. Savino<sup>a</sup>, L. Lanzoni<sup>b,c</sup>, A.M. Tarantino<sup>c</sup>, M. Viviani<sup>a</sup>

<sup>a</sup>*HEIG-VD - Haute Ecole d'Ingénierie et de Gestion du Canton de Vaud, Route de Cheseaux 1, CH-1401 Yverdon*

<sup>b</sup>*DESD - Dipartimento di Economia, Scienze e Diritto, University of San Marino, Salita alla Rocca 44, Republic of San Marino, 47890 San Marino*

<sup>c</sup>*DIEF-Department of Engineering "Enzo Ferrari", University of Modena and Reggio Emilia, 41125 Modena, Italy*

---

## Abstract

Fiber reinforced mortars (FRMs) and high performance fiber reinforced concretes (HPFRCs) are today widely used as repair and strengthening composites for existing structures, with particular reference to bridges and other concrete structures. Once a strengthening material is available for the market, there is the need of quickly and inexpensively testing its physical characteristics to ensure engineers that the product meets the designer's requirements. Such testing methods, although simple, must be able to correctly characterize the ductility, the tensile strength, the flexural strength as well as to establish a correlation between such parameters. This paper deals with the characterization of the mechanical properties and the ductility indexes of three commercial FRCs. The study focuses on the correlations between the ductility and both the direct and indirect tensile strengths. The obtained experimental data were compared with those provided by a model based on the "Composite Material Theory" (CMT), showing good agreement.

*Keywords:* Metal-matrix composites; Mechanical properties; Damage mechanics; Mechanical testing

---

*Email address:* `luca.lanzoni@unimo.it` (L. Lanzoni)

## 1. Introduction

The technology of steel fiber reinforced concrete (SFRC) has much evolved in the past years thanks to a great number of researches performed both on the fiber-based materials [6, 13, 27, 29, 15, 32, 26, 28, 39] and on the applications [16, 1, 3, 23, 2]. New shapes and types of steels have been developed to improve the fibers and, in turn, the SFRC performances[4, 5, 12, 31]<sup>1</sup>. The mixing procedures of SFRC have been shortened and simplified over the years. SFRC is today largely used to repair and strengthen existing structures (e.g. bridge decks, piers, etc.)<sup>2</sup>. When SFRCs are used as repair and strengthening materials, their mechanical behavior must be known a priori by the designer. The success of the SFRC has promoted the development of a number of commercial products that nowadays are commercially available and produced on large scale and according to precise technical data sheets. The mechanical performances of these commercial FRCs are affected by many parameters, with particular reference to the type of fibers, fibers dosages, geometry of tested specimens, fibers aspect ratio and the test methods also [14, 38, 25, 11, 10, 30]. Laboratory tests characterizing certain properties of SFRC (such as tensile strength, ductility and flexural strengths) are often time consuming and expensive. These characterization procedures are therefore ill-adapted to the need of a rapid and cheap performances control and validation that are usually required by the designers. To aid the practitioners and simplify the procedures, some technical guidelines have been developed and recently adopted in codes [33, 40, 41, 36, 37, 42, 43, 44, 45, 46, 47].

This paper deals with the mechanical characterization of three different SFRCs, two of which are UHPFRCs. The experimental tests have been made according to standard protocols in order to assess the compressive strength, direct tensile strength and flexural strength of the composites. In particular, compressive strength tests on cubic specimens, direct tensile tests on dog-bone specimens and four-point bending flexural tests, performed both on notched and un-notched prismatic specimens, were carried out. The behavior in terms of ductility was assessed accordingly.

The paper is organized as follows. A description about the procedures used to perform the investigation is provided in Section 2. In particular, in

---

<sup>1</sup>Recent studies about FRCs based on synthetic fibers can be found in [17, 24].

<sup>2</sup>An important issue concerning the mechanical interaction in time between the hosting structure and the reinforcement is addressed in [8, 7, 9].

34 such a Section the experimental tests are described in some detail. Informa-  
 35 tion about the concrete mixtures tested are also given in this Section. The  
 36 main results provided by the experimental tests are reported and discussed  
 37 in Section 3. Finally, conclusions are drawn in Section 4.

## 38 2. Materials and methods

39 In the present work the mechanical behavior of three different types of  
 40 commercial FRCs (termed hereinafter “A”, “B” and “C” ), are investigated  
 41 through a standard procedure. For each type of mortar the producer pre-  
 42 scribes in detail the following items<sup>3</sup>:

- 43 • Dosages of admixtures (inerts, water/cement ratio, plasticizers, accel-  
 44 erators, etc.);
- 45 • Quantity and type of steel fibers (hooked or straight, aspect ratio, etc.);
- 46 • Mixing, flowing and curing procedures.

47 The mix design for getting 1 m<sup>3</sup> of each mortar under testing is shown in  
 48 Table 1. The solid part (binders and granular skeleton) of each mortar is  
 49 different even if all of them were named “premix” by the producers. In fact,  
 50 the producers supply the premix in sacks or big-bags that have to be used  
 51 as a unit, i.e. a sack cannot be partially used.

material	kg in 1 m <sup>3</sup> of composite		
	A (HPC)	B (UHPC)	C(UHPC)
Premix	2226	2296	1970
Superplasticizer	22.3	43.13	39
Accelerator	-	10	-
Water	231	184	195
Hooked steel fibers 30/0.35 mm	130 (1.7 %)	-	-
Straight steel fibers 20/0.3 mm	-	195 (2.5 %)	-
Straight steel fibers 13/0.175 mm	-	-	296 (3.8 %)

Table 1: Mix design for the SFRCs under testing.

52 A premix usually contains cement, sands and pozzolans. The companies  
 53 producing these mortars do not provide the composition of the premix and

<sup>3</sup>A recent study about a polymer-based mortar for retrofitting is performed in [20].

54 the nature of the compounds and chemicals. Each premix comes with a  
 55 superplasticizer used to obtain the target water/cement ratio (that can be  
 56 as low as 0.2). The superplasticizer is used at high dosage to increase the  
 57 strength, enhance the durability and give high workability [48]. In the “B”  
 58 mortar the manufacturers advise to add a set and hardening accelerator to  
 59 shorten the dormant period and to speed up the hydration process. The type  
 60 of fibers used for these concretes is usually chosen according to the target  
 application.



Figure 1: Steel fibers used in the mortars: a) Hooked fibers 30/0.35 mm; b) Straight fibers 20/0.3 mm; c) Straight fibers 13/0.175 mm.

61

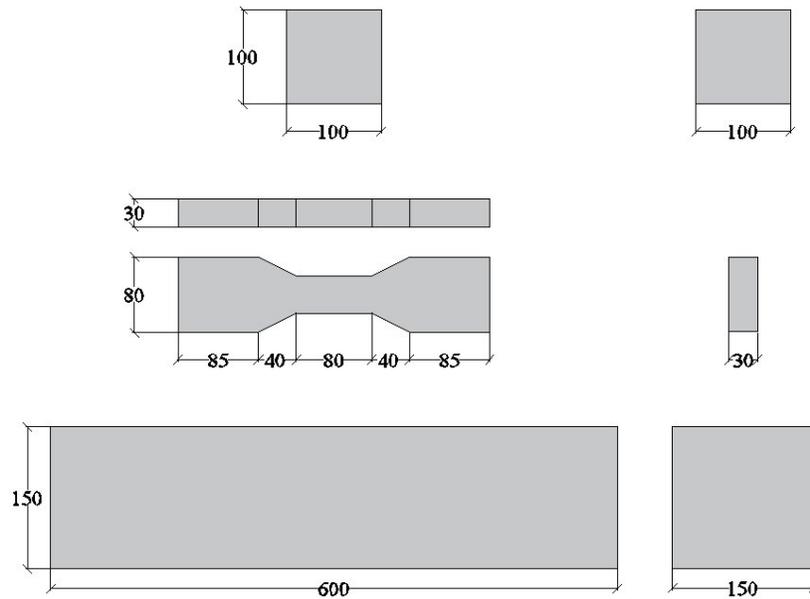


Figure 2: Specimen dimensions.

62 For structural applications steel fibers are usually preferred (see Fig. 1).  
 63 Mixing HPC and UHPC requires the use of high intensity mixers. In this  
 64 study a 1.5 kW high shear Zyklos rotating pan mixer was used to manufacture  
 65 standard specimens to test tensile, flexural and compressive strengths. The  
 66 types of tested specimens and their dimensions are shown in Fig. 2.

67 Compressive strength test on cubic specimens, direct tensile test on dog-  
 68 bone specimens and four-point bending flexural tests, both on notched and  
 69 un-notched prismatic specimens, were carried out. Standard tests configura-  
 70 tions are shown in Fig. 3.

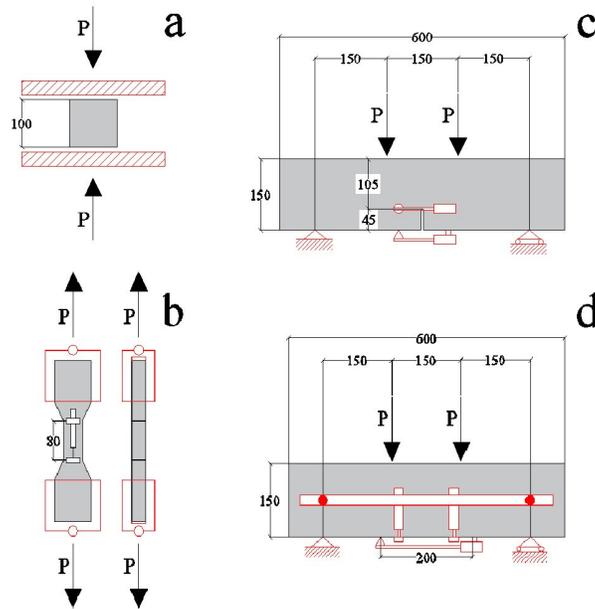


Figure 3: a) Compression test on cubic specimen; b) Direct tensile test on dog-bone specimen; c) Four-point bending test on notched beam specimen; d) Four-point bending test on un-notched beam specimen.

71 *2.1. Compression tests*

72 A compression machine Perrier type 138-5000 kN, as shown in Fig. 4(a),  
 73 was used for the compression tests according to [49, 50, 51]. The loading  
 74 speed of the compression machine was set to 6 kN/sec.

75 *2.2. Direct tensile test*

76 A machine Walter Bai type LVF-200 kN was used for the direct tensile  
 77 test on dog-bone specimens. The rate of the axial elongation was set equal

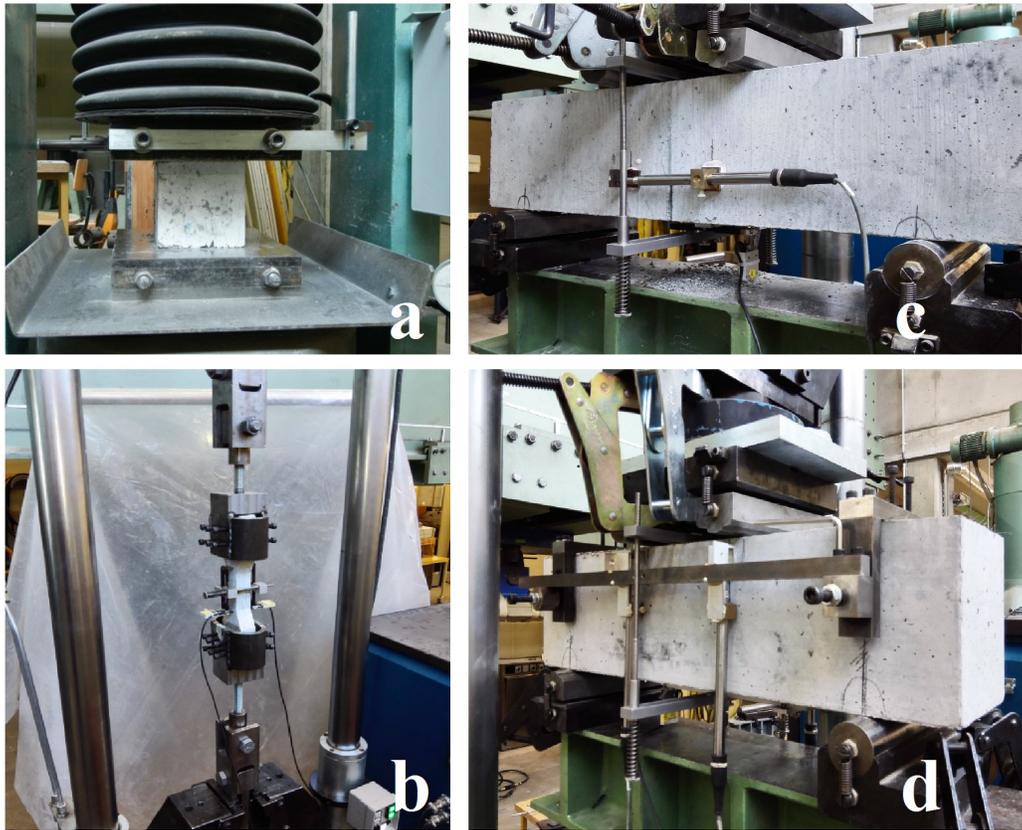


Figure 4: Test machines: a) Machine Perrier type 138-5000 kN used to perform compression tests on cubic specimens; b) Machine Walter Bai type LVF-200 kN used to direct tensile test; c) four-point bending test on notched beams; d) four-point bending test on un-notched beams.

78 to  $0.05 \pm 0.01$  mm/min according to the guidelines and codes employed for  
79 performing the experimental investigation [42, 52, 53]. Carrying out direct  
80 tensile tests, the specimens must be loaded along their main axis. This is  
81 necessary to prevent that bending occurs. A double hinge mechanism was  
82 designed specifically to keep the specimen and the load axis aligned. Fur-  
83 thermore, it is important to avoid any sliding between the machines clamps  
84 and the specimens in order to ensure that the entire load is transferred to the  
85 specimen. For this purpose an aluminium plate was glued to the specimen's  
86 surface as proposed by [53] and a special clamp was designed to have the  
87 maximum grip on the aluminium plate as illustrated in Fig. 4(b). When

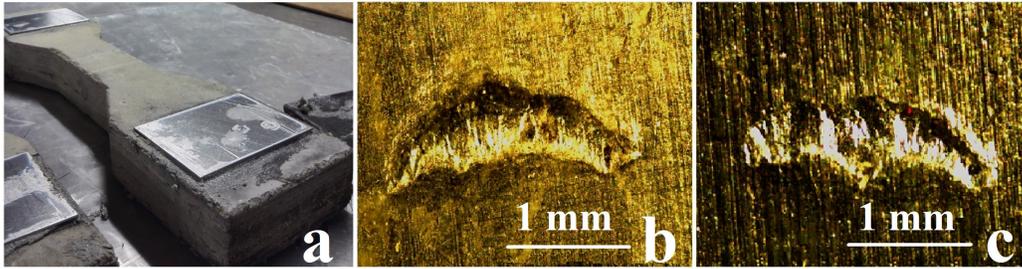


Figure 5: Some pictures related to the direct tensile test: a) Aluminum plate glued on the dog-bone specimen; b) Holes in the aluminum plate before tensile test; c) in the aluminum plate after tensile test.



Figure 6: Some pictures related to the extensometers used during the experimental tests: a) The extensometer is placed along the 80 mm characteristic lengths specimen; b) The extensometer is placed at the bottom of the notched beam along 100 mm characteristic length and two LVDT are placed on the sides; c) The extensometer is placed at the bottom of the un-notched beam along 200 mm characteristic length and four LVDT are placed under the rollers loading.

88 comparing the grooves on the aluminium plate glued on specimens prior and  
 89 after a direct tensile test (Fig. 5) it can be noted that there are no signs  
 90 of groove deformation and therefore no sign of slipping. As previously men-  
 91 tioned, the test was carried out by controlling the specimen's axial elongation  
 92 rate. A high precision extensometer was placed in each side of the specimens.  
 93 The average of the real time elongations measured by the two sensors per-  
 94 mitted to the traction machine to keep the programmed rate of deformation  
 95 according to [42], see Fig. 6(a).

### 96 2.3. Four-point bending flexural test

97 The machine Walter Bai type LVF-200 kN was used for carrying out the  
 98 four-point bending flexural tests as well. A standard prismatic specimen

99 is placed on two standard cylindrical supports (see Fig. 4(c)) and loaded  
 100 through two top rollers placed above the specimen. The distances of the  
 101 rollers and the supports were compliant with standards [51, 52, 54]. The  
 102 specimens were sawed in the middle section in order to produce a 45 mm deep  
 103 v-shaped notch according to [42, 51, 54]. Notched specimens are preferred  
 104 by norms for the softening materials because the position of the flexural  
 105 crack is known. The crack opening is measured both in the bottom and  
 106 in the side of the specimen. Bottom measurement is called Crack Mouth  
 107 Opening Displacement (CMOD), and side measurement is called Crack Tip  
 108 Opening Displacement (CTOD). The loading speed is controlled by setting  
 109 a rate of 0,05 mm/min for the CMOD is imposed until reaching a value of  
 110 crack opening of 3.5 mm. Sensors used to monitor the CMOD and CTOD  
 111 are respectively one extensometer and two LVDT displacement transducers  
 112 with a 100 mm gauge length (Fig. 6(b)). Tests on prismatic un-notched  
 113 specimen (Fig.4(d)) have been performed as well according to [52]. In these  
 114 tests the gauge length of the sensor was 200 mm and four LVDTs were used  
 115 to measure the localized displacement under the top rollers, see Fig. 6(c).

### 116 3. Results and discussion

117 Each test series encompasses several tests. To avoid confusion, a system  
 118 of samples identification was conceived and is displayed in Table 2 with an  
 119 example. Three FRC types have been investigated and they are labeled  
 120 “A”, “B” and “C”. All of these mortars are commercially available. “A”  
 121 and “C” specimens were cast in four different days and on slightly different  
 122 environmental conditions. “B” specimens were cast in two different days. The  
 123 samples are shown in Fig. 2.

C. . . .	type of FRC	A, B, C
. . 2. .	day of casting	1, 2, 3, 4, 5
	beam specimen	1, 2, 3
. . . .8	dog-bone specimens	4, 5, 6
	cube specimen	7, 8, 9

Table 2: Specimen identification.

124 *3.0.1. Compressive behavior*

125 Strength was tested on 100 x 100 x 100 mm standardly cured cubes. The  
126 peak value, the average value and the coefficient of variation (COV) of the  
127 measured compressive stress acting on the sample have been calculated (see  
128 Table 3):

$$f_{ci} = \frac{P_i}{A_i}, \quad f_{Cm} = \frac{\sum_{i=1}^n f_{Ci}}{n}, \quad COV = 100 \cdot \frac{\sigma}{n}, \quad (1)$$

129 being  $f_{Ci}$  the cubic compression in tension of a specimen;  $P_i$  the maximum  
130 load recorded during the test;  $A_i$  the cross section area of the uniformly  
131 loaded specimen;  $R_{cm}$  the average value of the tension in a campaign of  $n$   
132 specimens;  $\sigma$  the standard deviation and  $n$  the number of specimens.

133 The samples of all series shown a similar type of failure. As shown in Fig.  
134 7 the fibers, randomly oriented in the cement matrix, have a higher level of  
135 confinement which prevented the classical pyramid failure (usual for plane  
136 concrete cubes). It is important to note the beneficial effect of the fibers  
137 which avoid the separation of the specimen into separate pieces<sup>4</sup>.

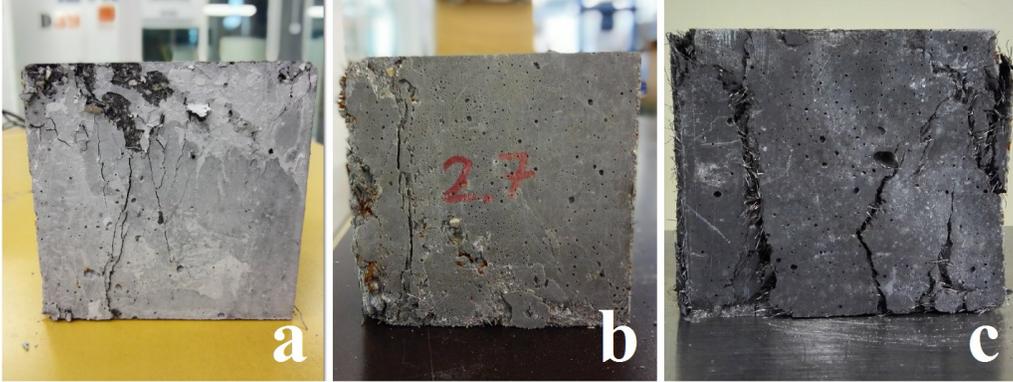


Figure 7: Cracking of samples subjected to the compression test: a) “A” mortar; b) “B” mortar; c) “C” mortar.

---

<sup>4</sup>Fracture can be modelled, in a nonlinear framework, following [34], whereas recent contributes about damage mechanics can be found in [18, 19, 21]. Bifurcation and stability in the context of homogeneous finite deformations can be found in [35].

138 *3.0.2. Tensile behavior in direct tensile test*

139 Table 3 reports the peak load, the average tension as well as the COV  
140 of the cross cracking section of the dog bone specimens. The tensile stress  
141  $\sigma_N$  is obtained by dividing the measured tensile load by the area of the  
142 nominal cross section at the crack location (30x40 mm<sup>2</sup>, in this case). The  
143 total elongation  $\delta$  (deformation) is obtained by multiplying the axial strain  
144 (specific deformation) measured by the characteristic length of the sensors  
145 (80 mm). The tensile load-elongation curves  $\sigma_N - \delta$ , obtained by testing dog-  
146 bones specimens, are shown in Fig. 8. It can be observed a series of sudden  
147 drops of tensile stress during the load tests in a number of specimens of  
148 mortars B and C. These crests and troughs in the stress-strain graphs often  
149 occur in SFRC tests and they are caused by one or a combination of the  
150 following mechanisms which can be recognized during the crack generation  
151 and growth in FRC:

- 152 • *Fiber failure.* It is the less common phenomenon and it happens usually  
153 in long fibers, where the bond is so high that the fiber can reach rupture.
- 154 • *Fiber pull-out.* This is the mechanism that dissipates less energy and  
155 it's the most common in straight fibers.
- 156 • *Fiber bridging.* Fibers link the two edges of the crack and the stress can  
157 be transferred through them. This is the most common phenomenon  
158 in non-straight fibers. The energy is absorbed by the deformation of  
159 the fiber.
- 160 • *Fiber/matrix debonding.* Like the fiber failure, this happens usually in  
161 long and well bonded fibers.
- 162 • *Matrix crack.* The fiber can contrast the propagation of micro-cracking  
163 through the matrix spreading the energy of the crack on a wider surface.

164 By observing the graphs in Fig. 8 it might be concluded that the crests  
165 and troughs affecting the experimental curves about mortars “B” and “C”  
166 are linked to the pull-out of the straight fibers used in these SFRCs. The  
167 pull-out mechanism was prevented in SFRC “A” by the use of hooked fibers.  
168 Dog-bone specimens of “A” SFRC specimens showed also a multi cracking  
169 behavior (Fig. 9), probably ascribable to the use of hooked fibers [22]. The  
170 hooked part of the fibers tend to prevent the total extraction of the fibers and  
171 ensures a higher degree of load redistribution. A multi cracking behavior is

Specimen	f <sub>i</sub> (MPa)	f <sub>m</sub> (MPa)	COV (%)	Specimen	f <sub>i</sub> (MPa)	f <sub>m</sub> (MPa)	COV (%)	Specimen	f <sub>i</sub> (MPa)	f <sub>m</sub> (MPa)	COV (%)
A.1.1	26.51			A.1.4	6.52			A.1.7	88.47		
A.1.2	29.00			A.1.5	5.60			A.1.8	82.75		
A.1.3	30.45			A.1.6	5.79			A.1.9	86.29		
A.2.1	43.68			A.2.4	7.51			A.2.7	81.85		
A.2.2	32.87	34.52	16.25	A.2.5	6.77			A.2.8	92.38		
A.2.3	39.03			A.2.6	6.90			A.2.9	95.89		
A.3.1	30.50			A.3.4	5.42	6.30	11.52	A.3.7	90.21	86.42	5.06
A.3.2	40.54			A.3.5	5.94			A.3.8	83.19		
A.3.3	38.08			A.3.6	6.38			A.3.9	81.41		
A.4.1	26.61			A.4.4	6.15			A.4.7	83.27		
A.4.2	30.67	29.27	6.44	A.4.5	5.20			A.4.8	87.21		
A.4.3	30.54			A.4.6	7.46			A.4.9	84.12		
B.1.1	60.36			B.1.4	10.31			B.1.7	154.25		
B.1.2	58.18			B.1.5	10.44			B.1.8	156.52		
B.1.3	67.63			B.1.6	10.02			B.1.9	151.32		
B.2.1	75.92	66.69	9.31	B.2.4	14.80	11.84	15.40	B.2.7	151.95	151.08	2.56
B.2.2	65.72			B.2.5	11.76			B.2.8	147.24		
B.2.3	72.32			B.2.6	13.73			B.2.9	145.22		
C.1.1	62.08			C.1.4	14.45			C.1.7	148.65		
C.1.2	77.72			C.1.5	11.76			C.1.8	153.98		
C.1.3	65.75			C.1.6	11.28			C.1.9	149.11		
C.2.1	82.16			C.2.4	12.43			C.2.7	144.24		
C.2.2	75.63			C.2.5	11.55			C.2.8	146.58		
C.2.3	82.81			C.2.6	14.49			C.2.9	140.87		
C.3.1	77.72	71.52	10.20	C.3.4	13.85	13.06	8.49	C.3.7	148.51	148.61	2.54
C.3.2	62.53			C.3.5	13.80			C.3.8	153.53		
C.3.3	63.83			C.3.6	13.67			C.3.9	149.04		
C.4.1	71.50			C.4.4	13.80			C.4.7	147.08		
C.4.2	71.47			C.4.5	13.58			C.4.8	147.58		
C.4.3	65.05			C.4.6	12.11			C.4.9	154.16		

Table 3: Results.

172 always desirable in concrete since it indicates an augmented ductility of the  
173 specimen under tension and bending. It is also well-known that an improved  
174 ductility and a multi cracking behavior are necessary to have high rotational  
175 capacity and good stress redistribution in certain structural elements. Direct  
176 tensile tests carried out on dog-bones specimens are initially characterized  
177 by an elastic phase. In this phase the contribution of the fibers could not  
178 be decoupled, since the matrix characteristics are governing the behavior  
179 of the SFRC. Instead, the post-peak phase was strongly influenced by the  
180 contribution of the fibers. After the appearance of the first crack, located in  
181 the central area of the specimen in B and C series and diffused along the mid-  
182 length of the specimen in the series A (Fig. 9), the matrix does not contribute  
183 any longer to the resistance. The stresses are transferred completely from the  
184 matrix to the (activated) fibers and from the activated fibers to the matrix

185 thanks to the interfacial bond strength. The fibers, bridging the cracks,  
186 transfer a part of the load in areas of the specimens that are not yet cracked.  
187 Thanks to their slow pull-out resistance and their high ductility, during the  
188 growth of the crack opening, a considerable amount of energy is dissipated.  
189 This energy dissipation is highlighted by the length of the post-peak curve  
190 as well as by the large areas under the graphs (see Fig. 8).

### 191 *3.1. Tensile behavior by four-point bending flexural test*

192 It is common practice to evaluate the tensile strength of concrete by  
193 means of flexural tests. Although well established, this type of test has some  
194 limitation when it comes to define the real tensile strength of materials such  
195 as SFRC. In this study the flexural strength was evaluated by tensile and  
196 flexural tests and a correlation between the two was established by means of  
197 the “Composite Material Theory”. Table 3 shows the peak load, the average  
198 and the COV of the flexural tests. These values and in particular the nominal  
199 tension stress  $\sigma_N$  were determined assuming that all the materials remained  
200 in the elastic field until failure [42]. The test campaign of SFRC “A” included  
201 both notched and un-notched specimens: The tests series A.4 was carried  
202 out on un-notched specimens. Most of the tests in all series were carried-  
203 out on notched specimens (see Table 3). Results of flexural tests on notched  
204 specimens are shown in Fig. 10. Fig. 10 presents the nominal tension-CTOD  
205 ( $\sigma_N - w$ ) curves of notched prismatic specimens.

206 A comparison between notched and un-notched prisms of the test se-  
207 ries A.4 permitted to highlight that the un-notched specimens had a multi-  
208 cracking behavior (although not pronounced) while notched specimens pre-  
209 sented a single crack localized in the middle-span (Fig. 11).

210 Even though the notched specimens did not show a multi-cracking be-  
211 havior, the great contribution of the fibers to the ductility was evident. The  
212 contribution of fibers to the ductility is easier to be observed in bending rather  
213 than in direct tensile test since the flexural test promote the formation of a  
214 plastic region in the neighborhood of the middlespan, thus permitting to the  
215 specimen important rotations. This plastic behavior is obtained only if the  
216 fiber volume exceeds the critical value  $V_{f,crit}$ . The critical volume of the fibers  
217 varies with the matrix and fibers characteristics.

#### 218 *3.1.1. The composite material theory applied to SFRC*

219 According to the Composite Material Theory (CMT), the critical volume  
220 of fibers that might promote a strain-hardening behavior can be obtained

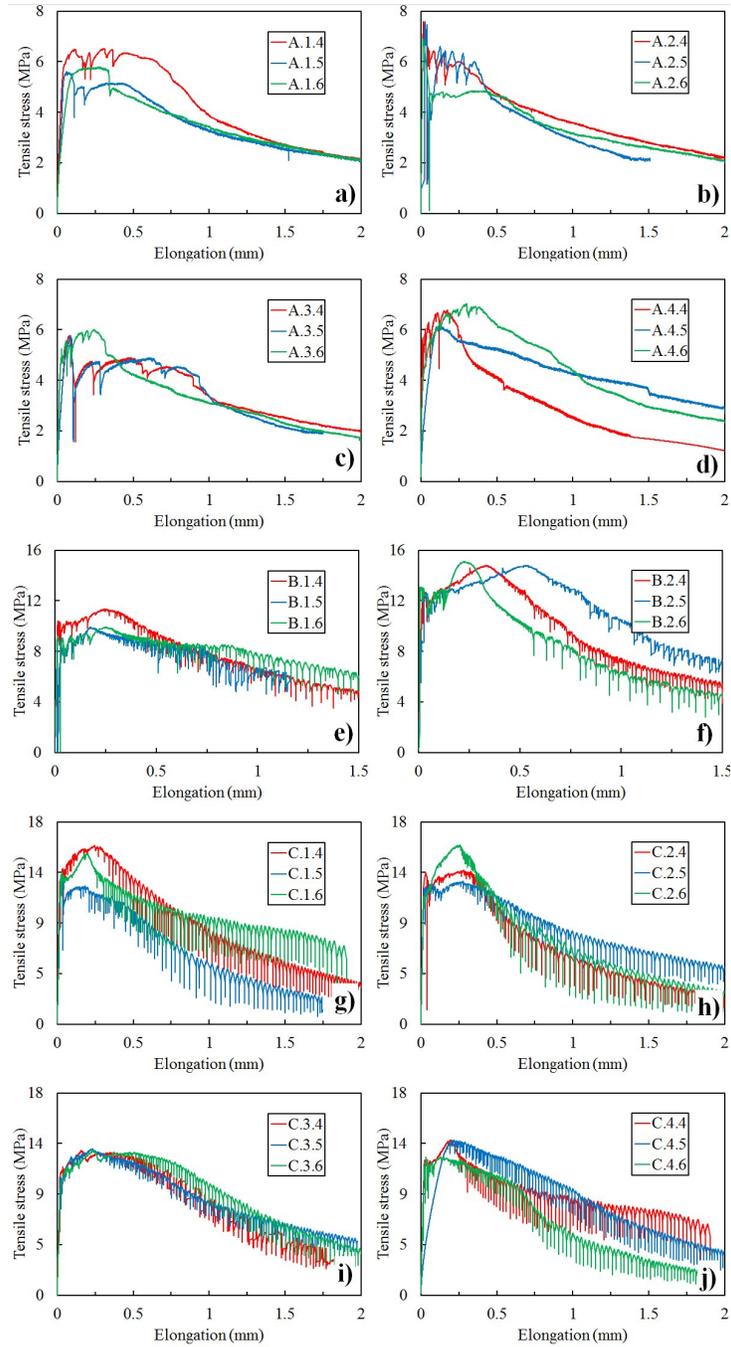


Figure 8: Tensile tension-elongation curves from direct tensile test on the dog-bone specimen series.

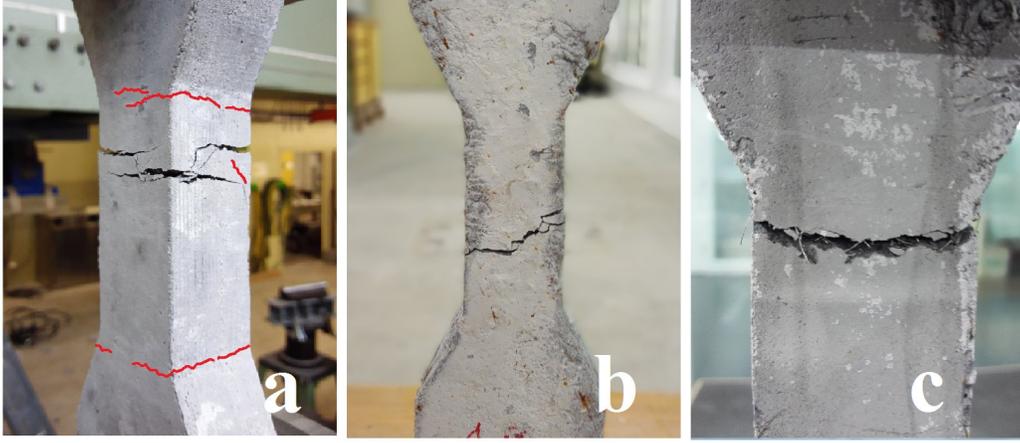


Figure 9: Tensile cracking: a) “A” mortar; b) “B” mortar; c) “C” mortar.

221 computationally by equating the normal stress acting on the composite  $\sigma_c$   
 222 before and after cracking. Before cracking the normal stress acting on the  
 223 composite is

$$\sigma_c = V_{f,crit} E_f (1 - V_{f,crit}) \sigma_{mu} \quad (2)$$

224 whereas after the specimen cracks, the matrix does not contribute any longer  
 225 to sustain the load and stresses are transferred to the active fibers crossing  
 226 the cracked section. Then, in this case the tension of the composite becomes  
 227

$$\sigma_c = V_{f,crit} \sigma_{fu}. \quad (3)$$

228 By equating (2) and (3) the critical fiber volume that promotes a hardening  
 229 behavior under direct tensile stress is obtained

$$V_{f,Cr} = \frac{\sigma_{mu}}{\sigma_{fu} - E_f \epsilon_{mu}}, \quad (4)$$

230 where  $\sigma_{mu}$  is the maximum matrix tension just before the cracking;  $\epsilon_{mu}$  is  
 231 the corresponding deformation which, according to the assumption of perfect  
 232 adhesion, is the same for steel fibers;  $\sigma_{fu}$  is the failure tension of the fibers  
 233 (rupture or pull-out);  $E_f$  is the elastic modulus of the steel fibers.

234 If the volume of the fibers in the composite exceeds the critical threshold  
 235  $V_{f,crit}$ , a hardening post-peak behavior will be expected. If the volume of  
 236 fibers is below the critical threshold, then a softening post-peak behavior  
 237 will be observed. In order to calculate the critical volume of fibers  $V_{f,crit}$ ,

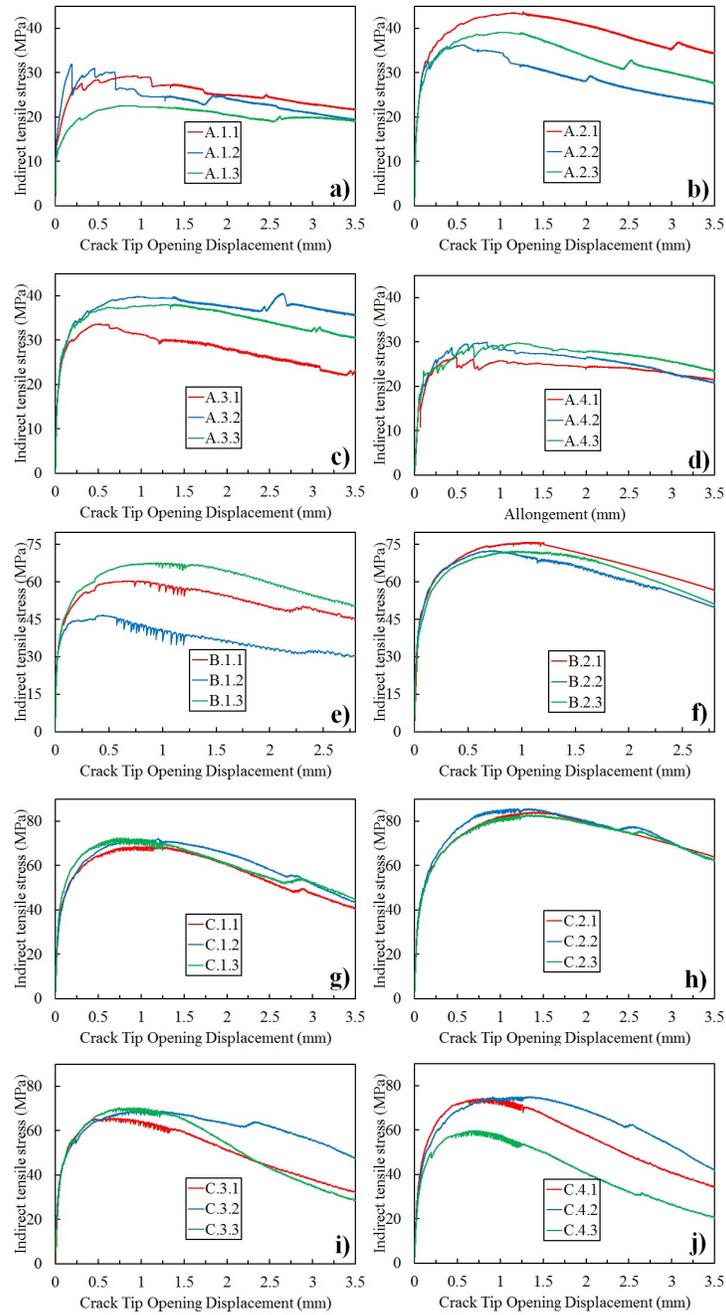


Figure 10: Tension-CTOD by flexion curves from four-point bending flexural test on prismatic specimens.

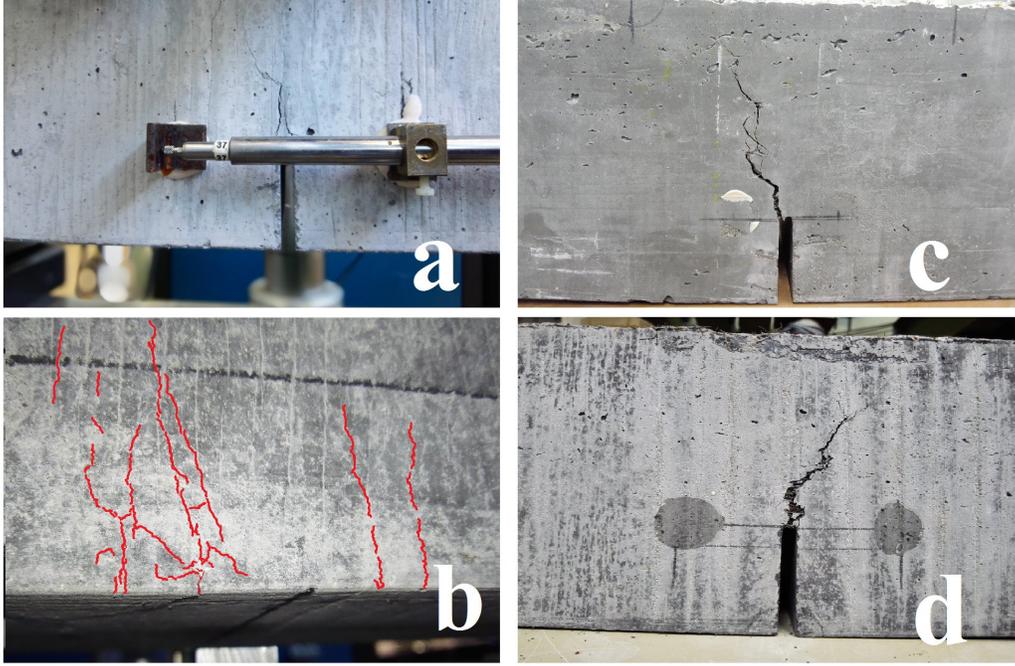


Figure 11: Failure modes observed during the loading bending tests: a) Mono-cracking behavior of notched beam A; b) multi-cracking behavior of un-notched beam A; c) mono-cracking behavior of notched beam B; d) mono-cracking behavior of notched beam C.

238 it is firstly necessary to model the stress distribution in the cross-section.  
 239 The pre-peak stress distribution in the cross section can be modeled as in  
 240 Fig. 12(a) with the tension stresses  $\sigma_{mu}$  and  $\sigma_{comp}$  having the same value.  
 241 The post peak behavior can be modeled as in Fig. 12(b), where  $\sigma_{cu}$  is the  
 242 ultimate strength of the composite reached in post-cracking phase. If the  
 243 neutral axis depth is defined to be equal to  $H/4$ ; the two ultimate bending  
 244 moments (pre-peak  $M_{R,a}$  and post-peak  $M_{R,b}$ ) can be expressed as follows:

$$M_{R,a} = \frac{\sigma_{mu}BH^2}{6}, \quad M_{R,b} = \frac{\sigma_{cu}BH^2}{32}, \quad (5)$$

245 where  $H$  and  $B$  are the dimensions of the cross section.

246 By equating the moments it follows:

$$\sigma_{cu} = 0.41\sigma_{mu} \leftrightarrow \sigma_{mu} = 2.44\sigma_{cu}, \quad (6)$$

247 and

$$M_{R,b} \geq M_{R,a} \leftrightarrow \sigma_{cu} \geq 0.41\sigma_{mu}. \quad (7)$$

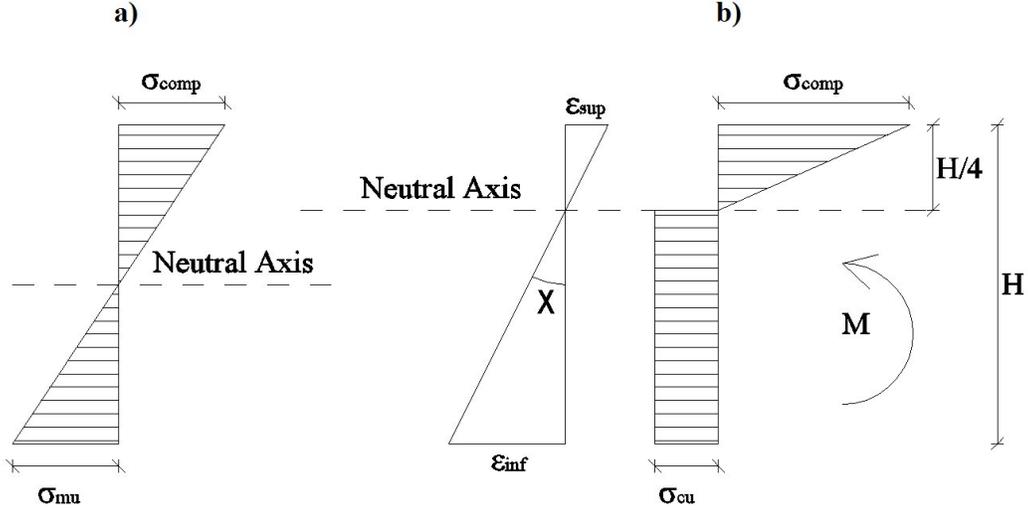


Figure 12: Stress and strain distributions on the cross section of a bent beam: a) Stress distribution before the matrix cracking; b) stress distribution in a post-cracking phase.

248 The fibers volume that is the boundary between hardening and softening  
 249 behavior in flexion can be obtained by inserting (6) into (3)

$$V_{f,crit,flexion} = 0.41 \frac{\sigma_{mu}}{\sigma_{fu}} \cong 0.41 V_{f,crit,traction}. \quad (8)$$

250 Eq. (8) proves that the hardening behavior in bending stress is also ob-  
 251 tained reducing the volume of fibers request to obtain the hardening behavior  
 252 under tensile load. Fig. 13(a) shows the tensile stress-strain curve obtained  
 253 by applying (2) and (3) under the hypothesis that the volume of the fibers  
 254 is equal to  $V_{f,crit,traction}$ . Fig. 13(b) shows the tensile stress-strain curve ob-  
 255 tained by applying (5)a and (5)b under the hypothesis that the volume of  
 256 the fibers is equal to  $V_{f,crit,traction}$ . Fig. 13(a) shows that, under pure ten-  
 257 sion, a post peak plastic plateau will appear, since the volume of fibers is the  
 258 critical volume. Fig. 13(b) shows a hardening behavior post cracking and  
 259 maximum tension strength equal to 2.44 times the peak obtained in direct  
 260 traction. Fig. 14(a) shows the tensile stress-strain curve obtained by apply-  
 261 ing (2) and (3) under the hypothesis that the volume of the fibers is equal  
 262 to  $V_{f,crit,flexion}$ . Fig. 14(b) shows the tensile stress-strain curve obtained by  
 263 applying (5)a and (5)b under the hypothesis that the volume of the fibers  
 264 is equal to  $V_{f,crit,flexion}$ . In this case a softening behavior is seen under pure  
 265 tensile stresses with the stress decreasing to 0.41 the max stress. A plastic

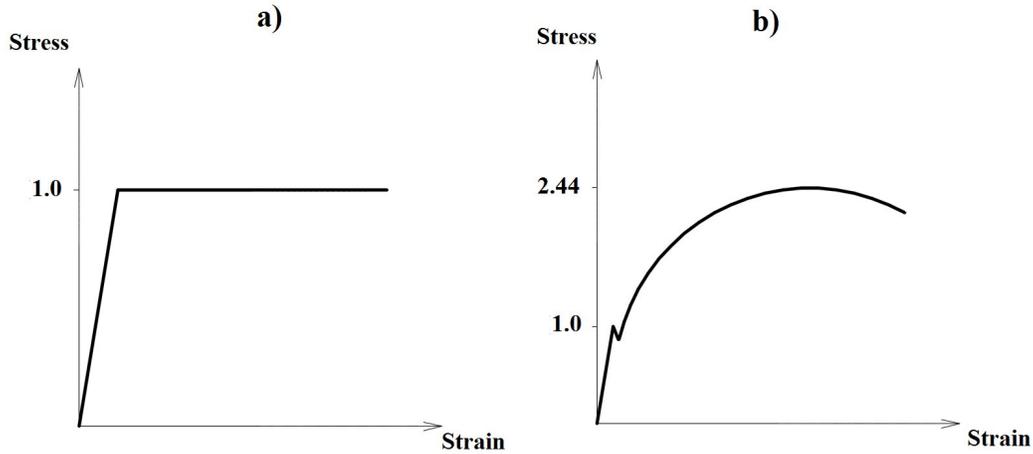


Figure 13: a) Direct tensile curve; b) indirect tensile curve by flexion test with a volume of fibers equal to  $V_{f,crit,traction}$ .

266 plateau is observed under bending (Fig. 14(b)). This simple application of  
 267 the Composite Material Theory (CMT) is sufficient to explain how the tensile  
 268 strength obtained by flexural tests is always higher than the value obtained  
 269 in direct tensile tests. The CMT, due its simplicity might be misleading since  
 270 many assumptions are made. Nevertheless the CMT builds a link between  
 271 flexural and tensile strength of a SFRC that might prove useful is carefully  
 272 verified for a specific commercial SFRC. Fig. 15 shows the direct tensile and  
 273 flexural stress-strain curves obtained by testing specimens in tension and in  
 274 bending. The similarity of diagram of Fig. 14 and Fig. 15 is evident.

### 275 3.1.2. Tensile stress-strain models under bending and direct tensile load

276 According to the standard used in this study [52], a bilinear tensile stress-  
 277 strain model can be derived from test data based on three parameters: The  
 278 tensile strength at the end of the elastic phase, the corresponding strain and  
 279 the tensile strength at the end of the softening branch. Fig. 16 shows the  
 280 test data (dog bones) and the curve obtained by the bilinear model derived  
 281 according to the standard. On SFRC A model, due to the high COV, first  
 282 and ultimate tensile strength values have been penalized more than 50%. In  
 283 SFRC B model the reduction of 52% for the first and 37% for the last tensile  
 284 strength values was caused by a COV of 16% and 8%, while in SFRC C a  
 285 37% and 52% of reduction is due to COV of 9% and 18%. The different  
 286 COVs and behavior of the three SFRCs can be explained by a number of

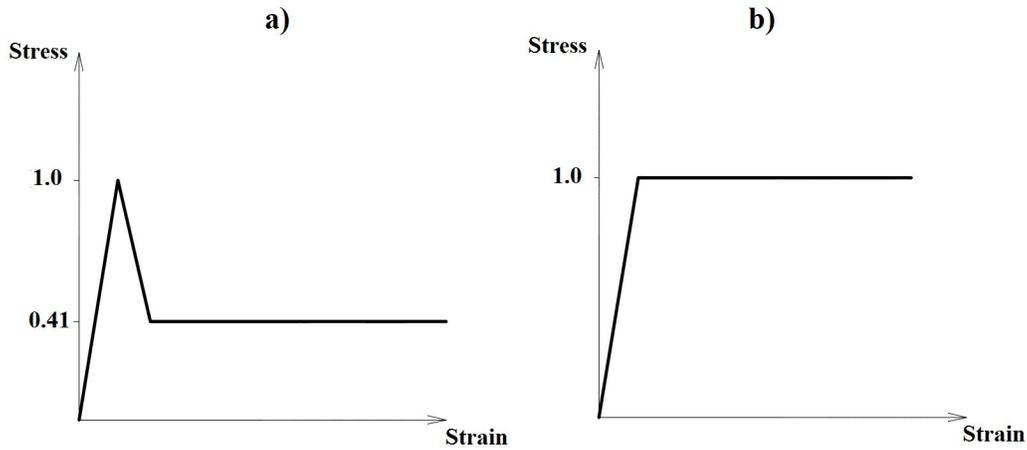


Figure 14: a) Direct tensile curve; b) indirect tensile curve by flexion test with a volume of fibers equal to  $V_{f,crit,flexion}$ .

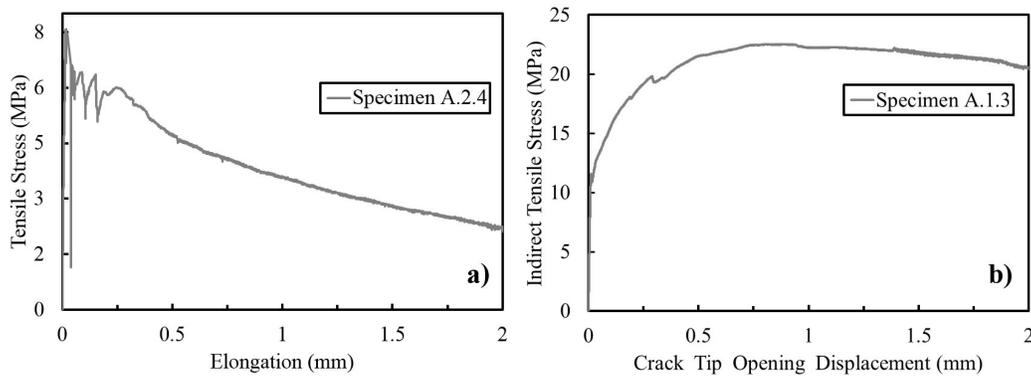


Figure 15: Experimental curves: a) direct tensile test on the specimen A.2.4; b) indirect tensile test provided by bending test on the specimen A.1.3.

287 factors. The most important ones, according to [55], are the fibers dosage,  
 288 the geometry of the tested specimen (thickness, in particular), aggregate,  
 289 fiber size and fiber orientation. Fiber dosage is one of the most important  
 290 factors of COV variation since a higher dosage of fibers should promote a  
 291 more even distribution of fiber [11]. As shown in Table 1 the fibres dosage in  
 292 SFRCs is 1.7%, 2.5% and 3.8% for mortars A, B and C respectively. In fact  
 293 the COV related to mortar A is slightly larger than that concerning mortar  
 294 C. Fiber dosage seems to be less significant for indirect tensile strength for  
 295 which the COV was about 20% for all notched specimens of A, B and C

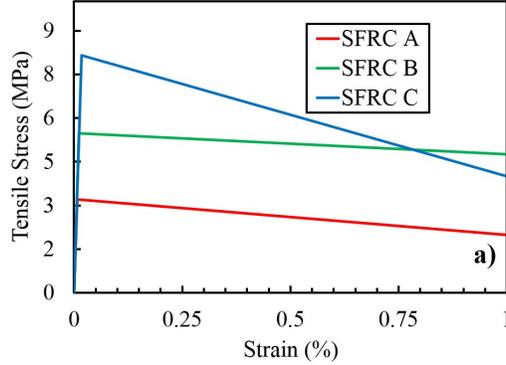


Figure 16: Standard bilinear tensile model.

296 mortars [25]. The COV in un-notched beams (SFRC A) is less than 6%.  
 297 The ductility indexes and the tensile strength class, that serves to compare  
 298 and choose between different commercial SFRCs, can be assessed according  
 299 to [54]. These indexes are obtained from the models derived by the method  
 300 proposed by the same standard. Table 4 shows the classification according to  
 301 ductility and tensile strength standard indexes [54]. Data obtained by testing  
 302 can be used to construct bilinear tensile constitutive law based on regression  
 303 analysis. This model was developed for each SFRC and is shown in Fig. 17.  
 304 Using this bilinear constitutive law, the CMT presented in section 3.3 and  
 305 the parabola-rectangle compression law according to [56] an indirect tensile  
 306 stress-strain model was derived for each SFRC. Fig. 18 shows a comparison  
 307 of the experimental (bending) indirect tensile stress - CTOD results with the  
 308 results of the analytic model for each concrete. Test results and predictions  
 309 obtained with the CMT significantly agree.

Class	F2.0	F2.5	F3.0	F3.7	F4.5	F5.5	F6.5	F7.7	F9.0
$f_{11k,min}$ (Mpa)	2	2.5	3	3.7	4.5	5.5	6.5	7.7	9
Index of ductility	Softening			Plastic			Hardening		
$D_{0k,min}$	$D_{S0}$	$D_{S1}$	$D_{S2}$	$D_P$		$D_{H0}$	$D_{H1}$	$D_{H2}$	
	$\leq 0.5$	$\geq 0.5$	$\geq 0.7$	$\geq 0.9$		$\geq 1.1$	$\geq 1.3$	$\geq 1.55$	
Classification							A	B	C
SFRC							(HPC)	(UHPC)	(UHPC)
Class F							F6.5	>F9.0	F7.7
Class $D_0$							$D_{S1}$	$D_{S1}$	$D_{S1}$
Class $D_1$							$D_{S0}$	$D_{S2}$	$D_{S2}$

Table 4: Classification of the SFRCs according to UNI 11039-2:2003.

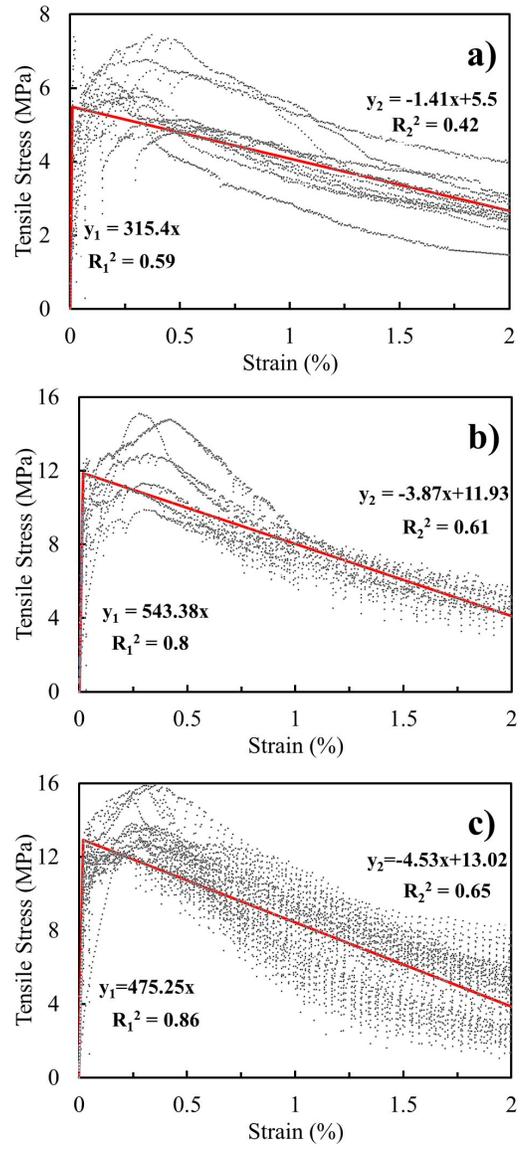


Figure 17: Standard Experimental bilinear tensile model for a) SFRC A; b) SFRC B; c) SFRC C.

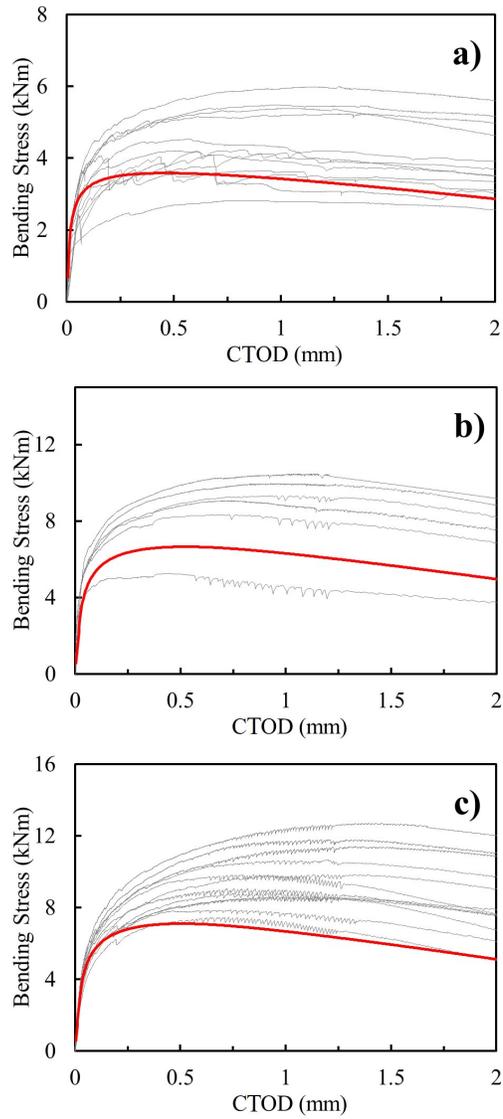


Figure 18: Predictive model Bending Stress-CMOD for a) SFRC A, b) SFRC B, c) SFRC C

## 310 4. Conclusion

311 This paper presents the mechanical characterization of three different  
312 SFRCs, two of which are UHPFRCs. The tests have been made according  
313 to recent standard protocols for compressive strength, direct tensile strength  
314 and flexural strength. The behavior in terms of ductility was assessed accord-  
315 ing to standards and rated. A tensile constitutive law have been constructed  
316 for each concrete and used to predict the indirect tensile stress-strain be-  
317 havior. Based on the experimental evidence and on the data analyses, the  
318 following conclusions can be drawn.

319 Stress-strain models proposed by codes are sensible to the COV of the tests  
320 results. Therefore, a concrete with a greater dispersion of the test results  
321 might be rated lower, in term of ductility and strength, that a less perfor-  
322 mant concrete.

323 Hooked fibers and higher aspect ratios promote a more ductile behavior and  
324 they are proved to be more efficient in providing ductility than a stronger  
325 matrix bonding straight fibers w/o higher aspects ratios. This conclusion is  
326 drawn on a limited number of tests and therefore must be further confirmed.  
327 All test results obtained with four points bending tests were compared with  
328 a model based on the CMT, showing good agreement. The CMT clearly  
329 explains the differences of the post-peak behavior of the tests carried out in  
330 a four-point bending and under direct tensile stress.

331 The purpose of the standards used in this study is to supply a fast and in-  
332 expensive method for conformity control of a SRFC to be used in field. All  
333 tested commercial SFRCs have been classified as low ductility. This might be  
334 the effect of the test protocols that might not to be able to fully characterize  
335 the stress-strain and ductility characteristics of all SFRC. Nevertheless the  
336 principle that it exist a mixture-intensive correlation between the direct and  
337 indirect tensile strengths of a SFRC has been found to be true.

## 338 Acknowledgments

339 Authors gratefully acknowledge the financial support provided by HEIG-  
340 VD. Financial support from the Italian Ministry of Education, University and  
341 Research (MIUR) in the framework of the Project PRIN “COAN 5.50.16.01”  
342 (code 2015JW9NJT) is gratefully acknowledged.

## Supplementary data

The raw/processed data required to reproduce these findings cannot be shared at this time due to time limitations. However the Authors will provide them soon.

## References

- [1] Ahlborn TM, Puese EJ, Misson DL. *Ultra-High Performance Concrete for Michigan Bridges*. Material Performance-Phase I 2008:190
- [2] Bastien-Masse M, Brühwiler E. *Concrete bridge deck slabs strengthened with UHPFRC*. IABSE Conference Rotterdam Assessment, Upgrading and Refurbishment of Infrastructures 2013;99:236237
- [3] Bencardino, F., Condello, A., Ashour, A.F. *Single-lap shear bond tests on Steel Reinforced Geopolymeric Matrix-concrete joints*. Composites Part B, 110, 2017, 62–71.
- [4] Caratelli, A., Imperatore, S., Meda, A., Rinaldi, Z. *Punching shear behavior of lightweight fiber reinforced concrete slabs*. Composites Part B, 99, 2016, 257–265.
- [5] Conforti, A., Tiberti, G., Plizzari, G.A. *Splitting and crushing failure in FRC elements subjected to a high concentrated load*. Composites Part B, 105, 2016, 82–92.
- [6] Choi W.-C., Seok-Joon J., Hyun-Do Y. *Bond and cracking behavior of lap-spliced reinforcing bars embedded in hybrid fiber reinforced strain-hardening cementitious composite (SHCC)*. Composites Part B, 108, 2017, 35–44
- [7] Dezi L, Menditto G, Tarantino AM. *Homogeneous structures subjected to successive structural system changes - ASCE Journal of Engineering Mechanics*, 116(8), 1990, 1723-1732.
- [8] Dezi L, Tarantino AM. *Time dependent analysis of concrete structures with variable structural system*. ACI Materials Journal, 88(3), 1991, 320-324.

- [9] Dezi L, Menditto G, Tarantino AM. *Viscoelastic heterogeneous structures with variable structural system* - ASCE Journal of Engineering Mechanics, 119(2), 1993, pp. 238-250
- [10] Eldin HKS, Mohamed HA, Khater M, Ahmed S. *Mechanical Properties of Ultra-High Performance Fiber Reinforced Concrete*. Int J Eng Innov Technol 2014;4:4-10
- [11] Finazzi S, Paegle I, Fischer G. & Minelli F. *Influence of bending test configuration on cracking behavior of FRC*. Proceedings of the 3rd All-Russia (International) Conference on Concrete and Reinforced Concrete 2014;3:196-205
- [12] Germano, F., Tiberti G., Plizzari, G. *Experimental behavior of SFRC columns under uniaxial and biaxial cyclic loads*. Composites Part B, 85 (2016) 76–92
- [13] Gesoglu, M., Gneyisi, E., Muhyaddin, G.F., Asaad, D.S. *Strain hardening ultra-high performance fiber reinforced cementitious composites: Effect of fiber type and concentration*. Composites Part B 103, 2016, 74–83
- [14] Hannawi, K., Bian, H., Prince-Agbodjan, W., Raghavan, B. *Effect of different types of fibers on the microstructure and the mechanical behavior of Ultra-High Performance Fiber-Reinforced Concretes*. Composites Part B 86, 2016, 215–220
- [15] Jiang L, Niu DT, Bai M. *Experiment Study on the Frost Resistance of Steel Fiber Reinforced Concrete*. Adv Mater Res 2010;150-151:243-6. doi:10.4028/www.scientific.net/AMR.150-151.243
- [16] Lackey T, Desgagne G, Benmokrane B, El-Salakawy E, El-Ragaby A. *Design, Construction and monitoring of four innovative concrete bridge decks using non-corrosive FRP composite bars*. Annu Conf Transp Assoc Canada 2004:1-20
- [17] Lanzoni L, Nobili A, Tarantino AM. *Performance evaluation of a polypropylene-based draw-wired fibre for concrete structures*. Construct. Build. Mater. 28 (2012) 798-806.
- [18] Lanzoni L, Tarantino AM. *Damaged hyperelastic membranes*. Int. J. NonLinear Mech. 60 (2014) 9-22.

- [19] Lanzoni L, Tarantino AM. *Equilibrium configurations and stability of a damaged body under uniaxial tractions*. ZAMP Zeitsc. Angew. Math. Phys. 66(1) (2015) 171-190.
- [20] Lanzoni L, Soragni M, Tarantino AM, Viviani M. *Concrete beams stiffened by polymer-based mortar layers: Experimental investigation and modeling*. Construction and Building Materials 105 (2016) 321-335.
- [21] Lanzoni L, Tarantino AM. *A simple nonlinear model to simulate the localized necking and neck propagation*. Int. J. NonLinear Mech. 84 (2016) 94-104.
- [22] Naaman AE, Najm H. *Bond-slip mechanisms of steel fibers in concrete*. ACI Mater J 1991;88:13545
- [23] Nazar S., Ismaiel MA., Ahmed M. *Possibility to Improve Strength and Structural Stability of Bridge Deck Slabs by Using Ultra High Performance Fiber Reinforced Concrete*. IOSR J Mech Civ Eng 2014;11:79-87
- [24] Nobili A, Lanzoni L, Tarantino AM. *Experimental investigation and monitoring of a polypropylene-based fiber reinforced concrete road pavement*. Construct. Build. Mater. 47 (2013) 888-895.
- [25] Parmentier B, De Grove E, Vandewalle L, Van Rickstal F. *Dispersion of the mechanical properties of FRC investigated by different bending tests*. Tailor Made Concrete Structure Walvraven & Stoelhorst 2008;507512
- [26] Popa M, Corbu O, Kiss Z, Zagon R. *Achieving Mixtures of Ultra-High Performance Concrete*. Constructi 2013;1:40-6
- [27] Ramli M, Dawood, E. *High-Strength Flowable Mortar Reinforced by Steel Fiber*. Slovak Journal of Civil Engineering 2011;19(3): 10-16
- [28] Sanal, I., Ozyurt, N., Hosseini, A. *Characterization of hardened state behavior of self compacting fiber reinforced cementitious composites (SC-FRCC's) with different beam sizes and fiber types*. Composites Part B, 105, (2016) 30–45.
- [29] Scott DA, Long WR, Moser RD, Green BH, Daniel JLO, Williams BA. *First International Interactive Symposium on UHPC*. 2016 2016:1-12

- [30] Shah, Surendra P, Brandt AM, Ouyang C, Baggot R, Eibl J, Glinicki MA, Krenchel H, Lambrechts A, Li VC, Mobasher B, and Taerwe L. *High performance fibre reinforced cement composites Chapter 6 2* 1996
- [31] Simoes, T., Octavio, C., Valena, J., Costa, H., Dias-da-Costa, D., Jlio, E. *Influence of concrete strength and steelfibre geometry on the fibre/matrix interface*. Composites Part B 122 (2017) 156–164
- [32] Slater E, Moni M, Alam MS. *Predicting the shear strength of steel fiber reinforced concrete beams*. Constr Build Mater 2012;26:423-36
- [33] Stålfibertong, *rekommendationer för konstruktion, utförande och provning Betongrapport n.4*. Svenska Betongföreningen 1995
- [34] Tarantino AM. *Nonlinear fracture mechanics for an elastic Bell material*. Quart. J. of Mech. and Appl. Math. 50(3), 1997, 435-456.
- [35] Tarantino AM. *Homogeneous equilibrium configurations of a hyperelastic compressible cube under equitriaxial dead-load tractions* - Journal of Elasticity, 92, 2008, pp. 227-254.
- [36] Vanderwalle L. et al. Recommendation of Rilem TC162-TDF. *Test and design methods for steel fibre reinforced concrete. Design of steel fibre reinforced concrete using the s-w method: principles and applications*. Materials and Structures 2002;35:262-278
- [37] Vanderwalle L. et al. Recommendation of Rilem TC162-TDF. *Test and design methods for steel fibre reinforced concrete: sigma-epsilon, Design method (final recommendation)*. Materials and Structures 2003;36:560-567
- [38] Yazıcı Ş, İnan G, Tabak V. *Effect of aspect ratio and volume fraction of steel fiber on the mechanical properties of SFRC*. Constr Build Mater 2007;21:12503. doi:10.1016/j.conbuildmat.2006.05.025
- [39] Yoo, D.Y., Banthia, N., Yoon, Y.S. *Predicting service deflection of ultra-high-performance fiber reinforced concrete beams reinforced with GFRP bars*. Composites Part B, 99 (2016) 381–397
- [40] ACI Committee 544. *ACI 544.4R88 Design consideration for steel Fiber Reinforced Concrete*. ACI 544.4R-88 America Concrete Institute. ACI Farmington Hills, MI 1996

- [41] AFGCSETRA. *Ultra High Performance Fibre-Reinforced Concretes*. Interim Recommendations. AFGC Publication. France 2002
- [42] CNR-DT 204. *Guidelines for design, construction and production control of fiber reinforced concrete structures*. National Research Council of Italy 2006
- [43] Deutscher Ausschuss für Stahlbeton (DafStb). *Guidelines for steel fiber reinforced concrete*. 23rd draft, richtlinie Stahlfaserbeton, DIN 1045 annex, parts 2007:1-4
- [44] ACI Committee 318. *Report ACI 31808/318R-08*. Building code and commentary. Report ACI 318-08/318R-08 American Concrete Institute, Farmington Hills, MI, 2008
- [45] JSCE. *Recommendations for Design* Recommendations for Design and Construction of High Performance Fiber Reinforced Cement Composites with Multiple Fine Cracks (HPFRCC). Rokug, K. Concrete Engng. Series 2008:82
- [46] AENOR UNE 83515. *Hormigones con fibras, Determinación de la resistencia a fisuración, tenacidad y resistencia residual a tracción. Metodo Barcelona*. Asociación Española de Normalización, Madrid, 2010
- [47] Fib Model Code for Concrete Structures 2010
- [48] Symposium SI, Tecnology C, February S. *CHEMICAL ADMIXTURES TODAY* Mario Collepardi, Enco, Engineering Concrete, Ponzano Veneto (Italy) 2005:527-41
- [49] UNI EN 206-1. *Concrete Part 1: Specification, performance, production and conformity* 2006
- [50] UNI EN 12390-3. *Testing hardened concrete Compressive strength of test specimens* 2003
- [51] UNI 11039-1. *Steel fibre reinforced concrete Definitions, classification and designation* 2003
- [52] UNI 11188. *Steel fibres reinforced concrete structural elements Design, execution and control* 2007

- [53] PrSIA 2052. *Béton fibré ultra-performant (BFUP)-Matériaux, dimensionnement et exécution* 2015-05
- [54] UNI 11039-2. *Steel fibre reinforced concrete Test method for determination of first crack strength and ductility indexes* 2003
- [55] Brite-Euram Project BRPRCT980813. *Test and Design methods for Steel Fibre Reinforced Concrete* 2001
- [56] EN 1992-1-1. *Design of concrete structures General rules and rules for buildings* 1992

## List of Figures

1	Steel fibers used in the mortars: a) Hooked fibers 30/0.35 mm; b) Straight fibers 20/0.3 mm; c) Straight fibers 13/0.175 mm. . . . .	4
2	Specimen dimensions. . . . .	4
3	a) Compression test on cubic specimen; b) Direct tensile test on dog-bone specimen; c) Four-point bending test on notched beam specimen; d) Four-point bending test on un-notched beam specimen. . . . .	5
4	Test machines: a) Machine Perrier type 138-5000 kN used to perform compression tests on cubic specimens; b) Machine Walter Bai type LVF-200 kN used to direct tensile test; c) four-point bending test on notched beams; d) four-point bending test on un-notched beams. . . . .	6
5	Some pictures related to the direct tensile test: a) Aluminum plate glued on the dog-bone specimen; b) Holes in the aluminum plate before tensile test; c) in the aluminum plate after tensile test. . . . .	7
6	Some pictures related to the extensometers used during the experimental tests: a) The extensometer is placed along the 80 mm characteristic lengths specimen; b) The extensometer is placed at the bottom of the notched beam along 100 mm characteristic length and two LVDT are placed on the sides; c) The extensometer is placed at the bottom of the un-notched beam along 200 mm characteristic length and four LVDT are placed under the rollers loading. . . . .	7
7	Cracking of samples subjected to the compression test: a) "A" mortar; b) "B" mortar; c) "C" mortar. . . . .	9
8	Tensile tension-elongation curves from direct tensile test on the dog-bone specimen series. . . . .	13
9	Tensile cracking: a) "A" mortar; b) "B" mortar; c) "C" mortar. . . . .	14
10	Tension-CTOD by flexion curves from four-point bending flexural test on prismatic specimens. . . . .	15
11	Failure modes observed during the loading bending tests: a) Mono-cracking behavior of notched beam A; b) multi-cracking behavior of un-notched beam A; c) mono-cracking behavior of notched beam B; d) mono-cracking behavior of notched beam C. . . . .	16

12	Stress and strain distributions on the cross section of a bent beam: a) Stress distribution before the matrix cracking; b) stress distribution in a post-cracking phase. . . . .	17
13	a) Direct tensile curve; b) indirect tensile curve by flexion test with a volume of fibers equal to $V_{f,crit,traction}$ . . . . .	18
14	a) Direct tensile curve; b) indirect tensile curve by flexion test with a volume of fibers equal to $V_{f,crit,flexion}$ . . . . .	19
15	Experimental curves: a) direct tensile test on the specimen A.2.4; b) indirect tensile test provided by bending test on the specimen A.1.3. . . . .	19
16	Standard bilinear tensile model. . . . .	20
17	Standard Experimental bilinear tensile model for a) SFRC A; b) SFRC B; c) SFRC C. . . . .	21
18	Predictive model Bending Stress-CMOD for a) SFRC A, b) SFRC B, c) SFRC C . . . . .	22