Effective thermal properties of fibre reinforced materials

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The thermal behaviour of an elastic matrix reinforced with synthetic micro or macro fibres subjected to a constant heat flow is investigated in the present work. Steady-state condition for the heat flux is considered and isotropic thermal conductivity for both the matrix and fibres is assumed. Owing to the geometry of the system, reference is made to bipolar cylindrical coordinates (α, β), which are linked to the Cartesian coordinates (x_1, x_2) through the map [1]

$$\alpha = \mathrm{Re}\left[\mathrm{Log}\frac{a + x_1 + i\,x_2}{-a + x_1 + i\,x_2}\right], \qquad \beta = -\mathrm{Im}\left[\mathrm{Log}\frac{a + x_1 + i\,x_2}{-a + x_1 + i\,x_2}\right], \tag{1}$$

being ($\pm a$, 0) the focal points of the bipolar coordinate system.
Temperature $T(\alpha, \beta)$ is related to a given heat flow $q(\alpha, \beta)$ propagating within the matrix through the Fourier law

$$q = -k\ \nabla T, \tag{2}$$

being k the thermal conductivity of the matrix.

Furthermore, according to the stationary condition for the heat flow, the temperature field $T(\alpha, \beta)$ must obey to the Laplace equation

$$\nabla^2 T = 0. \tag{3}$$

Different boundary conditions (BC)s can be considered on the contours of the fibres. In particular, for a matrix reinforced with two fibres taken as insulated inclusions, a vanishing heat flow ($q_i = 0$, $i = 1, 2$) across the contour of the fibres must be imposed.

By integrating eq (2), the fundamental temperature field $T^0(\alpha, \beta)$ is determined. However, such a field does not accomplish the BCs on the contours of the fibres. In order to fulfil the BCs, an extra-term $T^1(\alpha, \beta)$ is then introduced, thus finding the complete solution of the problem. The total temperature field T in then obtained as $T^0 + T^1$ [2]. Once the temperature field is known, a homogeneization procedure is performed in order to find the equivalent thermal properties of the matrix reinforced with fibres, following the procedure reported in [3].

References

[1] *Korn, G.A. and Korn, T.M., Mathematical handbook for scientists and engineers. Definitions, Theorems and Formulas for Reference and Review, Dover, New York (1968).*

[2] *Radi, E., Morini, L. and Sevostianov I., "Conservation integrals for two circular holes kept at different temperatures in a thermoelastic solid," International Journal of Solids and Structures,* ***85-86****, 1-14 (2016).*

[3] *Radi, E. and Sevostianov, I., "Toroidal insulating inhomogeneity in an infinite space and related problems," Proc. R. Soc. A ,* ***472****, 20150781 (2016).*