Requalifying public buildings and utilities with a group decision support system

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DOI: 10.1016/j.jclepro.2017.07.031

Link: http://linkinghub.elsevier.com/retrieve/pii/S0959652617314592

Abstract

Public buildings and utilities are responsible for a significant share of energy consumption and other related CO_2 emissions. There is therefore an acute need for energy requalification interventions. Unfortunately, municipalities are under tight budget constraints – but decisions have to be made. A new hybrid group decision support system has been proposed in a bid to provide them with firm, transparent support. The system is based on a combination of the analytic hierarchy process, the K-means algorithm, and the 0-1 knapsack model. First, the system aims at sorting alternatives into ordered classes of importance. To help in this task, the Bezier curve-fitting approach is used to construct the preference functions of decision-makers based on reference points. Then, the knapsack model selects the alternatives from the generated classes while complying with the budget constraints. A case study carried out in an Italian municipality allowed us to verify the validity of our new method in a real setting, and to highlight the advantages of an automatic sorting procedure in practice.

Keywords: energy requalification; decision support system; AHP; clustering; knapsack

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1. Introduction

Municipalities are constantly involved in proactive actions geared towards improving the welfare of the community under tight budget constraints. Amidst a wide set of possible interventions, the energy requalification of public buildings and utilities is an important action as they are responsible for the largest share of both energy consumption and other related CO_2 emissions (Pérez-Lombard et al., 2008). In 2007, the European Commission published the 2020 Energy Strategy ("20-20-20"), which aims to achieve three goals by 2020: a 20% reduction in CO_2 emissions, a 20% improvement in energy processes, and a 20% replacement of non-renewable energy with renewable energy (European Commission, 2007). The member states of the European Union (EU) have thus released national documents based on different strategies for satisfying the EU's 20-20-20 goals.
Municipalities play a relevant role for the streamlining of public resources and, in particular, for selecting the best choice of interventions for the energy requalification of public buildings and utilities (see for example Fiaschi et al., 2012). Several criteria can be taken into consideration in classifying the most urgent interventions: some are quantitative (e.g. CO_2 emissions and investment return), while others are qualitative (e.g. the impact on tourism, the return on municipality image, and the occupational effects). This type of problem is solved with methods based on group decisions given that, in municipalities, decision-making bodies are always composed of several actors (Poplawska et al., 2015). In addition, they often have divergent objectives and different skills, making the decision on interventions among a wide set of alternatives difficult. In synthesis, a multi-criteria sorting method for group decisions is required to tackle this problem. Once the interventions are classified, financial aspects are assessed from the most to the least urgent class. If there is sufficient funding to cover all the interventions of one class, the next class is assessed. If there is not enough funding to cover all the interventions of the class being analysed, an optimisation model with the knapsack resource allocation is applied.
In a bid to solve multi-criteria sorting group decisions, we have enhanced the existing AHP-K method (Lolli et al., 2014) for group decisions. AHP-K is a hybrid method based on AHP for the evaluation of the weights and the K-means for the sorting of alternatives into K-ordered classes. The aim of the resulting Analytic Hierarchy Process-K-Group Decision Support System (AHP-K-GDSS) is to incorporate all the actors' opinions, to mitigate their subjectivity and to evaluate objective and subjective criteria in one model. Moreover, a Bezier curve-fitting approach initially proposed by Sarfraz and Khan (2002, 2003, 2004) for the boundary approximation of bitmap characters is applied for the first time to define the utility functions on the objective criteria. It is an extension of the traditional multi-attribute utility theory approach (Keeney and Raiffa, 1979) and more recent variants (e.g. Rebai et al., 2016), demonstrating promising results. Finally, the 0-1

knapsack algorithm is used to solve the financial allocation problem. This new methodology was developed to solve a real case study of energy requalification of pebble buildings and utilities.
The paper is structured as follows: Section 2 contains the notations; Section 3 reports some of the recent major contributions on this research field; Section 4 explains the proposed approach step by step; Section 5 outlines the case study analysed; and Section 6 contains some conclusions and the further research agenda.

2. Notation

$\boldsymbol{I}$ = Set of criteria such that $\boldsymbol{I} = \boldsymbol{I^O} \cup \boldsymbol{I^S}$, with $\boldsymbol{I^O} \cap \boldsymbol{I^S} = \emptyset$

$\boldsymbol{I^O}$ = Set of objective criteria

$\boldsymbol{I^S}$ = Set of subjective criteria

J = Total number of alternatives

K = Total number of decision-makers

c_i = Criterion, with $i = 1, \dots, I$

j = Alternative, with $j = 1, \dots, J$

k = Decision-maker, with $k = 1, \dots, K$

$v_{i,j}^k$ = Score of alternative j on criterion $c_i \in \boldsymbol{I}$ for decision-maker k

$v_{i,j}$ = Value of alternative j on criterion $c_i \in \boldsymbol{I^O}$

$\boldsymbol{v_{i,j}^k}$ = Vector of score and value for alternative j on criterion $c_i \in \boldsymbol{I^O}$ for decision-maker k

$\boldsymbol{\beta^{i,k}}$ = Bezier curve for criterion $c_i \in \boldsymbol{I^O}$ for decision-maker k

$t_{i,q}^k = \boldsymbol{\beta^{i,k}}$ Curve parameter for order indicator q on criterion $c_i \in \boldsymbol{I^O}$ for decision-maker k

$\boldsymbol{P_{cp}^{i,k}}$ = Control point $cp \in \{0,1,2,3\}$ of $\boldsymbol{\beta^{i,k}}$

$\phi_{k,i}$ = Preference function on criterion $c_i \in \boldsymbol{I^O}$ for decision-maker k

p_j^k = Global priority of alternative j for decision-maker k

$\boldsymbol{\widehat{P}_j}$ = Vector of p_j^k

P_j = Global priority of alternative j

$\boldsymbol{\overline{P}_j}$ = Vector of P_j

$e_{ii'}^k$ = Decision-maker weight for criterion c_i compared to criterion $c_{i'}$

w_i^k = Weight of criterion $c_i \in \boldsymbol{I}$ for decision-maker k

u^k = Weight of decision-maker k

N = Number of individual and global classes

C_n^k = Class n for decision-maker k, with $n = 1, \dots, N$ and $k = 1, \dots, K$

C_n = Class n on final score, with $n = 1, \dots, N$

n^k = Class order of C_n^k, with $n = 1, \dots, N$ and $k = 1, \dots, K$

n = Class order of C_n, with $n = 1, \dots, N$

κ_j = Cost of alternative j

D = Budget

3. Literature review

Multi-criteria decision-making can solve many problems with regard to selection, ranking, sorting, etc. (Roy, 1981). However, specific methods need to be developed to deal with each type of problem. The sorting methods fall under two groups. The first group is based on direct elicitation. It requires decision-makers to provide parameters that represent their preferences as inputs to the methods. The method then classifies the alternatives based on these preferences. Conversely, the second group is based on indirect elicitation. It requires outputs, which means that the final result (i.e. the class of assignment) is required. Decision-makers are asked to classify a small subset of alternatives that they know very well. Classification rules are then inferred and applied in order to classify the remaining alternatives.

The direct elicitation models encompass several contributions, mainly extensions of ELECTRE-TRI (Yu, 1992) for group decisions. Dias and Clímaco (2000) dealt with imprecise parameter values (criteria weightings, thresholds for alternative sorting classes, etc.). Each decision-maker provides constraints bounding rather than fixing precise figures. De Morais Bezerra et al. (2014) integrated ELECTRE-TRI with Visual and Interactive Comparative Analysis (VICA). VICA allows for the comparison of results and encourages consensus-building by proposing different scenarios. Jabeur and Martel (2007) complement ELECTRE-TRI with AL3, enabling the creation of a consensual preference table by minimising the distance between individual preferences. PROMETHEE (Brans and Vincke, 1985), another outranking method, has been extended with FlowSort (Nemery and Lamboray, 2008) and GAIASort (Nemery et al., 2012) for sorting problems, and then in FlowSort-GDSS for group sorting problems (Lolli et al., 2015).

Decision-makers sometimes find it difficult to express their preferences with a precise numerical value, and the role they play in the model. Indirect elicitation models have been developed for this reason. Damart et al. (2007) proposed using the aggregation/disaggregation approach for the ELECTRE TRI method as implemented on the IRIS Decision Support System, where the group discusses how to sort examples of actions (some of which may be fictitious), instead of discussing what values the model parameters should take. In this case, the weights and cutting levels of

ELECTRE TRI are inferred. The other parameters of the model (profiles, veto, preference and indifference thresholds) are directly elicited.

The Dominance-based Rough Set Approach (DRSA) is based on the same principle, in which the decision-maker assigns some examples to classes. Rules in the form of “if PREMISE then CLASS” are then inferred (Greco et al., 2001).

Different variants of the DRSA for group decisions have been created. Greco et al. (2006) extend the concepts of the DRSA to deal with decision tables having multiple decision attributes, thus allowing comprehensive collective decision rules to be generated. Chakhar and Saad (2012) propose an aggregation algorithm, based on the majority principle and supporting the veto effect, allowing consensual decision rules to be inferred. Chakhar et al. (2016) introduce a more advanced version of the aggregation algorithm proposed in Chakhar and Saad (2012). Chen et al. (2012) use the Dempster-Shafer theory of evidence (Dempster, 1968; Shafer, 1976) to combine individual rules provided by the DRSA.

Cai et al. (2012) work on the same principle of assignment examples given by decision-makers to derive class thresholds and weights of criteria for an additive linear value function. Greco et al. (2012) propose using the robust ordinal regression for UTADIS^{GMS}-GROUP. Liu et al. (2015) combine the disaggregation-aggregation paradigm with the evidential reasoning approach to address the uncertainty of assigning alternatives to classes.

Both groups assume that the decision-maker (DM) is able to provide some information on the classes either directly as thresholds or indirectly by classifying some exemplary alternatives. This paper aims to deal with cases in which the DM is unable to provide any information on the classes, for example in the case of an entirely new problem with no previous experience collected.

4. Group Decision Support System

The developed Group Decision Support System (GDSS) consists of two stages. The first stage (see Lolli et al., 2016) involves sorting alternatives into ordered classes (Section 4.1). The second stage involves selecting the alternatives to be funded under the constraints of a limited budget (Section 4.2).

4.1 Sorting stage with AHP-K-GDSS

This stage is completed by following the steps below.

Step 1: Score assignment on qualitative criteria

A qualitative criterion does not have a measurement system. All the J alternatives have to be evaluated by the K decision-makers individually on all the qualitative criteria belonging to $\boldsymbol{I^S}$. Moreover, due to the high number of alternatives that typically arise in the field of energy requalification, the pairwise comparison method (e.g. AHP, MACBETH) would be an excessively time-consuming activity. A direct evaluation on a predefined scale $[\mathrm{Min}, \mathrm{Max}]$ is therefore preferred. The higher the score assigned to the alternative, the higher its preference degree. These preference scores are assigned by DMs to all the alternatives on all the qualitative criteria, and are named $v_{i,j}^k$, with criterion $i \in \boldsymbol{I^S}$, alternative $j = 1, \dots, J$ and DM $k = 1, \dots, K$.

Step 2: Score assignment on quantitative criteria

Alternatives have values $v_{i,j}$ on the quantitative criteria $\boldsymbol{I^O}$ that are physically measured. However, they have to be commensurable with the qualitative criteria (Step 1) to be aggregated. A simple linear normalisation of the values $v_{i,j}$between the worst and the best alternative on each criterion would solve this issue, but might violate the preference structure of the DMs. In fact, as experienced in real settings and universally confirmed in the literature, DMs often express nonlinear preferences among alternatives. In order to elicit the preference functions on the quantitative criteria, a marginal preference function $\phi_{k,i}: v_{i,j} \rightarrow v_{i,j}^k$ is elicited for each DM k and criterion i though a procedure similar to the well-known five-point approach adopted in the Multi-Attribute Utility Theory (Keeney and Raiffa, 1979). In particular, the DM is asked to evaluate $v_{i,j}$ on the same scale [Min, Max] as in the qualitative criteria for the lowest and highest $v_{i,j}$ and three intermediate points. The main differences with the standard approach of Keeney and Raiffa (1979) is that, instead of defining a piecewise linear function between each pair of consecutive points, all five points concur to identify a unique best-fitting curve. Such an approach is advantageous as it allows for the identification of a customised preference function for each DM, and greatly simplifies the elicitation of $v_{i,j}^k$, especially in case of a high number of alternatives. However, choosing the curves among a predefined set might affect the fitting goodness as not all possibilities are available. Another approach would be a polynomial fitting with the aim of minimising the mean square error. A polynomial fitting would certainly reach an exact fitting, i.e. with a null mean square error. However, the exact polynomial fitting has the inconvenience of the local minima that could appear among couples of consecutive points, which could be inconsistent with the human preference functions. Conversely, a good candidate for solving this fitting problem is the Bezier curve, as it avoids local minima among consecutive points.

The procedure for assigning the scores to the alternatives, along with the fitting approach of Sarfraz and Khan (2002, 2003, 2004), is explained below.

1. The five well-distributed reference points are obtained for each quantitative criterion as follows:

$$\hat{v}_{i,1} = \min_{j=1,\dots,J}\{v_{i,j}\} \tag{1}$$

$$\hat{v}_{i,2} = \frac{1}{4}\left[\max_{j=1,\dots,J}\{v_{i,j}\} - \min_{j=1,\dots,J}\{v_{i,j}\}\right] \tag{2}$$

$$\hat{v}_{i,3} = \frac{1}{2}\left[\max_{j=1,\dots,J}\{v_{i,j}\} - \min_{j=1,\dots,J}\{v_{i,j}\}\right] \tag{3}$$

$$\hat{v}_{i,4} = \frac{3}{4}\left[\max_{j=1,\dots,J}\{v_{i,j}\} - \min_{j=1,\dots,J}\{v_{i,j}\}\right] \tag{4}$$

$$\hat{v}_{i,5} = \max_{j=1,\dots,J}\{v_{i,j}\} \tag{5}$$

where $\hat{v}_{i,1}$and $\hat{v}_{i,5}$ correspond to real alternatives, respectively the biggest and the smallest on criterion c_i, whilst $\hat{v}_{i,2}$, $\hat{v}_{i,3}$, and $\hat{v}_{i,4}$ are intermediate points that do not necessarily match with real alternatives. Then, each DM $k = 1, \dots, K$ provide a score $\hat{v}_{i,l}^{k}$ on the scale $[\mathrm{Min}, \mathrm{Max}]$ for these points on each criterion $c_i \in I^{O}$.

2. For each criterion $c_i \in I^{O}$ and DM $k = 1, \dots, K$, a set of vectors is generated. Each vector is defined by a reference point $\hat{v}_{i,l}$, with $l \in \{1,2,3,4,5\}$, and its corresponding score:

$$\boldsymbol{\hat{v}_{i,l}^{k}} = \left(\hat{v}_{i,l}; \hat{v}_{i,l}^{k}\right) \qquad l \in \{1,2,3,4,5\} \tag{6}$$

3. Each vector in the set is assigned the corresponding value of the Bezier curve parameter $t_{i,l}^{k}$:

$$t_{i,l}^{k} \begin{cases} 0 & if\ l = 1 \\ \frac{\sum_{q=2}^{l}\left|\hat{v}_{i,q}^{k} - \hat{v}_{i,q-1}^{k}\right|}{\sum_{q=2}^{5}\left|\hat{v}_{i,q}^{k} - \hat{v}_{i,q-1}^{k}\right|} & if\ l \in \{2,3,4\} \\ 1 & if\ l = 5 \end{cases} \tag{7}$$

Thus, for each vector $\boldsymbol{\hat{v}_{i,l}^{k}}$ a corresponding point $\boldsymbol{\beta}^{\boldsymbol{i,k}}\left(t_{i,l}^{k}\right)$ on the bi-dimensional Bezier curve can be identified.

4. The Bezier curve assumes the following form:

$$\boldsymbol{\beta}^{\boldsymbol{i,k}}(t) = \boldsymbol{P}_{\boldsymbol{0}}^{\boldsymbol{i,k}} \cdot (1-t)^3 + \boldsymbol{P}_{\boldsymbol{1}}^{\boldsymbol{i,k}} \cdot 3t(1-t)^2 + \boldsymbol{P}_{\boldsymbol{2}}^{\boldsymbol{i,k}} \cdot 3t^2(1-t) + \boldsymbol{P}_{\boldsymbol{3}}^{\boldsymbol{i,k}} \cdot t^3 \quad t \in [0,1] \quad (8)$$

where t is the parameter of the curve.

It is characterised by four control points where the first $\boldsymbol{P}_{\boldsymbol{0}}^{\boldsymbol{i,k}}$ and last $\boldsymbol{P}_{\boldsymbol{3}}^{\boldsymbol{i,k}}$ correspond respectively to:

$$\boldsymbol{P}_{\boldsymbol{0}}^{\boldsymbol{i,k}} = \widehat{\boldsymbol{v}}_{\boldsymbol{i,1}}^{\boldsymbol{k}} \quad (9)$$

$$\boldsymbol{P}_{\boldsymbol{3}}^{\boldsymbol{i,k}} = \widehat{\boldsymbol{v}}_{\boldsymbol{i,5}}^{\boldsymbol{k}} \quad (10)$$

Thus, the only parameters that can be modified are the positions of the second $\boldsymbol{P}_{\boldsymbol{1}}^{\boldsymbol{i,k}}$ and third $\boldsymbol{P}_{\boldsymbol{2}}^{\boldsymbol{i,k}}$ control points.

Given a set of vectors $\widehat{\boldsymbol{v}}_{\boldsymbol{i,l}}^{\boldsymbol{k}}$, the sum of squared distances between each vector and the corresponding Bezier point $\boldsymbol{\beta}^{\boldsymbol{i,k}}\left(t_{i,l}^{k}\right)$ can be minimised by modifying the Bezier curve parameters. The minimum square error is given by:

$$MSE_{i,k} = \sum_{l\in\{1,2,3,4,5\}} \left| \widehat{\boldsymbol{v}}_{\boldsymbol{i,l}}^{\boldsymbol{k}} - \boldsymbol{\beta}^{\boldsymbol{i,k}}\left(t_{i,l}^{k}\right) \right|^2 \quad (11)$$

The following values for $\boldsymbol{P}_{\boldsymbol{1}}^{\boldsymbol{i,k}}$ and $\boldsymbol{P}_{\boldsymbol{2}}^{\boldsymbol{i,k}}$ can be obtained:

$$\boldsymbol{P}_{\boldsymbol{1}}^{\boldsymbol{i,k}} = \frac{A_2^{i,k} \cdot \boldsymbol{C}_{\boldsymbol{1}}^{\boldsymbol{i,k}} - A_{1,2}^{i,k} \cdot \boldsymbol{C}_{\boldsymbol{2}}^{\boldsymbol{i,k}}}{A_1^{i,k} \cdot A_2^{i,k} - \left(A_{1,2}^{i,k}\right)^2} \quad (12)$$

$$\boldsymbol{P}_{\boldsymbol{2}}^{\boldsymbol{i,k}} = \frac{A_1^{i,k} \cdot \boldsymbol{C}_{\boldsymbol{2}}^{\boldsymbol{i,k}} - A_{1,2}^{i,k} \cdot \boldsymbol{C}_{\boldsymbol{1}}^{\boldsymbol{i,k}}}{A_1^{i,k} \cdot A_2^{i,k} - \left(A_{1,2}^{i,k}\right)^2} \quad (13)$$

where the vectors $\boldsymbol{C}_{\boldsymbol{1}}^{\boldsymbol{i,k}}$ and $\boldsymbol{C}_{\boldsymbol{2}}^{\boldsymbol{i,k}}$ are defined as:

$$\boldsymbol{C}_{\boldsymbol{1}}^{\boldsymbol{i,k}} = \sum_{l\in\{1,2,3,4,5\}} \left[B_1\left(t_{i,l}^{k}\right) \cdot \left(\widehat{\boldsymbol{v}}_{\boldsymbol{i,l}}^{\boldsymbol{k}} - B_0\left(t_{i,l}^{k}\right) \cdot \boldsymbol{P}_{\boldsymbol{0}}^{\boldsymbol{i,k}} - B_3\left(t_{i,l}^{k}\right) \cdot \boldsymbol{P}_{\boldsymbol{3}}^{\boldsymbol{i,k}} \right) \right] \quad (14)$$

$$\boldsymbol{C}_{\boldsymbol{2}}^{\boldsymbol{i,k}} = \sum_{l\in\{1,2,3,4,5\}} \left[B_2\left(t_{i,l}^{k}\right) \cdot \left(\widehat{\boldsymbol{v}}_{\boldsymbol{i,l}}^{\boldsymbol{k}} - B_0\left(t_{i,l}^{k}\right) \cdot \boldsymbol{P}_{\boldsymbol{0}}^{\boldsymbol{i,k}} - B_3\left(t_{i,l}^{k}\right) \cdot \boldsymbol{P}_{\boldsymbol{3}}^{\boldsymbol{i,k}} \right) \right] \quad (15)$$

and the scalars $A_1^{i,k}$, $A_2^{i,k}$ and $A_{1,2}^{i,k}$ are defined as:

$$A_1^{i,k} = \sum_{l \in \{1,2,3,4,5\}} \left[B_1\left(t_{i,l}^k\right)\right]^2 \tag{16}$$

$$A_2^{i,k} = \sum_{l \in \{1,2,3,4,5\}} \left[B_2\left(t_{i,l}^k\right)\right]^2 \tag{17}$$

$$A_{1,2}^{i,k} = \sum_{l \in \{1,2,3,4,5\}} \left[B_1\left(t_{i,l}^k\right) \cdot B_2\left(t_{i,l}^k\right)\right]^2 \tag{18}$$

The scalars $B_0(t)$, $B_1(t)$, $B_2(t)$, $B_3(t)$ in (14) and (15) are given by:

$$B_0(t) = (1-t)^3 \tag{19}$$

$$B_1(t) = 3t(1-t)^2 \tag{20}$$

$$B_2(t) = 3t^2(1-t) \tag{21}$$

$$B_3(t) = t^3 \tag{22}$$

5. The Bezier function defined for each criterion $c_i \in I^O$ and DM $k = 1, \dots, K$ can be used to obtain for each score $v_{i,j}$ a preference $v_{i,j}^k$, with $j = 1, \dots, J$.
   Given a curve parameter value t of the Bezier function, the alternative value and the corresponding score are known. A numerical inversion is needed in order to obtain the inverse function, capable of taking the alternative value as input and the related score in output:

$$v_{i,j}^k = \phi_{k,i}\left(v_{i,j}\right) \qquad k = 1, \dots, K \qquad j = 1, \dots, J \qquad a_i \in I^O \tag{23}$$

Step 3: Pairwise comparison between criteria

The I criteria are now pairwise compared by each DM in order to calculate their relative weights. DMs k indicate their evaluation $e_{ii'}^k$ on the 1-9 scale defined by Saaty (1980), which represents the pairwise comparison between criteria i and i'.

It follows that K preference matrixes A^k of the following form have to be compiled:

$$A^k = \begin{pmatrix} 1 & \dots & e_{1I}^k \\ \dots & \dots & \dots \\ \frac{1}{e_{1I}^k} & \dots & 1 \end{pmatrix}_{I \times I} \tag{24}$$

In order to obtain the weights w_i^k of the criteria for the DM k, the eigenvalue method is adopted (Saaty, 1980).

Step 4: Individual ranking

The global priority of alternative j for DM k is thus achieved by the weighted sum of the $v_{i,j}^k$:

$$p_j^k = \sum_{i \in \boldsymbol{I}} w_i^k v_{i,j}^k \tag{25}$$

Let $\widehat{\boldsymbol{P}}_{\boldsymbol{k}}$ be a vector containing the individual ranking for DM k:

$$\widehat{\boldsymbol{P}}_{\boldsymbol{k}} = \left(p_1^k, \dots, p_J^k\right) \tag{26}$$

Hence, K different rankings are now available on the basis of the DMs' judgements both on the qualitative (Step 1) and quantitative (Step 2) criteria.

Step 5: Global ranking

After assigning the weights u^k to the K DMs by the owner of the decisional process such that $\sum_{k=1}^{K} u^k = 1$, the global ranking of alternative j is obtained by means of a weighted sum of p_j^k as follows:

$$P_j = \sum_{k=1}^{K} u^k \, p_j^k \tag{27}$$

Let $\overline{\boldsymbol{P}}$ be the vector of P_j, i.e. the global ranking:

$$\overline{\boldsymbol{P}} = \left(P_1, \dots, P_J\right) \tag{28}$$

Step 6: Global sorting

In order to sort the alternatives into N classes, the K-means algorithm is applied on the global ranking calculated in Step 5. This group sorting form of the AHP-K (Lolli et al., 2014) is based on the K-means algorithm (see e.g. Jain, 2010; Madhulatha, 2012). The aim is to create N compact and well-separated classes of alternatives by minimising the sum of the squared distances between the centroid of each class and the items in the class. Let μ_n be the centroid of the class C_n, where n is a generic class between 1 and N. The squared distance between μ_n and the items of the class C_n is defined as:

$$SD(C_n) = \sum_{P_j \in C_n} \left(P_j - \mu_n\right)^2 \tag{29}$$

The goal of the K-means algorithm is to minimise the sum of the squared distances for all N classes:

$$SD(\overline{\boldsymbol{P}}) = \sum_{n=1}^{N} \sum_{P_j \in C_n} \left(P_j - \mu_n\right)^2 \tag{30}$$

This is known to be an NP-hard problem (Drineas et al., 2004). The K-means is a greedy algorithm that converges to a local minimum starting with an initial partition of N classes and then assigning the values belonging to $\overline{\boldsymbol{P}}$ classes, in order to reduce the $SD(\overline{\boldsymbol{P}})$. In synthesis, this step allows for the classification of all the alternatives into classes with different importance levels.

Step 7: Veto system

This further step aims at comparing the global sorting achieved through Step 6 with the individual sorting. In fact, the alternatives could be sorted differently by the group (global sorting) than by one or more decision-makers (individual sorting). In order to find the individual sorting, the K-means is applied to $\widehat{\boldsymbol{P}}_{\boldsymbol{k}}$ (Step 4), with $k = 1, \ldots, K$, with the aim to minimise:

$$SD_k\left(\widehat{\boldsymbol{P}}_{\boldsymbol{k}}\right) = \sum_{n=1}^{N} \sum_{p_j^k \in C_n^k} \left(p_j^k - \mu_n^k\right)^2 \tag{31}$$

where μ_n^k indicates the centroid of class C_n^k, where n^k is its class order between 1 and N.

It must be noted that an alternative j may be classified into a global class C_n very far from several individual classes C_n^k. In this step, these divergent opinions may be taken into account with a veto. In order to apply this veto system, the additional parameter m is required, representing the veto activation threshold as the absolute sum of the number of classes between individual and global classes. Given an alternative $j \in C_n$ (globally) and $j \in C_n^k$ (individually) with $k = 1, \ldots, K$, n^k is not necessarily equal to n. The veto system may lead to reclassifying j into a global class $C_{n'}$ with $n' \neq n$ as follows:

$if\ |\sum_{k=1}^{K}(n^k - n)| \geq m$

$\{if\ \sum_{k=1}^{K}(n^k - n) < 0$

$n^{'} = n - 1;$

$else\ \ n^{'} = n + 1;\}$

$else\ \ n^{'} = n;$

Of course, DMs who assign j to individual classes as to the global one ($n^k = n$) are not contributing to claim a veto. In the particular case where two DMs assign an alternative to individual classes $C_n^k \neq C_n$ equidistant from the global class C_n, their individual sorting compensates each other ($|\sum_{k=1}^{K}(n^k - n)| = 0$) and the veto is not activated independently from m. It must be noted that in the AHP-K-Veto (Lolli et al., 2014), which is a mono decision-maker sorting method, the K-means is applied to the criteria to prevent an alternative that is highly (badly) classified on a criterion from being globally badly (highly) classified. Conversely, vetoes are applied to the global sorting from the individual sorting.

4.2 Selection stage

In the first stage, the alternatives were sorted into N ordered classes. This means that all alternatives in class C_n must necessarily be handled before those on class C_{n+1} since they are more important. Therefore, the budget allocation starts from C_1 and progresses to the next class until C_n when all alternatives of the current class cannot be funded. Hence, the first step searches for the C_n such that:

$$\sum_{n'=1}^{n-1} \sum_{j \in C_{n'}} \kappa_j \leq D < \sum_{n'=1}^{n} \sum_{j \in C_{n'}} \kappa_j \quad (32)$$

All the alternatives belonging to the classes C_1 to C_{n-1} are thus funded. The remaining budget $D - \sum_{n'=1}^{n-1} \sum_{j \in C_{n'}} \kappa_j$ has to be optimised in order to select alternatives maximising the sum of their global priorities P_j. This problem can be solved with a 0-1 knapsack formulation. It must be noted that if $\sum_{n'=1}^{n} \sum_{j \in C_{n'}} \kappa_j \leq D$, all the alternatives will be funded and a knapsack model is not necessary.

The 0-1 knapsack problem is formulated as follows:

$$\max \sum_{j \in C_n} P_j \cdot x_j \quad (33)$$

Subject to:

$$\sum_{j \in C_n} \kappa_j \cdot x_j \leq D - \sum_{n'=1}^{n-1} \sum_{j \in C_{n'}} \kappa_j \quad (34)$$

$$\begin{cases} x_j = 1 & if\ j\ is\ funded \\ x_j = 0 & otherwise \end{cases} \quad (35)$$

5. Case study

The proposed decision support system was validated in an Italian municipality of about 30,000 inhabitants. The aim of the study was to choose between 34 interventions of energy requalification $j = 1, \ldots, 34$ given the budget constraint of $D = €430{,}000$. These interventions can be grouped by typology and summarised as follows:

- Wrap insulation: responsible for thermal exchanges, and thermal coal represents an energy-efficient solution year-round.
- Heating system replacements: more efficient condensing generator systems could replace obsolete technologies.
- Photovoltaics: photovoltaic systems for the production of electric energy.
- Flux regulator: a device that regulates the supply voltage of lighting, with the aim of reducing the luminous flux and thus energy consumption. A typical use includes sodium lamps for street lighting (lowering the luminous flux at night).
- Astronomical time switches: used to programme the light switching on and off depending on the actual hours of daylight throughout the year. These are more accurate than twilight switches, which, in the absence of maintenance, tend to get dirty and control the switching in advance (or be affected by adverse weather conditions such as cloudiness), thus resulting in an unjustified increase in consumption.
- Aeolian: Aeolian systems for the production of electric energy.
- Fixtures: old fixtures could be replaced by new ones with aluminium thermal breaks and double glazing.
- Pool covering: covering swimming pools with insulating towels when they are not being used reduces the power consumption involved in maintaining water temperature.
- Cogeneration systems: replacing a boiler with natural gas CHP (combined heat and power), i.e. a system for the combined production of electricity and heat, allows for a higher return by consuming the electricity produced, and to value the excess fed into the grid.

- Reduction of pool water temperature: reducing pool water temperature reduces power consumption.
- Water heat recovery: a system that recovers part of the heat of water, which is changed daily in part, and thus transfers it to clean make-up water.
- Air circuit insulation: thermal insulation of the ducts of the ventilation treatment.
- Water circuit insulation: thermal insulation of the ducts of the water treatment.
- Low-flow dispensers: the installation of low-flow dispensers, which are devices that mix water with the air, reduces consumption of the water and energy required to heat it, without reducing comfort for the users.
- Efficient lighting technologies: more efficient optical systems either increase the amount of light flux available to the user or ensure that the same level of lighting is provided with less power.
- Advanced control systems for luminous flux regulation: devices that automatically regulate the luminous flux based on actual environmental conditions such as the availability of natural lighting, the presence of users, etc.
- Air handling units (AHU): AHUs are devices that recover part of the heat contained in the expelled air and transfer it to the renewal air, reducing energy consumption given the high rates of air exchange in public buildings to ensure the correct dehumidification.

Three council members served as decision-makers, with the mayor leading the decision process. In particular, DM_1 is a budget representative, DM_2 is responsible for social policies and DM_3 is an environmental expert.

The mayor assigned weights to the DMs on the basis of their experience and skills. In this case, the weights were set equal, i.e. $u^1 = u^2 = u^3 = 0.33$. The decision-makers were asked to directly assign scores on a scale of 1-20 to the alternatives as regards to the quantitative criteria. Section 5.1 contains the list of criteria used for selecting the energy requalification interventions and Section 5.2 solves the group sorting problem step by step by means of the proposed model (Section 4).

5.1. Criteria

The choice of criteria for evaluating the interventions represents a crucial stage of the case study. Nevertheless, the literature does not help in this choice as energy requalification was never faced as a multi-criteria problem involving both qualitative and quantitative criteria. Hence, the criteria were chosen according to the specific environment under analysis based on the opinion of the decision-makers. Three quantitative $\{c_1, c_2, c_3\} \in \boldsymbol{I^O}$, and five qualitative criteria $\{c_4, c_5, c_6, c_7, c_8\} \in \boldsymbol{I^S}$ were adopted for evaluating the alternatives.

Annual CO_2 savings (c_1)

Fossil fuel consumption is responsible for the majority of CO_2 emissions [ton/year]. The installation of photovoltaic cells for producing energy is considered one of the most attractive interventions of the energy requalification in respect to this environmental criterion. This criterion is quantifiable in a continuous dominium and by benefit type (i.e. must be maximised).

Annual monetary savings (c_2)

The primary item leading to monetary savings [€/year] is a decrease in energy consumption. Annual monetary savings are a quantifiable criterion defined on a continuous dominium and by benefit type.

Financial payback time (c_3)

The minimum number of operative years it takes for the annual monetary savings to cover the initial financial investment. This criterion is quantifiable, defined on a continuous dominium and by cost type (i.e. must be minimised).

Comfort improvement (c_4)

Thermal comfort (temperature and humidity), air purity and lighting conditions inside a building pertain to the broad field of risk assessment for the health and safety of workers. This benefit-type criterion is regarded as qualitative.

Image toward citizens (c_5)

Making the energy requalification visible to the community represents strategic involvement for administrative bodies. The energy requalification of institutional buildings such as schools, city halls and healthcare facilities shows locals that actions for rationalising public spending and "greening" their infrastructures are being taken. This is a qualitative criterion that needs to be maximised.

Educational value (c_6)

The interventions realised on educational facilities, such as primary schools, high schools and universities, also have an educational impact on pupils and students. This is a qualitative criterion that needs to be maximised.

Local employment development (c_7)

The city council shall commission requalification actions to local or non- local (i.e. outside the jurisdiction) companies, thereby creating employment. This criterion is considered qualitative and needs to be maximised.

Increase in energy self-sufficiency (c_8)

The production of energy from non-renewable resources is not sustainable. It is even worse if the resources (e.g. coal) are imported. Storage and transportation costs as well as pollution need to be added to the bill. It must be noted that some requalification interventions lead to complete energy self-sufficiency, e.g. photovoltaic cells. This criterion is considered qualitative and needs to be maximised.

5.2. Analysis

All the steps of the methodology described in Section 4 are applied below. DMs are asked to score the 34 alternatives on a scale of 1-20 as regards each qualitative criterion c_4, c_5, c_6, c_7, and c_8 (Step 1). Then, the decision-makers are asked to assign a score on the same scale to five reference points ($l \in \{1,2,3,4,5\}$) on the three quantitative criteria (Step 2). Table 1 shows the 1-20 scores assigned by DM_1, DM_2, and DM_3 to the reference points for criteria c_1, c_2, and c_3. The values of these points ($v_{1,l}$, $v_{2,l}$ and $v_{3,l}$) are outlined as well, where $l = 1$ and $l = 5$ correspond to the real alternatives having the minimum and the maximum values on c_1, c_2, and c_3, respectively.

	Annual CO2 savings (c1)				***Annual monetary savings (c_2)***				***Financial payback time (c3)***			
l	$v_{1,l}$	DM_1	DM_2	DM_3	$v_{2,l}$	DM_1	DM_2	DM_3	$v_{3,l}$	DM_1	DM_2	DM_3
1	546	20	20	18	51960	18	20	18	16	3	5	10
2	409.50	15	13	14	38938.50	16	17	15	12	5	8	12
3	273	10	8	8	25959	10	11	10	8	15	10	17
4	136.50	5	4	4	12979.50	5	5	6	4	17	12	18
5	0.01	0	0	0	42	0	3	2	0	20	20	20

Table 1. Scores assigned by the decision-makers to the five reference points.

The Bezier curve-fitting approach is then applied for each DM and quantitative criterion. For instance, the four control points (Equations 8, 9, 12, and 13) for DM_1 and c_2 are: $\boldsymbol{P_0^{2,1}} = (42,0)$; $\boldsymbol{P_1^{2,1}} = (17348,4.52)$; $\boldsymbol{P_2^{2,1}} = (34654,17.62)$; and $\boldsymbol{P_3^{2,1}} = (51960,18)$. Figure 1 shows the nine best-fitting Bezier curves, i.e. preference functions, where the crosses indicate the reference points and the asterisks are the values converted on the 1-20 scale. An almost linear shape is achieved only on c_1 for DM₁, while non-linear shapes are achieved for the remaining criteria and DMs. In particular, decreasing functions are obtained for c_3, as a cost-type criterion.

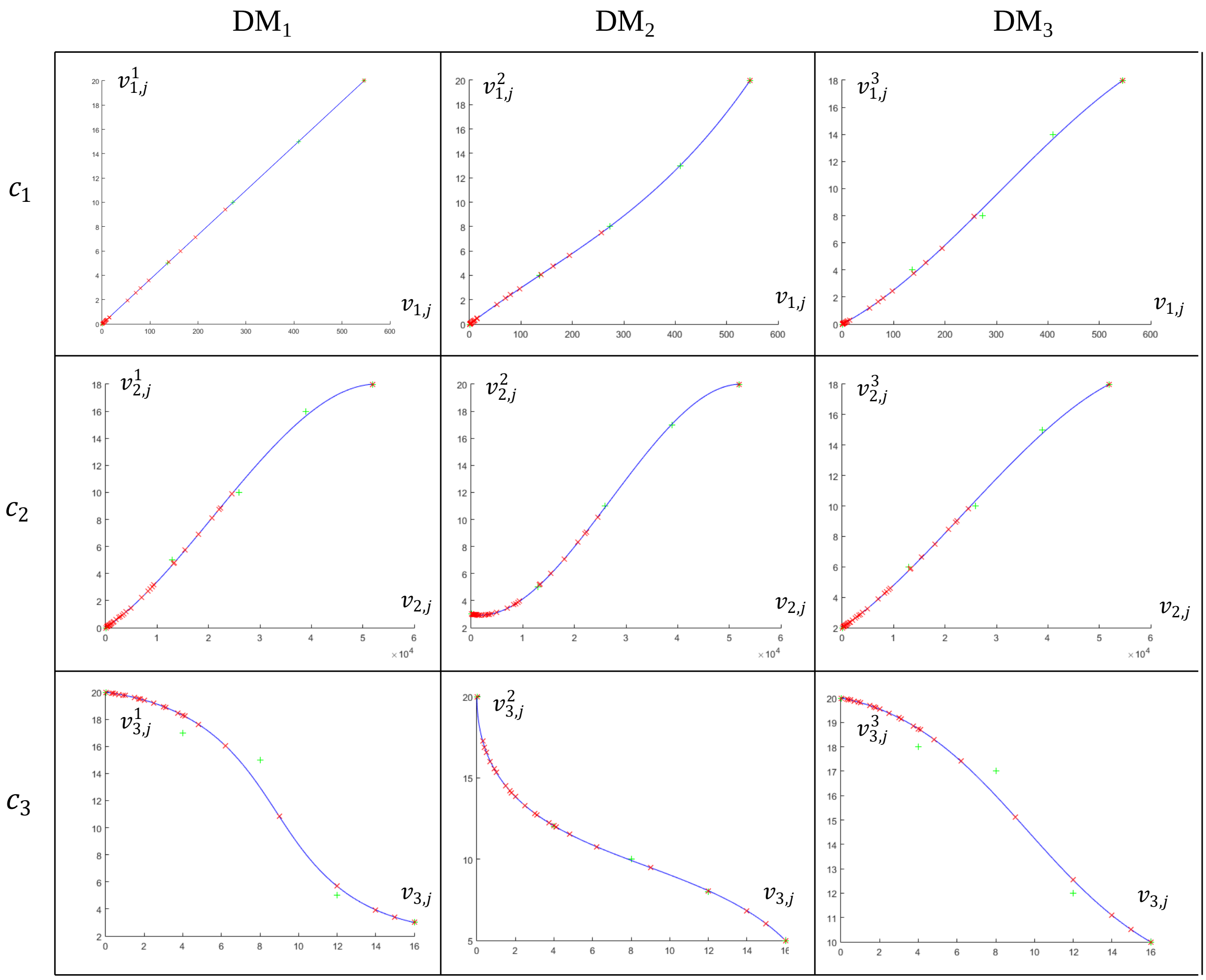

Figure 1. The best-fitting Bezier curves for the elicitation of the preference functions on quantitative criteria.

The Appendix outlines the scores assigned by DMs to the alternatives on each criterion.

The eight criteria have been pairwise compared, and their weights have been calculated by means of the eigenvalue method (Step 3). Table 2 shows these weights for each DM along with the consistency ratios (CRs), which are all lower than the threshold value of 0.1. As predicted, the strictly environmental criteria (c_1 and c_8) are more important for DM₃, while strictly economic (c_2 and c_3) and social (c_5 and c_6) criteria are key to DM₁ and DM₂ respectively.

	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	**CR**
DM$_1$	0.071	0.231	0.331	0.048	0.024	0.033	0.157	0.106	0.03
DM$_2$	0.054	0.023	0.018	0.153	0.385	0.223	0.105	0.038	0.09
DM$_3$	0.459	0.12	0.082	0.056	0.02	0.028	0.038	0.197	0.06

Table 2. Weights assigned by DMs to the criteria.

The individual rankings (Step 4) were achieved by multiplying the scores (see Appendix) by the weights of the criteria (Table 2) for each DM. The global rankings were then obtained by aggregating these individual rankings through a weighted sum over DMs (Step 5), with DMs equally weighted in this case.

The K-means algorithm was launched on the global ranking in order to achieve the global sorting into three ordered clusters named C_1, C_2 and C_3 from the most to least preferred (Step 6). As the K-means is a greedy algorithm, the final partition depends on both the number of iterations and the starting partition. In order to verify the robustness of the solution, the algorithm was launched 20 times with 100 iterations, each time starting from a different random partition. It has been found that the solution does not change over launches, and can therefore be considered robust. In order to apply the veto system, the global sorting approach has to be compared with the individual sorting, which was also obtained with the K-means algorithm (Step 7). Table 3 shows the individual and the global rankings of the alternatives, with individual and global priorities reported in brackets, along with the individual and the global sorting. Furthermore, the last column on the right contains the final classes after the veto application by fixing $m = 2$ (Step 7), where the symbol "-" indicates that no change in sorting occurred. Since the number of classes and decision-makers is three in this case, the veto is simply expressed by the following two conditions: i) if an alternative is globally sorted as C_1 (C_3), but at least one DM classifies it as C_3 (C_1), then it is reclassified into C_2; and ii) if an alternative is globally sorted as C_2, but two DMs classify it as C_3 (C_1), then it is reclassified into C_3 (C_1). In our case, only alternative 17 was reclassified into C_2 for condition i).

Alternatives	DM1		DM2		DM3		*Global Ranking*	*Global sorting*	*Veto*
	Ranking	*Sorting*	*Ranking*	*Sorting*	*Ranking*	*Sorting*			
1	33 (5.81)	C3	15 (11.07)	C3	32 (6.54)	C3	30 (7.73)	C3	-
2	34 (4.91)	C3	14 (11.20)	C3	33 (5.41)	C3	34 (7.10)	C3	-
3	32 (7.53)	C3	10 (11.89)	C3	10 (6.95)	C3	18 (8.71)	C3	-
4	17 (9.75)	C2	25 (8.88)	C3	13 (6.03)	C3	24 (8.14)	C3	-
5	30 (8.28)	C2	32 (8.80)	C3	9 (5.61)	C3	33 (7.48)	C3	-
6	21 (9.57)	C2	26 (8.87)	C3	7 (5.91)	C3	25 (8.03)	C3	-
7	29 (8.82)	C2	30 (8.81)	C3	8 (5.69)	C3	31 (7.70)	C3	-
8	24 (9.27)	C2	28 (8.84)	C3	12 (5.79)	C3	26 (7.88)	C3	-
9	27 (9.02)	C2	29 (8.82)	C3	30 (5.73)	C3	27 (7.78)	C3	-
10	20 (9.59)	C2	23 (9.76)	C3	15 (6.41)	C3	21 (8.50)	C3	-
11	18 (9.65)	C2	21 (9.79)	C3	11 (6.44)	C3	19 (8.54)	C3	-
12	19 (9.63)	C2	22 (9.78)	C3	1 (6.43)	C3	20 (8.53)	C3	-
13	22 (9.57)	C2	24 (9.75)	C3	3 (6.41)	C3	22 (8.49)	C3	-
14	10 (10.56)	C2	7 (12.90)	C2	34 (7.58)	C3	9 (10.24)	C2	-
15	4 (12.48)	C1	13 (11.64)	C3	26 (7.12)	C3	7 (10.31)	C2	-
16	9 (11.20)	C2	11 (11.83)	C3	22 (7.17)	C3	11 (9.96)	C3	-
17	1 (13.91)	C1	33 (7.37)	C3	2 (16.07)	C1	1 (12.32)	C1	C2
18	12 (10.27)	C2	18 (10.11)	C3	18 (6.72)	C3	16 (8.94)	C3	-
19	23 (9.57)	C2	34 (6.61)	C3	17 (7.28)	C3	29 (7.74)	C3	-
20	28 (8.95)	C2	17 (10.59)	C3	20 (7.46)	C3	17 (8.91)	C3	-
21	7 (11.26)	C2	16 (10.94)	C3	19 (7.81)	C3	12 (9.90)	C3	-
22	15 (9.94)	C2	20 (9.82)	C3	14 (8.87)	C2	14 (9.45)	C3	-
23	14 (9.98)	C2	12 (11.78)	C3	16 (8.59)	C2	10 (10.01)	C3	-
24	6 (11.51)	C1	9 (11.92)	C3	25 (10.99)	C2	5 (11.36)	C2	-
25	25 (9.26)	C2	27 (8.85)	C3	23 (5.39)	C3	28 (7.75)	C3	-
26	26 (9.13)	C2	31 (8.81)	C3	29 (5.36)	C3	32 (7.69)	C3	-
27	2 (12.54)	C1	3 (14.83)	C2	21 (8.05)	C3	3 (11.69)	C2	-
28	3 (12.53)	C1	2 (15.60)	C1	31 (8.54)	C2	2 (12.10)	C1	-
29	5 (11.70)	C1	1 (15.62)	C1	27 (8.06)	C3	4 (11.68)	C2	-
30	16 (9.89)	C2	19 (10.05)	C3	28 (5.54)	C3	23 (8.41)	C3	-
31	11 (10.33)	C2	6 (13.00)	C2	5 (7.78)	C3	8 (10.27)	C2	-
32	8 (11.25)	C2	4 (14.05)	C2	24 (6.81)	C3	6 (10.60)	C2	-
33	13 (10.08)	C2	8 (12.80)	C3	4 (6.16)	C3	13 (9.58)	C3	-
34	31 (8.21)	C2	5 (13.04)	C2	6 (7.08)	C3	15 (9.35)	C3	-

Table 3. Individual and global rankings and sorting.

After the final sorting has been achieved, the subsequent stage (Section 4.2) aims at selecting the alternatives in observance of the budget constraints. Since the sum of the investments required for the alternatives belonging to C_1 and C_2 is lower than the budget, i.e. $409{,}274 < 430{,}000$, all these interventions are selected. However, the remaining budget of €$20{,}726$ is not sufficient to cover all interventions of C_3. The 0-1 knapsack model was therefore launched. Table 4 shows the selected alternatives from C_3 with their priorities P_j and costs κ_{a_j}, and the total budget allocated. Overall, 23 alternatives were selected, i.e. the nine alternatives from C_1 and C_2 and 14 from C_3.

Selected alternatives	$\boldsymbol{P_j}$	$\boldsymbol{\kappa_{a_j}}$
7	7.695284	1405.9
8	7.884687	2154
9	7.780169	6932.2
10	8.503183	427
11	8.537888	300
12	8.526525	300
13	8.49009	300
19	7.739919	20
20	8.911368	2500
22	9.447074	3000
25	7.75285	319.2
26	7.687882	200
30	8.410329	1500
33	9.583048	1000
Total budget allocated = $20{,}358.3$		

Table 4. The selected alternatives.

6. Conclusions

The selection of the interventions for energy requalification in public buildings and utilities represents a crucial problem for municipalities, whose main criticality is the need for a transparent group procedure of selection in compliance with a budget constraint. In particular, a group multi-criteria selection problem arises with regard to aggregating qualitative and quantitative criteria into a global score. It could be observed that a simple ranking approach does not solve this problem directly, as it would lead to selecting the alternatives from the most to the least preferred without maximising the total priority of the selected alternatives. Conversely, a sorting approach combined with a knapsack model allows for the selection of alternatives from the most to the least preferred classes. The group sorting is a well-established problem in the multi-criteria decision-making field. In this paper, an automatic procedure based on the K-means algorithm is adopted for solving the group sorting problem without requiring any intervention from the decision-makers either for eliciting the limiting/central profiles of the classes or for providing an assignment example, which is the main novelty of our proposal. Moreover, the K-means algorithm applied also to the individual rankings allows for the introduction of a veto system. In particular, the individual scores assigned to the alternatives on the quantitative criteria are obtained by means of a Bezier curve-fitting procedure aimed at searching for the best individual preference functions, enriching the traditional multi-attribute utility approaches with promising results. Finally, a 0-1 knapsack model aims at selecting the interventions by maximising the total priority within a class of importance under the budget constraints.

The application of this group decision support system to a real case study has validated the approach and confirmed its usefulness in real settings, where transparent procedures are required in decision-making groups.

As regards limitations, it must be underlined that the qualitative scores were provided by the decision-makers in a crisp form that does not represent the uncertainty of human judgements. In order to overcome this limitation, fuzzy logic and the Dempster-Shafer theory of evidence could enrich our proposal by dealing with the vagueness and incompleteness of human judgements more robustly.

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APPENDIX: Scores assigned by DMs to alternatives.

Alternative	Description	c_1			c_2			c_3			c_4			c_5			c_6			c_7			c_8		
		DM_1	DM_2	DM_3	DM_1	DM_2	DM_3	DM_1	DM_2	DM_3	DM_1	DM_2	DM_3	DM_1	DM_2	DM_3	DM_1	DM_2	DM_3	DM_1	DM_2	DM_3	DM_1	DM_2	DM_3
1	Wrap insulation	0.38	0.32	0.21	0.26	2.95	2.23	3.00	5.00	10.00	10	18	10	16	8	10	15	13	17	10	17	15	17	10	18
2	Heating syst. replacement	0.16	0.14	0.09	0.11	2.97	2.10	3.39	6.03	10.50	15	17	14	8	8	10	16	14	8	8	16	18	10	14	12
3	Photovoltaics	0.16	0.14	0.09	0.38	2.94	2.33	3.94	6.83	11.09	18	16	16	20	10	14	19	13	17	13	18	19	20	16	17
4	Flux regulator	0.51	0.44	0.29	1.47	3.11	3.25	19.53	14.22	19.63	10	13	12	10	10	7	9	6	11	8	10	16	6	8	11
5	Flux regulator	0.08	0.07	0.04	0.21	2.95	2.19	16.05	10.77	17.41	10	13	12	10	10	7	9	6	11	8	10	16	6	8	11
6	Flux regulator	0.30	0.26	0.17	0.82	2.95	2.71	19.49	14.09	19.60	10	13	12	10	10	7	9	6	11	8	10	16	6	8	11
7	Flux regulator	0.11	0.09	0.06	0.27	2.95	2.24	17.63	11.53	18.31	10	13	12	10	10	7	9	6	11	8	10	16	6	8	11
8	Flux regulator	0.16	0.14	0.09	0.42	2.93	2.37	18.88	12.74	19.15	10	13	12	10	10	7	9	6	11	8	10	16	6	8	11
9	Flux regulator	0.13	0.11	0.07	0.32	2.94	2.28	18.22	11.98	18.69	10	13	12	10	10	7	9	6	11	8	10	16	6	8	11
10	Astronomical time switch	0.03	0.03	0.02	0.08	2.98	2.07	19.84	16.01	19.88	18	10	11	13	11	10	14	9	15	6	13	14	4	7	14
11	Astronomical time switch	0.07	0.06	0.04	0.18	2.96	2.16	19.94	17.27	19.95	18	10	11	13	11	10	14	9	15	6	13	14	4	7	14
12	Astronomical time switch	0.05	0.05	0.03	0.14	2.97	2.12	19.91	16.89	19.93	18	10	11	13	11	10	14	9	15	6	13	14	4	7	14
13	Astronomical time switch	0.03	0.02	0.01	0.06	2.98	2.06	19.79	15.55	19.83	18	10	11	13	11	10	14	9	15	6	13	14	4	7	14
14	30 kW photovoltaics	0.21	0.18	0.11	2.24	3.43	3.87	10.83	9.48	15.13	18	14	15	19	12	15	18	15	16	16	18	18	19	17	18
15	5 kW Aeolian	0.27	0.24	0.15	2.70	3.69	4.24	18.30	12.05	18.74	14	12	7	20	9	15	19	15	17	14	19	18	17	18	16
16	170m2 fixtures	0.55	0.47	0.31	3.16	3.97	4.61	18.30	12.05	18.74	12	17	7	11	12	15	10	8	9	15	19	17	8	13	17
17	25 m pool covering	20.00	20.00	18.00	18.00	20.00	18.00	19.76	15.35	19.81	8	10	5	6	4	8	8	6	10	5	8	4	2	8	16
18	36 kWe CHP	0.21	0.18	0.11	4.83	5.21	5.90	18.30	12.05	18.74	10	12	9	10	10	8	11	10	14	8	14	16	7	10	14
19	1°C water heat reduction	5.97	4.75	4.53	5.75	6.01	6.62	20.00	20.00	20.00	6	2	3	10	12	8	9	3	10	1	1	5	2	4	10
20	25 m pool-water heat recovery	3.55	2.89	2.42	0.74	2.94	2.64	19.76	15.35	19.81	10	9	11	5	10	13	10	14	13	2	13	16	7	10	13
21	60 kWe CHP	1.94	1.62	1.21	8.87	9.08	9.04	18.47	12.23	18.86	11	8	10	9	13	14	13	10	8	8	14	16	5	13	15
22	50 m pool heat recovery	7.11	5.63	5.61	0.93	2.97	2.80	19.76	15.35	19.81	10	7	9	12	8	13	10	14	13	4	13	17	9	14	13
23	Air circuit insulation	5.09	4.07	3.73	4.76	5.15	5.85	19.60	14.50	19.69	14	9	11	9	12	15	9	16	10	2	11	16	5	12	14

24	Water circuit insulation	9.41	7.50	7.93	9.87	10.18	9.83	19.60	14.50	19.69	15	8	11	9	12	15	9	16	10	2	11	16	5	12	14
25	Low flow dispenser in 38 showers	0.00	0.00	0.00	0.13	2.97	2.12	19.89	16.56	19.92	8	12	13	15	8	12	16	10	15	4	8	9	7	13	9
26	Low-flow dispenser in 21 washbasin	0.00	0.00	0.00	0.00	3.00	2.00	19.60	14.50	19.69	8	12	13	15	8	12	16	10	15	4	8	9	7	13	9
27	Efficient lighting technologies	2.56	2.11	1.65	8.73	8.94	8.93	18.93	12.82	19.19	16	14	9	17	16	10	13	17	18	9	15	18	10	16	14
28	Efficient lighting technologies	2.56	2.11	1.65	6.89	7.07	7.50	19.19	13.29	19.38	18	13	9	16	17	10	16	18	14	10	18	17	10	17	18
29	Luminous flux regulators and advanced control systems	2.93	2.40	1.93	0.58	2.93	2.50	19.60	14.50	19.69	15	15	10	19	18	12	18	16	14	13	16	19	10	18	17
30	AHUs	0.31	0.27	0.17	1.17	3.02	3.00	19.19	13.29	19.38	14	15	8	6	10	8	10	10	13	8	10	15	8	8	10
31	40 kW photovoltaics	0.33	0.28	0.18	3.00	3.87	4.48	10.83	9.48	15.13	17	16	16	19	12	16	18	14	17	15	18	18	17	17	18
32	48 replacements of 1,000 W lightspots with LEDs	0.24	0.21	0.13	8.13	8.31	8.46	19.49	14.09	19.60	18	10	17	16	18	13	7	13	16	5	18	8	6	9	11
33	8 replacements of 2,000 W headlights and 12 420 W headlights with LEDs	0.10	0.09	0.05	2.86	3.78	4.36	19.41	13.84	19.54	17	12	15	7	16	14	8	10	15	5	18	8	9	9	11
34	15 kW photovoltaics	0.07	0.06	0.04	1.02	2.99	2.88	5.68	8.06	12.54	16	12	9	18	13	16	19	15	17	15	19	17	18	17	19