Introduction to the papers of TWG04 : Geometrical thinking

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INTRODUCTION

TWG04 on geometrical thinking had more than 22 participants from 10 countries. During the sessions, the participants discussed thirteen papers and three posters (one paper and two posters were submitted but not discussed in the working group).

TWG04 about geometrical thinking has been working on this topic for many CERME sessions, and mostly studied what geometrical thinking would mean. It is concerned with research and development of geometrical thinking from pre-school up to University teaching and learning, including any type of geometry. In line with the previous sessions, CERME8 tried to identify four competencies that helped describe geometrical thinking: reasoning, figural, operational and visual (Maschietto et al., 2013). The discussion paper focused on educational aspects related to the development of these geometrical competencies for students, the difficulties of learning and the design of curricula and their implementation. These four poles were very helpful to use a common language and shared reference concepts. They also allowed to show how the many faces of geometrical activity are intertwined, which made necessary a clarification of what was meant by these poles.

In this working group at CERME 9, there were more specific contributions about the way geometry is, or should be, taught: for instance, in-class observations, pre-service teachers education, going from everyday concepts to geometrical knowledge, specific issues in geometry. As a consequence, the four competencies were used as a general way of describing the geometrical activity and for creating links between different points of view. For instance, line symmetry was studied either in a didactical engineering perspective or to characterize ordinary classrooms practices. Then, the papers and posters contributed to study what is at stake in the teaching and learning processes.

During the discussions, the work was structured by four specific themes, specifically studied in the contributions:

- Initial geometrical knowledge
- Visualization
- Transformations and 3D
- Actions on objects (material and mental)

We will emphasize here how the papers about these four themes deal with the following questions, and more specifically:

- Who is the research about?
  - Pupils
  - Teachers
  - Teachers educators
- What are the aims of the research?

 - In the description of existing teaching and learning phenomena
 - In the design of new tasks
- How do we study this?
 - Theoretical tools
 - Methodology
- What are the interactions with the other themes? This question also concerned the role of language and social interactions that appeared as a common issue to several papers.

THE ROLE OF INITIAL GEOMETRICAL KNOWLEDGE

We chose to use this notion, instead of pre-concepts, to emphasize a general issue in geometry: going to everyday concepts to geometrical ones does not only concern early ages geometry. The participants showed that "intuitive notions" are very important for the construction of geometrical knowledge. This is true for many topics as rotation (Swoboda), line symmetry (Chesnais & Mathé), polygons (Ulusoy), polyhedra (Mithalal-Le Doze & Papadaki) and at every age from the beginning of the primary school (Rodrigues & Serrazina) to prospective teachers' education (Brunheria & da Ponte, Kuzniak & Nechache).

This idea of initial geometrical knowledge is linked to the idea of "geometrization", seen as a dynamical and continuous process that turns it into an institutionalized geometrical knowledge, with the development of geometrical competencies. This process is a fundamental part of geometrical thinking, like the more scientific knowledge it helps building. We could see that this process involves at the same time pupils, teachers, teacher educators, and researchers.

It is necessary for teachers to be aware of how pupils' personal previous experience may influence their learning geometry, in order to design efficient teaching sequences (Loureiro & Serrazina, Herendiné-Kónya). Therefore, they need theoretical tools, especially pedagogical and didactical knowledge, to adapt the in-class experience to the pupils and the topics.

Many frameworks were evoked - and had been mentioned in the previous CERME works. Some of those frameworks are very specific to geometry, as Van Hiele levels (Papadaki), Geometrical Working Spaces (Kuzniak & Nechache), concept image and concept definition (Rodrigues & Serrazina, Ulusoy), figural concept (Ulusoy), visualization (Papadaki, Mithalal-Le Doze). Other frameworks are more general in mathematics education, as Theory of Didactical Situation (Douaire & Emprin) or from psychologists (mainly Battista or Gagatsis' works exploited by Loureiro and Serrazina). The papers have shown that in this case, it is possible to design tasks involving outdoor activities, real-life experiences, with high didactical potential (Douaire & Emprin).

ACTION ON OBJECTS

The first point concerns the role of action on objects, both mental ones and material ones. A paradoxical situation was raised, as it is at the same time very natural to pupils, and quite difficult for the teachers to develop their geometrical competencies from it and not to use it only with young pupils to increase their engagement.

"Action on objects" is quite confusing, and we decided to reduce its meaning to action performed by hand on material or tangible objects -- which includes the use of instruments (like transparent paper by Chesnais and Mathé, or Uygan et al.) or Dynamic Geometry Software (Mithalal-Le Doze). We studied the many functions of action (Pytlak): it helps the pupils to develop intuition, concept image and definition, geometrical imagination, and at the same time it makes children's knowledge more visible to the teachers.

This function strongly depends on specific conditions, and for many of us it was essential that manipulation came first, to create a need for anticipation, validation or control that may justify geometrical knowledge. This articulation is organized by the tasks themselves, the backing of the teacher and the social interactions.

TRANSFORMATION AND 3D GEOMETRY

These two topics were developed in many papers, which gave good examples of how complex the relations between action, visualization and geometrical knowledge, are.

The learning of geometrical transformations (rotation by Swoboda, line symmetry by Chesnais & Mathé, or isometrics by Thaqi et al. and Uygan et al.) depends on linking multiple contexts and representations. It also requires articulating a global and a punctual point of view and going from perception to geometrical properties.

The same questions were discussed for 3D geometry, and we showed that the greatest issues were not only "sight" issues. Indeed, a psychological point of view shows the role of getting better "images" to act on (physically or mentally), but many of the previously mentioned mathematical aspects are part of the visualization process. For instance, Dynamic Geometry Software, and more generally geometrical tools, were seen as ways of making geometrical knowledge useful for a better control of the actions and visualization. This knowledge was at the heart of the visualization process when using only the sense of touch: linking the subparts that were perceived by touch is a mathematical process linked to what Duval calls deconstructions (see Mithalal-Le Doze, Papadaki). At the end, we showed that visualization depends on perceptual, psychological, but also – and this is fundamental – on mathematical aspects.

VISUALIZATION

Eventually, we had to clarify what we called visualization. It was involved in studies of mental abilities (especially for initial geometrical knowledge and action on objects), classification (how do we discriminate the information while seeing), and analysis of drawings in a deductive geometry context (Brunheria & da Ponte, Herendiné-Kónya). We underlined that, in this case, visualizing aims at being able to solve geometry problems, so that it both depends on very particular cognitive processes and mathematical knowledge. Therefore, a great difficulty is the gap between visualization by teachers, based on categories and geometrical properties, and by pupils, often based on prototypes. The main theoretical tools used were Laborde's work about drawings and figures (Mithalal-Le Doze), and Duval's distinction between iconic and non-iconic visualization (see Mithalal-Le Doze).

CONCLUSION

The discussions in TWG04 confronted very different points of view on geometry teaching and learning, with complementary issues (e.g., teaching practices studies vs. task design), very different cultural and educational contexts that change the role of geometry in the curricula and the way it is presented. It also appeared that these contexts had an influence on our researcher positioning, which not only concerns geometry: Does it mean our research practice or the teachers' education practice that most of us share? What can we learn from research on everyday practices? How is it possible to better combine our teacher, teacher educators, and researcher positions?

Eventually, let us mention the new issue of the role of language and social interactions in the teaching and learning geometry processes. This has been little discussed during the previous sessions, but it appeared that this could play a great role in each of the topics mentioned above and that some of the phenomena are very specific to geometry.

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