

# Dynamics and Stability of Carbon Nanotubes

Matteo Strozzi, Marco Barbieri, Antonio Zippo, Francesco Pellicano  
*Department of Engineering "Enzo Ferrari", University of Modena and Reggio Emilia,  
via Pietro Vivarelli 10/1, 41125 Modena, Italy  
E-mail: matteo.strozzi@unimore.it*

*Keywords:* nanotubes, nonlinear dynamics.

The low-frequency oscillations and energy localization of Single-Walled Carbon Nanotubes (SWNTs) are studied in the framework of the Sanders-Koiter shell theory. The circumferential flexure modes (CFMs) are analysed. Simply supported, clamped and free boundary conditions are considered. Two different approaches are proposed, based on numerical and analytical models.

The numerical model [1] uses in the linear analysis a double mixed series expansion for the displacement fields based on Chebyshev polynomials and harmonic functions. The Rayleigh-Ritz method is applied to obtain approximate natural frequencies and mode shapes. In the nonlinear analysis, the three displacement fields are re-expanded by using approximate eigenfunctions. An energy approach based on Lagrange equations is considered in order to obtain a set of nonlinear ordinary differential equations, which is solved by the Runge-Kutta numerical method.

The analytical model [2] considers a reduced version of the Sanders-Koiter shell theory obtained by assuming small circumferential and tangential shear deformations. These two assumptions allow to condense the longitudinal and circumferential displacement fields into the radial one. A nonlinear fourth-order partial differential equation for the radial displacement field is derived, which allows to calculate the natural frequencies and to estimate the nonlinearity effect. An analytical solution of this equation is obtained by the multiple scales method.

The previous models are validated in linear field by means of comparisons with experiments, molecular dynamics simulations and finite element analyses retrieved from the literature [3].

The concept of energy localization in SWNTs is introduced, which is a strongly nonlinear phenomenon. The low-frequency nonlinear oscillations of the SWNTs become localized ones if the intensity of the initial excitation exceeds some threshold which depends on the SWNTs length. This localization results from the resonant interaction of the zone-boundary and nearest nonlinear normal modes leading to the confinement of the vibration energy in one part of the system [4].

The value of the initial excitation corresponding to this energy confinement is referred to as energy localization threshold. The effect of the aspect ratio on the analytical and numerical values of the energy localization threshold is investigated; different boundary conditions are considered.

## *References*

- [1] Strozzi, M., Manevitch, L.I., Pellicano, F., Smirnov, V.V. and Shepelev, D.S., "Low-frequency linear vibrations of single-walled carbon nanotubes: Analytical and numerical models", *Journal of Sound and Vibration*, **333**, 2936-2957 (2014).
- [2] Manevitch, L.I., Smirnov, V.V., "Limiting phase trajectories and origin of energy localization in nonlinear oscillatory chains", *Physical Review E*, **82**, 1-9 (2010).
- [3] Gupta, S.S., Bosco, F.G. and Batra, R.C., "Wall thickness and elastic moduli of single-walled carbon nanotubes from frequencies of axial, torsional and inextensional modes of vibration", *Computational Materials Science*, **47**, 1049-1059 (2010).
- [4] Smirnov, V.V., Shepelev, D.S. and Manevitch, L.I., "Localization of bending vibrations in the single-walled carbon nanotubes", *Nanosystems: Physics, Chemistry, Mathematics*, **2**, 102-106 (2011).