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decomposition analysis**

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Abstract

In this paper we investigate short-run co-movements before and after the Lehman Brothers' collapse among the volatility series of US and a number of European countries. The series under investigation (implied and realized volatility) exhibit long-memory and, in order to avoid misspecification errors related to the parameterization of a long memory multivariate model, we rely on wavelet analysis.

More specifically, we retrieve the time series of wavelet coefficients for each volatility series for high frequency scales, using the Maximal Overlapping Discrete Wavelet transform and we apply Maximum Likelihood for a factor decomposition of the short-run covariance matrix.

The empirical evidence shows an increased interdependence in the post-break period and points at an increasing (decreasing) role of the common shock underlying the dynamics of the implied (realized) volatility series, once we move from the 2-4 days investment time horizon to the 8-16 days. Moreover, there is evidence of contagion from the US to Europe immediately after the Lehman Brothers' collapse, only for realized volatilities over an investment time horizon between 8 and 16 days.

JEL: C32, C38, C58, G13

Keywords: Implied volatility, Realized Volatility, Co-movements, Long Memory, Wavelets.

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1. Introduction

In this paper we study implied and realized volatility co-movements, taking into account the long-memory properties of the series. Implied volatility embed the investor's perception on future uncertainty, whereas realized volatility measures the actual volatility experienced in the market. Evidence of long memory in volatility measures is well documented. The studies of Baillie et al. (1996), Andersen and Bollerslev (1997), Comte and Renault (1998) give evidence of long-run dependencies, described by a fractionally integrated process, in GARCH, realized volatilities, and stochastic volatility models, respectively. More recently, empirical studies show that the volatility implied from option prices exhibits properties well described by a fractionally integrated process. The long run relationship between implied and realized volatilities is analyzed through fractional cointegration by Bandi and Perron (2006) and Christensen and Nielsen (2006), focusing on the stock market; by Kellard et al. (2010), focusing on the currency market. Moreover, Bollerslev et al. (2013) employ a co-fractional VAR to model long run and short run dynamics of realized variance, implied variance and stock return in the US market. All the aforementioned studies focus on the US stock market. The only study analyzing spillovers effects across different volatility indices for the US and Europe (with emphasis on the role played by news) is the one by Jiang et al. (2012). However, the authors' focus is on first differences of implied volatilities.

Given that the implied volatility indexes represent a measure of market expectations of near-term volatility of the underlying stock index, conveyed by option prices, they are deemed by market participants to capture the so-called "market fear": high index values are associated with high uncertainty in the underlying market, low index values with stable conditions (Muzzioli, 2013). Therefore, our first contribution to Jiang et al. (2012) is to focus on the levels of such "market fear" indices and not on their first order differences. Second, we extend the analysis to the actual ex-post realized volatilities. Third, to our knowledge, we are the first to explore co-movements in implied and realized volatility in terms of interdependence and contagion. According to Forbes and Rigobon

(2002), interdependence is the co-movement driven by common shock; (shift) contagion is defined as a significant and temporary increase in cross-market linkages (beyond the one driven by common shocks). We are particularly interested in analyzing whether contagion from the volatility of the US stock market to the one of European countries (UK, Germany, France, Netherlands and Switzerland), occurred during the period immediately after the Lehman Brothers' collapse (between mid-September 2008 and the end of year 2008).

In the first stage of the analysis we confirm (using daily data) the findings of Bandi and Perron (2006) regarding the existence of long memory in the level of implied and realized volatilities not only for the US, but also for the European stock markets. In a second stage, we provide a methodological contribution to modelling the dynamics of long-memory time series in a multivariate setting. More specifically, we prefer not to rely on a Fractionally Integrated Vector Autoregressive Model (e.g. Bollerslev et al. (2013)) or a Vector Autoregressive Model with a common factor following an ARFIMA process (see Cassola and Morana, 2008) to explore co-movements, since we want to avoid model misspecification errors related to the lag order of the VAR model or to the use of a biased estimate of the fractional integration parameters. Therefore, we use wavelet analysis to explore short run (i.e. the ones associated to high frequencies) volatility co-movements both in terms of contagion and interdependence.

In line with Percival and Walden (2000) we use the Maximal Overlapping Discrete Wavelet Transform, MODWT, to estimate the covariance matrix of a pair of fractionally integrated time series at different scales (each associated with a given frequency range). Then, we explore the contribution of common and idiosyncratic shocks to the variability of the level of each volatility index at different scales through a factor decomposition of a given scale covariance matrix. The factor decomposition is obtained by Maximum Likelihood and inference is carried out via bootstrap. Moreover, since the highest frequency range considered by wavelet decomposition is

between two and four days, we are also able to overcome the problem of asynchronous data, without losing any observation¹.

The structure of the paper is as follows. Section 2 describes the empirical methodology; Section 3 provides the empirical evidence and Section 4 concludes.

2 Univariate and multivariate analysis of long memory series

2.1 Long memory definition and univariate analysis

Let the implied volatility series, imp_t , be described by an $ARFIMA(p,d,q)$ process:

$$\Phi(L)(1-L)^d imp_t = \Theta(L)\varepsilon_t \quad (1)$$

where ε_t is an *iid* Gaussian process with variance σ_ε^2 . The AR component is given by a polynomial of degree p (with roots outside the unit circle):

$$\Phi(L) = 1 + \varphi_1 L + \varphi_2 L^2 + \dots + \varphi_p L^p \quad (2)$$

and the MA component is described by a polynomial of degree q (with roots outside the unit circle):

$$\Theta(L) = 1 + \theta_1 L + \theta_2 L^2 + \dots + \theta_q L^q \quad (3)$$

The fractional differencing operator $(1-L)^d$ can be derived from a power series expansion as follows:

$$(1-L)^d = 1 + \sum_{z=0}^{\infty} \frac{\Gamma(z-d)}{\Gamma(-d)\Gamma(z+1)} L^z \quad (4)$$

It turns out that, for $0 < d < 0.5$, the process imp_t is stationary and invertible. For such processes, the effect of a shock ε at time t on imp at time $t+h$ decays as h increases, but the rate of decay is much lower than for a process integrated of order zero, hence the autocorrelation function for a fractionally integrated process decays hyperbolically. If $0.5 < d < 1$, then the process is long-memory non-stationary and it is characterized by an infinite variance.

¹ Forbes and Rigobon (2002) use a moving average across two consecutive days to circumvent the problem of asynchronicity, halving the number of available observations.

The most prevalent method for estimating the fractional differencing parameter is the method proposed by Geweke and Porter-Hudak (1983, hereafter GPH) which is based on the low frequency spectral behavior of the time series, exploiting the property that the spectral density of a long memory processes is infinite at frequency zero. In practice, the GPH estimator is simply the slope of the sample log periodogram:

$$\ln I_{x,T}(\omega_j) = c - 2d \ln(2 \sin(\omega_j / 2)) + \varepsilon_j \quad (5)$$

where $I_{x,T}(\omega_s)$ is the sample periodogram at the j^{th} Fourier frequency $\omega_j = \frac{2\pi j}{T}$, with $j=1, \dots, T/2$, and T is the sample size. The log-periodogram regression uses observations pertaining to frequencies ranging from j equal to 1 up to m . In line with the study of Bandi and Perron (2006), the maximum number of frequencies m involved in log periodogram regression is set either to

$$T^{0.5}, \text{ or to } T^{0.6}, \text{ or to } T^{0.7}. \text{ The asymptotic standard error of the parameter } d, \text{ equal to } \sqrt{\left(\frac{\pi^2}{24}\right) \frac{1}{\sqrt{m}}},$$

was obtained by Robinson (1995a) in the presence of stationary data and by Velasco (1999) in the presence of non-stationary data with $\frac{1}{2} \leq d < \frac{3}{4}$.

The local Whittle estimator developed by Kunsch (1987) and by Robinson (1995b) maximizes a frequency-domain Gaussian likelihood for frequencies in the neighborhood of zero, i.e.:

$$\log \left[\frac{1}{m} \sum_{s=1}^m \omega_s^{2d} I_{x,T}(\omega_j) \right] - \frac{2d}{m} \sum_{s=1}^m \omega_s \quad (6)$$

The asymptotic standard error of the parameter d has been derived by Robinson (1995b) and it is equal to $\sqrt{\frac{1}{4} \frac{1}{\sqrt{m}}}$.

2.2 Multivariate analysis: an introduction to multi-resolution analysis

Once we have investigated the long memory properties of the implied volatility series, we turn our focus on multivariate analysis. A Fractionally Integrated Vector Autoregressive Model (e.g. Bollerslev et al. (2013)) or a Vector Autoregressive Model with a common factor following an ARFIMA process (see Cassola and Morana, 2008) could then be employed to explore co-movement and causal linkages between long memory time series. The choice of the lag length and the estimation of the fractional integration parameters could be a potential source of model misspecification. We circumvent the problem using wavelet analysis.

Since we need to retrieve information both from time and frequency domain, we suggest the use of a multi-scale decomposition of each volatility time series through wavelet analysis. Frequency domain approaches provide an insightful representation of econometric data by decomposing it into sinusoidal components at various frequencies, which have intensities that vary across the frequency spectrum. The shortcoming of Fourier analysis is related to the assumption of intensities constant through time. This feature makes Fourier methods ineffective in analyzing signals containing local irregularities, such as spikes or discontinuities, which, we argue, are a feature of financial time series. Wavelets can be a particular useful tool when the signal is localized in time as well as frequency. Consequently, wavelet transforms can localize a process in time and scale, revealing long-run, or high-scale, features of the process in a more flexible manner than Fourier analysis.

The wavelet transform (see Appendix for more details) decomposes a time series into time scale components, each reproducing the evolution over time of the original series for a particular level j of decomposition, associated to a given frequency range. In particular, at level j and scale $\lambda_j = 2^{j-1}$, the time series of wavelet coefficients are able to capture frequencies spanning cycles with periodicity between 2^j and 2^{j+1} . The lower scales are associated to the highest frequency range and the highest scales (up to maximum level of decomposition J) correspond to the lowest frequency range. Given the definition of financial contagion as a temporary, hence short-term, phenomenon

occurring during a period of financial turmoil, we investigate co-movements for investment time horizon between two and four days, between four and eight days and between eight and sixteen days (hence we do not go beyond level 3 of decomposition). Since the highest frequency range considered is between two and four days, then we are able to bypass the issue of asynchronous data without losing any observation.

2.3 Factor model description and estimation

In order to examine co-movements, in terms of interdependence and contagion, among the volatilities of the US market and of the European markets, before and after the Lehman Brothers' collapse, we suggest to use two different factor models. For the pre-crisis period, we model the volatility (either implied or realized) for the i^{th} country stock market at time t and for scale j , using the following factor model specification:

$$vol_{it}^j = \gamma^j u_t + \sigma_i^j \eta_{it} \quad (8)$$

where u is the common shock with the associated factor loading γ^j , and η_{it} is the idiosyncratic shock for market i whose size (proxied by the corresponding standard deviation) is given by σ_i^j . The unobservable common shock u is meant to proxy macroeconomic news (e.g. shocks to fundamentals) driving market interdependencies (see for instance Jiang et al., 2012).

For the post-crisis period, co-movements are analyzed through the following factor model for all countries (except the US):

$$vol_{it}^j = \gamma^j u_t + \sigma_i^j \eta_{it} + \delta_i^j \sigma_{us}^j \eta_{us,t} \quad (9)$$

where δ_i^j measures the contagion from the US to European country i , for an investment time horizon associated with the j th level of decomposition. In this setting, in case of contagion from the US market to a European market, we expect a positive δ . Given the short time period considered

for the crisis regime, we treat US as the only potential originator of contagion, implying a zero coefficient δ when equation (9) applies to the dynamics of the US volatility index.

Using matrix notation, if we define \bar{vol}_t^j the 6×1 vector of volatilities (for the US, UK, Germany, France, Netherlands and Switzerland stock markets) at time t and for scale j , the following factor model specification applies to the non-crisis ($k=nc$) period:

$$\bar{vol}_t^j = \Gamma_k^j u_t + \Omega_k^j \bar{\eta}_t$$

and the following factor model specification is defined for the crisis ($k=c$) period:

$$\bar{vol}_t^j = \Gamma_k^j u_t + \Delta^j \Omega_k^j \bar{\eta}_t$$

where $k=c, nc$; u is the common shock with the associated factor loadings:

$$\Gamma_k = \begin{bmatrix} \gamma_k \\ \gamma_k \\ \gamma_k \\ \gamma_k \\ \gamma_k \\ \gamma_k \end{bmatrix}$$

$\bar{\eta}_t$ is the 6×1 vector of idiosyncratic shocks with covariance matrix Ω_k^{2j} , where Ω_k^j is defined as:

$$\Omega_k^j = \begin{bmatrix} \sigma_{us,k}^j & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{uk,k}^j & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{ger,k}^j & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{fra,k}^j & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{ned,k}^j & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{swi,k}^j \end{bmatrix}$$

and Δ^j is the spillover effects matrix given by:

$$\Delta^j = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \delta_{uk} & 1 & 0 & 0 & 0 & 0 \\ \delta_{ger} & 0 & 1 & 0 & 0 & 0 \\ \delta_{fra} & 0 & 0 & 1 & 0 & 0 \\ \delta_{ned} & 0 & 0 & 0 & 1 & 0 \\ \delta_{swi} & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Therefore, before 15 September 2008 volatility co-movements are shaped only by a common shock; after the Lehman Brothers' collapse, both a common shock and a shock to the US stock market, drive co-movements.

The estimation is carried through the following steps. In the first stage, we apply the Maximal Overlapping Discrete Wavelet Transform, MODWT (see Percival and Walden, 2000; Whichter et al., 2000) to obtain a decomposition of each time series into different scales (each associated to a given frequency range) localized in time (see Appendix for more details). Unlike the Discrete Wavelet Transform (DWT), the MODWT, by producing a decomposition of a given time series into components having the same size as the original time series, is better suited than DWT to locate potential structural breaks.

In the second stage of the analysis, we employ a factor decomposition of the covariance matrix for the i th volatility series across different scales. As shown by Percival and Walden (2000) (see also Whichter et al, 2000), the wavelet covariance between two fractionally integrated time series X and Y (with the orders of integration d_1 and d_2 , respectively) for scale $\lambda_j = 2^{j-1}$ is defined as $\gamma(\lambda_j)$ and it is given by:

$$\frac{1}{N_j} \sum_{l=L_j-1}^{N-1} w_{j,t}^X w_{j,t}^Y \quad (10)$$

where $w_{j,t}$ are the non-boundary (stationary) wavelet coefficients for scale λ_j , obtained from a wavelet transform using a filter of length L ; $N_j = N - L_j + 1$, $L_j = (2^j - 1)(L - 1) + 1$ is the filter length at level j .²

The existence of a finite variance-covariance equation as given by (10) between two fractionally integrated time series relies on the stationary property of the wavelet coefficients. The choice of filter length to achieve stationarity in the wavelet coefficients depends on the trade-off between leakage and boundary affected coefficients: the longer the filter, the closer to an ideal high pass filter, but also the higher the number of boundary coefficients. For that reason, in presence of time series exhibiting an high degree of persistence, the condition suggested by Percival and Walden (2000), $L > 2d$ (where d is the fractional integration parameter), ensuring stationary wavelet coefficients would suggest the use of a filter with length L , at least equal to two. However, the longer is the filter, the higher is the number of boundary affected coefficients. In this study we use both the Haar filter (i.e. L equal to 2) and the Least Asymmetric filter of length L equal to 4 (e.g. the LA4 filter).

In the final stage of the analysis we apply a factor decomposition of level j covariance matrix through Maximum Likelihood estimation. More specifically, we maximize the following Gaussian log-likelihood function fitted to the 6×1 vector of wavelet coefficients \bar{W}_{jt} :

$$\sum_{t=1}^{T_1} L(\bar{W}_{jt}; \Omega^j + \Gamma^j \Gamma^{j'}) + \sum_{t=T_1+1}^T L(\bar{W}_{jt}; \Delta^j \Omega^j \Delta^{j'} + \Gamma^j \Gamma^{j'}) \quad (11)$$

where $L(\cdot)$ is the multivariate Gaussian log-density at time t and for scale j . The first addend of $L(\cdot)$ involves observations for the tranquil period (starting from 2/1/2002 to 14/9/2008) and the second

² Non boundary coefficients are those not influenced by the end-effect problem. The wavelet transform, and its variants, such as the MODWT, makes use of circular filtering. The series under investigation, x , is treated as if it is a portion of a periodic sequence with period N . In other words, the transform considers x_{N-1}, x_{N-2}, \dots as useful surrogates for the unobserved x_1, x_2, \dots . A problem with the periodic extension can occur when there is a large discontinuity between the end of one replication of the sample and the beginning of the next. In such cases the coefficients produced by the transform result remarkably high and the reconstructed details are affected.

addend involves observations for the crisis period (from the 15th of September 2008 to the 31st of December 2008). Given that the focus is on the short-run horizon, we focus on the ML estimation for the first three scales (that is, for a level of decomposition j equal to 1, 2 and 3). The total number of coefficients is 19, whereas we have a total of 42 moment conditions (21 per regime given six endogenous variables), giving 23 over-identifying restriction.

Inference on the coefficients and a likelihood ratio test for the over-identifying restriction are carried through a block bootstrap, which is obtained by re-sampling (with replacement) the wavelet coefficients associated to each regime (either tranquil or crisis period). The re-sampling is obtained from random draws from a uniform distribution and it is repeated 250 times. At each replication, the structural form parameters in eq. (10) and the maximized log-likelihood function are estimated by ML. Then, the p-value for the test of the null of a zero coefficient against the alternative of a positive one is obtained by counting the number of replications for which the ML estimate exceed zero and dividing by the total number of replications. The p-value for the likelihood ratio test for the over-identifying restrictions is obtained by counting the number of replications for which the likelihood ratio statistics is below the one associated to the point estimates³.

3 Data and empirical evidence

The volatility series for US, UK, Germany, France, Netherlands and Switzerland are observed at daily frequency from 2/1/2002 till 31/12/2008. The annualized implied volatilities indices (in percentage values) are the risk neutral expectations for near term volatility (e.g. for the next 30 days) and they are available from DASTREAM. The annualized realized volatility (in percentage values) series have been obtained from the daily realized variances in (hundreds) available from the OXMAN Realized library⁴.

³ The log-likelihood for the reduced form model is obtained by using the sample covariance matrix of the wavelet coefficients (for a given scale) in the log-likelihood function given by eq. (10).

⁴ We use the daily realized variance obtained from 5minute squared returns using 1-minute subsamples. The dataset is available from the website <http://realized.oxford-man.ox.ac.uk/>

Descriptive statistics for pre and post break (that is over the 2/1/2002-14/9/2008 sub-sample and over the 15/9/2008-31/12/2008 sub-sample, respectively) are reported in Table 1. We can observe that the US implied volatility index is the one experiencing the largest increase (equal to 195%), in its mean value after the Lehman Brothers' collapse. In particular, there is a rise from a pre break mean value of 18.541 to a post break mean value of 54.718, which is the largest among the countries under investigation. Similarly the largest increase in the standard deviation of the implied volatility index, equal to 88%, is for the US, rising from a pre-break value of 6.751 to a post break value of 12.702. These findings are confirmed by the realized volatilities series: there is a rise from a pre break mean value of 12.460 to a post break mean value of 48.889, and a rise from a pre break standard deviation value of 6.510 to a post break standard deviation value of 20.773, which is the largest among the countries under investigation. The lowest increment in implied and realized volatility from pre to post break is attained for Germany. For all the countries implied volatility is fairly higher than realized volatility both in the pre and in the post break period, showing the existence of a negative variance risk premium (i.e. investors are willing to pay a high fixed rate in order to be hedged against peaks of realized volatility, which usually are associated with bad market conditions). If implied volatility is perceived as the investors' sentiment on the future value of realized volatility, by looking at co-movements in implied or realized volatility, we are able to see if they concern more the investors' perceptions on volatility (implied volatility) or the actual volatility experienced in the market (realized volatility).

The empirical evidence given in Tables 2 and 3, where we report the GPH and Local Whittle estimates of the long memory parameter using the full sample, suggests the existence of non-stationary long memory for both implied and realized volatilities series across all markets. In particular, the realized volatilities series exhibit a smaller degree of persistence when compared to the implied volatilities series (as in Bandi and Perron, 2006). Although results in Tables 2 and 3 point at non stationary long memory, we can observe a variety of point estimates for the fractional integration parameters of each series. Therefore, we prefer to bypass the use of a parametric long

memory multivariate model relying on the fractional difference operator d estimated in the first stage of the analysis.

We now turn our focus on the multivariate analysis based on the time scale decomposition via wavelets. From Tables 4-7 we can notice that the bootstrapped p-values for the likelihood ratio test suggest that the over-identifying restriction are not rejected. As for the implied volatilities indices, we can observe from Table 4 and 5 that there is a pronounced increase in the influence of the common shock once we move from lower to higher scales, and this is especially true once the focus shifts from tranquil to crisis regime. In particular, from Table 4, the empirical evidence based on the Haar filter shows that, as for the tranquil period, the common factor loading increases from 0.521 to 0.917 once the focus shifts from an investment time horizon between two and four days (e.g. scale 1), to investment time horizon between eight and sixteen days (e.g. scale 3). These findings are even more pronounced when we concentrate on the crisis period: the common factor loading increases from 2.209 to 3.826 once the focus shifts from an investment time horizon between two and four days to investment time horizon between eight and sixteen days. The use of the LA4 filter (see Table 5) confirms the empirical findings obtained from the use of the Haar filter (although the point estimates of the common factor loading are lower than the ones associated to the shorter filter).

For a given regime (either tranquil or crisis period), the role of the idiosyncratic shocks to the implied volatility of US decreases once we move from the first to the third scale. In particular, immediately after the Lehman Brothers' collapse, the size of the US shock (proxied by the standard deviation) decreases from a value of 2.068 to 1.438 once the focus shifts from an investment time horizon between two and four days to an investment time horizon between eight and sixteen days (Haar filter). A similar pattern for the crisis period is shared by the idiosyncratic shocks to UK (even if the decrease is non-monotonic with the time scale, displaying a hump shape) Germany and France (this latter country is the one experiencing the largest decrease, from 2.215 to 0.831, in the size of the idiosyncratic shock once the focus shifts from scale 1 to scale 3). Switzerland and

Netherlands are the only countries where the role of idiosyncratic shocks increases when we move towards a longer investment horizon and to the crisis period. The use of the LA4 filter (see Table 5) confirms the empirical findings regarding the size of idiosyncratic shocks obtained from the use of the Haar filter. Finally, there is no evidence of contagion from the US implied volatility to the other European implied volatilities, given that the bootstrapped p-values of the coefficients δ_i^j 's show a very low probability of getting a positive value (indicating contagion).

The empirical findings for realized volatilities are somehow different from those obtained for the implied volatility series. From Tables 6 and 7 we can observe that, for a given regime, the loadings of the idiosyncratic and of the common shock decrease once we move from level 1 to level 3 of the decomposition (the only exception being Germany in the post break period). However, for a given scale, in a way similar to the implied volatility case, there is an increased loading of the idiosyncratic and of the common shock once we move from the pre-break to the post break regime. Finally, there is evidence of contagion from the US to other markets realized volatilities if we refer to level 3 of the decomposition.

4. Conclusions

In this paper we investigate short-run co-movements before and after the Lehman Brothers' collapse among long memory time series: the implied and realized volatility series of US, UK, Germany, France, Netherlands and Switzerland. Our contribution to previous studies of volatility co-movements is in avoiding the potential miss-specification errors (due to the choice of wrong lag order and the use of biased estimate of the fractional integration parameter) which might be associated to the use of a Fractionally Integrated Vector Autoregressive Model (e.g. Bollerslev et al., 2013) or a Vector Autoregressive Model with a common factor following an ARFIMA process (see Cassola and Morana, 2008). For this purpose we employ a two stage analysis. In the first one, we use the Maximal Overlapping Discrete Wavelet transform to obtain the wavelet coefficients for

each volatility series and for high frequency scales. Since, as shown by Percival and Walden (2000), the variance-covariance matrix of each pair of (fractionally integrated) raw data for different scales is obtained from the associated (stationary) wavelet coefficients, we apply, in a second stage, Maximum Likelihood for a factor decomposition of the covariance matrices for the high frequency scales. To our knowledge, we are the first to explore co-movements in implied and realized volatility in terms of interdependence and contagion. Since the highest frequency range considered is between two and four days, we are able to circumvent the issue of asynchronous data, without losing any observation.

The empirical findings show that in the crisis period there is evidence of increased co-movements among European countries and the US both in the investor's perception on future uncertainty and in the actual realized volatility. However, contagion from the US stock market is evident only in the actual realized volatilities for an investment time horizon between 8 and 16 days. Moreover, the higher the frequency, the higher are the co-movements in realized volatility (which can be explained as in Blasco et al., 2012, with herding behavior detected at high frequency). Finally, the higher the frequency, the lower are the co-movements in investor's sentiment on future uncertainty. These empirical findings from multivariate analysis confirm the univariate results (e.g. the long memory parameter for implied volatility is higher than the one for realized volatility). This is what we could expect given that implied volatility is an average consensus on future realized volatility.

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Table 1: Descriptive Statistics for pre and post break volatilities

Implied Volatilities						
	US	UK	GER	FRA	NED	SWI
Mean	18.541 54.718	19.220 49.885	22.228 46.656	22.207 49.850	23.542 55.921	19.214 49.631
Std dev	6.751 12.702	8.355 12.272	9.717 11.722	9.631 12.143	11.245 12.421	8.288 12.897
Min	9.890 30.300	9.099 30.950	10.980 26.060	9.242 26.640	10.121 32.180	9.239 27.894
Max	45.080 80.860	57.137 75.540	58.250 74.000	61.463 78.050	65.669 81.220	53.037 84.896
Realized Volatilities						
	US	UK	GER	FRA	NED	SWI
Mean	12.460 48.889	11.574 35.417	17.213 44.113	15.089 42.904	14.774 42.295	12.848 35.050
Std dev	6.510 20.774	7.173 13.467	11.024 19.362	9.008 17.651	9.485 15.106	7.388 12.745
Min	3.378 17.343	3.294 14.294	3.418 12.707	3.890 16.62	4.316 13.987	4.790 16.050
Max	56.087 146.739	89.284 88.339	94.096 110.789	86.755 108.767	85.373 97.373	65.960 77.142

Footnote: The top number of each entry gives the value of the pre-break (e.g. over the 2/1/2002-14/9/2008 sub-sample) descriptive statistic. The bottom number of each entry gives the value of the post-break (e.g. over the 15/9/2008-31/12/2008 sub-sample) descriptive statistic.

Table 2: GPH estimates of parameter d

Implied Volatilities					
US	UK	GER	FRA	NED	SWI
			$m = T^{0.5}$		
0.844	0,821	0,948	0,840	0,820	0,919
			$m = T^{0.6}$		
0.902	0.765	0.859	0.838	0.865	0.854
			$m = T^{0.7}$		
0.911	0.915	0.889	0.867	0.991	0.975
Realized Volatilities					
US	UK	GER	FRA	NED	SWI
			$m = T^{0.5}$		
0.659	0.6128	0.7257	0.6335	0.6681	0.7482
			$m = T^{0.6}$		
0.6852	0.7106	0.8629	0.7266	0.8290	0.7446
			$m = T^{0.7}$		
0.6424	0.6504	0.6353	0.6531	0.7219	0.6621

Footnote: the maximum number of frequencies ω involved in the estimation of the fractional integration parameter is given by m . The asymptotic standard errors are equal to 0.101, 0.069, 0.048 for $m=T^{0.5}$, $m=T^{0.6}$, and $T^{0.7}$, respectively.

Table 3: Whittle estimates of parameter d

Implied Volatilities					
US	UK	GER	FRA	NED	SWI
$m = T^{0.5}$					
0.937	0.880	0.942	0.846	0.890	0.940
$m = T^{0.6}$					
1.006	0.879	0.929	0.862	0.955	0.947
$m = T^{0.7}$					
0.994	0.953	0.922	0.900	0.998	1.016

Realized Volatilities					
US	UK	GER	FRA	NED	SWI
$m = T^{0.5}$					
0.747	0.693	0.750	0.725	0.697	0.751
$m = T^{0.6}$					
0.752	0.743	0.819	0.747	0.813	0.784
$m = T^{0.7}$					
0.660	0.639	0.625	0.648	0.689	0.666

Footnote: the maximum number of frequencies ω involved in the estimation of the fractional integration parameter is given by m . The asymptotic standard errors are equal to 0.078, 0.054, 0.037 for $m = T^{0.5}$, $m = T^{0.6}$, and $T^{0.7}$, respectively.

Table 4: Factor decomposition of scale covariance matrices for the implied volatilities time series (Haar filter)

Parameter	SCALE 1		SCALE 2		SCALE 3	
	Pre break	Post break	Pre break	Post break	Pre break	Post break
σ_{US}	0.610 (0.000)	2.068 (0.000)	0.516 (0.000)	1.515 (0.000)	0.535 (0.000)	1.438 (0.000)
σ_{UK}	0.375 (0.000)	1.598 (0.000)	0.312 (0.000)	1.662 (0.000)	0.343 (0.000)	1.142 (0.000)
σ_{GER}	0.229 (0.000)	1.451 (0.000)	0.263 (0.000)	1.126 (0.000)	0.316 (0.000)	1.023 (0.000)
σ_{FRA}	0.513 (0.000)	2.215 (0.000)	0.450 (0.000)	1.929 0.000	0.523 0.000	0.831 0.000
σ_{NED}	0.400 (0.000)	0.814 (0.000)	0.462 (0.000)	0.818 (0.000)	0.569 (0.000)	0.916 (0.000)
σ_{SWI}	0.376 (0.000)	1.113 (0.000)	0.281 (0.000)	1.571 (0.000)	0.330 (0.000)	1.517 (0.000)
γ	0.521 (0.000)	2.209 (0.000)	0.653 (0.000)	2.882 (0.000)	0.917 (0.004)	3.826 (0.000)
δ_{UK}		0.109 (0.296)		-0.690 (0.984)		0.004 (0.860)
δ_{GER}		-0.048 (0.668)		-0.295 (0.808)		-0.414 (0.992)
δ_{FRA}		-0.032 (0.700)		-0.939 (0.996)		-0.487 (0.992)
δ_{NED}		0.053 (0.496)		-0.513 (0.984)		-0.082 (0.980)
δ_{SWI}		-0.296 (0.940)		-0.877 (0.976)		-0.675 (0.992)
<i>Test over-identifying restrictions (bootstrapped p-value):</i>		0.852		0.628		0.208

Note: Bootstrapped p-values for testing the null of zero against the alternative hypothesis of a positive coefficient are given in parenthesis. The p-value for the over-identifying restriction test is obtained by computing the number of bootstrap replications for which the likelihood ratio statistics exceed the one associated with the point estimates. The pre-break sample period runs from 2/1/2002 until 14/9/2008. The post-break sample period runs from 15/9/2008 until 31/12/2012

Table 5: Factor decomposition of scale covariance matrices for the implied volatilities time series (LA4 filter)

Parameter	SCALE 1		SCALE 2		SCALE 3	
	Pre break	Post break	Pre break	Post break	Pre break	Post break
σ_{US}	0.532 (0.000)	2.083 (0.000)	0.490 (0.000)	1.657 (0.000)	0.513 (0.000)	1.021 (0.000)
σ_{UK}	0.328 (0.000)	1.750 (0.000)	0.293 (0.000)	1.651 (0.000)	0.323 (0.000)	1.117 (0.000)
σ_{GER}	0.218 (0.000)	1.514 (0.000)	0.240 (0.000)	1.065 (0.000)	0.291 (0.000)	0.948 (0.000)
σ_{FRA}	0.418 (0.000)	2.630 (0.000)	0.416 (0.000)	1.979 (0.000)	0.521 (0.000)	0.788 (0.000)
σ_{NED}	0.356 (0.000)	0.702 (0.000)	0.415 (0.000)	0.685 (0.000)	0.540 (0.000)	0.988 (0.000)
σ_{SWI}	0.274 (0.000)	1.614 (0.000)	0.256 (0.000)	1.534 (0.000)	0.283 (0.000)	1.607 (0.000)
γ	0.445 (0.000)	1.836 (0.000)	0.558 (0.000)	2.472 (0.000)	0.776 (0.000)	3.284 (0.004)
δ_{UK}		-0.120 (0.652)		-0.581 (0.972)		-0.337 (0.676)
δ_{GER}		-0.183 (0.884)		-0.163 (0.800)		-1.008 (0.992)
δ_{FRA}		-0.251 (0.804)		-0.780 (0.988)		-1.071 (0.992)
δ_{NED}		-0.101 (0.722)		-0.412 (0.980)		-0.478 (0.976)
δ_{SWI}		-0.235 (0.944)		-0.734 (0.984)		-1.472 (0.996)
<i>Test over-identifying restrictions (bootstrap d p-value):</i>		0.500		0.436		0.288

Note: see note to Table 4

Table 6: Factor decomposition of scale covariance matrices for the realized volatilities time series (Haar filter)

Parameter	SCALE 1		SCALE 2		SCALE 3	
	Pre break	Post break	Pre break	Post break	Pre break	Post break
σ_{US}	1.958 (0.000)	8.246 (0.000)	1.432 (0.000)	6.558 (0.000)	1.232 (0.000)	6.601 (0.000)
σ_{UK}	1.409 (0.000)	1.555 (0.000)	1.093 (0.000)	0.970 (0.000)	0.785 (0.000)	0.954 (0.000)
σ_{GER}	1.362 (0.000)	3.499 (0.000)	1.047 (0.000)	4.213 (0.000)	0.798 (0.000)	3.692 (0.000)
σ_{FRA}	0.769 (0.000)	2.763 (0.000)	0.607 (0.000)	2.426 (0.000)	0.470 (0.000)	1.751 (0.000)
σ_{NED}	0.926 (0.000)	2.336 (0.000)	0.791 (0.000)	2.206 (0.000)	0.641 (0.000)	1.783 (0.000)
σ_{SWI}	1.042 (0.000)	2.216 (0.000)	0.780 (0.000)	1.452 (0.000)	0.651 (0.000)	1.283 (0.000)
γ	1.948 (0.004)	6.396 (0.000)	1.839 (0.000)	5.479 (0.000)	1.734 (0.004)	4.822 (0.000)
δ_{UK}		0.047 (0.476)		-0.017 (0.571)		0.259 (0.096)
δ_{GER}		0.181 (0.296)		0.126 (0.200)		0.541 (0.056)
δ_{FRA}		0.178 (0.312)		0.097 (0.286)		0.421 (0.076)
δ_{NED}		0.101 (0.344)		-0.016 (0.600)		0.292 (0.092)
δ_{SWI}		0.121 (0.248)		0.131 (0.114)		0.358 (0.088)
<i>Test over-identifying restrictions (bootstrap p-value):</i>		0.544		0.743		0.524

Note: see note to Table 4

Table 7: Factor decomposition of scale covariance matrices for the realized volatilities time series (LA4 filter)

Parameter	SCALE 1		SCALE 2		SCALE 3	
	Pre break	Post break	Pre break	Post break	Pre break	Post break
σ_{US}	1.938 (0.000)	8.037 (0.000)	1.396 (0.000)	6.233 (0.000)	1.197 (0.000)	6.658 (0.000)
σ_{UK}	1.399 (0.000)	1.563 (0.000)	1.085 (0.000)	1.102 (0.000)	0.768 (0.000)	1.099 (0.000)
σ_{GER}	1.344 (0.000)	3.135 (0.000)	1.062 (0.000)	4.228 (0.000)	0.786 (0.000)	3.672 (0.000)
σ_{FRA}	0.756 (0.000)	2.651 (0.000)	0.605 (0.000)	2.631 (0.000)	0.469 (0.000)	1.740 (0.000)
σ_{NED}	0.902 (0.000)	2.253 (0.000)	0.794 (0.000)	2.205 (0.000)	0.643 (0.000)	1.731 (0.000)
σ_{SWI}	1.032 (0.000)	2.194 (0.000)	0.775 (0.000)	1.398 (0.000)	0.643 (0.000)	1.202 (0.000)
γ	1.862 (0.004)	6.124 (0.000)	1.800 (0.000)	5.450 (0.000)	1.690 (0.004)	4.512 (0.000)
δ_{UK}		0.035 (0.436)		-0.080 (0.766)		0.256 (0.028)
δ_{GER}		0.172 (0.288)		0.035 (0.489)		0.600 (0.012)
δ_{FRA}		0.162 (0.300)		0.042 (0.383)		0.426 (0.028)
δ_{NED}		0.094 (0.308)		-0.082 (0.660)		0.277 (0.028)
δ_{SWI}		0.098 (0.280)		0.100 (0.213)		0.362 (0.028)
<i>Test over-identifying restrictions (bootstrap d p-value):</i>		0.580		0.426		0.512

Note: see note to Table 4

Appendix

In case of a dyadic multi-resolution analysis, the dilated and translated family of wavelets functions can be defined as⁵:

$$\psi_{j,k}(t) = 2^{-j/2} \psi(2^{-j}t - k); j, k \in I \quad (\text{A.1})$$

Where j and k are the integer parameters governing the scale resolution (*i.e.* 2^{-j}) and translation in time, respectively. All the wavelet basis functions, $\psi_{j,k}$, are self-similar, namely, they differ only by translation and change of scale from one another. These functions result from a *mother* wavelet, $\psi(t)$, which is any oscillating function with zero mean, finite support and unit energy, *i.e.*:

$$\begin{aligned} \int_{-\infty}^{+\infty} \psi(t) dt &= 0 \\ \int_{-\infty}^{+\infty} |\psi(t)|^2 dt &= 1 \end{aligned} \quad (\text{A.2})$$

The object of a wavelet analysis is to associate an amplitude (wavelet) coefficient w to each of the wavelet. The task is accomplished by the Discrete Wavelet Transform which is implemented via the pyramid algorithm of Mallat (1987). If certain conditions are satisfied, these coefficients completely characterize the signal which is resolved in terms of a coarse approximation and the sum of fine details:

$$x(t) = \sum_k v_{J,k} \phi_{J,k}(t) + \sum_j \sum_k w_{j,k} \psi_{j,k} \quad (\text{A.3})$$

Here J is the highest possible level of decomposition; $\phi_{J,k}$ is the set of translated orthogonal scaling functions spanning the lower frequency range $[0, \pi/2^{(J)}]$. Therefore, the first term $\sum_k v_{J,k} \phi_{J,k}(t)$ in eq.

⁵Given a time series with T observations, conventional dyadic multi-resolution analysis applies to a succession of frequency intervals in the form of $(\pi/2^{(j)}, \pi/2^{(j-1)})$, with the decomposition level j running from 1 to J . The bandwidths are halved (down-sampled by 2) repeatedly descending from high to low frequencies. By the j^{th} round, there will be j wavelet bands and one accompanying scaling function band. At the decomposition level j , one obtains a set of $T/2^j$ mutually orthogonal wavelets functions given by equation (7), separated from each other by 2^j points.

(A.3) is the coarse approximation of the signal, and the second term $\sum_j \sum_k w_{j,k} \psi_{j,k}$ in eq. (A.3) is the sum of fine details.

The scaling and wavelet coefficients $v_{j,k}$ and $w_{j,k}$ are the following projections of $x(t)$ on the bases $\phi_{j,k}$ and $\psi_{j,k}$ respectively:

$$v_{j,k} = \int x(t) \phi_{j,k}(t) dt \quad (\text{A.4})$$

$$w_{j,k} = \int x(t) \psi_{j,k}(t) dt \quad (\text{A.5})$$

The signal can then be written as a set of orthogonal components at resolutions 1 to J:

$$x(t) = S_J + D_J + D_{J-1} + \dots + D_1 \quad (\text{A.6})$$

At level j the detail component D_j captures frequencies spanning cycles with periodicity between 2^j and 2^{j+1} and the smooth S_j captures cycles with periodicity greater than 2^{j+1} periods.

A disadvantage of the conventional dyadic wavelet analysis is the restriction on the sample size T which has to be a power of 2. A further problem lies in the fact that the DWT depends upon an non-symmetric filter that is liable to induce a phase lag in the processed data. These difficulties can be circumvented by means of the Maximum Overlapping Discrete Wavelet Transform (MODWT), through which, the filtered output at each stage of the pyramid algorithm is not subjected to down-sampling, as in DWT analysis. As a consequence, the number of coefficients generated at the j -th stage of the decomposition, are in number equal to the sample size, T , instead that equal to $T/2^j$.

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