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by Silvia Muzzioli

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CEFIN – Centro Studi di Banca e Finanza
Dipartimento di Economia Aziendale – Università di Modena e Reggio Emilia
Viale Jacopo Berengario 51, 41100 MODENA (Italy)
tel. 39-059.2056711 (Centralino) fax 059 205 6927

Towards a volatility index for the Italian stock market

S. Muzzioli[1]
Department of Economics and CEFIN
University of Modena and Reggio Emilia

Abstract.

The aim of this paper is to analyse and empirically test how to unlock volatility information from option prices. The information content of three option based forecasts of volatility: Black-Scholes implied volatility, model-free implied volatility and corridor implied volatility is addressed, with the ultimate plan of proposing a new volatility index for the Italian stock market. As for model-free implied volatility, two different extrapolation techniques are implemented. As for corridor implied volatility, five different corridors are compared.

Our results, which point to a better performance of corridor implied volatilities with respect to both Black-Scholes implied volatility and model-free implied volatility, are in favour of narrow corridors. The volatility index proposed is obtained with an overall 50% cut of the risk neutral distribution. The properties of the volatility index are explored by analysing both the contemporaneous relationship between implied volatility changes and market returns and the usefulness of the proposed index in forecasting future market returns.

Keywords: volatility index, Black-Scholes implied volatility, model-free implied volatility, corridor implied volatility, implied binomial trees.

JEL classification: G13, G14.

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[1] Department of Economics and CEFIN, University of Modena and Reggio Emilia, Viale Berengario 51, 41100 Modena (I), Tel. +390592056771 Fax +390592056947, e-mail: silvia.muzzioli@unimore.it. The author thanks Diego Covizzi for research assistance, Andrea Cipollini and George Skiadopoulos for helpful comments and suggestions. The author gratefully acknowledges financial support from MIUR and Fondazione Cassa di Risparmio di Modena. Usual disclaimer applies.

1. Introduction.

Volatility is a key variable for portfolio selection models, option pricing models and risk management techniques. Volatility can be estimated and forecasted by using either historical information or option prices. The present paper focuses on option based volatility forecasts for three main reasons. First, for the forward-looking nature of option based forecasts (as opposed to the backward-looking nature of historical information); second, for the average superiority, documented in the literature, of option based estimates in forecasting future realized volatility (see e.g. Poon and Granger (2003)); third, for the widespread use of option prices in the computation of the most important market volatility indexes (see e.g. the VIX index for the Chicago Board Options Exchange).

Among option based volatility forecasts we find Black-Scholes (BS) implied volatility, that is a "model-dependent" forecast since it relies on the Black and Scholes (1973) model, the so called "model-free" implied volatility (MF), proposed by Britten-Jones and Neuberger (2000), which does not rely on a particular option pricing model, being consistent with several underlying asset price dynamics (see e.g. Jiang and Tian (2005)) and corridor implied volatility (CIV), introduced in Carr and Madan (1998), and recently implemented in Andersen and Bondarenko (2007), which is obtained from model-free implied volatility by truncating the integration domain between two barriers.

The three volatility forecasts present some drawbacks arising from the discrepancy from the theoretical underpinnings of the measures and the reality of financial markets. BS option pricing model is derived under the assumption of a constant volatility. However, BS implied volatility differs depending on strike price of the option (the so-called smile effect), time to maturity of the option (term structure of volatility) and option type (call versus put). Nonetheless at-the-money Black-Scholes implied volatility is widely recognized by market participants as a good predictor of future realised volatility.

The theoretical definition of MF implied volatility supposes the availability of a continuum of option prices in strikes, ranging from zero to infinity. As in the market only a limited number of strike prices are quoted, both truncation and discretization errors occur (see e.g. Jiang and Tian (2007)). Truncation errors are faced since a limited range of strike prices is used. Discretization errors are due to the fact that only a finite number (instead of a continuum) of strike prices are used. In order to overcome this limits interpolation and extrapolation techniques have been proposed (see e.g. Jiang and Tian, 2005 and 2007).

CIV measures are implicitly linked with the concept that the tails of the risk-neutral distribution are estimated with less precision than central values, due to the lack of liquid options for very high and very low strikes. The theoretical definition of CIV implied volatility supposes the availability of a continuum of option prices in strikes, between the two barriers. Therefore if the barriers are set within the quoted domain of strikes, only discretization errors are faced. However, in order to truncate the strike price domain, CIV needs a costly estimation of the risk-neutral distribution of the underlying asset and a subjective choice of the barriers, which render its forecasting performance mainly an empirical question.

Carr and Wu (2006) highlight that MF implied volatility should be theoretically superior to BS implied volatility. They show that at-the-money BS implied volatility can be considered as a proxy for a volatility swap rate, while model-free variance can be considered as a proxy for a variance swap rate. While the payoff on a volatility swap is difficult to replicate, the payoff of a variance swap rate is easily replicable by using a static position in a continuum of European options and a dynamic position in futures (for more details see Carr and Wu (2006)). Given the more concrete economic meaning of model-free implied volatility, the most important market volatility indexes (see e.g. the VIX index for the Chicago Board Options Exchange, or the V-DAX New for the German stock market) have switched from an old version based on an average of at-the-money BS implied volatilities to a formula based on MF implied volatility.

However, the latter market volatility indexes are computed with the use of quoted strike prices only, as such, they can be considered as a CIV measure with barriers set at the minimum and maximum strike price quoted (which fulfil some liquidity constraints, as will be explained in Section 5). Jiang and Tian (2007) point out how the latter choice, which makes the barriers stochastically dependent on the range of quoted strike prices may affect the usefulness of the volatility indexes, which can severely underestimate or overestimate the true volatility.

Nonetheless, the VIX index methodology has been widely used in order to compute the volatility indexes gradually introduced in various European exchanges. The VDAX New for the German market, the VSMI for the Swiss market, and the VSTOXX volatility indices with their respective sub-indices were launched on 20, April, 2005. The VAEX Volatility Index for the Dutch market, the VBEL Volatility Index for the Belgian market and the VCAC Volatility Index for the French market started to be traded on 3 September 2007. On 12 June 2008, VFTSE, the volatility index of FTSE 100 British market index has been launched. Surprisingly, a volatility index for the Italian market has not been introduced yet.

At the empirical level, the forecasting power of both MF and CIV implied volatilities has not been extensively tested yet and the superiority w.r.t. BS volatility is questioned. As for MF implied volatility, some papers find that it is an unbiased and an efficient forecast of future realised volatility (see e.g. Lynch and Panigirtzoglou (2003), Jiang and Tian (2005), Bollerslev et al. (2009)). However, its superiority w.r.t. BS is questioned (see e.g. Taylor et al. (2006), Becker at al. (2007), Muzzioli (2010)). To the best of our knowledge, the performance of CIV implied volatility has been empirically tested only in Andersen and Bondarenko (2007) and Tsiaras (2009). Both studies are in favour of CIV implied volatility with respect to BS and MF. However, opposite results regarding the optimal corridor width are found. Andersen and Bondarenko (2007), by using options on the S&P500 futures market, find that narrow corridor measures, closely related to BS implied volatility are more useful for volatility forecasting than broad corridor measures, which tend to model-free implied volatility as the corridor widens. Tsiaras (2009), by using options on the 30 components of the DJIA index, concludes that CIV forecasts are increasingly better as long as the corridor width enlarges.

In light of the above, the paper supplements existing literature by analysing and empirically testing the three option-based measures of volatility, with the ultimate plan of devising a volatility index for the Italian market. As for model-free implied volatility, we consider two different implementation techniques that vary in the extrapolation of the strike price domain. As for BS implied volatility, we use a weighted average of implied volatilities backed out from different option classes. As for CIV implied volatility, the enhanced Derman and Kani (1994) (EDK) method proposed in Moriggia et al. (2009) is used in order to derive the risk-neutral distribution of the underlying asset. Relatively to other methodologies, the use of the EDK method presents several advantages. First, it fits existing option prices ensuring positive risk-neutral probabilities, i.e. absence of arbitrage opportunities, second it correctly models the tails of the distribution, fundamental for the computation of CIV implied volatilities with broad corridors. Following the industry standard for volatility indexes, a CIV measure based only on quoted strike prices is also obtained.

This paper makes at least two contributions to the ongoing debate on the information content of option implied measures of volatility and the construction of volatility indexes. First, unlike previous studies (Andersen and Bondarenko (2007), Tsiaras (2009)) which address the information content of CIV measures by using settlement prices, it uses the more informative intra-daily synchronous prices between the options and the underlying asset. This is important to stress, since the implied volatilities obtained are real "prices", as determined by synchronous no-arbitrage relations. Second, it is the first contribution aimed at devising a volatility index for the Italian stock market, which is one of the most important European markets.

The plan of the paper is as follows. Section 2 presents the volatility measures used. Section 3 illustrates the computational methodology of European and US volatility indexes. Section 4 briefly reviews the different approaches for estimating the risk-neutral distribution of the underlying asset and Section 5 recalls the EDK methodology used in the paper. Section 6 presents the data set used. Section 7 illustrates the computation of the volatility measures. Section 8 evaluates the forecasting performance of the different volatility measures for two forecasting horizons. Section 9 illustrates the properties of the implied volatility index proposed. The last Section concludes. The Appendix recalls the relationship between model-free implied volatility and the computational methodology of traded volatility indexes.

2. Variance and Volatility measures.

Assume that the stock price evolves as a diffusive process (no jumps allowed), as follows:

$$\frac{dS_t}{S_t} = \mu(t,...)dt + \sigma(t,...)dZ_t \tag{1}$$

Realized variance (also called integrated variance) in the period 0-T is given by:

$$V = \frac{1}{T}\int_0^T \sigma^2(t,...)dt \tag{2}$$

If we assume absence of arbitrage opportunities and the existence of a unique risk-neutral measure, the fair price of variance is the risk-neutral expectation of future integrated variance:

$$\widehat{E}(V) = \frac{1}{T}\widehat{E}\left[\int_0^T \sigma^2(t,...)dt\right] \tag{3}$$

Note that equation (1) includes implied tree models (see e.g. Derman and Kani (1994)) as a special case, if volatility is assumed as a deterministic function of asset price and time. In implied tree models the so called local volatility $\sigma(S,t)$ is obtained by calibrating the implied tree to quoted option prices. Local volatility (also known as forward volatility) was introduced by Dupire (1994) and Derman and Kani (1994) as the market expectation (or the fair value as it is implied from actual implied volatilities) of instantaneous volatility for a future market level of K at some future date T, $\sigma(K,T)$. While implied volatilities are global measures of volatility, since under certain conditions (no strike dependence of the implied volatility) can be considered as the market's estimate of expected average of volatilities up to expiry, local volatilities are a local measure since they give a volatility forecast for a couple of K and T. As noted in Demeterfi et al. (1999) as we do not know the value of future volatility, we can resort to simulation in order to compute the fair price of the variance. In particular, we can use local volatility, in order to proxy for realised volatility and compute the fair price of the variance by averaging across simulated underlying paths consistent with the implied tree.

Local volatilities could be useful to value the contract, but not to replicate it. Demeterfi et al. (1999) and Britten-Jones and Neuberger (2000), show how to replicate the risk-neutral expectation of variance with a portfolio of options with strike price ranging from zero to infinity, as follows:

$$\widehat{E}(V) = \frac{1}{T}\widehat{E}\left[\int_0^T \sigma^2(t,...)dt\right] = \frac{2e^{rT}}{T}\int_0^\infty \frac{M(K,T)}{K^2}dK \tag{4}$$

where *M(K,T)* is the minimum between a call or put option price, with strike price *K* and maturity *T*, i.e. only out-of-the-money options are used.

Equation (4) is also known as model-free implied variance, and its square root as model-free implied volatility, since, differently from Black-Scholes implied volatility it does not rely on any particular option pricing model. Demeterfi et al. (1999) and Britten-Jones and Neuberger (2000) assumed only a diffusion process for the underlying asset; Jiang and Tian (2005) extended to jump-diffusion process the derivation of model-free implied variance.

A practical limitation of model-free implied volatility is that in the reality of financial markets only a limited and discrete set of strike prices are quoted, therefore interpolation and extrapolation are needed in order to compute model-free implied volatility (see Jiang and Tian (2005) and (2007)).

Carr and Madan (1998) and Andersen and Bondarenko (2007) introduce the notion of "corridor variance". A corridor variance contract pays realised variance only if the underlying asset lies between two specified barriers B_1 and B_2, Therefore corridor integrated variance can be defined as follows:

$$CIV(0,T) = \frac{1}{T}\int_0^T \sigma^2(t,...)I_t(B_1,B_2)dt \tag{5}$$

where $I(B_1,B_2)$ is the indicator function that is equal to 1 only when the underlying is inside the two barriers and determines if variance is accumulated or not. If $B_1=0$ and $B_2=\infty$, then corridor variance coincides with model-free variance. Therefore if a CIV measure is used as a forecast of integrated variance, there is a mismatch between the forecast and the forecasted quantity.

Carr and Madan (1998) and Andersen and Bondarenko (2007) show that it is possible to compute the expected value of corridor variance under the risk-neutral probability measure, by using a portfolio of options with strikes ranging from B_1 to B_2, as follows:

$$\widehat{E}(CIV(0,T)) = \widehat{E}\left[\frac{1}{T}\int_0^T \sigma^2(t,...)I_t(B_1,B_2)dt\right] = \frac{2e^{rT}}{T}\int_{B_1}^{B_2} \frac{M(K,T)}{K^2}dK \tag{6}$$

Equation (6) is known as corridor implied variance and its square root as corridor implied volatility. By choosing different levels for the barriers, we obtain CIV measures with wider or narrower corridors. CIV measures are implicitly linked with the concept that the tails of the risk-neutral distribution are estimated with less precision than central values, due to the lack of liquid options for very high and very low strikes.

In the following, volatility measures are taken as the square root of variance measures.

3. Volatility indexes.

Volatility indexes are deemed by market participants to capture the so-called "market fear": high index values are associated with high uncertainty in the underlying market, low index values with stable conditions. Volatility indexes serve as underlying assets for volatility derivatives, which have been introduced in various exchanges in order to make pure volatility tradable. The possibility to trade volatility as a separate asset class has at least three advantages. First, it enables to better hedge the portfolio with a pure position in volatility, in contrast with an impure hedging usually pursued with options that are sensitive to both volatility and the underlying asset and thus require continuous delta hedging. Second, it permits to diversify the portfolio by adding a new asset class: volatility derivatives are ideal to hedge downside equity market risk and are also viewed as an important tool for disaster hedges, given their quasi perfect negative correlation with the market. Last, it allows speculating on future volatility levels by exploiting the mean-reversion nature of volatility. In this Section we briefly illustrate the characteristics of the main volatility indexes traded in Europe and U.S.A, in order to enucleate the best practices to be applied to the Italian volatility index.

The CBOE Volatility Index (VIX) is a key measure of market expectations of near-term volatility conveyed by S&P500 stock index option prices. Since its introduction in 1993, VIX has been considered by many to be the world's premier barometer of investor sentiment and market volatility. On September 22, 2003 the old VIX index (renamed as VXO) that was based on an average of at-the-money implied volatilities of S&P100 (OEX) options has been substituted by the new VIX, based on model-free implied volatility computed from S&P500 (SPX) options. CBOE continues to calculate and disseminate also the original-formula index VXO and calculates also other important volatility indexes such as the DJIA Volatility index, NASDAQ-100 Volatility index, Russell 2000 Volatility index, S&P500 3-Month Volatility index.

The VDAX New, VSMI and VSTOXX volatility indices with their respective sub-indices were jointly developed by Goldman Sachs and Deutsche Börse and launched on 20, April, 2005. VDAX New is the volatility index of the DAX index, representative of the 30 largest companies of the German stock market, VSMI is the volatility index of the SMI index, representative of the 40 largest companies of the Swiss market, VSTOXX is the volatility index of the Dow Jones EURO STOXX 50, a blue chips index of the top 50 stocks of the Euro zone. These volatility indices capture the volatility expectations over the following 30 days and are based on a formula similar to the one used in the computation of the VIX index of the CBOE. However, differently from the VIX, sub-indices are calculated also for different times-to-maturity. The VDAX-New replaces the old V-DAX index, still computed and disseminated by Deutsche Börse. The old V-DAX index expresses the volatility to be expected in the next 45 days for the DAX and is based on an average of at-the-money implied volatilities of DAX-index options.

NYSE EURONEXT has issued on 3 September 2007 the VAEX Volatility Index that represents the implied volatility of the AEX Dutch market index, the VBEL Volatility Index for the BEL 20 Belgian market index and the VCAC Volatility Index for the CAC40 French market index. On 23 June 2008, VFTSE, the volatility index of FTSE 100 British market index has been introduced. All these indexes follow the computational methodology of the VIX index, adapted to European markets.

All the above mentioned volatility indexes are based on the following formula (see Demeterfi et al. (1999)) computed for the nearest (*i=1*) and the next term (*i=2*) expiries:

$$\sigma_i^2 = \frac{2}{T_i}\sum_j \frac{\Delta K_{i,j}}{K_{i,j}^2} \mathrm{e}^{\mathrm{r_i T_i}} M(K_{i,j}) - \frac{1}{T_i}\left[\frac{F_i}{K_{i,0}} - 1\right]^2 \tag{7}$$

where:

T_i = time to expiry of the *i*-th maturity options, expressed in fraction of year,

F_i = forward price derived from the prices of the *i*-th maturity for which the absolute difference between call and put prices is smallest:

$$F_i = K_{\min|C - P|} + \mathrm{e}^{\mathrm{r_i T_i}} (C - P),$$

$K_{i,j}$ = exercise price of the j-th out-of-the-money option of the *i*-th expiry month in ascending order,

$\Delta K_{i,j}$ = interval between strike prices (computed as half the interval between the one higher and the one lower strike prices or the simple difference between the highest and the second highest strike prices or lowest and second lowest): $\Delta K_{i,j} = \frac{K_{i,j+1} - K_{i,j-1}}{2}$,

$K_{i,0}$ = highest exercise price below forward price,

e^{riTi} = refinancing factor for the *i*-th expiry,

r_i = risk free interest rate for the *i*-th expiry,

$M(K_{i,j})$ = call or put option price with strike price $K_{i,j}$, with $K_{i,j} \neq K_{i,0,}$

$M(K_{i,0})$ = average of call and put prices at exercise price $K_{i,0,}$

$$M\left(K_{i,j}\right)=\begin{cases} Put : K_{i,j} < K_{i,0} \\ \dfrac{Put + Call}{2} : K_{i,j} = K_{i,0} \\ Call : K_{i,j} > K_{i,0} \end{cases}$$

The volatility index is computed by linear interpolation of the two sub-indices which are the nearest to the remaining time to expiry of 30 days (σ_1 is for the near-term maturity and σ_2 for the next-term), as follows:

$$VOLIND = 100\sqrt{\left\{T_1\sigma_1^2\left[\frac{N_{T_2} - N_T}{N_{T_2} - N_{T_1}}\right] + T_2\sigma_2^2\left[\frac{N_T - N_{T_1}}{N_{T_2} - N_{T_1}}\right]\right\} \times \frac{N_{365}}{N_T}}$$

where

N_{Ti} = number of seconds /minutes to expiry of the i-th maturity index option

N_T = number of seconds/minutes in 30 days

N_{365} = number of seconds/minutes in a year

Note that calendar time is measured in seconds for VDAX-New family of indexes and NYSE EURONEXT indexes, while in minutes for CBOE indexes.

Even if all traded indexes are computed on the basis of the same formula, important differences among indexes hold for the filters applied to the data set. Regarding the options' expiry, the VDAX-New family of indexes uses only options with at least two days to expiration, whereas the VIX family of indexes retains options with at least seven days to expiration and the NYSE EURONEXT family of indexes utilizes options with at least eight days to expiration. Regarding liquidity constraints, once two consecutive call (put) options are found to have zero bid prices, no calls (puts) with higher (lower) strike are considered in the computation of the VIX family of indexes. For the NYSE EURONEXT indexes, prices are excluded if the bid-ask spread is too wide, in particular if $\frac{Ask - Bid}{0.5(Ask + Bid)} > 50\%$. For the V-Dax New, all option prices that are one-sided (with either a bid or an ask price only), options that are not quoted within the established maximum spread for EUREX market makers and options whose price is lower than 0.5 index points are excluded from the index computation.

4. Recovering risk-neutral distribution from option prices.

As for the implementation of corridor implied volatility we need the estimation of the risk-neutral distribution, in this Section we briefly review the numerous methodologies proposed in the literature for recovering the risk-neutral distribution from option prices at one particular date in the future. For a complete survey we refer the interested reader to Jackwerth (1999).

We distinguish among parametric and non-parametric methods. Parametric methods start from a basic distribution and generalize it with the help of additional parameters. Parametric methods include expansion methods, generalized distribution methods and mixture methods. Starting from a basic distribution, such as the normal or the log-normal one, expansion methods (see e.g. Rubinstein (1998)) use correction terms in order to make the basic distribution more flexible. A drawback of expansion methods is that the positivity of the risk-neutral density is not guaranteed. Generalized distribution methods (see e.g. Aparicio and Hodges (1998)) use family of distributions that may contain basic distributions as special cases and are therefore inherently more flexible. Mixture methods (see e.g. Melick and Thomas (1997)) generate new distributions from mixture of two or more basic distributions. Parametric methods are not suitable for small samples where data over-fitting can be serious.

Non-parametric methods are data-driven methods that make no assumptions on the parametric form of the distribution. In this category we find kernel methods, maximum entropy methods, curve fitting methods and implied trees. Kernel methods (see e.g. Ait Sahalia and Lo (1998)) try to fit a function to the data, without specifying the parametric form of the function. A modified kernel method is the positive convolution approximation method, developed by Bondarenko (2003) and implemented in Andersen and Bondarenko (2007), which ensures no-arbitrage. Maximum entropy methods try to find a non parametric probability distribution that is as close as possible to a prior distribution, while ensuring some no-arbitrage pricing constraints (see e.g. Stutzer (1996)). Curve fitting methods interpolate implied volatilities between strike prices (see e.g. Shimko (1993)) or the risk-neutral distribution itself see e.g. Rubinstein (1994), by some general function, such as the class of polynomials. In the implied volatility interpolation, in order to increase the fit to the data, splines (see e.g. Campa et al. (1998) can be used in order to connect the knots smoothly. Tsiaras (2009) uses a variation of Shimko (1993) by interpolating implied volatilities in the delta space rather than in strikes. Note that curve-fitting methods do not guarantee the positivity of the risk-neutral probabilities (see e.g. Monteiro et al. (2008)).

Implied trees are discretizations of one or two dimension diffusions, aimed at introducing non-constant volatility in an option pricing model. We can distinguish between deterministic and stochastic models, depending on whether volatility is assumed to be a deterministic function of asset price and time or it is assumed to follow a stochastic process. These models are also called smile-consistent models since the market price of options is taken as given and used to infer the underlying asset distribution. Deterministic volatility models (see e.g. Derman and Kani (1994), Barle and Cakici (1998), Rubinstein (1994), Jackwerth (1997), Dupire (1994)) derive endogenously from European option prices the instantaneous volatility as a deterministic function of asset price and time. Stochastic volatility models (see e.g. Derman and Kani (1998), Britten-Jones and Neuberger (2000)) allow for a no-arbitrage evolution of the implied volatility surface.

As for the accuracy of the different methodologies, Jackwerth and Rubinstein (1996) argue that if we observe a sufficient number of option prices (10-15, as it is the case in our dataset), then all the different methods tend to be rather similar, except in the modelling of the tails of the distribution. The same conclusion has been reached in Campa et al. (1998), where three different methods are used: a mixture of lognormals, an implied binomial tree of Rubinstein (1994) and cubic splines. They find similar probability density functions in any of the methods, but they prefer the implied tree approach for its flexibility and good representation of the data. As for the problem of modelling the tail distribution, they point out that one drawback of the cubic splines interpolation is the potential explosion of implied volatility outside the quoted range of strike prices. In their implementation of the Rubinstein (1994) binomial tree many outliers were generated in the tails. In order to avoid outliers, they trim the implied binomial tree with a 20% cut of the possible outcomes in the top and bottom terminal nodes.

5. The EDK implied tree and the risk-neutral probabilities computation.

In our dataset made of almost 15 strike prices quoted each day, we expect the different methodologies to be rather similar. However, three are our goals: fitting quoted option prices, ensuring positivity of the risk-neutral probabilities, i.e. absence of arbitrage opportunities and correctly modelling the tails of the distribution, fundamental for the computation of CIV measures with wide corridors. We prefer non parametric methods, since parametric methods are not flexible enough to fit exactly a given number of option prices. Following Campa et al. (1998), we concentrate on implied trees; however, with the aim of correctly modelling the tails of the distribution, we focus on forward induction implied trees. In particular, we use the Derman and Kani (1994) algorithm with the modifications proposed in Moriggia et al. (2009), (the so called Enhanced Derman and Kani implied tree (EDK)) which are fundamental both to avoid arbitrage opportunities and to correctly model the tails of the distribution. In fact the Derman and Kani (1994) implied tree, even with the Barle and Cakici (1998) modifications, is not free from arbitrage, in particular at the boundary of the tree and may become numerically unstable, when the number of steps becomes large. The advantages of the proposed methodology are at least three. First, it is a methodology that fits the data well without imposing a rigid parametric structure. Second, it does not require any costly estimation of the risk-neutral probability by entropy maximization or distance minimization from the a-priori distribution with subjective choice of the loss function used. Last, it ensures positivity of the risk-neutral probabilities.

The Derman and Kani (1994) implied tree is computed as follows (for more details see Derman and Kani (1994)). Let $j=0,\dots,n$ be the number of levels of the tree, that are spaced by Δt. As the tree recombines, $i=1,\dots,j+1$ is the number of nodes at level j. Forward induction is used to compute level j variables given level $j-1$ variables as inputs. The initial inputs are the risk-less interest rate, the stock price at time zero and the smile function. The latter is used to determine the price of the appropriate ATM call and put prices.

The Derman and Kani (1994) methodology assumes that the tree has been implied out to level $j-1$. The known stock price $S_{i,j-1}$, can evolve into $S_{i+1,j}$ in state up and $S_{i,j}$, in state down. The risk-neutral probability of an up jump is $p_{i,j}$. The Arrow-Debreu price, $\lambda_{i,j}$, is computed by forward induction as the sum over all paths leading to node (i,j) of the product of the risk-neutral probabilities discounted at the risk-free rate at each node in each path.

The problem is how to imply nodes at level j. There are $2j+1$ unknowns: $j+1$ stock prices $(S_{i,j})$ and j risk-neutral probabilities of an up move $(p_{i,j})$. Hence, $2j+1$ equations are needed: the first $2j$ equations require the theoretical value of j forwards and j options expiring at time j to match their market values (for the upper part of the tree call options are used, while for the lower part of the tree, put options), the remaining degree of freedom is used to require the tree to develop around the current stock price (centring condition). The centring condition is given by equation (8) if the level is even and by equation (9) if the level is odd:

$$S_{\frac{j}{2}+1} = S_{0,0} \tag{8}$$

$$S_{\frac{j+3}{2}} S_{\frac{j+1}{2}} = S_{0,0}^{\;2} \tag{9}$$

where $S_{0,0}$ is the initial stock price.

For the upper part of the tree the recursive equation to compute $S_{i+1,j}$ given $S_{i,j}$ is:

$$S_{i+1,j} = \frac{S_{i,j}\left[e^{r\Delta t} C_{i,j-1} - \sum\ \right] - \lambda_{i,j-1} S_{i,j-1}\left(F_{i,j-1} - S_{i,j}\right)}{\left[e^{r\Delta t} C_{i,j-1} - \sum\ \right] - \lambda_{i,j-1}\left(F_{i,j-1} - S_{i,j}\right)} \tag{10}$$

where $\sum = \sum_{k=i+1}^{j} \lambda_{k,j-1}\left(F_{k,j-1} - S_{i,j-1}\right)$, $F_{i,j-1}$ is the forward value of $S_{i,j-1}$ and $C_{i,j-1}$ is the price at time 0 of a call with strike $S_{i,j-1}$ and maturity j. In order to use equation (10), an initial node $S_{i,j}$ is needed. If the number of nodes is even, the central node is chosen to be equal to the current spot (equation (8)); if the number of nodes is odd, combining equations (9) and (10) yields the following equation:

$$S_{i+1,j} = \frac{S_{0,0}\left[e^{r\Delta t}C_{0,j-1} + \lambda_{i,j-1}S_{0,0} - \sum\right]}{\lambda_{i,j-1}F_{i,j-1} - e^{r\Delta t}C_{0,j-1} + \sum} \qquad (11)$$

For nodes in the lower part of the tree, a put with strike $S_{i,j-1}$ instead of a call, is used. The recursive formula that provides $S_{i,j}$ given $S_{i+1,j}$ is obtained:

$$S_{i,j} = \frac{S_{i+1,j}\left[e^{r\Delta t}P_{i,j-1} - \sum'\right] + \lambda_{i,j-1}S_{i,j-1}\left(F_{i,j-1} - S_{i+1,j}\right)}{\left[e^{r\Delta t}P_{i,j-1} - \sum'\right] + \lambda_{i,j-1}\left(F_{i,j-1} - S_{i+1,j}\right)} \qquad (12)$$

where $\sum' = \sum_{k=1}^{i-1} \lambda_{k,j-1}\left(S_{i,j-1} - F_{k,j-1}\right)$ and $P_{i,j-1}$ is the price at time 0 of a call with strike $S_{i,j-1}$ and maturity j and it is computed using a j step tree with constant volatility obtained from the smile function. By repeating this process at each level, the entire tree is generated.

The EDK methodology is aimed at ensuring the absence of no-arbitrage violations in the DK implied tree (for more details see Moriggia et al. (2009)). To this end it provides no-arbitrage checks and proposes no-arbitrage replacements for all the nodes in the tree. The no-arbitrage condition and the relative replacements are summarized in Table 1.

[Table 1 about here]

Transition probabilities $p_{i,j}$ for $i=1,\dots,j+1, j=1,\dots,100$, are computed as follows:

$$p_{i,j} = \frac{F_{i,j} - S_{i,j+1}}{S_{i+1,j+1} - S_{i,j+1}}$$

Arrow-Debreu prices at the boundary of the tree ($i=1$ and $i=j+1$, j=$1,\dots,100$) are computed as follows:

$$\lambda_{1,j} = (1 - p_{1,j-1})\lambda_{1,j-1}e^{-rdt}$$

$$\lambda_{j+1,j} = p_{j+1,j-1}\lambda_{j+1,j-1}e^{-rdt}$$

Arrow-Debreu prices in the remaining part of the tree ($i=2,\dots j-1,\ j=1,\dots,100$) are computed as follows:

$$\lambda_{i,j} = \left(p_{i-1,j-1}\lambda_{i-1,j-1} + (1 - p_{i,j-1})\lambda_{i,j-1}\right)e^{-r^{*}dt}$$

The final risk-neutral probabilities ($P_{i,n}$) are obtained by multiplying the final Arrow-Debreu prices by e^{rT}:

$$P_{i,n} = \lambda_{i,n} e^{rT} \quad i = 1,\dots,n+1$$

For the current implementation, the initial node is taken as the average value of the underlying asset recorded in the hour of trades, corrected for the dividend yield. We build an implied tree with 100 steps.

6. The Data set.

The data set consists of intra-daily data on FTSE MIB-index options (MIBO), recorded from 1 June 2009 to 30 November 2009. Each record reports the strike price, expiration month, transaction price, contract size, hour, minute, second and centisecond. MIBO are European options on the FTSE MIB index, which is a capital weighted index composed of 40 major stocks quoted on the Italian market. FTSE MIB options quote in index points, representing a value of 2.5 €, with 10 different expirations (the 4 three-monthly expiries in March, June, September and December, the 2 nearest monthly expiry dates, the 4 six-month maturities (June and December) of the two years subsequent the current year, the 2 annual maturities (December) of the third and fourth years subsequent the current year). The contract expires on the third Friday of the expiration month at 9.05 am. If the Exchange is closed that day, the contract expires on the first trading day preceding that day. For each maturity up to twelve months (monthly and three-month maturities), exercise prices are generated at intervals of 500 index points. At least 15 exercise prices are quoted for each expiry: one at-the-money, seven in-the-money and seven out-of-the-money strikes. The daily closing price is established by the clearing and settlement organisation Cassa di Compensazione e Garanzia.

As for the underlying asset, intra-daily prices of the FTSE MIB-index recorded from 1 June 2009 to 31 December 2009 are used. The FTSE MIB is the primary benchmark Index for the Italian equity market and seeks to replicate the broad sector weights of the Italian stock market. It is adjusted for stocks splits, changes in capital and for extraordinary dividends, but not for ordinary dividends. Therefore, the daily dividend yield is used in order to compute the appropriate value for the index, as follows:

$$\widehat{S}_t = S_t e^{-\delta_t \Delta t}$$

where S_t is the FTSE MIB value at time t, δ_t is the dividend yield at time t and Δt is the time to maturity of the option.

As a proxy for the risk-free rate, Euribor rates with maturities one week, one, two and three months are used. Appropriate yields to maturity are computed by linear interpolation. The data-set for the FTSE MIB index and the MIBO is kindly provided by Borsa Italiana S.p.A, Euribor rates and dividend yields are obtained from Datastream.

Several filters are applied to the option data set. First, in order not to use stale quotes, we eliminate options with trading volumes of less than one contract. Second, we eliminate options near to expiry which may suffer from pricing anomalies that might occur close to expiration. In order to keep the outline similar to the computation methodology of quoted volatility indexes, we choose to use the most conservative filter that eliminates options with time to maturity of less than 8 days. Third, following Ait-Sahalia and Lo (1998) only at-the-money and out-of-the-money options are retained (call options with moneyness $K/S > 0.97$ and put options with moneyness $K/S < 1.03$). Fourth, option prices violating the standard no-arbitrage constraints are eliminated: $P \geq \max(Ke^{-r(T-t)} - Se^{-\partial(T-t)}, 0)$, $C \geq \max(Se^{-\partial(T-t)} - Ke^{-r(T-t)}, 0)$. Finally, in order to reduce computational burden, we only retain options that are traded in the last hour of trade, from 4:40 to 5:40 (the choice is motivated by the high level of trading activity in this interval).

7. The computation of the volatility measures.

In order to keep the computation of the volatility measures as much similar as possible to the computation of traded market volatility indexes, the volatility measures are computed by linear interpolation of the two sub-indices which are the nearest to the remaining time of expiry of 30 days (σ_1 is for the near-term maturity and σ_2 for the next-term), as follows:

$$VOL\,MEASURES = 100\sqrt{\left\{\frac{T_1}{365}\sigma_1^2\left[\frac{T_2-30}{T_2-T_1}\right] + \frac{T_2}{365}\sigma_2^2\left[\frac{30-T_1}{T_2-T_1}\right]\right\}\times\frac{365}{30}}$$

where:

T_i= number of days to expiry of the i-th maturity index option, $i=1,2$.

To this end, each day of the sample, we divided quoted option prices in two sets: near term and next term options, in order to compute the two volatility sub-indices.

We compute four volatility measures: realised volatility (σ_r), BS implied volatility (σ_{BS}) model-free implied volatility (σ_{MF}) and corridor implied volatility (σ_{CIV}). Realised volatility is computed, in annual terms, as the squared root of the sum of five-minute frequency squared index returns over the next 30 days:

$$\sigma_r = \sqrt{\sum_{t=1}^{n}\left[\ln\left(\frac{S_{t+1}}{S_t}\right)\right]^2 * \frac{365}{30}}.$$

where n is the number of index prices spaced by five minutes in the 30 days period. The choice of using five-minute frequency is made following Andersen and Bollerslev (1998) and Andersen et al. (2001) who showed the importance of using high frequency returns in order to measure realised volatility and point out that returns at a frequency higher than five minutes are affected by serial correlation.

BS implied volatility (σ_{BS}) is defined as the weighted average of the two implied volatilities that correspond to the two strikes that are closest to being at-the-money, with weights inversely proportional to the distance to the moneyness (for example if the FTSE MIB index is 20600 and the closest strikes are 21000 and 20500 the implied volatility of the 21000 strike will be weighted 1/5 against the implied volatility of the 20500 strike which is weighted 4/5). As we are using one hour of data, the underlying used is the average in the hour of trades. Similarly, for the two strikes closest to the at-the-money value, the average of the corresponding implied volatilities in the hour of trades is taken.

Model-free implied volatility (σ_{MF}) is computed as follows:

$$\sigma_{MF} = \sqrt{\frac{2e^{rT}}{T}\int_0^\infty \frac{M(T,K)}{K^2}dK} \approx \sqrt{\frac{e^{rT}}{T}\sum_{i=1}^{m}\left[g(T,K_i)+g(T,K_{i-1})\right]\Delta K}$$

where $\Delta K = (K_{\max} - K_{\min})/m$, m is the number of abscissas, $K_i = K_{\min} + i\Delta K$, $0 \le i \le m$, $g(T,K_i) = [\min(C(T,K_i), P(T,K_i)]/K_i^2$, and the trapezoidal rule is used in order to increase the accuracy of integration.

Model-free implied volatility has been computed with the following procedure. First, we recover the Black-Scholes implied volatilities by using synchronous prices of the option and the underlying that are matched in one minute interval. These implied volatilities are averaged for each strike in the hour of trades resulting in a matrix of quoted strike prices and corresponding implied volatilities. Second, as only a discrete number of strikes are available, we need to interpolate and extrapolate option prices in order to generate the missing prices that are input to the model-free implied volatility formula. As for the interpolation, following Shimko (1993) and Ait-Sahalia and Lo (1998) we choose to interpolate implied volatilities between strike prices, rather than option prices. With the aim of having a smooth function, following Campa et al. (1998), we use cubic splines to interpolate implied volatilities.

As for the extrapolation methodology, we compute model-free implied volatility in two different ways. First, we compute model-free implied volatility (σ_{MF1}) by following the methodology in Jiang and Tian (2005): we suppose that for strikes below (above) the minimum (maximum) value, implied volatility is constant and equal to the volatility of K_{min} (K_{max}). Second, as the latter methodology may underestimate implied volatility in the tails of the strike price domain, we compute model-free implied volatility (σ_{MF2}) by following the methodology in Jiang and Tian (2007): we extrapolate volatilities outside the listed strike price range by using a linear function that matches the slope of the smile function at K_{min} and K_{max}. This methodology has the advantage that the smile function remains smooth at K_{min} and K_{max}. As this latter methodology of extending the strike price domain by a segment that matches the slope of the smile function at K_{min} and K_{max} may generate implied volatilities that are artificially too high (in case the slope is positive) or too low (in case the slope is negative), we have imposed both a lower and an upper bound to implied volatilities equal to 0.001 and 0.999 respectively. Recall that σ_{MF1} is by construction subject to possible no-arbitrage violations (associated to kinks in the smile function), nonetheless Muzzioli (2010), by investigating the DAX-index options market, found it to perform better than σ_{MF2}.

In order to extrapolate implied volatilities outside the minimum and the maximum strike price quoted, we extend the strike price domain by using a factor u such that: $S/(1+u) \leq K \leq S(1+u)$. For the current implementation u has been chosen to be equal to *10*, since in Muzzioli (2010) it has been shown that the truncation bias is likely to be negligible for values of u greater than *0.3*. Therefore we expect our results not to be affected by truncation errors. In order to have a sufficient discretization of the integration domain, given that FTSEMIB index values in the observed time period are greater than 17673 index points, we compute strikes spaced by an interval $\Delta K = 10$, since in Muzzioli (2010) it has been shown that a strike price discreteness of *1%* is enough to ensure an insignificant discretization error. Finally, we use the Black and Scholes formula in order to convert implied volatilities into call prices.

Corridor implied volatility (σ_{CIV}) is computed as follows:

$$\sigma_{CIV} = \sqrt{\frac{2e^{rT}}{T}\int_{B_1}^{B_2}\frac{M(K,T)}{K^2}dK} \approx \sqrt{\frac{e^{rT}}{T}\sum_{i=1}^{m}\left[g(T,K_i)+g(T,K_{i-1})\right]\Delta K}$$

where $\Delta K = (K_{\max} - K_{\min})/m$, m is the number of abscissas, $K_i = K_{\min} + i\Delta K$, $0 \leq i \leq m$, $g(T,K_i) = [\min(C(T,K_i), P(T,K_i)]/K_i^2$, $K_{\max} = B_2$, $K_{\min} = B_1$, $B_1 = H_0^{-1}(p)$ and $B_2 = H_0^{-1}(1-p)$ where p = *0.25, 0.10, 0.05, 0.025* respectively for CIV1, CIV2, CIV3, CIV4, and the trapezoidal rule is used in order to increase the accuracy of integration. From CIV1 to CIV4 we explore wider corridor implied volatility measures and we expect CIV4 to be the more similar to model-free implied volatility. The barriers B_1 and B_2 are computed by looking at the risk-neutral distribution obtained by fitting an implied binomial tree with 100 levels to quoted option prices, as described in Section 5. As the implied tree yields a discrete cumulative distribution, $H(x) = P(X \leq x) = \sum_{t \leq x} p(t)$, the barrier level x has been chosen to be the average between x_1 and x_2, where x_1 and x_2 are the barrier levels such that $P(X \leq x_1)$ and ($P(X \leq x_2)$) are the closest to the desired p. The implied tree is fitted to the volatility smile computed by using the same interpolation-extrapolation technique used in Jiang and Tian (2007), the same used for the computation of σ_{MF2}. In fact the extrapolation technique used for σ_{MF1}, that supposes constant volatility outside quoted strike prices, yields several no-arbitrage replacements, since kinks in the smile function are usually associated to no-arbitrage violations. Last, in order to have an estimate consistent with the computational methodology of traded volatility indexes, we compute a corridor measure (σ_{CIV5}) with barriers equal to the lowest and highest strike price quoted.

We report descriptive statistics for the volatility series in Table 2. Figures 1 and 2 plot the volatility series in our sample period. On average realised volatility is lower and less volatile than option based volatility estimates, indicating that variance risk is priced with a substantial risk premium (Carr and Wu, 2009). Model-free implied volatilities are on average higher than BS which is higher than CIV measures (as in Andersen and Bondarenko (2007). As expected, CIV measures are higher as long as the corridor width increases, CIV5, obtained with only quoted strike prices is much higher than CIV4, obtained with a cut of 0.025, indicating that deep out-of-the-money options carry a very high implied volatility. Among model-free measures, MF1, obtained with natural splines extrapolation, is lower than MF2, obtained with clamped splines extrapolation. The volatility series are on average skewed (long right tail except realised volatility) and leptokurtic (except CIV1 and CIV5) and the hypothesis of a normal distribution is rejected for realised volatility and BS, indicating the presence of extreme movements in volatility.

[Table 2 about here]

[Figures 1 and 2 about here]

8. The results.

In order to gauge the forecasting performance of the different volatility measures, we resort to popular evaluation metrics[2] widely used in the literature (see e.g. Poon and Granger (2003)). In particular, as indicators of the goodness of fit, we use the MSE, the RMSE, the MAE, the MAPE and the QLIKE, defined as follows:

$$MSE = \frac{1}{m}\sum_{i=1}^{m}(\sigma_i - \sigma_r)^2$$

$$RMSE = \sqrt{\frac{1}{m}\sum_{i=1}^{m}((\sigma_i) - (\sigma_r))^2}$$

$$MAE = \frac{1}{m}\sum_{i=1}^{m}\left|\sigma_i - \sigma_r\right|$$

$$MAPE = \frac{1}{m}\sum_{i=1}^{m}\left|\frac{\sigma_i - \sigma_r}{\sigma_r}\right|$$

$$\text{QLIKE} = \frac{1}{m}\sum_{i=1}^{m}\left(\ln(\sigma_i) + \frac{\sigma_r}{\sigma_i}\right)$$

where σ_i is the volatility forecast (*i=CIV1, CIV2, CIV3, CIV4, CIV5, BS, MF1, MF2)*, σ_r is the subsequent realised volatility, *m* is the number of observations. The MSE, RMSE and the MAE are indicators of absolute errors, while the MAPE indicates the percentage error. The QLIKE discriminates between positive and negative errors by assigning a larger penalty if the forecast underestimate realised volatility. Since a higher volatility is usually associated with negative market returns, the QLIKE function considers more important the correct estimation of volatility peaks than volatility minima.

An important issue in ranking volatility forecasts is that the forecasted quantity is not observable, even ex-post. As a proxy for the true volatility is used, the substitution of a different volatility proxy may change the ranking of the different volatility forecasts. On this point, we remark that our computation methodology that exploits intra-daily five-minutes squared returns, has been shown to provide large gains in terms of consistent ranking with respect to other more noisy volatility proxies (Patton and Sheppard (2007), Hansen and Lunde (2006)).

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[2] Mincer-Zarnowitz regressions are widely used in the literature in order to assess the unbiasedness and efficiency (with respect some historical measure of volatility) of the volatility forecasts. In order to avoid the telescoping overlap problem described in Christensen et al. (2001) forecasts are usually sampled at a monthly frequency (see e.g. Jiang and Tian, 2005). Given the limited sample at our disposal, we leave the investigation of the unbiasedness and efficiency of the volatility forecasts for future research.

8.1 The results for the 30-day horizon.

The results for the 30-day forecast horizon are reported in Table 3. According to all the indicators, CIV measures perform better than both BS implied volatility and MF implied measures. The best performance is obtained by CIV1 (narrowest corridor) and as long as the corridor width becomes larger the fit gradually deteriorates. CIV5 (the worst among CIV measures) is inferior to BS implied volatility, but superior to MF measures. Overall, BS implied volatility performs better than model-free measures. Among model-free measures the best implementation technique is the natural splines extrapolation (a similar result was obtained in Muzzioli (2010)). All option based volatility measures substantially over predict subsequent realised volatility. The narrowest the corridor of strike prices used, the best the forecasting performance. This points out to a very low degree of information of deep-out-of-the-money options.

[Table 3 about here]

In order to see if the differences in forecasting performance are significant from a statistical point of view, we compare the predictive accuracy of the forecasts by computing the Diebold and Mariano test statistic (for more details see Diebold and Mariano (1995)). We concentrate the attention on two ranking functions (the MSE and the QLIKE) that are considered as robust to the presence of noise in the volatility proxy (Patton (2010)).

[Tables 4 and 5 about here]

The pair-wise comparisons are reported in Tables 4 and Table 5 for the MSE and the QLIKE ranking functions respectively (t-statistics along with the p-values). Note that a positive (negative) t-statistic indicates that the row model produced larger (smaller) average loss than the column model. The Diebold and Mariano test statistic under the null of equal predictive accuracy is distributed as a $N(0,1)$. The null of equal predictive accuracy is strongly rejected at the 1% level in all cases, except for BS and CIV4 which are not clearly distinguishable. Both the MSE and the QLIKE point to the same ranking and thus corroborate the results found in Table 3. As an additional test, the modified Diebold and Mariano test (Harvey et al. (1997)), which is useful in moderate-sized samples has been implemented (the results are available upon request) and the findings remain unaltered.

8.2. The results for the one-day horizon.

In order to see if the implied volatility conveyed by option prices can also be considered as forward looking indicator of next-day realised volatility, in this Section we compare the predictive accuracy of the volatility measures as forecasts of realised volatility in the subsequent day. This can be very important in a Value at risk (VaR) framework, where the implied volatility measures can be used directly to proxy for volatility in the VaR specification (see e.g. Giot (2005a)).

In order to measure daily realised volatility, we compute the squared root of the sum of five-minute frequency squared index returns over the next day:

$$\sigma_r = \sqrt{\sum_{t=1}^{n}\left[\ln\left(\frac{S_{t+1}}{S_t}\right)\right]^2} .$$

where n is the number of index prices spaced by five minutes in the next day.

Following Giot (2005a), we compute implied volatility measures for the forward looking time horizon of 1-day from the 30-day measures expressed in annualized terms as:

$$\sigma_{i,1,t} = \sqrt{\frac{1}{365}}\sigma_{i,t}$$

for i=BS, MF1, MF2, CIV1, CIV2,CIV3, CIV4, CIV5, where $\sigma_{i,1,t}$ is the expected volatility over the next day.

[Table 6 about here]

[Figures 3 and 4 about here]

We report descriptive statistics for the volatility series in Table 6. Figures 3 and 4 plot the volatility series in our sample period. On average realised volatility is much more volatile than option based volatility estimates since option based estimates reflect the implied volatility over the next 30-days and can be considered as a smoothed expectation of realised volatility. Model-free implied volatility is on average higher than BS, which is higher than CIV measures. As expected, CIV measures are higher as long as the corridor width increases, with CIV5 being on average the highest. The volatility series are positively skewed and leptokurtic (except MF1 and CIV1) and the hypothesis of a normal distribution is rejected for both realised volatility and BS.

[Table 7 about here]

The results reported in Table 7 almost confirm the 30-day horizon results obtained in the previous Section, even if some differences emerge. CIV1 is the best forecast only for the MAPE, for the other indicators the preferred forecast is CIV2, closely followed by CIV3. BS obtains a better performance than in the 30-day horizon case, since it is on average one of the best forecasts. CIV5 is the worst forecasts among CIV measures. Model-free forecasts obtain the worst performance, with MF1 better than MF2. Looking at the Diebold and Mariano tests, reported in Table 8, that are performed with respect to the MSE loss function, we can see that Black Scholes implied volatility is better (almost at the 1% level) than CIV5 and model-free measures, corridor implied volatilities (unique exception CIV1) are better than model-free measures. However, it is impossible to clearly distinguish among BS and CIV measures (except for CIV5 that obtains clearly a worse performance). Among model-free measures, the best performance at the 1% level is obtained by MF1 that uses natural splines, the worst by MF2 that extrapolates with clamped splines. The Diebold and Mariano tests have been performed also with respect to the QLIKE loss function and similar results, reported in Table 9, have been obtained. The findings are confirmed by the modified Diebold and Mariano test (Harvey et al. (1997)) which has been implemented (the results are available upon request).

[Tables 8 and 9 about here]

9. Properties of the implied volatility index

From the analysis conducted in Section 8, CIV1 emerges as the best volatility index on a 30-day horizon and one of the best indexes for next-day subsequent realised volatility. As we deem more important the results obtained in the 30-day horizon, given the natural interpretation of implied volatility as the expected value of realised volatility over the life time of the option, we choose CIV1 as the suggested implied volatility index. In this Section we investigate the relationship between the suggested implied volatility index and the returns of the underlying stock index. Following Skiadopoulos (2004), who conducts a similar analysis for the proposed Greek volatility index, we investigate first if the volatility index is an indicator of current risk; second we investigate if the volatility index can be considered as an indicator of future market returns. On the first issue, several papers (see e.g Whaley (2000), Skiadopoulos (2004), Giot (2005b)) have found a negative relationship between volatility changes and index returns, since negative returns are usually associated with an increase in volatility. In fact, bad news as measured by negative returns, increase the risk perception, boosting the purchase of put options and therefore augmenting implied volatility. Thus the volatility index is expected to spike upward (downward) during periods of market turmoil (calm).

Figure 5 shows the evolution of the proposed index and the FTSE-MIB index over the sample period. As expected, there seems to be a negative relationship between changes in the market index and changes in the volatility index: when the FTSE-MIB index increases (decreases) the volatility index decreases (increases). Table 10 reports the descriptive statistics of FTSE-MIB daily index returns (continuously compounded) and daily changes in CIV1 ($\Delta CIV1 = CIV1_t - CIV1_{t-1}$). The correlation coefficient between FTSE-MIB index returns and changes in CIV1 is -0,478, thus suggesting the existence of some leverage effect.

[Table 10 about here]

[Figure 5 about here]

In order to investigate the relationship between FTSE-MIB index returns and changes in CIV1, we regress:

$$R_t = \alpha + \beta_1 \Delta CIV1_t + \varepsilon_t$$

The regression results are (t-values in brackets):

$$R_t = 0.00\,(0.34) - 0.42(-3.47)\Delta CIV1_t$$

The R^2 is 0.23 and the slope coefficient is highly significant. By looking at the sign of the slope coefficient we can observe that there is a negative relationship between the volatility index and the FTSE-MIB: if the volatility index rises (decreases) by 0.01, the FTSE-MIB decreases (rises) by 0.0042.

In order to control for asymmetric effects, we divide the changes in CIV1 in positive ($\Delta CIV1_t^+ = \Delta CIV1_t \; if \; \Delta CIV1_t > 0, \Delta CIV1_t^+ = 0$) and negative ones ($\Delta CIV1_t^- = \Delta CIV1_t \; if \; \Delta CIV1_t < 0, \Delta CIV1_t^- = 0$) as follows:

$$R_t = \alpha + \beta_1 \Delta CIV1_t^- + \beta_2 \Delta CIV1_t^+ + \varepsilon_t$$

The regression results are (t-values in brackets):

$$R_t = 0.00\,(1.42) - 0.25\,(-2.00)\Delta CIV1_t^- - 0.59\,(-3.02)\Delta CIV1_t^+$$

The R^2 is 0.24; the slope coefficient of positive changes in the volatility index is highly significant, while the slope coefficient of negative changes is significantly different from zero only at the 5% level. As the slope coefficient of positive changes in the volatility index is more than twice the slope coefficient in negative changes, a rise in implied volatility affects the returns more than twice than a decrease in implied volatility. As expected, an upward volatility spike is more important than a downward volatility spike. Therefore CIV1 can be considered as a measure of market fear more than a measure of market excitement, since the market reacts more negatively to increases in volatility than it reacts positively when the volatility index decreases.

As for the second goal of our investigation, we want to assess if the volatility index can be considered as an indicator of future market returns. Skiadopulos (2004) found that investors can use past market returns in order to forecast future changes in implied volatility. Giot (2005b) has argued that positive returns are to be expected as a consequence of high levels of implied volatility. A possible explanation is that if volatility is high investors are over-reacting selling stocks without a clear rationale, as a consequence stocks could be undervalued and therefore high volatility can be viewed as a "buy" signal. To this end we estimate a Vector Autoregression (VAR) model as follows:

$$\Delta CIV1_t = c + \sum_{l=1}^{K} a_l \Delta CIV1_{t-l} + \sum_{l=1}^{K} b_l R_{t-l} + u_t$$

$$R_t = c + \sum_{l=1}^{K} a_l R_{t-l} + \sum_{l=1}^{K} b_l \Delta CIV1_{t-l} + u_t$$

with k=2, chosen in order to keep the model as parsimonious as possible, according to the Schwarz criterion.

We perform a Granger causality test, the null hypothesis is $b_i=0$, $i=1,2$ in order to see if R does not Granger cause $\Delta CIV1$ in the first regression and $\Delta CIV1$ does not Granger cause R in the second regression. The results are reported in Table 11. We can observe that while returns can not be explained by past returns or past differences in CIV1, changes in CIV1 can be explained by past changes in CIV1 (the negative sign indicates the mean reverting nature of CIV1) and only marginally by past returns (that are significant only at the 10% level). From the Granger causality test we can see that the returns are marginally (only at the 10% level) useful in order to forecast future volatility, while the changes in CIV1 do not contain any information about future returns of the FTSE-MIB. Therefore, our results suggest that an investor can use the returns on the FTSE-MIB index in order to predict future movements of the implied volatility and set up an appropriate option strategy, but changes in implied volatility have no explanatory power in predicting the underlying FTSE-MIB returns.

[Table 11 about here]

10. Conclusions

In this paper several option based measures have been analysed in the Italian FTSE-MIB index options market in order to devise a volatility index for the latter. Black-Scholes implied volatility, two model-free measures (obtained with different extrapolation techniques), four corridor implied volatility measures that vary in the choice of the corridor width and one corridor measure which closely mimics the construction methodology of traded volatility indexes, have been compared. Two forecasting horizons have been analysed: the 30-day and the one-day horizon. The volatility forecasts have been ranked on the basis of popular evaluation measures and the difference in the forecasting performance has been statistically scrutinised on the basis of the Diebold and Mariano test of equal predictive accuracy with the use of robust loss functions.

As for the 30-day horizon, according to all the indicators, CIV measures perform better than both BS implied volatility and MF implied measures. The best performance is obtained by CIV1 (narrowest corridor, that corresponds to an overall 50% cut) and as long as the corridor width becomes larger the fit gradually deteriorates. CIV5 (the one obtained with a methodology similar to the one used for traded volatility indexes) is inferior to BS implied volatility, but superior to MF measures. Overall, BS implied volatility performs better than model-free measures. Among model-free measures the best implementation technique is the natural splines extrapolation (a similar result was obtained in Muzzioli (2010)). As for the one-day horizon the results are quite similar, even if in this case the narrowest CIV measure is the best only for one ranking measure. According to the Diebold and Mariano test CIV5 and model-free measures still obtain clearly the worst performance, however it is impossible to discriminate among corridor implied volatilities and Black-Scholes implied volatility. Overall, the results point out to a very low degree of information of deep out-of-the-money options, which are notoriously less liquid than around-the-money options. In order to devise an implied volatility index for the Italian stock market the results do not suggest the use of quoted strike price in order to cut the integration domain: an average of at-the-money Black-Scholes implied volatility would be better. Rather, the results suggest to “cut the wings” with some corridor implied volatility: the 50% cut (CIV1) is the best in our sample. As in Andersen and Bondarenko (2007) we find that stretching the corridor yields to a better forecasting performance.

On the basis of our analysis CIV1 has been selected as the suggested volatility index for the Italian market. As for the properties of the proposed volatility index, we have analysed both the contemporaneous relationship between implied volatility changes and market returns and the usefulness of the proposed index in forecasting future market returns. Our results advise that the suggested volatility index can be considered more as a measure of market fear than a measure of market excitement, since the market reacts more negatively to increases in volatility than it reacts positively when the volatility index decreases. Moreover, the proposed volatility index can not be used in order to predict future market movements; rather there is weak evidence that an investor can use the returns on the FTSE-MIB index in order to predict future movements of the implied volatility and set up an appropriate option strategy.

The present paper lends itself to be extended in many directions. High on the research agenda are the investigation of other corridor measures with asymmetric cuts of the risk neutral distribution and the use of a longer dataset in order to investigate the unbiasedness and efficiency of the different volatility forecasts.

Appendix. The VIX index and the theoretical definitions of variance.

In this Appendix we clarify the link between model-free implied volatility developed by Britten-Jones and Neuberger (2000), fair value of future variance developed in Demeterfi et al. (1999) and the VIX index formula. We follow Jiang and Tian (2007).

In order to show that the fair value of future variance developed in Demeterfi et al. (1999) coincides with the Britten-Jones and Neuberger variance equation we start from Britten Jones and Neuberger (2000) equation in forward prices:

$$E^Q\left[\int_0^T\left(\frac{dS_t}{S_t}\right)^2\right]=\frac{2}{T}\int_0^\infty\frac{e^{rT}C(T,K)-\max(S_0e^{rT}-K,0)}{K^2}dK \qquad \text{(A1)}$$

Partitioning the integral into two segments at $K=F_0=S_0e^{rT}$ we get:

$$E^Q\left[\int_0^T\left(\frac{dS_t}{S_t}\right)^2\right]=\frac{2e^{rT}}{T}\left[\int_0^{F_0}\frac{C(T,K)-S_0+Ke^{-rT}}{K^2}dK+\int_{F_0}^\infty\frac{C(T,K)}{K^2}dK\right] \qquad \text{(A2)}$$

And using the put-call parity we obtain:

$$E^Q\left[\int_0^T\left(\frac{dS_t}{S_t}\right)^2\right]=\frac{2e^{rT}}{T}\left[\int_0^{F_0}\frac{P(T,K)}{K^2}dK+\int_{F_0}^\infty\frac{C(T,K)}{K^2}dK\right] \qquad \text{(A3)}$$

As in the reality of financial markets it is very unlikely to find a call or a put with strike price equal to the forward price, we partition the integral into three segments, using $K=K_0$, where K_0 is taken to be an arbitrary stock price, that defines the boundary between calls and puts, typically chosen to be the strike immediately below F_0 (at-the-money forward stock level):

$$E^Q\left[\int_0^T\left(\frac{dS_t}{S_t}\right)^2\right]=\frac{2e^{rT}}{T}\left[\int_0^{K_0}\frac{P(T,K)}{K^2}dK+\int_{K_0}^{\infty}\frac{C(T,K)}{K^2}dK+\int_{K_0}^{F_0}\frac{P(T,K)-C(T,K)}{K^2}dK\right] \tag{A4}$$

Using the put-call parity for the put in the last integral yields:

$$E^Q\left[\int_0^T\left(\frac{dS_t}{S_t}\right)^2\right]=\frac{2e^{rT}}{T}\left[\int_0^{K_0}\frac{P(T,K)}{K^2}dK+\int_{K_0}^{\infty}\frac{C(T,K)}{K^2}dK+\int_{K_0}^{F_0}\frac{Ke^{-rT}-S_0}{K^2}dK\right] \tag{A5}$$

Integrating out the last term yields:

$$E^Q\left[\int_0^T\left(\frac{dS_t}{S_t}\right)^2\right]=\frac{2e^{rT}}{T}\left[\int_0^{K_0}\frac{P(T,K)}{K^2}dK+\int_{K_0}^{\infty}\frac{C(T,K)}{K^2}dK+e^{-rT}\left(rT-(\frac{S_0}{K_0}e^{rT}-1)-\ln(\frac{K_0}{S_0})\right)\right] \tag{A6}$$

Equation (A6) is the fair value of future variance developed in Demeterfi et al. (1999). Note that the last term disappears if $K_0=F_0$.

In order to obtain the VIX index formula, we rewrite the last term in equation (A6) as follows:

$$\frac{2}{T}\left\{rT-\left[\frac{S_0}{K_0}e^{rT}-1\right]-\ln\left(\frac{K_0}{S_0}\right)\right\}=\frac{2}{T}\left[\ln\left(\frac{F_0}{K_0}\right)-\left(\frac{F_0}{K_0}-1\right)\right] \tag{A7}$$

Applying the Taylor series expansion and ignoring terms of order higher than the second we get:

$$\ln\left(\frac{F_0}{K_0}\right)\approx\left(\frac{F_0}{K_0}-1\right)-\frac{1}{2}\left(\frac{F_0}{K_0}-1\right)^2 \tag{A8}$$

and therefore:

$$\frac{2}{T}\left\{rT-\left[\frac{S_0}{K_0}e^{rT}-1\right]-\ln\left(\frac{K_0}{S_0}\right)\right\}\approx-\frac{1}{T}\left(\frac{F_0}{K_0}-1\right)^2 \tag{A9}$$

From equations (A6) and (A9) we get:

$$E^Q\left[\int_0^T\left(\frac{dS_t}{S_t}\right)^2\right]=\frac{2}{T}\left[e^{rT}\int_0^{K_0}\frac{P(T,K)}{K^2}dK+e^{rT}\int_{K_0}^{\infty}\frac{C(T,K)}{K^2}dK\right]-\frac{1}{T}\left(\frac{F_0}{K_0}-1\right)^2 \tag{A10}$$

The VIX index is computed as linear interpolation of the near (i=1) and next term (i=2) variances (σ_i^2):

$$\sigma_i^2=\frac{2}{T_i}\sum_j\frac{\Delta K_{i,j}}{K_{i,j}^2}\mathrm{e}^{\mathrm{r_i T_i}}M(K_{i,j})-\frac{1}{T_i}\left[\frac{F_i}{K_{i,0}}-1\right]^2 \tag{A11}$$

Equation (A11) is the discrete time version of equation (A10), where the integral is substituted by a discrete summation (discretization error) and the integration interval has been truncated to the limited range of strike prices that fulfil the liquidity constraints described in Section 3 (truncation error). Beside truncation and discretization errors, the VIX index is also subject to an approximation error given by the Taylor series expansion of the log function. This latter error is likely to be negligible since K_0 is chosen to be the closest strike below F_0.

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Figure 1. Realised volatility, Black-Scholes implied volatility and model-free measures (σ_{MF1} , σ_{MF2}) for the 30-days horizon.

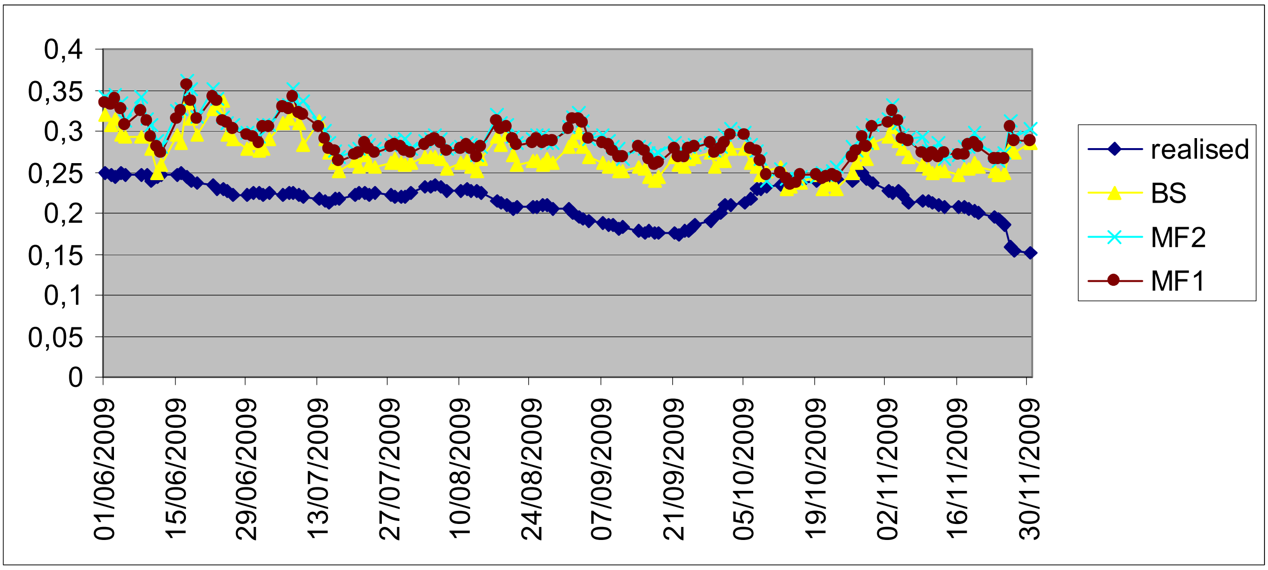

Figure 2. Realised volatility, CIV measures for the 30-days horizon.

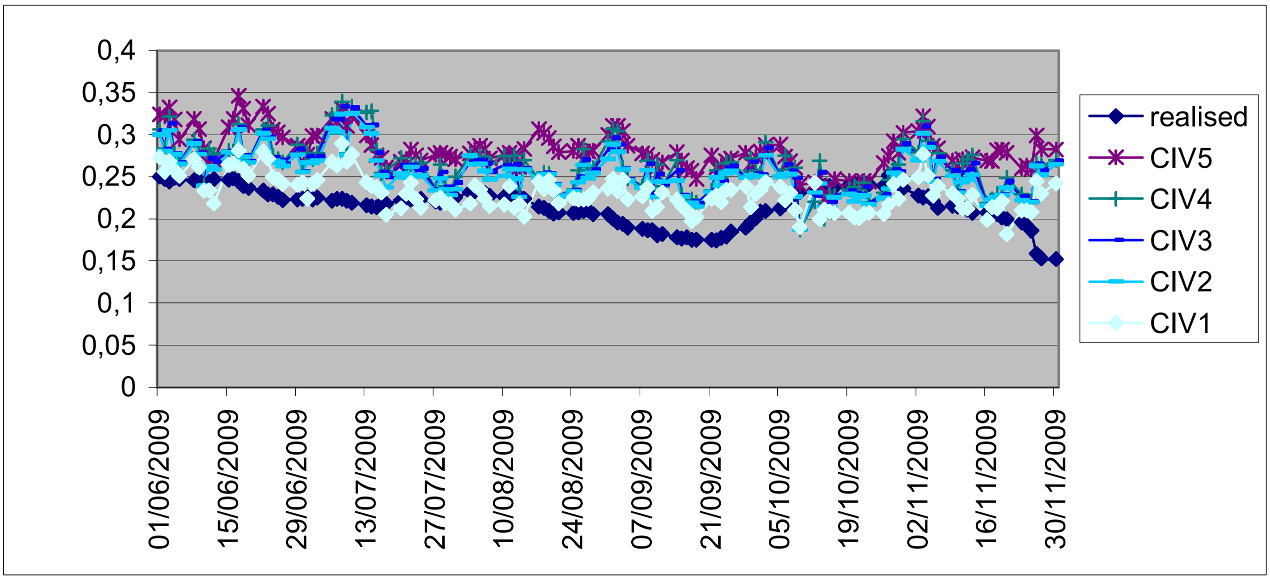

Figure 3. Realised volatility, Black-Scholes implied volatility and model-free measures (σ_{MF1}, σ_{MF2}) for the one-day horizon.

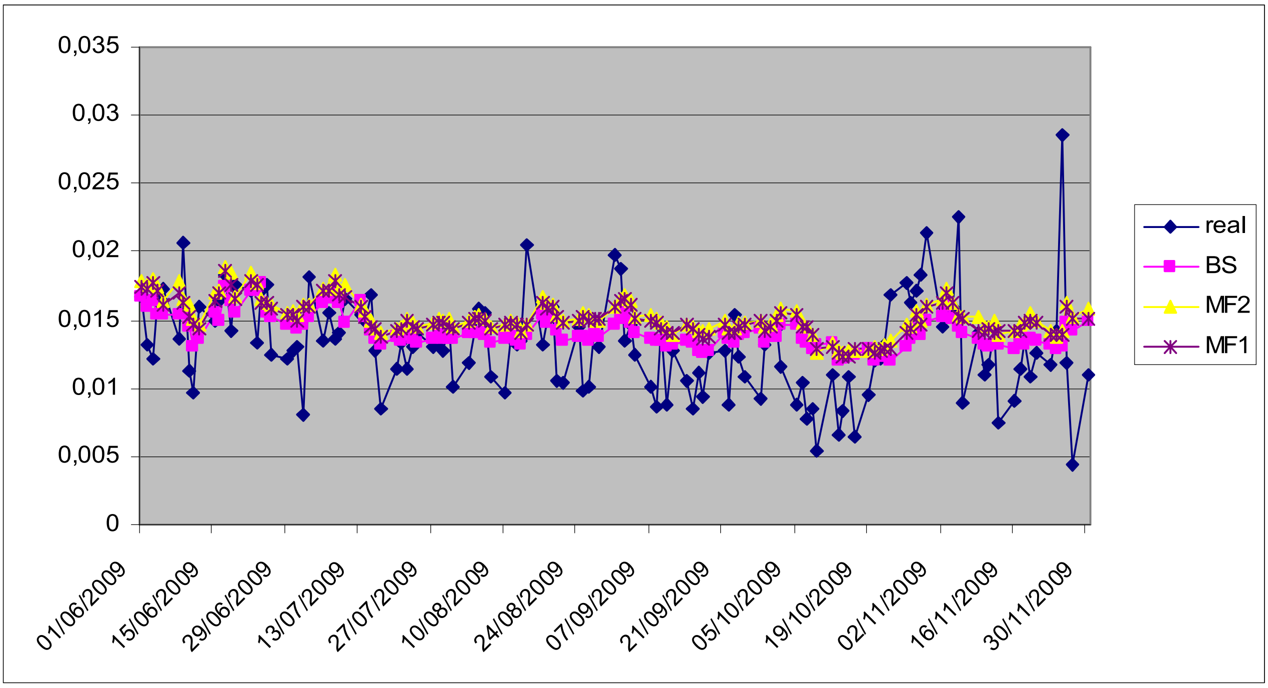

Figure 4. Realised volatility, CIV measures for the one-day horizon.

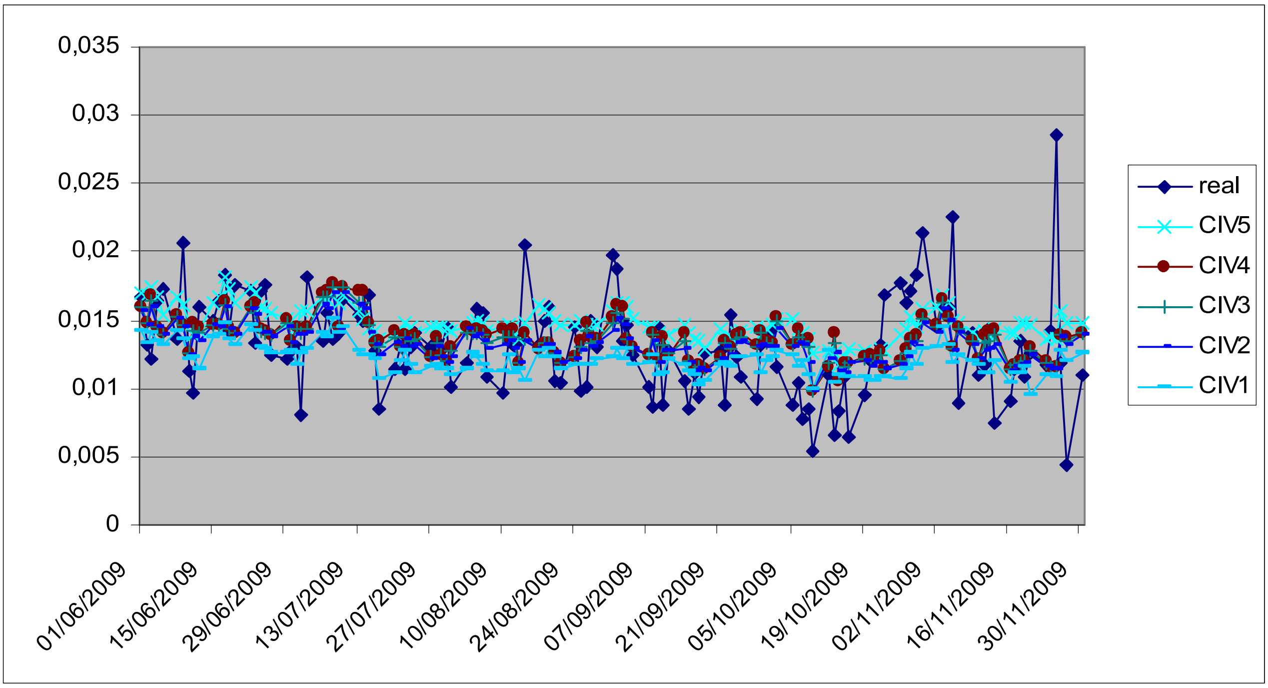

Figure 5. Corridor implied volatility CIV1 and the FTSE-MIB over the sample period.

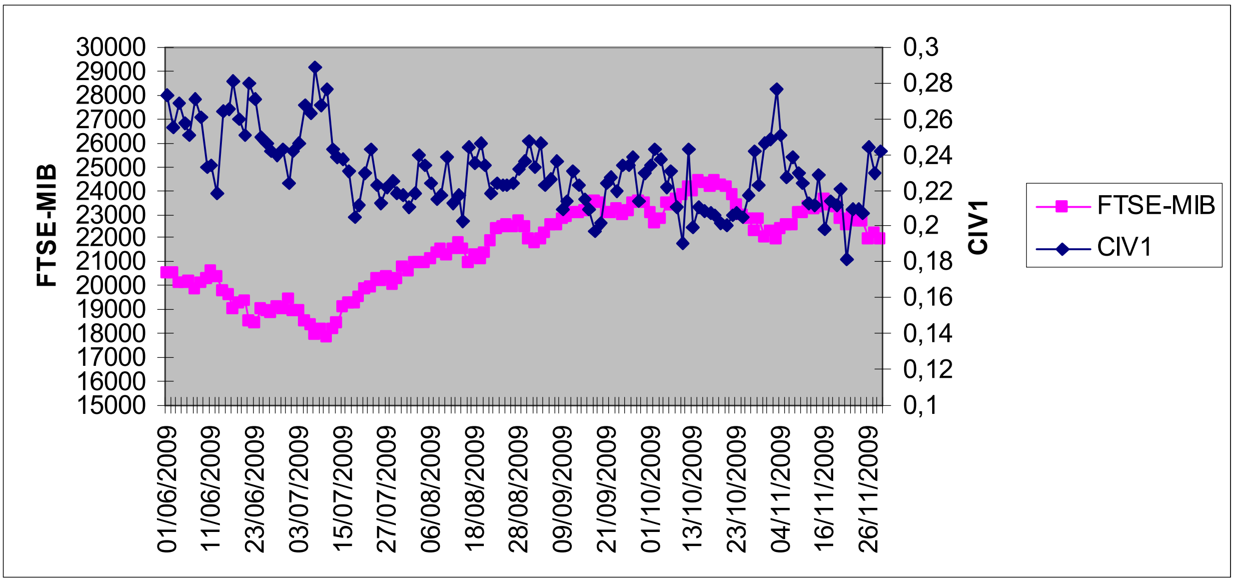

Table 1. No-arbitrage conditions and corresponding replacements.

	No-arbitrage condition	No-arbitrage replacement
Nodes in the upper part of the tree	$S_{i,j-1}/e^{(r-\delta)\Delta t} < S_{i,j} < F_{i.-1,j-1}$	$S_{i,j} = \dfrac{S_{i,j-1}/e^{(r-\delta)\Delta t} + F_{i.-1,j-1}}{2}$
Nodes in the lower part of the tree	$F_{i.,j-1} < S_{i,j} < F_{i.-1,j-1}$	$S_{i,j} = \dfrac{F_{i.,j-1} + F_{i.-1,j-1}}{2}$
Nodes at the boundary of the tree in the upper part	$S_{i,j-1}/e^{(r-\delta)\Delta t} < S_{i,j}$	$S_{i,j} = S_{i,j-1}/e^{2(r-\delta)\Delta t}$
Nodes at the boundary of the tree in the lower part	$S_{i-1,j-1} * e^{(r-\delta)\Delta t} > S_{i,j}$	$S_{i,j} = S_{i,j-1} * e^{2(r-\delta)\Delta t}$

Table 2. Descriptive statistics.

Statistic	σ_r	σ_{BS}	σ_{MF1}	σ_{MF2}	σ_{CIV1}	σ_{CIV2}	σ_{CIV3}	σ_{CIV4}	σ_{CIV5}
mean	0.216	0.270	0.287	0.292	0.231	0.254	0.258	0.263	0.282
std dev	0.022	0.023	0.024	0.025	0.021	0.025	0.027	0.028	0.023
skewness	-0.704	0.752	0.379	0.397	0.479	0.469	0.397	0.307	0.305
kurtosis	3.004	3.200	3.019	3.120	2.885	3.181	3.281	3.171	2.972
Jarque Bera	10.840	12.550	3.152	3.529	5.088	4.999	3.876	2.225	2.033
p-value	0.004	0.002	0.207	0.171	0.078	0.082	0.144	0.328	0.362

Table 3. Predictive accuracy of the different volatility measures, 30-day horizon.

	σ_{BS}	σ_{MF1}	σ_{MF2}	σ_{CIV1}	σ_{CIV2}	σ_{CIV3}	σ_{CIV4}	σ_{CIV5}
MSE	0.0037	0.0059	0.0067	0.0009	0.0023	0.0027	0.0032	0.0052
RMSE	0.0610	0.0767	0.0817	0.0306	0.0475	0.0519	0.0562	0.0719
MAE	0.0547	0.0708	0.0757	0.0251	0.0408	0.0451	0.0488	0.0661
MAPE	0.2645	0.3403	0.3637	0.1221	0.1977	0.2177	0.2353	0.3183
QLIKE	-0.5067	-0.4932	-0.4887	-0.5267	-0.5170	-0.5140	-0.5110	-0.4974

Table 4. Diebold and Mariano tests: pair-wise comparisons (MSE).

	σ_{BS}	σ_{MF1}	σ_{MF2}	σ_{CIV1}	σ_{CIV2}	σ_{CIV3}	σ_{CIV4}	σ_{CIV5}
σ_{BS}		-9.94	-10.79	6.39	5.78	3.86	1.88	-7.99
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.06)	(0.00)
σ_{CIV1}		-9.04	-9.39		-5.16	-5.57	-5.89	-8.84
		(0.00)	(0.00)		(0.00)	(0.00)	(0.00)	(0.00)
σ_{CIV2}		-16.81	-9.71			-9.24	-6.89	-8.48
		(0.00)	(0.00)			(0.00)	(0.00)	(0.00)
σ_{CIV3}		-8.12	-8.98				-6.53	-7.21
		(0.00)	(0.00)				(0.00)	(0.00)
σ_{CIV4}		-6.77	-7.85					-5.60
		(0.00)	(0.00)					(0.00)
σ_{CIV5}		-8.24	-9.45					
		(0.00)	(0.00)					
σ_{MF1}			-8.88					
			(0.00)					

Note. The Table reports the t-statistic and associated p-value for the Diebold and Mariano test of equal predictive accuracy for each couple of forecasts. The loss function used is the MSE.

Table 5. Diebold and Mariano tests: pair-wise comparisons (QLIKE).

	σ_{BS}	σ_{MF1}	σ_{MF2}	σ_{CIV1}	σ_{CIV2}	σ_{CIV3}	σ_{CIV4}	σ_{CIV5}
σ_{BS}		-9.91	-10.90	6.87	5.86	4.12	2.26	-8.02
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.03)	(0.00)
σ_{CIV1}		-8.43	-8.82		-5.23	-5.72	-6.16	-8.15
		(0.00)	(0.00)		(0.01)	(0.00)	(0.00)	(0.00)
σ_{CIV2}		-8.60	-9.23			-7.17	-7.74	-7.99
		(0.00)	(0.00)			(0.00)	(0.00)	(0.00)
σ_{CIV3}		-7.77	-8.54				-7.52	-6.93
		(0.00)	(0.00)				(0.00)	(0.00)
σ_{CIV4}		-6.60	-7.51					-5.58
		(0.00)	(0.00)					(0.00)
σ_{CIV5}		-8.63	-10.02					
		(0.00)	(0.00)					
σ_{MF1}			-9.36					
			(0.00)					

Note. The Table reports the t-statistic and associated p-value for the Diebold and Mariano test of equal predictive accuracy for each couple of forecasts. The loss function used is the QLIKE.

Table 6. Descriptive statistics, one day horizon.

Statistic	σ_r	σ_{BS}	σ_{MF1}	σ_{MF2}	σ_{CIV1}	σ_{CIV2}	σ_{CIV3}	σ_{CIV4}	σ_{CIV5}
mean	0.013	0.014	0.015	0.015	0.012	0.013	0.013	0.014	0.015
std dev	0.004	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
skewness	0.658	0.752	0.304	0.379	0.479	0.469	0.397	0.307	0.397
kurtosis	4.916	3.200	2.972	3.019	2.885	3.181	3.280	3.171	3.120
Jarque Bera	29.510	12.550	2.033	3.152	5.088	4.999	3.876	2.224	3.529
p-value	0.000	0.001	0.361	0.206	0.079	0.082	0.144	0.328	0.171

Table 7. Predictive accuracy of the different volatility measures, 1 day horizon.

	σ_{BS}	σ_{MF1}	σ_{MF2}	σ_{CIV1}	σ_{CIV2}	σ_{CIV3}	σ_{CIV4}	σ_{CIV5}
MSE	$0.118 \cdot 10^{-4}$	$0.142 \cdot 10^{-4}$	$0.151 \cdot 10^{-4}$	$0.126 \cdot 10^{-4}$	$0.113 \cdot 10^{-4}$	$0.118 \cdot 10^{-4}$	$0.123 \cdot 10^{-4}$	$0.131 \cdot 10^{-4}$
RMSE	$0.343 \cdot 10^{-2}$	$0.377 \cdot 10^{-2}$	$0.389 \cdot 10^{-2}$	$0.355 \cdot 10^{-2}$	$0.336 \cdot 10^{-2}$	$0.343 \cdot 10^{-2}$	$0.351 \cdot 10^{-2}$	$0.362 \cdot 10^{-2}$
MAE	$0.258 \cdot 10^{-2}$	$0.296 \cdot 10^{-2}$	$0.313 \cdot 10^{-2}$	$0.262 \cdot 10^{-2}$	$0.245 \cdot 10^{-2}$	$0.251 \cdot 10^{-2}$	$0.260 \cdot 10^{-2}$	$0.280 \cdot 10^{-2}$
MAPE	0.236	0.277	0.292	0.206	0.211	0.220	0.230	0.261
QLIKE	-3.332	-3.326	-3.324	-3.329	-3.333	-3.332	-3.331	-3.328

Table 8. Diebold and Mariano tests: pair-wise comparisons, one day horizon (MSE).

	σ_{BS}	σ_{CIV5}	σ_{MF1}	σ_{MF2}	σ_{CIV1}	σ_{CIV2}	σ_{CIV3}	σ_{CIV4}
σ_{BS}		-2.72	-3.87	-4.30	-0.55	0.67	0.04	-1.02
		(0.01)	(0.00)	(0.00)	(0.58)	(0.51)	(0.97)	(0.31)
σ_{CIV1}		-0.30	-0.79	-1.19		1.46	0.77	0.21
		(0.76)	(0.43)	(0.23)		(0.15)	(0.44)	(0.83)
σ_{CIV2}		-1.70	-2.35	-2.82			-2.07	-2.55
		(0.09)	(0.02)	(0.01)			(0.04)	(0.01)
σ_{CIV3}		-1.50	-2.28	-2.82				-2.96
		(0.14)	(0.02)	(0.01)				(0.00)
σ_{CIV4}		-1.00	-1.95	-2.59				
		(0.32)	(0.05)	(0.01)				
σ_{CIV5}			-5.65	-5.63				
			(0.00)	(0.00)				
σ_{MF1}				-4.59				
				(0.00)				

Note. The Table reports the t-statistic and associated p-value for the Diebold and Mariano test of equal predictive accuracy for each couple of forecasts. The loss function used is the MSE.

Table 9. Diebold and Mariano tests: pair-wise comparisons, one day horizon (QLIKE).

	σ_{BS}	σ_{CIV5}	σ_{MF1}	σ_{MF2}	σ_{CIV1}	σ_{CIV2}	σ_{CIV3}	σ_{CIV4}
σ_{BS}		-2.60	-3.62	-3.93	-0.59	0.63	0.13	-0.62
		(0.01)	(0.00)	(0.00)	(0.56)	(0.53)	(0.90)	(0.54)
σ_{CIV1}		-0.09	-0.47	-0.75		1.71	0.91	0.45
		(0.93)	(0.64)	(0.46)		(0.09)	(0.37)	(0.66)
σ_{CIV2}		-1.49	-1.99	-2.34			-1.96	-2.36
		(0.14)	(0.05)	(0.02)			(0.05)	(0.02)
σ_{CIV3}		-1.30	-1.90	-2.29				-2.74
		(0.20)	(0.06)	(0.02)				(0.01)
σ_{CIV4}		-0.90	-1.62	-2.08				
		(0.37)	(0.11)	(0.04)				
σ_{CIV5}			-5.23	-5.21				
			(0.00)	(0.00)				
σ_{MF1}				-4.07				
				(0.00)				

Note. The Table reports the t-statistic and associated p-value for the Diebold and Mariano test of equal predictive accuracy for each couple of forecasts. The loss function used is the QLIKE.

Table 10. Descriptive statistics of daily returns on the FTSE-MIB and daily changes in CIV1.

	FTSE-MIB Returns	**ΔCIV1**
mean	0.0005	-0.0002
std. Dev.	0.015	0.017
skewness	-0.374	0.261
kurtosis	2.815	3.494
Jarque-Bera	3.213	2.801
p-value	0.200	0.246
cross correlation	-0.478	

Table 11. VAR estimates and Granger causality test between daily returns on the FTSE-MIB and daily changes in CIV1 (t-values in parentheses).

	R	**ΔCIV1**
R(-1)	-0.04	-0.19
	(-0.42)	(-1.79)
R(-2)	0.13	-0.18
	(1.29)	(-1.71)
ΔCIV1(-1)	-0.04	-0.51
	(-0.42)	(-4.94)
ΔCIV1(-2)	-0.02	-0.21
	(-0.19)	(-2.04)
c	0.00	-0.00
	(0.47)	(-0.11)

Null Hp.	**F-stat**	**p-value**
R does not Granger cause ΔCIV	2.80	(0.06)
ΔCIV does not Granger cause R	0.09	(0.92)

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CEFIN – Centro Studi di Banca e Finanza
Dipartimento di Economia Aziendale – Università di Modena e Reggio Emilia
Viale Jacopo Berengario 51, 41100 MODENA (Italy)
tel. 39-059.2056711 (Centralino) fax 059 205 6927