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Michele Bruni*

Abstract
After recalling the classical definitions of stock and flow introduced by Fisher, the paper briefly summarizes Georgescu-Roegen's analysis of the production process and his definitions of the concepts of stock and flow, and also of those of fund and services necessary in his approach. The paper does then propose new definitions of the same four concepts and explore their implications for a different and more realistic approach to labor market analysis.

Keywords: Stock-flow model, production process, labor market,

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The classical definitions of the concepts of stock and flow

The definitions of the concepts of stock and flow generally adopted in economic literature, and more specifically in labor market literature, have been criticized in a very convincing way by Georgescu-Roegen, the author of some of the most relevant contributions to economic theory published in the twentieth century. These criticisms, however, have not received much attention and recognition.

One of the first problems raised by western philosophy was the problem of change. Heraclitus argued that "Everything is in flux and nothing is at rest". Therefore, *things are not really things, they are processes*. To Parmenides, the pupil of the monotheist Xenophanes, the world of change was an illusion: the world was motionless.

If we have to believe our perception, life moves at the speed that time has on our planet and is subject to a process of continuous transformation, creation and destruction. At the empirical level we are faced by the problem of measuring both *what is* and *what becomes*. To measure what is, it would be necessary to stop time, to measure what is becoming we need a time interval. Since, we cannot stop time, the only possible way to measure the being is to use the shortest interval possible[1], knowing that the shorter the interval the more precise the measure. In the second case, the problem is that of choosing a time interval coherent with the phenomenon we intend to measure.

Starting from this general perspective, economic analysis classifies variables into two groups: stock variables and flow variables. Fisher (1986), defined the firsts as those variables that do not have a time dimension, and can therefore be measured in an instant of time; the second as those variables that can be measured only in a given time interval: "stocks relates to a point of time, flows to a stretch of time".

Later (Fischer, 1919), however, he defined a flow (FL) as the change of a stock (S) measured in two different moments of time (t-1) and t.

[1] $${}_{(t-1)}FL_t = S_t - S_{(t-1)}$$

Georgescu-Roegen analysis

Georgescu-Roegen observed that this definition obliterates any antinomy between what flows and what does not flow; in fact, it cannot distinguish the difference between what is contained in a silos in two successive moments of time and the difference between the amounts of wheat contained in two silos in the same moment[2].

Fischer had tried to solve this problem opposing to stocks, not flows, but the rate of flow. According to Georgescu-Roegen, this does not solve the problem[3]: it simply reduces the antinomy to a problem of different time dimensionality[4].

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[1] According to dimensional analysis, a field of mathematical analysis which studies units of measurement, the smallest time measurement that will ever be possible is the Planck time that is the time required for light to travel, in a vacuum, a distance of 1 Planck length, roughly 10^{-43} seconds.

[2] "The difference between two quanta of, say, wheat is also a quantum of wheat whether the two quanta refer to the same storehouse at two different instants or to two storehouses at the same instant. It is because of this truism that we are apt to commit the error of confusing stock with flow. According to formula 1 both an income over any period and a bank balance consist of dollars indistinguishable from each other. Why should we treat income and wealth as two different essences?" (N. Georgescu-Roegen, 1971).

[3] Without forgetting that "To wit, an instantaneous flow rate also refers to a point in time", (N. Georgescu-Roegen, ibidem).

[4] "According to this line of thought, the only reason against confusing the monthly rate of income with the monthly bank balance is the general principle that concepts of different dimensionality must be kept separate in

The real problem is that interpreting a flow as a change of a stock is reductive: many flows, including those generated by production, are not the outcome of the de-accumulation of a stock, but the outcome of creative processes: *stocks are always the result of the accumulation of flows, but flows are not always the result of a process of de-accumulation of stocks*. Therefore according to Georgescu-Roegen; "a flow is a stock spread out over a time interval"[5], that is a flow is a stock with a time dimension.

The production process

This discussion of the concepts of stock and flow represents the starting point of the analysis of production. Since production is a process, it can be analyzed only in real time. In order to describe a process[6] it is therefore necessary to define its physical[7] and temporal[8] borders. The choice of the borders -that allows distinguishing the process from the surrounding environment and to define its duration- depends on the goal of the analysis.

According to Geogescu-Roegen the description of the production process employed by economic theory presents a series of shortcomings and incoherencies related both to the representation of the process itself and to the analytical categories employed.

Standard economic theory maintains that the production process[9] can be indifferently described by flow models and by stock models. In the first case, the process can be fully described by flows, namely "by the rate of flow per unit of time of each of the *n* goods that are involved in the process"[10]. The description of the process consists therefore in accounting for everything crossing the borders. In the second case[11], a process can be fully described by the quantities existing at the beginning and at the end of the process. Neither of these representations is complete: if both provided a complete representation of the process, the antinomy between Stocks and Flows would be apparent.

In order to provide a correct and complete representation of the production process it is necessary:

- To list all the factors involved;
- To describe all the phases of the process;
- To insert the different phases in the time frame of the process.

Inputs, outputs, flows, funds and services

Once the physical borders and the duration of the process have been defined, it is possible to distinguish the inputs (everything which crosses the border from the outside) and the outputs (everything which crosses the border from the inside). We can then classify the elements that are involved in the production process in three groups:

1. Elements that are only inputs or only outputs;
2. Elements that are both inputs and outputs
3. Elements that are both inputs and outputs, but that during the production

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our minds and in our operations". N. Georgescu-Roegen, ibidem

[5] N.Georgescu-Roegen, ibidem.

[6] Also the concept of process is very elusive. It is however connected to change, to the continuous flowing of reality.

[7] The physical borders can be traced in total freedom and according to the aims of the analysis. If it is true that in nature there are no predefined borders, it is also true that each discipline has a specific tradition of analytical borders that are part of its methodological toolbox and represent, therefore, both a strong point of the paradigm and an obstacle to innovation.

[8] Duration is defined by the starting and ending points of the process; the length of the interval must be greater than 0.

[9] The production function was introduced by Wicksteed in 1894

[10] T.C. Koopmans (1951); G.J. Stigler (1942). The most classical example is the input-output analysis; see W.W. Leontief (1951).

[11] A.L. Bowley (1924); Hicks (1932); P.A. Samuelson (1948); J. von Neumann (1945).

process are subject to qualitative changes, in the sense that they progressively deteriorate.

Solar energy and waste material are examples of the first type of elements, wheat in the production of wheat of the second, workers and utensils of the third.

From another perspective, the elements that enter in the production process can be classified in *flows* and *funds*. The firsts are elements that are destroyed during the production process; the seconds are elements that are used, but not consumed. Funds (land, capital, labor force) produce services, without being consumed and being subject only to qualitative changes.

On the basis of this analysis, Georgescu-Roegen reaches the conclusion that *funds (a machine, a man) are not stocks*. In fact, while stocks are the result of an accumulation process of its elementary units and can be instantaneously de-accumulated, a fund cannot be obtained through an accumulation process of the services that it will deliver and its de-accumulation takes time. Moreover, if a flow is a stock spread over time, *the concept of flow cannot be used to indicate the services provided by a fund*. The expression "the flow of services" is therefore to be avoided. In this way we will not make the mistake to maintain that services can be incorporated into the final product as we do with raw materials. Finally, services and funds have different dimensions. The quantity of a flow is measured in the appropriate measurement unit; the rate of flow is the quantity per unit of time. The quantity of services demanded is given by the number of units of the fund per unit of time (100 men per 100 days; 100 machines per one week). Since the rate of services is equal to the fund divided by the time, it will be equal to the number of units of the fund.

A redefinition of the concepts of stock, fund, flows and services

The analysis of the concepts of fund and flows developed by Georgescu Roegen presents a series of problems.

Flows are generated by a creative process or can be the outcome of the de-accumulation of an existing stock. A flow of cars can be the outcome of a production process of an automotive factory, but can also be generated by the exits from a parking lot.

Stocks can be generated *only* by progressive accumulation of flows or better they are the net result of processes of accumulation and de-accumulation that is of entry flows that, in their turn, can be the result of the de-accumulation of other stocks, but originally are always the result of a "creative" process, and of exit flows. The stock of cars present in a parking lot in a given moment is the result of entries and exits that have taken place since the moment the parking lot has been opened.

In conclusion, flow variables represent the original variables, while stock variables are the results of flows. From this perspective it would appear more correct to define a stock as a function of flows, than to think of a flow as a stock with a time dimension. Therefore a stock at time t (S_t) can be thought as the sum of previous entries and exits flows (FLe and FLx):

$$[2]\quad S_i = \sum_{-\infty} (FLe - FLx)_t$$

An implication of equation [2] is that a flow cannot be the difference between a stock measured in two different moments of time, a part from the special case of a stock interested only by entry flows or exit flows. In general:

$$[3]\quad S_t - S_{t-1} = \sum_{t-1} (Fle - Flu)_t$$

Let us now consider the funds, i.e. those elements that:

- Are both inputs and outputs;
- Are used, but not consumed;
- Have the capacity to generate services that transform inputs into outputs.

A man or a machine, are not stocks since they cannot be de-accumulated instantaneously or reconstructed from the services they provide and they are both the result of a creative process. This is not true for the set of men and machines that a firm uses in its production process. If we consider not only the ways in which men and machines enter into and contribute to the production process, but also the ways in which companies build the fund of men and machines, it is evident that men and capital are stocks since they are the historical outcome of processes of accumulation and de-accumulation.

If we did not take into account this point we would miss one of the main implications of Georgescu-Roegen analysis of the production process: to have shown that there is no analytical difference between men and machines in the production process. This, not only because they both produce the services required to transform inputs into outputs-a fact that is accepted also by the neoclassical production function- but because the production of these services requires not only the presence of a fund of machines, but also of a fund of men.

The analytical distinction between a factor of production and the services it produces has always been clear and explicit for machines. In the production function the amount of output is correctly a function of capital services, while investment theory deals with the problem of how a company should operate in order to have the right amount of capital stock, and to innovate its technology.

On the contrary, this analytical distinction has received very little attention in relation to labor. The main reason being that neoclassical theory has always considered labor as a variable factor, i.e. a factor whose quantity can be changed instantaneously and with no cost, according to the changes in the level of production. As a consequence, production theory relates output to labor services. The dependent variable of the derived demand for labor is represented by labor services and neoclassical theory has never developed a theory of the fund of labor in parallel to capital theory. This does also create a logical inconsistency between theoretical and empirical analysis of the labor market, the first being expressed in terms of labor services, while the second deals with people. Finally, this explains the theoretical difficulties of introducing flow analysis in the main body of labor market theory.

It must also be underlined that considering labor as a variable factor has the logical implication that firms can adapt the level and the structure of labor services to the level of production in any given moment of time. In other terms, firms can choose “every day” the amount and the typology of labor they need. Therefore, the level and the structure of employment are the result of a continuous process of choice. This logical construct has important implications both at the theoretical and empirical level. At the theoretical level it explains while mainstream labor market theory has never tackled the problem of how companies deal with the issue of labor turnover and plan their employment level and structure. At the empirical level it explains while labor market surveys are mainly concerned with stock data, flow data playing a marginal role and little attention being paid to their quality and their coherence with stock data.

In reality, the level and the structure of employment are the results of marginal adaptations while bigger firms have long-term employment plan pursued by carefully monitored turnover policies.

In summary: the assumptions that labor is a variable factor conceals the fact that the construction of a given stock of workers is one of the more important goals of a firm - as much

as the process of investing in machines - and that firms act continuously in order to bring the existing stock of labor to the desired level and structure.

Conclusion

Considering labor as a variable factor, as done by the Neoclassical model, does not leave any theoretical space to analyze neither the stock of labor, neither entry and exit flows.

This has made impossible to develop a theory of labor demand centered on men, their characteristics and, what is more important, their history; to link exits from education to entries into labor market, and exits from employment to entries into retirement[12]

The definition of the concept of stock provided by [2] implies the identification between the concept of stock and the concept of population. This should not be surprising given the fact that the two words are used in an interchangeable ways in many instances and that demographic tools have been used in many areas outside the study of human populations. It will suffice to remember the growing number of studies in which the stock (population) of firms is "explained" as the outcome of births and deaths and the fact that numerous capital theory models consider different generations of machines.

Moreover, in the last 60 years numerous labor market studies have used demographic tools not only to produce empirical analysis of labor turnover, but also to analyze, for instance, unemployment duration.

Since the concepts of stock and population coincide (they both refer to a set of individuals that have a common characteristic and are the result of processes of accumulation and de-accumulation) the tools developed by demographers to analyze population can be used to analyze labor market stock variables (employment and labor force. This opens the door to a more pronounced integration between demographic procedures and economic theory.

[12] For an effort in this direction see Bruni, (1988 and 1993) and the following applications of the stock-flow models in the areas of demography and labor market forecasting (Bruni, 2014, 2012)

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