Available online at www.eccomasproceedia.org
Eccomas Proceedia COMPDYN (2025) 4987-4998

ECCOMAS
Proceedia

COMPDYN 2025
10th ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, M. Fragiadakis (eds.)
Rhodes Island, Greece, 15-18 June 2025

IMPROVED MULTIPLICATION FACTORS FOR SIMPLIFIED ANALYSIS OF VERTICAL VIBRATIONS IN FOOTBRIDGES UNDER CROWD LOADS

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Abstract.

Modern architectural trends have led to the design of increasingly slender footbridges, which often have natural frequencies sensitive to pedestrian-induced vibrations. Ensuring comfort and performance under vertical vibrations is now critical in footbridge design and serviceability assessment, necessitating accurate yet efficient characterization of the pedestrian activity as the vibration source. While single-pedestrian loading is well understood and easy to model, crowd-induced loading is much more complex due to variability in gait, synchronization, and pedestrian interactions. These factors lead to fluctuations in walking behaviour, making crowd-induced loading difficult to model, further complicated by human-structure interaction effects. This paper proposes a simplified method based on the multiplication factor approach, which predicts vertical vibrations by amplifying the response exerted by an ideal virtual single pedestrian to represent crowd effects. The method is calibrated using detailed simulations covering a broad range of structure parameters and traffic levels, accounting for inter-subject variability, step-by-step fluctuations, and human interactions modelled through the social force model. It also allows for the incorporation of human-structure interaction by using the coupled crowd-structure system modal parameters rather than those of the empty footbridge. By reducing the computational complexity of crowd modelling, this method offers a practical tool for serviceability checks on footbridges, balancing simplicity and accuracy. It enables easier assessment of slender footbridges under realistic crowd-loading conditions, advancing modern footbridge design and evaluation. Results are compared with existing conservative codes of practice and experimental tests from literature, confirming the method effectiveness.

Keywords: Crowd Loading, Experimental Test, Footbridges, Vertical Vibrations, Multiplication Factor, Vibration Serviceability.

ISSN:2623-3347 © 2025 The Authors. Published by Eccomas Proceedia.
Peer-review under responsibility of the organizing committee of COMPDYN 2025.
doi: 10.7712/120125.12789.26001

1 INTRODUCTION

The growing demand for innovative and aesthetically appealing designs has driven the construction of increasingly slender footbridges. These structures often feature low natural frequencies that coincide with typical pedestrian walking frequencies, making vibration serviceability a critical factor in their design and assessment [1, 2]. This involves accurately characterizing the human-induced dynamic force as the vibration source. However, differently with undisturbed single pedestrians which are straightforward to model, crowd loading remains time-consuming and still under investigation [3]. The challenges stem from aspects such as density-dependent randomness in pedestrian gait (including inter- and intra-subject variabilities), human-human interaction (HHI), which affect individual movement within the crowd due to both physical and psychological factors, and human-structure interaction (HSI), where the crowd alters the footbridge dynamic properties and vibrations impact pedestrian behaviour. These factors lead to variations in walking speed, step frequency, path direction, and synchronization, which should be considered when modelling crowd-induced loads.

The computational complexity of accurate crowd modelling complicates its integration into the design process, leading to the absence of a universally accepted vibration serviceability code of practice. Consequently, only a few guidelines are available [4, 5, 6, 7], and these are often considered overly conservative [8, 9, 10]. Many of these rely on the multiplication factor approach, which amplifies single pedestrian responses to estimate crowd-induced vibrations. Despite their simplicity, the multiplication factors are often based on limited datasets regarding crowd density and/or dynamic parameters, making their broader applicability uncertain. Furthermore, these guidelines tend to overlook critical factors such as intra-subject variability, human-human and human-structure interactions, and the absence of perfect synchronization at higher crowd densities. Additionally, the lack of comprehensive experimental validation further complicates the reliability and applicability of methods for crowd modelling [11].

In this context, the authors research aims to provide a practical yet detailed model based on the multiplication factor approach for the vertical vibration serviceability analysis of footbridges. This method is applicable to footbridges with varying geometries and modal characteristics, and can accommodate a wide range of crowd densities. Designed for use in extensive case studies, the model relies on high-detail microsimulations to accurately represent crowd dynamics [12], including inter-subject variability, step-by-step fluctuations in pedestrian forces, and human-human interactions. Furthermore, the model accounts for human-structure interactions by using the equivalent modal properties of the coupled crowd-structure system, rather than those of the empty footbridge [13].

The proposed method is calibrated using extensive numerical simulations of pedestrian flows, which are converted into crowd loading and applied to simply supported footbridges to calculate the crowd-induced acceleration of the fundamental mode. Improved multiplication factors are then evaluated across various crowd and structural parameters, based on the ratio of crowd responses to those of a virtual single pedestrian representing the crowd. This virtual pedestrian is designed to reflect the traffic level, with gait characteristics corresponding to the crowd density. The term ’virtual’ distinguishes between structures crossed by crowds and those crossed by single pedestrians, providing a consistent approach for comparing their effects. The improved multiplication factor dataset is analytically modelled, using fitting procedures based on bell-shaped functions placed at crowd-dependent pacing frequencies and integer multiples, with parameters depending on structure modal properties, traffic level and deck geometries.

The paper is structured as follows: Section 2 provides an overview of the background simulations that underpin the method, including crowd flows and virtual single pedestrians; Section 4 details the calibration of the improved multiplication factors, aimed at their analytical formulation; Section 5 illustrates the step-by-step application of the method; finally, Section 6 concludes with the key findings and implications of the study.

2 BACKGROUND SIMULATIONS OF HUMAN-INDUCED VIBRATIONS

This section addresses simulated human-induced vibrations, including: crowd dynamics and flow translation into loading (Section 2.1); single pedestrian force (Section 2.2); structural response calculations (Section 2.3); and an improved multiplication factor definition (Section 3).

2.1 Simulated crowds

To simulate unidirectional pedestrian flows on footbridges, the study employs the Social Force Model (SFM), a widely used microscopic model in crowd dynamics. Originally proposed by Helbing and Molnár [14], the SFM describes pedestrian motion as influenced by 'social forces', motivations driven by personal and environmental stimuli. These forces include the desire for free movement (driving force), the need to avoid other pedestrians and obstacles (repulsive forces), and the tendency to stay with groups or move towards points of interest (attractive forces).

An updated version of the SFM [12] is adopted here, with meta-parameters calibrated based on reflecting the Weidmann's speed-density relation [15]:

$$v_s(\rho) = 1.34\left\{1 - \exp\left[-1.913\left(\frac{1}{\rho} - \frac{1}{5.4}\right)\right]\right\} \tag{1}$$

which represents the natural decline of mean crowd velocity with increasing density. The SFM in [12] assumes a homogeneous pedestrian population, excluding random fluctuations and attractive forces, as is typical in many SFM applications [14, 16]. Key improvements in [12] include the activation of pedestrian repulsion only when individuals are within a critical distance, preventing premature adjustments to their trajectories when far apart, and the consideration that pedestrians are more influenced by those ahead than behind, avoiding unrealistic acceleration in slower individuals approached from behind.

The footbridge used for crowd simulations has dimensions of 3 meters in width and 40 meters in length, giving a total area of 120 m^2. Each pedestrian is assigned initial conditions, including a starting position and an initial velocity, with positions randomly distributed along a 40-meter-long access route and velocities drawn from a typical pedestrian speed distribution [17], with a mean of 1.34 m/s and a standard deviation of 0.26 m/s. These initial conditions, along with social forces, are modelled using a system of first-order differential equations, which are solved numerically in MATLAB to track pedestrians positions and velocities over time.

To maintain a constant number of pedestrians on the footbridge, new individuals enter as others exit. The input to the SFM is the number of pedestrians, N, rather than the target crowd density. However, by keeping N constant, the desired density, $\rho = N/A$, can be set in advance, where A indicates the footbridge area. After an initial transient phase, the crowd density stabilizes, and the pedestrian average velocity conforms to the Weidmann's speed-density relationship, as the adopted SFM is calibrated accordingly [12]. Simulations are conducted for densities ranging from 0.2 to 1.5 ped/m^2 (at 0.1 ped/m^2 increments), corresponding to pedestrian group sizes between $N = 24$ and $N = 180$. Each scenario is repeated 150 times to ensure statistical reliability, with convergence assessed based on the resulting accelerations.

Once the crowd flow is simulated using the SFM, the real-time position and velocity of each pedestrian α can be utilized to reconstruct their individual loading in the time domain:

- Pedestrian body weight G_α is sampled from a log-normal distribution with mean 73.85 kg (724.2 N) and standard deviation 15.68 kg (153.8 N) [18].
- The 2-D trajectory is derived from SFM positions, the SFM velocities in the two dimensions are combined together to obtain the time-varying speed $v_{s,\alpha}$, and the pacing frequency $f_{s,\alpha}$ is calculated according to [19], as:

$$f_{s,\alpha}(t) = 0.35 v_{s,\alpha}(t)^3 - 1.59 v_{s,\alpha}(t)^2 + 2.93 v_{s,\alpha}(t) \tag{2}$$

- The application time of step forces is defined at discrete intervals $T^i_{s,\alpha} = 1/f_{s,\alpha}(t_{i-1})$, where $T^i_{s,\alpha}$ represents the step period for footfall i evaluated based on the pacing frequency of the previous step at time t_{i-1}.
- The footfall location is mapped as the SFM position at the corresponding application time instant projected onto the footbridge centreline, simplifying the problem to 1-D while ensuring consistency with bending mode analysis.
- Each i-th footfall force is modelled as a Fourier series, following the single-step force model by Li et al. [20]:

$$P_{\alpha,i}(t) = G_\alpha \sum_{k=1}^{5} \mathit{DLF}_k \sin\left(\frac{\pi k}{T^i_{c,\alpha}} t\right) \qquad 0 \leq t \leq T^i_{c,\alpha} \tag{3}$$

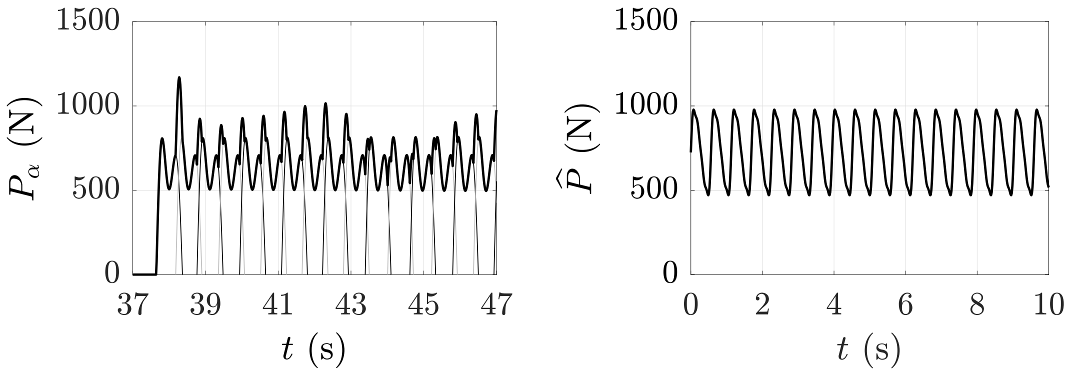
  where k is the harmonic counter, DLF_k (-) is the k-th pacing-frequency-dependent dynamic load factor, and $T^i_{c,\alpha} = T^i_{s,\alpha}/0.76$ is the foot-ground contact duration, all set according to [20].
- The pedestrian walking force $P_\alpha(t)$ is obtained as the sum of all single step forces, each applied at its specific location and time, as illustrated in Fig. 1(i).

The total crowd load $P(t)$ is then calculated by superimposing the individual forces. This approach is legitimate as the forces added together are triggered by pedestrians who interacted during the SFM simulations. However, note that if undisturbed, a pedestrian maintains a constant speed, resulting in uniform step forces. While the loading assembling procedure captures personal variability driven by flow dynamics, it does not account for intrinsic intra-personal variability, which refers to fluctuations in movement independent of external conditions.

2.2 Simulated single pedestrians

Each crowd density is paired with a representative single pedestrian, designed to reflect the average behaviour of the crowd. This pedestrian generates a periodic walking force modelled as a continuous multi-harmonic load [21], which minimizes computational complexity compared to step-by-step force modelling, as shown in Fig. 1. The following procedure ensures that the pedestrian walks at a velocity consistent with the paired crowd density and a physically sustainable step frequency:

- The pedestrian weight, $\widehat{G}$, is set to 725 N (73.85 kg), matching the crowd mean value.



(i) (ii)

Figure 1: Loadings: (i) step-by-step force from a pedestrian in a SFM simulated crowd (thick line), derived from superimposing left (black) and right (grey) step forces; (ii) continuous force from the undisturbed single pedestrian representing the crowd.

- The steady speed, $\widehat{v}_s$, is calculated from the density ρ using Eq. (1).
- The corresponding step frequency, $\widehat{f}_s$, is derived from $\widehat{v}_s$ via Eq. (2).
- The walking force is computed as:

$$\widehat{P}(t) = \widehat{G} + \widehat{G} \sum_{k=1}^{4} \widehat{DLF}_k \sin(2\pi k \widehat{f}_s t + \widehat{\varphi}_k); \tag{4}$$

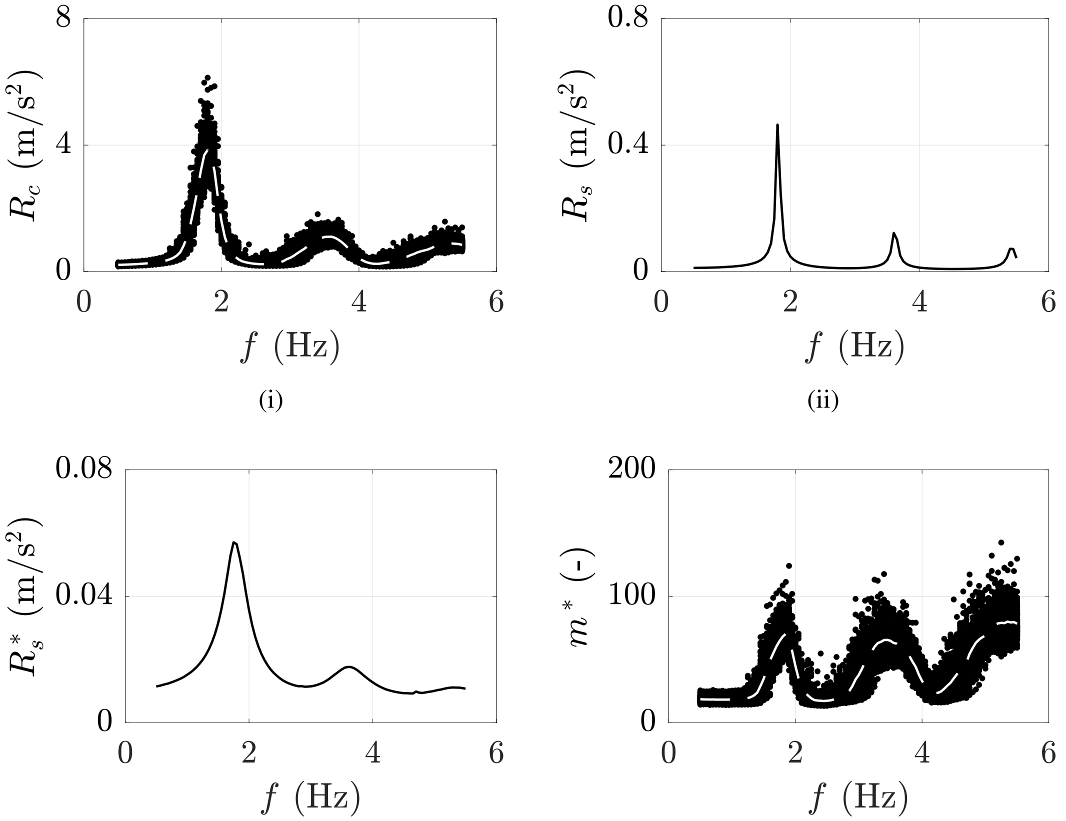
where k is the harmonic counter, $\widehat{DLF}_k$ (-) is the k-th harmonic dynamic load factor (dependent on the step frequency in keeping with [21]), and $\widehat{\varphi}_k$ (rad) is the k-th harmonic phase shift, set to zero here for exact reproducibility. This time-dependent load is treated as travelling at a constant speed $\widehat{v}_s$.

2.3 Simulated accelerations

Each crowd loading is applied to over a thousand structures, characterized by natural frequencies f ranging from 0.5 to 5.5 Hz at 0.05 increments, and damping ratios ξ of 0.1, 0.2, 0.5, 0.8, 1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0 and 10.0%. The inclusion of such high damping ratios is intended to account for HSI, whose main effect can be modelled by introducing the equivalent damping ratio of the crowd-structure system [13], higher compared to that of the unoccupied structure. Each structure under consideration is a simply supported beam measuring 40 m in length and 3 m in width, exhibiting linear dynamic behaviour with a modal mass of 25×10^3 kg.

Only the fundamental mode, namely the first bending mode having a half-sine mode shape, is considered. To calculate the modal force, the step loadings of pedestrians in crowds are weighted by the amplitude of the mode shape at each footfall location and normalized by modal mass. Then, the footbridge dynamic response $q(t)$ is determined by numerically integrating the modal equation of motion in the modal space, treating the footbridge as a single degree of freedom system and using time increments of 0.001 s:

$$\ddot{q}(t) + 2\xi\omega\dot{q}(t) + \omega^2 q(t) = \frac{P(t)\phi(t)}{M} \tag{5}$$



(i) (ii)

(iii) (iv)

Figure 2: Example case of 0.9 ped/m^2 density and 0.8% damping: (i) crowd-induced maximum accelerations (150 simulated runs) in black, mean trend in dashed white, (ii) maximum acceleration due to the crowd-associated single pedestrian, (iii) maximum acceleration due to the crowd-representing single pedestrian walking on the virtual footbridge, (iv) improved multiplication factors in black, mean trend in dashed white.

where $\omega = 2\pi f$ is the circular frequency, ξ the damping ratio, M the modal mass, and ϕ the mode shape. The crowd-induced structural response R_c is then selected as the maximum absolute value of the mid-span acceleration over the entire time history. A subset of the R_c simulated data is shown in Fig. 2(i), corresponding to the example cluster with 0.9 ped/m^2 crowd density and 0.8% structure damping.

For every density, the crowd behaviour is probabilistically modelled, representing an average of 150 simulations involving various participants. In contrast, the single pedestrian response is treated deterministically. Indeed, a probabilistic model for the individual pedestrian would be computationally demanding and impractical during the design phase. Therefore, an alternative approach has been developed to achieve computational efficiency, using virtual parameters to simulate the single pedestrian response. The method incorporates an additional damping mechanism for the footbridge crossed by the single pedestrian, in contrast to that subjected to the associated crowd. This supplementary damping, denoted as ξ^*, is designed to emulate the variability present in crowd dynamics even for single pedestrians. Since this variability is widely recognized as being dependent on traffic level, ξ^* (-) is defined to vary proportionally with the density-dependent collective pacing frequency variability recorded in SFM simulations, measured as the standard deviation of people average step frequencies:

$$\xi^* = 0.005595\rho^{-1.013} + 0.07885 \tag{6}$$

More details concerning the calibration of the extra damping ξ^* can be found in [22]. Fig. 2(ii) and Fig. 2(iii) show the maximum acceleration induced by the single pedestrian representing the example crowd walking on actual and virtual (i.e., overdamped with $\xi^* = 0.0859$) footbridges, respectively denoted as R_s and R_s^*. The increase in damping leads to a response that more closely resembles the behaviour of the crowd, as demonstrated by the aforementioned figure.

3 Simulated improved multiplication factors

For every run of any pedestrian group size crossing any footbridge, an improved multiplication factor m^* is derived, computed as the ratio of the crowd-induced acceleration R_c to the acceleration produced by the corresponding virtual single pedestrian R_s^*. In the cluster example case, this is done by dividing each R_c dot marker in Fig. 2(i) by the corresponding R_s^* value from Fig. 2(iii) yielding the m^* dot markers shown in Fig. 2(iv).

It is important to note that the modal mass impacts the modal force, and consequently the acceleration. However, since the modal mass equally affects both the crowd and virtual single pedestrian accelerations, its effect is elided while calculating m^*. Therefore, while R_c and R_s^* are dependent on the chosen modal mass, their ratio m^* is entirely independent of it. The same logic applies to the mode shape, which also equally influences both the crowd and single pedestrian modal forces (and accelerations). As a result, m^* is also unaffected by the mode shape: although the simulated improved multiplication factors are based on a fundamental bending mode (i.e., a half-sine mode shape), its application is suitable for any vertical mode.

4 CALIBRATION OF THE IMPROVED MULTIPLICATION FACTOR

This section presents the analytical development of the improved multiplication factors m^*, as previously simulated. Specifically, both the average and 95th percentile trends of the m^* dataset are analytically described to respectively predict average and 95th percentile crowd responses based on virtual single pedestrians. The analysis of the average crowd response is addressed first.

The definition of mean maximum accelerations induced by crowds is achieved by fitting the average trend of the m^* data for each density-damping cluster with a sum of three bell-shaped functions depending on density-dependent f_s and multiples, along with a constant d:

$$m^*(f) = d + \sum_{n=1}^{3} a_n \exp\left[-\left(\frac{f - nf_s}{c_n}\right)^2\right] \tag{7}$$

where a denotes the amplitude of the peaks, c controls the width of the bells, while the position of the peak centres is ruled by f_s, typical crowd step frequency calculated via Eq. (1) and Eq. (2) based on the traffic level ρ. Parameters are analytically calibrated as constants or functions of density ρ, deck area A and damping ξ as indicated in Table (1). The implementation of these parameters into Eq. (7) yields the analytical representation of m^*, which helps evaluating the average crowd-induced acceleration.

a_n	c_n	d
$a_1 = 0.4105\sqrt{\rho A}\,\xi^{-0.5021}$	$c_1 = 0.24$	$d = 1.868\sqrt{\rho A}\,\xi^{-0.01086}$
$a_2 = 0.9a_1$	$c_2 = 2c_1$	
$a_3 = 1.3a_1$	$c_3 = 3c_1$	

Table 1: Analytical definitions of model parameters.

For the forecast of design (95th percentile) crowd accelerations, the ratio of the 95th to 50th m^* simulated percentiles, denoted as Δ, is modelled using a power function of ξ:

$$\Delta = \xi^{-0.08098} - 0.05682 \tag{8}$$

In this way, the analytical definition for the 95th percentile of m^* is formulated as:

$$m^*_{95} = m^* \cdot \Delta \tag{9}$$

This provides a compact and efficient method for predicting both average and 95th percentile maximum crowd-induced accelerations, which can be calculated as summarised in the following.

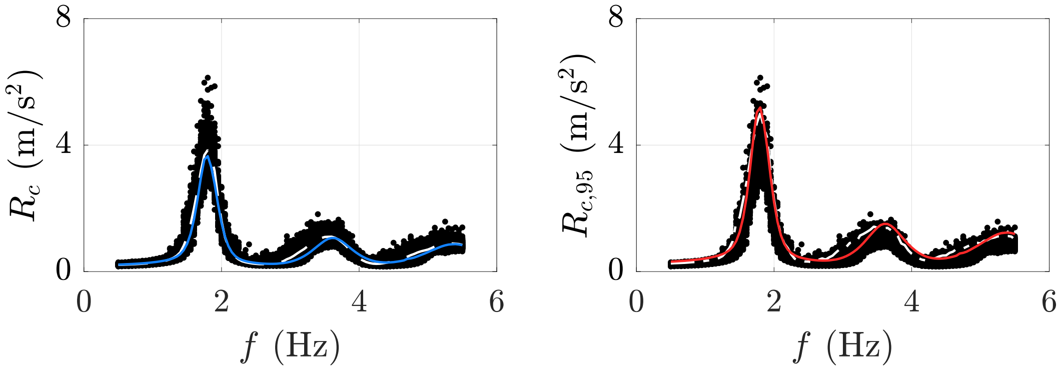
5 APPLICATION OF THE METHOD FOR SERVICEABILITY EVALUATIONS

This section outlines the application of the proposed method, detailing the procedure for its implementation and specifying the extent of its validity.

The analytical definitions of the improved multiplication factors m^* and m^*_{95} provide a straightforward method for estimating the average and 95th percentile maximum crowd-induced acceleration by calculating only the maximum acceleration from a single virtual pedestrian. When examining specific values for natural frequency $\bar{f}$ (Hz), damping ratio $\bar{\xi}$ (-), and crowd density $\bar{\rho}$ (ped/m^2) as in real serviceability assessments, the procedure below should be followed. The method also incorporates the effects of HSI on the structural dynamic properties, using the equivalent modal parameters of the coupled crowd-structure system (as determined, for instance, in [13]). As a result, $\bar{f}$ and $\bar{\xi}$ may represent either the properties of the empty footbridge or the equivalent ones, contingent upon whether the HSI is considered.

To estimate the average (50th percentile) maximum acceleration due to crowd excitation:

- Identification of $\widehat{v}_s$ and $\widehat{f}_s$: Evaluate the step frequency $\widehat{f}_s$ corresponding to a pedestrian velocity $\widehat{v}_s$ which is consistent with the crowd density $\bar{\rho}$, through Eq. (1) and Eq. (2).
- Definition of $\widehat{P}(t)$: Compute the single pedestrian multi-harmonic loading $\widehat{P}(t)$ according to Eq. (4).
- Definition of $\bar{\xi}_{\text{TOT}}$: To define the virtual structure, determine the extra damping $\bar{\xi}^*$ (-) at the examined crowd density $\bar{\rho}$ through Eq. (6), and calculate the total damping ratio of the virtual footbridge as the sum of structural and extra damping values, $\bar{\xi}_{\text{TOT}} = \bar{\xi} + \bar{\xi}^*$ (-), where $\bar{\xi}$ (-) may refer to either the empty footbridge or the equivalent damping ratio considering HSI.
- Calculation of R^*_s: Evaluate the maximum acceleration R^*_s induced by the single pedestrian, who represents the crowd density $\bar{\rho}$ and walks at constant speed $\widehat{v}_s$ on the virtual footbridge with total damping $\bar{\xi}_{\text{TOT}}$ and natural frequency $\bar{f}$ (see Eq. 5).



(i) (ii)

Figure 3: Example case of 0.9 ped/m^2 density and 0.8% damping, with simulated crowd induced maximum accelerations in black: (i) average trend (dashed white) and analytical prediction (solid blue); (ii) 95th percentile (dash-dotted white), analytical prediction (solid red)

- Evaluation of m^*: Determine the average improved multiplication factor m^* through Eq. (7) at $\bar{f}$, based on parameters derived from Table (1) depending on $\bar{\rho}$, $\bar{\xi}$, and the deck area A.
- Estimation of R_c^*: Compute the average maximum crowd-induced acceleration R_c^* by multiplying m^* by R_s^*.

To estimate the design (95th percentile) maximum acceleration:

- Estimation of R_s^* and m^*: As in the previous steps, determine m^* and R_s^*.
- Calculation of Δ: Compute the factor Δ, representing the ratio of the 95th to 50th percentile multiplication factors, based on Eq. (8) and depending on the damping ratio $\bar{\xi}$ (-).
- Evaluation of m_{95}^*: Compute the 95th percentile multiplication factor m_{95}^* through Eq. (9).
- Calculation of $R_{c,95}$: Determine the 95th percentile maximum crowd-induced acceleration $R_{c,95}$ by multiplying R_s^* by m_{95}^*.

Across all cases, including all the examined crowd densities and footbridge modal parameters, both the average and design crowd responses predicted by the method (blue and red lines in Fig. 3, respectively) show strong agreement with the numerically simulated accelerations derived from the SFM flows (white lines in Fig. 3), exhibiting coefficients of determination around 0.90.

The improved multiplication factor, based on $N = \rho A$, accounts for both crowd density and footbridge area, ensuring proper reflection of deck size in the assessment. Additional analyses tested the method adaptability across footbridges with varying lengths (20–60 m) and widths (2.5–3.5 m), using 50 SFM crowd simulations per density, confirming the method effectiveness in adapting to diverse geometries.

The model is independent of modal mass and unaffected by mode shapes, making it applicable to bridges with multiple spans and different flexural modes. Accelerations from various modes can be combined using methods like the Square Root of the Sum of the Squares (SRSS). The method also predicts maximum Root Mean Square (RMS) accelerations by multiplying single-pedestrian RMS values by the improved multiplication factor, with prediction accuracy comparable to maximum accelerations.

6 CONCLUSIONS

This paper proposes a simplified model for predicting vertical responses of footbridges under crowd excitation, by amplifying the acceleration induced by a representative undisturbed virtual single pedestrian with an analytical improved multiplication factor based on nearly 3 million simulations. These simulations account for a wide range of traffic conditions and pedestrian variability, including inter-subject and step-by-step differences. The model incorporates the HHI phenomenon using the SFM to simulate diverse, interacting crowd flows, and also considers the HSI impact on structural dynamics by allowing the use of equivalent modal parameters of the coupled crowd-structure system.

The model calculates the acceleration induced by a virtual single pedestrian and amplifies it using improved multiplication factors, which reflect both the average behaviour and the 95th percentile of crowd-induced accelerations. These factors are derived from analytical models and are dependent on crowd density and structure parameters, including modal and geometric properties. The model covers a broad spectrum of traffic densities and is applicable to a variety of footbridge configurations, to evaluate the crowd-induced acceleration contribution to any bending mode. Its ease of use and broad applicability make it a useful tool for assessing the serviceability of footbridges under vertical vibrations.

In conclusion, the proposed method offers a robust and practical approach for assessing the vertical vibration serviceability of footbridges under bending motion, accurately accounting for crowd dynamics, while providing a reliable tool for evaluating the impact of different crowd densities and footbridge configurations. Future work will focus on expanding the model to include torsional effects, which, although typically secondary, may become significant when human loading or footbridge geometry exhibit eccentricities.

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