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Engineering Economics and Financial Modeling: accounting-and-finance engineering system for the study of industrial projects

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Summary

The project “Engineering Economics and Financial Modeling: accounting-and-finance engineering system for the study of industrial projects” is aimed at exploring the intricate relationships between the economic and financial figures that characterize industrial projects and at studying the creation of mathematical models. The abstract models are meant, on one hand, to organize data and knowledge and, on the other hand, to further investigate through simulations and experiments.
The tools used in this path are i) financial modeling and its implementation in spreadsheets, ii) sensitivity analysis, mainly with the “Finite Change Sensitivity Indices (FCSI)” methodology (Borgonovo 2010), iii) comparison of scenarios, with design and implementation of algorithms that feed mathematical models, iv) Split-Screen Matrix (Magni 2020), used to automate the model calculation engine, and v) linear programming, derived from the disciplines of operations research.
Thanks to these tools, the research activity has made it possible to achieve several results in the fields of Engineering Economics, Finance and Accounting, including i) the development of new evaluation models, ii) the identification of new criteria for investment decisions, iii) the clarification of how operational estimates can affect financial decisions and iv) the ex-post decomposition of the decisions' effects, even in the different life periods of a project.
The main application fields explored are the creation of value in photovoltaic systems (results exposed discussed in the conferences “37th EU PVSEC – European PV Solar Energy Conference, September 7-11, 2020” and “MIC2020 – 20th Management International Conference, November 12-15, 2020” and in an article published in the *International Journal of Production Economics*), the decomposition of added value in financial investments (results discussed in an article submitted to *European Journal of Operational Research*, second revision submitted), the use of the Split-Screen Matrix in financial modeling (with a methodology, described in a paper not yet submitted, aimed at defining an algorithmic system applicable in spreadsheets, which is being introduced in the companies of the GRAF Group) and economic optimization in a project to replace water meters (first results presented at the “EURO2021 – 31st European Conference on Operational Research, July 11-14, 2021”).
This doctoral thesis is structured as a collection of academic papers co-authored during the doctoral course.

Keywords: engineering economics, financial modeling, project evaluation, value creation, Split-Screen Matrix

Title and summary in Italian

Engineering Economics e Financial Modeling:
sistema di ingegnerizzazione contabile-finanziaria per lo studio di progetti industriali

Il progetto "Engineering Economics e Financial Modeling: sistema di ingegnerizzazione contabile-finanziaria per lo studio di progetti industriali" è volto all'esplorazione delle complesse relazioni tra le grandezze economiche e finanziarie che caratterizzano i progetti industriali e a studiare la creazione di modelli matematici che hanno lo scopo, da un lato, di organizzare i dati e le conoscenze e, dall'altro, di indagare ulteriormente attraverso simulazioni e sperimentazioni.
Gli strumenti utilizzati in questo percorso sono i) la modellazione finanziaria e la sua implementazione in fogli di calcolo elettronici, ii) l'analisi di sensibilità, condotta principalmente con la metodologia dei "Finite Change Sensitivity Indices (FCSI)" (Borgonovo 2010), iii) il confronto di scenari, ottenuto grazie alla progettazione e implementazione di algoritmi che alimentano i modelli matematici, iv) la Split-Screen Matrix (Magni 2020), utilizzata per automatizzare il motore di calcolo dei modelli, e v) la programmazione lineare, mutuata dalle discipline della ricerca operativa.
Grazie a questi strumenti, l'attività di ricerca ha consentito di raggiungere vari risultati nel campo dell'ingegneria economica, della finanza e della contabilità, tra cui i) lo sviluppo di nuovi modelli di valutazione, ii) l'individuazione di nuovi criteri per le decisioni di investimento, iii) l'esplicitazione di come le stime operative possano influire sulle decisioni finanziarie e iv) la scomposizione a posteriori degli effetti delle decisioni, anche nei diversi periodi di vita di un progetto.
I principali campi applicativi esplorati sono la creazione di valore in impianti fotovoltaici (risultati esposti nei convegni "37th EU PVSEC – European PV Solar Energy Conference, September 7-11, 2020" e "MIC2020 – 20th Management International Conference, November 12-15, 2020" e in un articolo pubblicato in *International Journal of Production Economics*), la scomposizione del valore aggiunto negli investimenti finanziari (risultati discussi in un articolo di cui è stata sottoposta la seconda revisione a *European Journal of Operational Research*), l'impiego della Split-Screen Matrix nella modellazione finanziaria (con una metodologia, descritta in un articolo non ancora sottoposto per la pubblicazione, volta a definire un sistema algoritmico applicabile nei fogli di calcolo, che si sta introducendo nelle aziende del Gruppo GRAF) e l'ottimizzazione economica in un progetto di sostituzione dei contatori dell'acqua (i cui primi risultati sono stati esposti nel convegno "EURO2021 – 31st European Conference on Operational Research, July 11-14, 2021").
Questa tesi di dottorato è strutturata come raccolta di articoli scientifici co-autorati durante il corso di dottorato.

Parole chiave: ingegneria economica, modello finanziario, valutazione di progetto, creazione di valore, Split-Screen Matrix

Introduction

The present thesis is the collection of academic papers co-authored by me during the doctoral course together with:

- Carlo Alberto Magni, professor of Engineering Economics at the University of Modena and Reggio Emilia
- Andrea Marchioni, doctoral student at the University of Modena and Reggio Emilia
- Manuel Iori, professor of Operations Research at the University of Modena and Reggio Emilia
- Dario Vezzali, doctoral student at the University of Modena and Reggio Emilia

The project "Engineering Economics and Financial Modeling: accounting-and-finance engineering system for the study of industrial projects" is aimed at exploring the complex relationships between the economic and financial quantities that characterize industrial projects and at studying the creation of mathematical models that have the purpose, on the one hand of organizing data and knowledge and, on the other, to investigate further through simulations and experiments.
The tools identified for this path are:

- financial modeling and its implementation in spreadsheets,
- sensitivity analysis, conducted mainly with the methodology of "Finite Change Sensitivity Indices (FCSI)" (Borgonovo 2010),
- the comparison of scenarios obtained thanks to the design and implementation of algorithms feeding mathematical models,
- the Split-Screen Matrix (Magni 2020), which is intended to be used to automate the model calculation engine,
- linear programming, borrowed from the discipline of operations research.

These tools have made it possible to achieve the objectives of

- evaluating financing and distribution policy on industrial projects,
- decomposing effects of manager's and client's decisions in an active investment portfolio in the different periods,
- studying new operative tools,
- understanding how to use Operational Research tools and their application to the field of Engineering Economics.

The main application fields explored are

(i) the creation of value in solar photovoltaic systems,
(ii) the decomposition of added value in financial investments,
(iii) the use of the Split-Screen Matrix in financial modeling (with a methodology aimed at defining an algorithm applicable in spreadsheets), and
(iv) the economic optimization in a project to replace water meters.

An aspect of particular interest of this industrial doctorate project is the double exchange between the research aspect and the industrial aspect. This exchange allowed, on the one hand, the use of real cases in the theoretical study (introducing the complexity of reality in theoretical models) and, on the other hand, the application of research results in everyday business life (obtaining precious opportunities to validate theories).

i. Value creation in solar photovoltaic (PV) plants

Publications

(1) Baschieri, D., Magni, C.A., Marchioni, A. (2020). Comprehensive financial modeling of solar PV systems. *37th EU PVSEC – European Photovoltaic Solar Energy Conference*, Lisbon, September 7-11. ISBN: 3-936338-73-6.

(2) Marchioni, A., Magni, C.A., Baschieri, D. (2020). Investment and financing perspectives for a solar photovoltaic project. *MIC2020 – 20th Management International Conference*, Ljubljana, November 12-15. ISBN: 978-961-293-025-7.

(3) Magni, C.A., Marchioni, A., Baschieri, D. (2022). Impact of financing and payout policy on the economic profitability of solar photovoltaic plants. *International Journal of Production Economics*, Volume 244, February, 108338.

Spreadsheet models and Visual Basic software (not published)

(a) "Baschieri Magni Marchioni - EUPVSEC 2020.xlsm", research results presented at *37th EU PVSEC – European Photovoltaic Solar Energy Conference* (1).

(b) "Marchioni Magni Baschieri - MIC2020.xlsx", research results presented at *MIC2020 – 20th Management International Conference* (2).

(c) "Magni Marchioni Baschieri - IJPE 2022.xlsm", research results published in *International Journal of Production Economics* (3).

This project aims to analyze how the introduction of a solar photovoltaic system creates value for the investor. We applied the models to a real-life photovoltaic project, designed by GRAF Industries SpA (the company where I am project analyst), in Northeast Italy. In Baschieri, Magni, and Marchioni (2020), we apply the Finite Change Sensitivity Index methodology to the economic evaluation of the real photovoltaic plant, measuring the contribution of any input variation to the NPV variation. In Marchioni, Magni, and Baschieri (2020), we show the connections between estimated data and decision variables, studying the role of the distribution policy in the photovoltaic plant. In Magni, Marchioni, and Baschieri (2022), we present the comprehensive evaluation model for appraising the investment. It illustrates the contribution of technical (estimated) variables and financial (decision) variables in the shareholder value creation. A significative result is "the logical loop for calculating the project's cash flows", an eight-steps algorithm, immediately appliable to real-life projects, which offers simple rules to obtain the cash flow, starting from industrial and financial assumptions.

ii. Decomposition of Value Added in financial investments

Publications (submitted)

(4) Magni, C.A., Marchioni, A., Baschieri, D. (2022). The Attribution Matrix and the joint use of Finite Change Sensitivity Index and Residual Income for value-based performance measurement. Submitted to *European Journal of Operational Research* (second revision submitted).

Spreadsheet models and Visual Basic software (not published)

(d) "Magni Marchioni Baschieri - Attribution Matrix 2022.xlsm"; research results submitted to *European Journal of Operational Research* (4).

This project is aimed to study how to decompose the added value of an active investment portfolio. The focus is on differentiating the effects of the investment manager choices (selection and allocation choices) and the investor's decisions (drawdowns and injections).
In Magni, Marchioni, and Baschieri (2022), we introduce the Attribution Matrix, a new approach for performance measurement, aimed at detecting the decision effects (measuring the impact of manager/investor choices) and the period effects (measuring the impact of each period). Moreover, in this paper, we apply the Attribution Matrix to an Italian fund, Anima Italia "A", between 2013 and 2020.

iii. New Modeling Tool: Split-screen Spreadsheet Algorithmic System

Publications (ready to be submitted)

(5) Baschieri, D., Magni C.A. (2022). The Split-Screen approach for project appraisal (Part II: Spreadsheet modelling). Ready to be submitted to *The Engineering Economist*.

Spreadsheet models

(e) Baschieri, D., Magni C.A. (2022). "CAD.xlsx", Microsoft Excel file ready to be submitted to *The Engineering Economist*, associated with the paper (5).

This project studies an algorithmic system for the transposition of the theory presented in Magni (2020) on spreadsheets, to obtain its practical application. This technique

- allows to give the financial model a simple, linear structure, repeated always the same simple, linear structure in every period; the structure is schematic, organized and ordered; the user can easily insert in it the inputs and quickly extract from it the outputs;
- supports the organization of models in modules;
- organizes the disordered information obtained from a deconstructed data collection in the three categories (capital, income, cash flow), allowing the use of mixed inputs, and completing the information with the values missing (thanks to the laws of motion and conservation, again from Magni 2020);
- enables the identification of repetitive operations and proposes techniques to reproduce until the final results are obtained (repetition of the same operations in all periods).

This project allows the structuring and dissemination of application methods of the Split-Screen Matrix.

Baschieri and Magni (2022) is the applicative part of the associated first part, Magni (2022, The split-screen approach for project appraisal (Part I: The theory)), which describes the theoretical framework. In Baschieri and Magni (2022), we propose a guide step by step, cell by cell, for the creation of the split-screen matrices describing the project film. We show how, using the theory of Magni 2020, it is possible to implement a financial model with frugal use of the most common spreadsheet functions, fulfilling the requirements of clearness, transparency, consistency, and easiness-to-use.

This theory has found an application in the budgeting system of the GRAF group enterprises, realized with the Split-Screen Strip technique. Furthermore, this project achieves the integration between the accounting modeling and financial evaluation. This unification is currently non-existent: business

plan and financial evaluation are seen as two completely different things, according to the classic separation between accounting and finance.

iv. Economic optimization of water-meter replacement

Publications

(6) Vezzali, D., Iori, M., Magni, C.A., Marchioni, A., Baschieri, D. (2021). Smart-Meter Installation Scheduling in the Context of Water Distribution. *EURO2021 – 31st European Conference on Operational Research*, Athens, July 11-14. ISBN: 978-618-85079-1-3.

Spreadsheet models (not published)

(f) "Vezzali Iori Magni Marchioni Baschieri 2021 – IRETI.xlsx", spreadsheet model and Visual Basic software; research results presented at *EURO2021 – 31st European Conference on Operational Research* (6).

The purpose of this project is the economic analysis of a massive replacement of water meters in the province of Reggio Emilia, where IRETI group manages the water services. Currently, the users have traditional volumetric water-meters; IRETI intends to evaluate the profitability of their replacement with a "walk-by" meter that allows proximity remote-reading (i.e., the capability to read the measures through a radio connection). In Vezzali, Iori, Magni, Marchioni, and Baschieri (2021), we propose a Mixed Integer Linear Programming (MILP) formulation to model the smart-meter installation scheduling problem. The project proposes a formulation aimed to define a scheduling for the massive substitution plan that satisfies a number of operational constraints and produces the maximum NPV. My contribution in this project was in the study of objective functions that can satisfy the economic and financial rules and the constraint of the Operations Research.
This project is still in progress and the final result will be delivered to IRETI and will be discussed in a new future paper.

In the following, we provide the full-text papers:

(1) Baschieri, D., Magni, C.A., Marchioni, A. (2020). Comprehensive financial modeling of solar PV systems. *37th EU PVSEC – European Photovoltaic Solar Energy Conference*, Lisbon, September 7-11. ISBN: 3-936338-73-6.

(2) Marchioni, A., Magni, C.A., Baschieri, D. (2020). Investment and financing perspectives for a solar photovoltaic project. *MIC2020 – 20th Management International Conference*, Ljubljana, November 12-15. ISBN: 978-961-293-025-7.

(3) Magni, C.A., Marchioni, A., Baschieri, D. (2022). Impact of financing and payout policy on the economic profitability of solar photovoltaic plants. *International Journal of Production Economics*, Volume 244, February, 108338.

(4) Magni, C.A., Marchioni, A., Baschieri, D. (2022). The Attribution Matrix and the joint use of Finite Change Sensitivity Index and Residual Income for value-based performance measurement. Submitted to *European Journal of Operational Research* (second revision submitted).

(5) Baschieri, D., Magni C.A. (2022). The Split-Screen approach for project appraisal (Part II: Spreadsheet modelling). Work in progress, Ready to be submitted to *The Engineering Economist*.

(6) Vezzali, D., Iori, M., Magni, C.A., Marchioni, A., Baschieri, D. (2021). Smart-Meter Installation Scheduling in the Context of Water Distribution. *EURO2021 – 31st European Conference on Operational Research*, Athens, July 11-14. ISBN: 978-618-85079-1-3.

Comprehensive financial modeling of solar PV systems

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COMPREHENSIVE FINANCIAL MODELING OF SOLAR PV SYSTEMS

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ABSTRACT. The adoption of a photovoltaic system has positive environmental effects, but the main driver of the choice in the industrial and commercial sector is economic profitability. Switching from acquisition of energy to production of energy is an investment with costs (e.g. leasing annual payment, O&M costs, capital expenditure) and benefits (e.g. savings in the electric bill, sale of the energy exceeding consumptions). In this work, we use an accounting-and-finance model to calculate the Equity Net Present Value in different scenarios and a sensitivity-analysis method (Finite Change Sensitivity Index) to explain the reasons for differences in results. This technique enables identifying the contribution of any input factor in the output value variation. In this way, the investor can draw attention on the most significant critical variables in the initial estimations to ensure success in forecasting.

Keywords: photovoltaic, economic analysis, financial modelling, financing, estimation, decision.

1 AIM AND APPROACH USED

Solar energy undeniably brings about environmental benefits, but the adoption of solar energy by the industrial, commercial, and residential sectors is strongly affected by economic considerations (e.g., Cucchiella et al 2018 [3], Dong et al 2017 [4]). The mapping which links the key performance drivers and the investment's economic profitability entails understanding of the intricate network of relations among technical aspects, accounting magnitudes, forecasting of financial data, and assumptions on financing decisions, which makes the determination of economic profitability particularly complex. It is then important to provide decision-aiding tools capable of measuring the investment return, taking into account uncertainty and providing insights on possible managerial actions that may affect the decision to adopt solar energy.

Building upon Magni and Marchioni (2019) [8], we propose a comprehensive framework for modeling investment decisions in solar photovoltaic (PV) systems, aimed at helping analysts, advisors, firms' managers to assess the economic impact of solar energy, manage uncertainty, distinguish the high-impact drivers from the low-impact drivers, calibrate the structure of the model (increasing the depth of analysis for those drivers which have major effects on the investment financial efficiency), and choose various alternative proposals (e.g., alternative capturing technologies).

Specifically, the proposed model makes use of Magni's (2020) [6] accounting-and-finance system to engineering economic decisions. It accomplishes a detailed analysis of the sources of value creation in both absolute and relative terms, always supplying the net present value (NPV), the rate of return, and the financial efficiency, thereby overcoming the limitations of the internal rate of return (IRR), usually recommended in benefit-cost analysis (Sartori et al 2014 [10], Mangiante et al 2020 [9]), but most likely to be undetermined in this kind of projects.

The model acknowledges the distinction between *estimation variables* and *decision variables* on one hand and between *operating variables* and *financial variables* on the other hand: The estimation variables necessitate some estimation process to be determined (e.g., operating and maintenance costs, disposal costs, interest rate on debt financing) while the decision variables are under the managers' control (e.g., timing and size of distributions to shareholders, recourse to debt borrowing or to cash withdrawals for covering the financial needs). The operating variables express the factors which have a direct impact on the firm's costs and revenues as a result of the adoption of solar energy (e.g., solar panel efficiency, the avoided electric bill, energy price, amount of self-consumption, credit terms for energy sales to the grid). The financial variables regard the factors which affect the mix of financing sources and the amount of incremental liquid assets in the firm's balance sheets (e.g., interest rate on liquid assets, risk-adjusted cost of capital, distribution to equityholders).

We also aim at validating the model by means of sensitivity analysis (SA), which confirms that the presence or absence of relevant drivers may affect the increase in investors' wealth and may affect the decision. In particular, we assess the contribution of financial variables and decision variables to the output variability. With the aid of the recently developed Clean FCSI (Magni et al 2019 [7]), based on Borgonovo's (2010) [2] FCSI, we aim to detect the most critical drivers and understand which driver is more likely to cause a change in the decision. SA will also be of help to analysts for calibrating the model: if the contribution to value of some parameters is small, then there is no need of modeling those inputs in more detail;

in contrast, if some parameters contribute significantly to value creation, then the analyst may consider a further development of the model for gaining deeper insights. Clean FCSI will also be of help to show that interactions among all the variables substantially affect the investment's economic profitability. This testifies to the importance of modeling the project to take account of all relevant value drivers and to make analysts aware of the effect of estimation process on the accept/reject decision.

2 SCIENTIFIC INNOVATION AND RELEVANCE

This work presents a comprehensive approach to financial modeling of investments in solar energy which differentiates itself from the traditional financial modeling derived from finance. The innovation of the approach may be summarized as follows:

1. as opposed to traditional models, the proposed model acknowledges that the investment value (and related decision) depends on both operating variables and financial variables. Also, it depends on decision variables such as the distribution of cash to shareholders and the reinvestment of cash, which may affect the return on solar investment. The proposed model is transparent, for it takes distribution policy in explicit consideration as well as borrowing policy, and appraises the interaction with the operating variables, reflecting their impact on the firm's pro forma financial statements and, hence, on the investment value and return
2. in real life, a substantial amount of solar PV plants is financed by firms with internal funds (i.e., cash withdrawals from bank accounts) and/or by debt, with no recourse to equity issuance. In traditional financial modeling, this form of financing is not taken into explicit account. The proposed model takes account of any mix of financing sources, either internal (cash withdrawals) or external (debt and/or equity)
3. contrary to traditional financial modeling, the proposed model apportions the overall investment value according to the various sources of value, namely, the operating activities, the financial activities (reinvestment of excess cash and cash withdrawals), and the debt borrowing
4. in this kind of investments, it is likely that financial efficiency may not be determined with traditional tools such as the internal rate of return (IRR) (see Magni and Marchioni 2019 [8]). Equipped with Magni's (2010) [5] Average Internal Rate of Return, the proposed model always provides an appropriate measure of financial efficiency, in terms of Return On Investment (entity perspective) or Return On Equity (equity perspective)
5. we validate the model with the aid of SA, which also supplies helpful information to calibrate the model for a more careful treatment of the highest-impact value drivers and confirm the relevance of the interaction effects and the importance of fine-tuning the estimation process.

3 RESULTS

The accounting-and-finance model we propose is able to make a thorough evaluation of the various aspects of the option of switching to solar energy for an agent (e.g., a firm) currently importing energy from electric grid. Switching to a solar PV system entails cost savings equal to the electric bill and incremental costs due to the purchase of the solar PV system. This may be purchased with an upfront payment or, as frequently occurs, with lease contracts (or power purchase agreements); at the end of the contract, the lessee may pay a lump to acquire the plant. The lump sum will be financed either with debt, equity, or internal financing (withdrawal from liquid assets, i.e., cash and cash equivalents). The amount of power which will be produced in excess of self-consumption will be sold to the grid operator, generating cash inflows after some period (depending on the credit terms); in contrast, if energy consumption is smaller than energy production, the firm will buy the residual energy from the grid. For example, consider the case of a ground-mounted solar panel system to be installed in a currently rented land, associated with a lease contract and with no equity financing. We use data for a solar PV plant proposed by GRAF Spa, a solar PV installer company, to an Italian firm located in Northern Italy.

Table I: Equity NPV in two different scenarios

Variables	Scenario 1	Scenario 2
Operating variables (estimation)		
Nameplate capacity [kWp]	92	92
Unit cost [€/kWp]	1,050	1,050
Useful life of PV plant [years]	22	28
Annual unit prod. (Y 1) [kWh/kWp/y]	1,000	1,130
Solar panel degradation rate [%/y]	1.15%	0.65%
Lease term length [years]	20	20
Lease interest rate [%]	4%	4%
Purchase price of plant (year 20) [€]	25,000	25,000
O&M, insurance, etc. [%]	4.00%	2.75%
Disposal costs [€]	3,000	2,500
Lost rent from land property [€/y]	1,500	1,250
Growth rate for costs [%]	1.50%	0.50%
Annual energy consumption [kWh/y]	62,500	87,500
Tax rate [%]	30%	20%
Energy purchase price [€/kWh]	0.140	0.180
Energy selling price [€/kWh]	0.105	0.155
Growth rate of energy price [%]	0.50%	2.00%
Credit terms for energy purchases [dd]	0	0
Credit terms for energy sales [dd]	365	365
Financial variables (estimation)		
Interest rate on liquid assets [%]	4.00%	-0.50%
Interest rate on debt [%]	6.00%	2.00%
Required return on oper. assets [%]	6.00%	6.00%
Required return on liquid assets [%]	2.00%	2.00%
Required return on debt [%]	3.00%	3.00%
Financial variables (decision)		
Internal financing (cash) [%]	60%	60%
Debt borrowing [%]	40%	40%
Equity financing [%]	0%	0%
First CFE distribution [y]	1	1
Payout ratio [%]	50%	50%
Equity NPV [€]	**−15,494.88**	**84,570.02**

In Table I, column 2 (scenario 1) reports the estimated input data, for a given set of financing and distribution policy. These input data are used for drawing up three pro forma financial statements (balance sheets, income statements, cash flow statements) which are logically interconnected in a non-trivial way, since decisions on financing and cash flow distribution will affect the amount

of liquid assets and debt outstanding in the firm. This in turn affects next-period interest on debt and on liquid assets, which in turn affects next-period income and, therefore, the equity. With these data, shareholders' wealth increase, as measured by the shareholder net present value (NPV), is negative and equal to −15,494.88, so the project is not worth undertaking. (It is worth noting that neither the project IRR nor the operating IRR nor the equity IRR exist).[i]

Consider now a different set of estimated parameters, as described in column 3 (scenario 2). Shareholder value created increases by almost 100,000 to 84,570, so making the project highly profitable.

Table II breaks down the equity NPV into operating NPV (i.e., NPV of the operating assets), non-operating NPV (i.e., NPV of the liquid assets), and debt NPV (i.e. NPV of the debtholders).

Table II: Equity NPV

	Scenario 1	Scenario 2
+ Operating NPV	−12,110.92	+108,603.47
+ Non-operating NPV	−3,142.14	−24,264.57
− Debt NPV	−(+241.83)	−(−231.12)
= Equity NPV	**−15,494.88**	**84,570.02**

The FCSI helps explain why this dramatic change occurs, providing the change in NPV due to the change in estimate of the drivers (columns 2 and 3 in table III. See Magni et al 2019 [7] for details on FCSI). It is worth noting that the most important driver of change is a financial driver, the interest rate on liquid assets (rank 1). This means that attention should be drawn on the estimation of such a variable and it is worth modeling such an aspect in greater detail and/or refining the estimation process. Energy prices and O&M (operating drivers) are next in importance (ranks 2, 3, and 4). Somewhat unexpected is the negligible effect of the efficiency loss (rank 12). Disposal costs are also negligible (rank 13). Even the sharp deviation of estimate in the interest rate on debt is irrelevant (rank 14), suggesting that, in this case, the conditions of the loan contract are non-significant.

Once calibrated the model and obtained a reliable set of estimated data, the analyst should fine- tune the borrowing policy and the distribution policy in order to increase the project's value and get the best output for the investors. Preliminary results show that a change in such policies may have a remarkable effect on the output and, in some cases, may even cause a change in the decision to adopt solar energy (and distribution policy may have an even greater effect than borrowing policy).

Table III: Changes in NPV (%) and Rank of input factors

Variable	Change in NPV (%)	Rank
Operating variables (estimation)		
Useful life of PV plant	−6.09%	9
Annual unit prod. (Y 1)	7.27%	8
Solar panel degradation rate	0.70%	12
O&M, insurance, etc.	13.10%	4
Disposal costs	0.16%	13
Lost rent from land property	3.28%	11
Growth rate for costs	5.61%	10
Annual energy consumption	10.17%	5
Tax rate	−9.04%	6
Energy purchase price	19.91%	2
Energy selling price	14.18%	3
Growth rate of energy price	8.79%	7
Financial variables (estimation)		
Interest rate on liquid assets	31.99%	1
Interest rate on debt	−0.03%	14

4 CONCLUSIONS

Since solar energy undeniably contributes to a sustainable economy, the decision of adopting a solar energy system by firms is important to achieve a substantial cumulative effect in the environment. However, firms' decisions are mostly motivated by financial efficiency and shareholder value creation. We present an operational tool increasing analysts' and managers' awareness on the financial impact of solar energy on these economic measures. This model blends accounting and finance and takes account of the subtle network of relations between operating variables and financial variables on one hand, and estimation variables

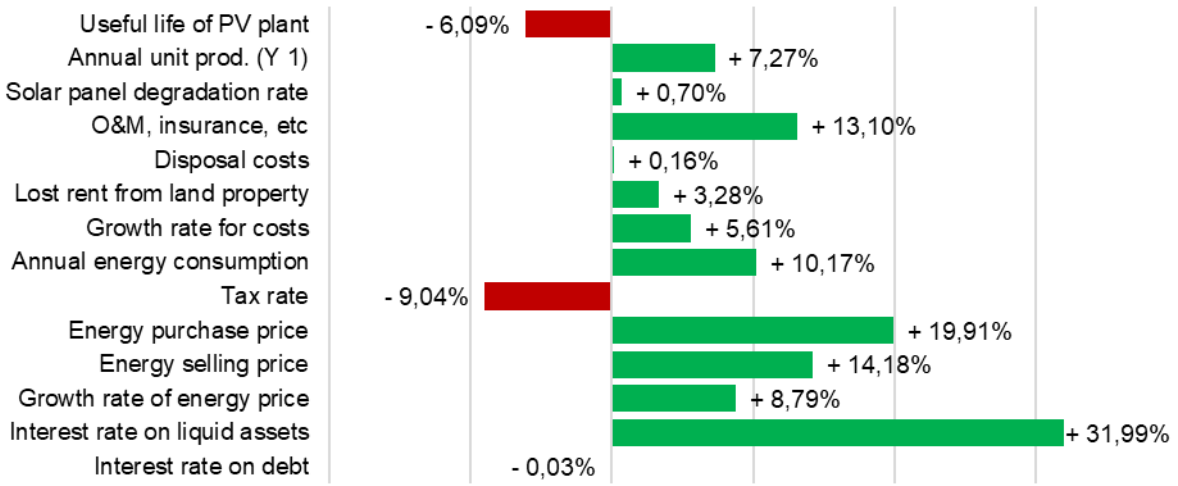

Figure I: Changes in NPV (%)

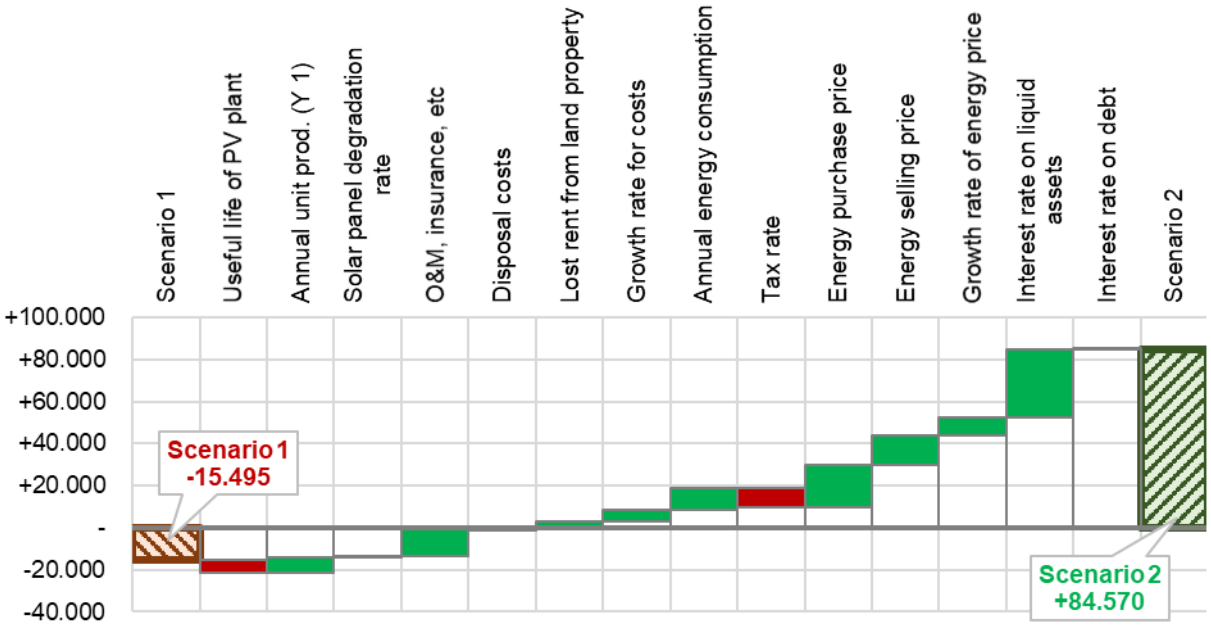

Figure II: Changes in NPV

and decision variables on the other hand. In particular, it explicitly takes account of the impact of internal financing as opposed to equity financing as well as of the reinvestment of retained cash as opposed to a full payout policy. The model is associated with a sensitivity-analysis technique which validates the model and provides managerial insights on the most critical drivers, which helps calibration of the model to the firm's needs. It also helps analysts to fine-tune the firm's borrowing and distribution, for any given set of estimated input data, in order to increase the financial benefits of solar energy.

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[i] It is interesting to note that the initial cash flow to equityholders is equal to 0 (they do not provide any capital injection) while it is positive in the following years. This is the reason why the equity IRR does not exist.

Investment and financing perspectives for a solar photovoltaic project

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Investment and Financing Perspectives for a Solar Photovoltaic Project

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Abstract. In this work we illustrate a simple logical framework serving the purpose of measuring value creation in a real-life solar photovoltaic project, funded with a lease contract, a loan contract and internal financing (i.e., withdrawal from liquid assets). We use the projected accounting data to compute the value created. We assess the project from both an investment perspective (operating assets and liquid assets) and a financing perspective (debt and equity). Furthermore, focusing on value creation for equityholders, we calculate the expected contribution on shareholders' wealth increase of operating and financing activity. In particular, we highlight the role of the distribution policy in financial modeling by describing the strict logical connections between estimated data and financial decisions.

Keywords: photovoltaic solar energy, project evaluation, net present value, distribution policy

1 Economic setting

Switching from traditional energy sources to renewable energy has a beneficial impact in terms of ecological sustainability (Ezbakhe and Pérez-Foguet 2021, Kang et al. 2020, Lei et al. 2019, Sinke 2019, Lupangu and Bansal 2017). However, firms willing to switch from retail energy to renewable energy are also concerned with the impact on economic profitability (Pham et al. 2019, Cucchiella et al. 2018, Dong et al. 2017). Therefore, an appropriate financial modeling and profitability metrics are required which correctly assess the effect on shareholders' wealth (Magni and Marchioni 2019, Baschieri, Magni and Marchioni 2020). In this study, we consider the appraisal of a solar photovoltaic (PhV) project proposed by an Italian installer company to a small firm, located in Northern Italy, which aims to switching from retail energy to solar energy and draw up a financial model which connects operating variables and financing variables.

Let Rev_t be the incremental revenues derived from the sale of excess energy, OpC_t be the incremental operational costs brought about by the plant, Dep_t be the depreciation charge of the solar PhV plant, I_t^l the interest income derived from reinvestment of liquid assets, I_t^d the interest expenses associated with debt, and τ the corporate tax rate. Formally, the project income is $I_t = \left(Rev_t - OpC_t - Dep_t + I_t^l\right)(1 - \tau) + \tau I_t^d$. As is standard in finance, the project's cash flows, F_t, can be computed by subtracting the

change in capital from the income, so that $F_t = I_t - \Delta C_t$. Let r_t be the project's cost of capital (minimum required rate of return).

The net present value (NPV) quantifies the net effect of the project on the investors' current wealth (Brealey, Myers and Allen 2011):

$$NPV = F_0 + \frac{F_1}{1+r_1} + \frac{F_2}{(1+r_1)(1+r_2)} + \cdots + \frac{F_n}{(1+r_1)(1+r_2)\ldots(1+r_n)}. \tag{1}$$

Capital amounts, incomes and cash flows of the project are intertwined in a non-trivial way via the pro forma financial statements, namely the balance sheets, the income statements and the cash-flow statements. These depend on estimated data regarding the operating activity but also on the firm's financing policy, that is, borrowing policy and distribution policy. Three sources of financing are possible:

- debt financing
- equity financing
- internal financing (i.e., withdrawal from liquid assets).

As for the distribution policy, the operating cash flows generated by the project may well be (wholly or partially) retained by the firm. and, if they are invested in financial assets, they produce interest incomes. Let $j = o, l, d, e$ be the operating assets, liquid assets, debt, and equity of the project, respectively. The first two components, o and l, represent the investment side of the project whereas the last two categories, d and e, describe its financing side. Each area is associated with its own net present value (NPV), as represented in Figure 1.

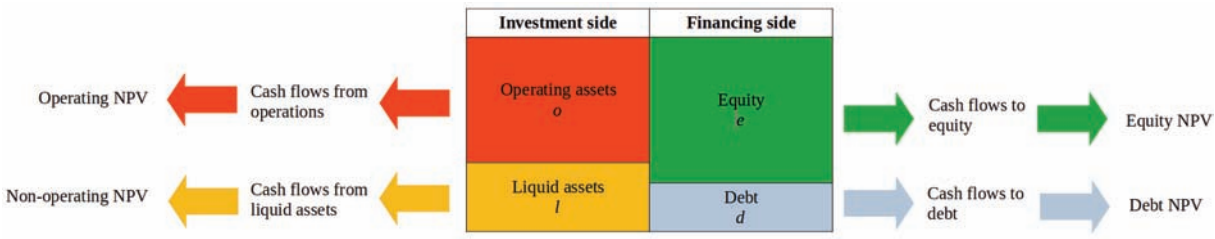

Figure 1: NPV of investments and financing sources

The NPV of each asset class j can be computed as

$$NPV^j = F_0^j + \frac{F_1^j}{1+r_1^j} + \frac{F_2^j}{(1+r_1^j)(1+r_2^j)} + \cdots + \frac{F_n^j}{(1+r_1^j)(1+r_2^j)\ldots(1+r_n^j)}$$

where F_t^j and r_t^j are the cash flows and costs of capital corresponding to each asset class. As shown in Magni (2020), the NPV of the project may be viewed under an investment perspective and a financing perspective:

$$\overbrace{NPV^o + NPV^l}^{\text{investment perspective}} = \overbrace{NPV}^{\text{project NPV}} = \overbrace{NPV^e + NPV^d}^{\text{financing perspective}}, \tag{2}$$

where

NPV^o = NPV of operating assets

NPV^l = NPV of liquid assets
NPV^e = NPV of equityholders
NPV^d = NPV of debtholders.

Since the managers' primary mandate is wealth increase of equityholders, the measure we focus on is the equity NPV, NPV^e. From (2),

$$NPV^e = NPV^o + NPV^l - NPV^d, \tag{3}$$

meaning that equityholders may benefit not just from a value-creating operating activity ($NPV^o > 0$), but also from an efficient management of liquid assets such that they are invested at a rate of return greater than the cost of capital of liquid assets ($NPV^l > 0$), and from the ability of borrowing at lower rate than the cost of debt, that is, the equilibrium rate prevailing in the capital markets ($NPV^d < 0$).[1]

In this work, we model the technical and financial description of a real-life case of solar PhV system. We measure the contribution of operating and financial areas on the overall value creation of the investment project and on the wealth increase for equityholders.

2 Solar PhV plant

We describe a real-life industrial case where an Italian company located in Northern Italy faces the opportunity of replacing a conventional retail electricity system (based on supplies from a grid operator) with a standalone solar PhV system purchased from an Italian producer and installer. The plant will be installed on a land property owned by the company and currently rented. With retail energy, the firm periodically pays a utility bill and receives a rental income from the rent of the land. The solar PhV plant implies a leasing contract whereby lease payments and operating and maintenance costs are made periodically. After several years, at the expiration date, the lessee will pay a lump sum to acquire the plant, and the system will continue to generate electric power for some years. The lump sum is paid through the issuance of new debt capital and withdrawal from liquid assets. At the end of its useful life, the plant will be removed, and the firm will incur disposal costs. If the retail system is replaced by the PhV plant, the incomes and cash flows will increase as a result of the ceased lease payment and the cost savings (the utility bill), but will increase as a result of operating and maintenance costs, the terminal outlay for acquiring the plant, and the lost rental income.

The model is described as follows: the quantity of energy consumed for the firm's operations is estimated to be constant through time and equal to q; the current purchase price of energy is p_p, growing at a constant rate g_p per year. The utility bill is payed periodically, in the same year in which energy is consumed. The leasing contract contains the following economic conditions: the lease payment, equal to L, is made periodically; at time m (expiration date) the firm may acquire the plant paying a lump sum equal to $CapEx$, and the system will keep producing electric power for some years, until time n. $CapEx$ represents the capital expenditure for buying the plant and is depreciated evenly from $t = m + 1$ until

1 The debt NPV is the part of the value generated by the project captured by debtholders: if it is negative, then equityholders grasp that value. Usually, such an NPV is zero or positive, so part of the value generated by the project is shared with the debtholders.

$t = n$, so that the depreciation charge is $Dep = CapEx/(n - m)$. As anticipated, the PhV plant is installed at $t = 0$ in a field owned by the firm, which could otherwise be rented on the property market at a rent equal to R growing at the constant annual rate g_c. The latter represents an opportunity cost for the firm (a foregone income).
Starting from the first period, the PhV plant requires operating, maintenance and insurance costs. Technical experts determine a suggested level of these costs for the first year in order to maximize the energy production, which we denote as $SuggO\&M$. We denote as $O\&M$ the actual expenses, which may be equal to or smaller than the suggested ones (i.e., $O\&M \leq SuggO\&M$), both assumed to grow at the constant annual rate g_c.

If $O\&M = SuggO\&M$, the PhV system will produce Q_{max} units of energy in the first year, which decrease every year at the rate g_Q. In contrast, if $O\&M = 0$ (i.e., the company is not willing to spend for operating and maintenance costs), the energy production suffers from a percentage loss due to lack of maintenance, denoted as $ProdLoss$. Furthermore, technical experts expect that the effective energy production in each period t, denoted as Q_t, is proportional to the level of actual $O\&M$ costs as compared to the suggested level. Specifically,

$$Q_t = Q_{max}(1 - g_Q)^{t-1} \cdot \left(1 - \max\left(ProdLoss \cdot \frac{SuggO\&M - O\&M}{SuggO\&M}, 0\right)\right).$$

If the energy produced by the plant, Q_t, is higher than the energy consumed by the firm, the firm sells the differential quantity to the Energy Service Operator at the energy selling price p_s, growing at a constant rate g_p per year, with payment in the following year. We assume that, at time $t = n$, the energy sold is paid immediately. Therefore, if the produced quantity is lower than the consumed energy in year t, that is, $Q_t < q$, energy costs savings arise equal to $Q_t \cdot p_p(1 + g_p)^{t-1}$; if the produced quantity is higher than the consumed one, that is, $Q_t > q$, energy costs savings arise equal to $q \cdot p_p(1 + g_p)^{t-1}$ as well as energy sales revenues equal to $(Q_t - q) \cdot p_s(1 + g_p)^{t-1}$, determining the presence of operating working capital. Hence, the income effect of the energy sales revenues and costs savings in the two different scenarios can be summarized with the expression

$$\min(q, Q_t) \cdot p_p(1 + g_p)^{t-1} + \max(0, Q_t - q) \cdot p_s(1 + g_p)^{t-1}$$

and the operating working capital can be represented with the formula $WC_t = max(0, Q_t - q) \cdot p_s(1 + g_p)^{t-1}$ and $WC_n = 0$. At time n, the plant is removed with disposal costs equal to H growing at the constant annual rate g_c.

To sum up, the firm-without-the-project pays the utility bills and receives the rent for the land (for the whole period); in contrast, the firm-with-the-project sustains the lease payments (until $t = m$), the operating and maintenance costs (until $t = n$), the lump sum (in $t = m$), and the disposal costs (in $t = n$), and receives payments for the energy sold to the Energy Service Operator. Considering that a project represents, by definition, the difference between the firm-with-the-project and the firm-without-the-project, the project's incomes are:

$$I_t = \Big[\min(q, Q_t) \cdot p_p\big(1+g_p\big)^{t-1} + \max(0, Q_t - q) \cdot p_s\big(1+g_p\big)^{t-1} - L - R \cdot (1+g_c)^{t-1} - O\&M \cdot (1+g_c)^{t-1} + I_t^l\Big](1-\tau) + \tau I_t^d \quad \text{for } 1 \le t \le m$$

$$I_t = \Big[\min(q, Q_t) \cdot p_p\big(1+g_p\big)^{t-1} + \max(0, Q_t - q) \cdot p_s\big(1+g_p\big)^{t-1} - R \cdot (1+g_c)^{t-1} - O\&M \cdot (1+g_c)^{t-1} - Dep + I_t^l\Big](1-\tau) + \tau I_t^d \quad \text{for } m+1 \le t \le n-1$$

$$I_t = \Big[\min(q, Q_t) \cdot p_p\big(1+g_p\big)^{t-1} + \max(0, Q_t - q) \cdot p_s\big(1+g_p\big)^{t-1} - R \cdot (1+g_c)^{t-1} - O\&M \cdot (1+g_c)^{t-1} - Dep - H \cdot (1+g_c)^{t-1} + I_t^l\Big](1-\tau) + \tau I_t^d \quad \text{for } t = n.$$

The project's assets are represented by working capital, liquid assets (C_t^l) and, from time m, fixed assets:

$$C_t = \overbrace{\max(0, Q_t - q) \cdot p_s\big(1+g_p\big)^{t-1}}^{\text{working capital}} + \overbrace{C_t^l}^{\text{liquid assets}} \quad \text{for } 1 \le t \le m-1$$

$$C_t = \overbrace{\max(0, Q_t - q) \cdot p_s\big(1+g_p\big)^{t-1}}^{\text{working capital}} + \overbrace{CapEx - Dep \cdot (t-m)}^{\text{fixed assets}} + \overbrace{C_t^l}^{\text{liquid assets}} \quad \text{for } m \le t \le n\text{-}1$$

$$C_t = 0 \quad \text{for } t = n$$

where the balance of liquid assets at the end of period t, C_t^l, is obtained from the liquid balance at the beginning of period, C_{t-1}^l, increased by the interest income I_t^l and by the cash contribution into the liquid assets account at time t, equal to $-F_t^l$, that is, $C_t^l = C_{t-1}^l + I_t^l - F_t^l$ (for the derivation of liquid assets see also the numerical application below). Finally, as already mentioned, the forecasted cash flows are obtained as $F_t = I_t - \Delta C_t,\ \forall t = 0,1,\dots,n$.

Considering the financing policy, until the expiration date of the leasing contract$m$, the project is fully financed with internal financing, that is, with retained cash. The rate of return on liquid assets is constant and equal to i^l, hence the interest income is $I_t^l = i^l \cdot C_{t-1}^l$. At time m, the operating disbursement is covered by absorbing resources from the liquid assets (internal financing), according to a proportion W, and by a loan contract for the complementary proportion $1 - W$. After time m, further disbursements are fully satisfied via internal financing.

The dividend distribution to equityholders, F_t^e, starts at a time d_m, according to the payout ratio α, to be applied to the smallest between the net income and the potential dividend (i.e., the difference between the operating cash flow and the cash flow to debt, $F_t^o - F_t^d$), provided that they are both positive, that is $F_t^e = \alpha \cdot \max\big[0, \min\big(I_t^e, F_t^o - F_t^d\big)\big]$. The cash contribution into the liquid assets account at time t, $-F_t^l$, is the retained cash, that is, the amount not distributed to the equityholders, therefore $-F_t^l = (F_t^o - F_t^d) - \alpha \cdot \max\big[0, \min\big(I_t^e, F_t^o - F_t^d\big)\big]$. At time n, the project is terminated, such that every asset and liability go back to zero.

The income statements, balance sheets, and cash-flow statements of the solar PhV plant are derived from the technical and financial model described above. The overall value creation is calculated via eq. (1) by discounting the cash flows F_t and, analogously, the NPVs of the asset classes $j = o, l, d, e$ are determined by considering the corresponding cash flows F_t^j. The decomposition of the project NPV and the explanation of the equityholders' value creation are computed via (2) and (3).

In the next section, we present the technical and financial data of the photovoltaic project and illustrate the practical applications of the financial measures for making a decision.

3 Value creation and NPV decomposition of the solar PhV plant

The industrial case of the solar PhV project is described with the following operating and financial input data.

Operating inputs:

- Useful life of PV plant: n =28 years
- Total cost of the plant = € 96,600.00
- Annual unit production in the first year at the technically suggested O&M (including insurance costs): Q_{max} =103,960 kWh
- Efficiency loss (per year): $g_Q = 0.65\%$
- Actual O&M and insurance: $O\&M = 2.75\%$ of total cost of the plant
- Technically suggested O&M and insurance: $SuggO\&M = 4\,\%$ of total cost of the plant
- Productivity loss due to lack of maintenance (with O&M=0): $ProdLoss = 15\,\%$
- Disposal costs: H = €2,500.00
- Lost rent from land property: R = €1,250.00
- Growth rate for costs: $g_c = 0.50\%$
- Lease term length: $m = 20$ years
- Purchase price of PV plant: $CapEx$ = €25,000.00
- Leasing annual payment: L = €6,268.45
- Annual energy consumption: q = 87,500 kWh
- Tax rate: $\tau = 20.00\%$
- Energy purchase price: $p_p = 0.180$(€/kWh)
- Energy selling price: $p_s = 0.155$ (€/kWh)
- Growth rate of energy price: $g_p = 2.00\%$

Financial inputs:

- First of year of CFE distribution: $d_m = 1^{st}$ year
- Payout Ratio: $\alpha = 50.0\%$ of the minimum between the net income and the potential dividends
- Internal financing: $W = 60\%$ of the purchase price of PhV plant
- Debt borrowing: $1 - W = 40\%$ of the purchase price of PhV plant
- Interest rate on liquid assets $i^l = 0\%$
- Interest rate on debt: $i^d = 2.00\%$
- Required return on operating assets (constant): $r^o = 6.00\%$
- Required return on liquid assets (constant): $r^l = 2.00\%$

- Required return on debt (constant): $r^d = 3.00\%$

The corresponding pro forma balance sheets, income statements and cash-flow statements are presented in Tables 1-3. Discounting the overall cash flows F_t, it results that the project NPV is $NPV = 84{,}338 > 0$, signaling that the PhV solar plant creates value. The decomposition of the value created under the investing and financing perspectives is described in the table below, via eq. (2).

Investment perspective			Financing perspective		
NPV^o	=	+108,125	NPV^e	=	+88,635
NPV^l	=	− 19,721	NPV^d	=	−231
NPV	=	88,404	NPV	=	88,404

According to the investment perspective (left side of the table), the operations create value by $NPV^o = 108{,}125 > 0$, which is partly offset by the significant value destruction due to the liquidity management with $NPV^l = -19{,}721 < 0$ (due to an inefficient allocation of capital with $i^l = 0\% < r^l = 2.00\%$).

Considering the financing perspective (right side of the table), equityholders increase their wealth by $NPV^e = 88{,}635 > 0$, higher than the project NPV, $NPV = 88{,}404$, due to a value-creating borrowing policy, such that $NPV^d = -231 < 0$ (because the loan interest rate i^d is lower than the cost of debt capital r^d). This means that equityholders gain value at the expense of the debt-holders, but this transfer of value is tiny, due to the very small difference between the interest rate on debt (2%) and the maximum acceptable financing rate (3%), as well as the limited scale of the debt.

Finally, we decompose the wealth increase of equityholders into the contributions of operations, liquidity and debt, according to (3), obtaining the following partition.

$+ NPV^o$	=	108,125
$+ NPV^l$	=	−19,721
$- NPV^d$	=	−(−231)
$= NPV^e$	=	88,635

The equity NPV is lower than the operating NPV because investments in liquid assets significantly destroy value whereas value transfer from debtholders to equityholders is almost irrelevant (as also depicted in Figure 2).

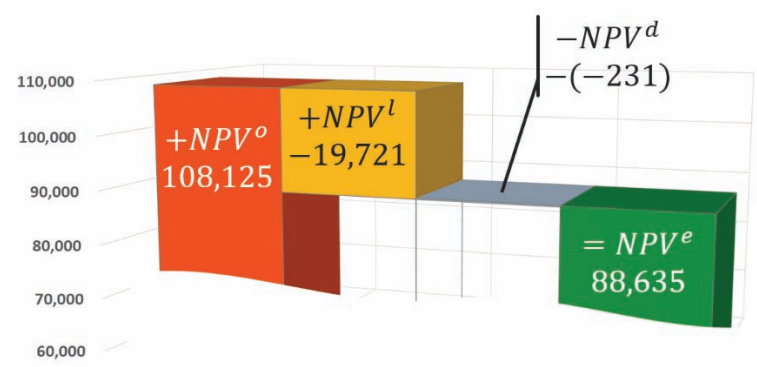

Figure 2: Decomposition of equity NPV

4 Financial efficiency of the solar PhV plant

As opposed to the NPV which does not suffer from any shortcoming, we note that the Internal Rate of Return (IRR), which is the most employed relative performance ratio in capital budgeting, does not exist for the overall project nor for the equity investment, as a consequence of the non-conventional cash flows streams $(F_0, F_1, \dots, F_n)$ and $(F_0^e, F_1^e, \dots, F_n^e)$, the first one having more than one change in sign and the second one having no change in sign.

Since the IRR fails, a viable solution for measuring the rate of return (and, therefore, the financial efficiency) of the project and of the equity investment is offered by the so-called average internal rate of return (AIRR) approach, introduced in Magni (2010, 2013), based on the estimated incomes and capital amounts, coherently defined as the ratio of the overall (discounted) income over the overall (discounted) capital. The AIRR of the project quantifies the project's rate of return over the total invested capital:

$$AIRR = \frac{\sum_{t=1}^{n} \frac{I_t}{(1+r_1)(1+r_2)\dots(1+r_t)}}{C_0 + \sum_{t=1}^{n} \frac{C_t}{(1+r_1)(1+r_2)\dots(1+r_t)}} = \frac{113{,}956}{589{,}145} = 19.34\% \tag{4}$$

and, analogously, the equity AIRR measures the relative performance for equityholders, expressed as the ratio of net income to total equity invested:

$$AIRR^e = \frac{\sum_{t=1}^{n} \frac{I_t^e}{(1+r_1)(1+r_2)\dots(1+r_t)}}{C_0^e + \sum_{t=1}^{n} \frac{C_t^e}{(1+r_1^e)(1+r_2^e)\dots(1+r_t^e)}} = \frac{113{,}717}{575{,}270} = 19.77\% \tag{5}$$

where r_t and r_t^e are explicitly derived from the costs of capital of operating assets, non-operating assets, and debt (see Magni 2020, Ch. 8 for details on the calculation of the project costs of capital).

Furthermore, Magni (2010, 2013) proves that the AIRR approach is NPV-consistent[2] and is possible to decompose the value creation of the project into a financial efficiency component (defined as the difference between the AIRR of the project and the average cost of capital r) and an investment scale component, therefore enriching the informational content of the valuation. More precisely,

$$NPV = \overbrace{(AIRR - r)}^{\text{financial efficiency}} \cdot \overbrace{\left(C_0 + \sum_{t=1}^{n} \frac{C_t}{(1+r_1)(1+r_2)\dots(1+r_t)}\right)}^{\text{scale}} \tag{6}$$
$$= (19.34\% - 4.34\%) \cdot 589{,}145 = 15.01\% \cdot 589{,}145 = €\,84{,}404$$

where r is the project's average cost of capital. Symmetrically, the equity NPV is decomposed via the AIRR approach as the product of financial efficiency for equityholders and the scale of the equity investment:

$$NPV^e = \overbrace{(AIRR^e - r^e)}^{\text{equity financial efficiency}} \cdot \overbrace{\left(C_0^e + \sum_{t=1}^{n} \frac{C^e{}_t}{(1+r^e{}_1)(1+r^e{}_2)\dots(1+r^e{}_t)}\right)}^{\text{equity scale}} \tag{7}$$
$$= (19.77\% - 4.66\%) \cdot 575{,}270 = 15.41\% \cdot 575{,}270 = €\,88{,}635$$

where r^e is the average cost of equity capital.

Considering the equityholders' perspective, each euro invested in the project produces an equity return equal to 19.77%, remarkably higher than the alternative return equal to 4.66% that could be obtained on the financial market for investments of comparable risk. The financial efficiency of equity is positive, equal to 15.41%, representing the relative advantage for equityholders in investing in the PhV plant instead of alternative available investments. Overall, the equityholders invest € 575,270 at an above-normal return of 15.41%, so realizing a wealth increase equal to €575,270 · 15.41% = €88,635.

5 The role of distribution policy

It is worth noting that, in such a model, the estimated data are logically chained to decisions regarding distribution policy and retained cash. For example, to build the balance of liquid assets at the end of period $t = 14$, C_{14}^l, one needs start from the balance at the beginning of that period, $C_{13}^l = €45{,}997$. Assuming that the cash retained in the firm will not generate any interest income, the balance will increase by the retained cash (i.e., the amount not distributed to the equityholders) at time $t = 14$, which is equal to

$$\overbrace{-F_{14}^l}^{\text{retained cash}} = \overbrace{(F_{14}^o - F_{14}^d)}^{\text{potential dividends}} - \overbrace{\alpha \cdot \max\left[0, \min\left(I_{14}^e, F_{14}^o - F_{14}^d\right)\right]}^{\text{cash flow to equity}} = €4{,}362.$$

Therefore, we obtain the balance of liquid assets at the end of period as

$$C_{14}^l = C_{13}^l - F_{14}^l = €45{,}997 + €4{,}362 = €50{,}358.$$

In this application, the distribution policy remarkably affects the economic results, with $NPV^l = -19{,}721$, because of high differences between the interest rate on liquid assets and minimum acceptable

[2]See also Marchioni and Magni (2018) for a definition of strong NPV-consistency of rates of return.

rate of return on liquid assets and high balances of liquid assets in several different periods of the investment. Only after computing the balance of liquid assets, the equity book value may be calculated as $C_{14}^{e} = C_{14}^{o} + C_{14}^{l} - C_{14}^{d}$.

Logically, the disregard of the distribution policy would have invalidated the logical consistency of the model. It is necessary to first calculate the potential dividends, then subtract the part of it which is not distributed and add it to the cash balance, as we have shown above. This brings about a network of complex relationships among the accounting magnitudes, which makes it necessary to draw up the cash-flow statement. The latter enables the analyst to calculate the cash flow associated with the liquid assets, F_t^l, which depends on the cash flow distributed to equityholders, F_t^e, which in turn depends on the operating cash flow. However, the latter can be computed only on the basis of elements of the income statement (the operating income) and elements of the balance sheets (operating assets). In turn, the balance sheet cannot be completed without the cash-flow statement, because, as we remind, the equity capital is equal to $C_t^e = C_t^o + C_t^l - C_t^d$ and C^l cannot be computed without computing F_t^l (i.e., without using the cash-flow statement). This nontrivial relationships among these three financial statements also testifies to the connections between estimated data (operating variables) and decision variables (distribution policy and reinvestment of retained cash). As a result, pro forma balance sheet and income statement are not sufficient; the cash flow statement is required for a sound and logically consistent model (and, therefore, a correct valuation of the project).[3]

6 Conclusions

In the current work we have provided a logically consistent model for the investment appraisal of a real-life photovoltaic energy project. Contrary to traditional modeling, we take account of the subtle relations interconnecting operating variables and financing variables, which depend on decisions (borrowing decision and distribution policy). We have considered the firm's decisions on distribution in the cash-flow statement, which is necessary to draw up the balance sheet (and, therefore, the income statement of the next period). We have decomposed the value created under two different perspectives, namely, the investment view which considers operating and liquid assets, and the financing view, which analyzes the equity and debt components, highlighting that the equity NPV may be significanty different from the operating NPV due to the remarkable role of financial decisions about liquid assets and debt.

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[3] Some authors discount the potential dividends, $(F_t^o - F_t^d)$ at the cost of equity capital r_t^e thereby avoiding the calculation of the balance of liquid assets. However, this does not produce the correct equity NPV, unless the retained cash is invested at the cost of equity, an often implausible assumption, or the firm distributes 100% of the potential dividends to equityholders, which is not always the case (and is not the case of the Italian company we consider) (see also Magni 2020, p. 344).

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Table 1: Balance sheets (thousands of Euro)

BALANCE SHEET	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
BS ASSETS																													
Operating Assets	-	1.8	1.7	1.7	1.6	1.5	1.4	1.4	1.3	1.2	1.1	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.1	25.0	21.9	18.8	15.6	12.5	9.4	6.3	3.1	-
Accounts receivable from grid operator	-	*1.8*	*1.7*	*1.7*	*1.6*	*1.5*	*1.4*	*1.4*	*1.3*	*1.2*	*1.1*	*1.0*	*0.9*	*0.8*	*0.7*	*0.6*	*0.5*	*0.4*	*0.3*	*0.1*	*0.0*	-	-	-	-	-	-	-	-
Net fixed assets	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	*25.0*	*21.9*	*18.8*	*15.6*	*12.5*	*9.4*	*6.3*	*3.1*	-
Liquid assets	-	2.1	5.2	8.4	11.7	15.1	18.6	22.2	25.9	29.7	33.6	37.6	41.7	46.0	50.4	54.8	59.4	64.1	69.0	73.9	69.0	77.2	85.5	93.9	102.4	111.1	119.8	128.6	-
ASSETS	**-**	**3.8**	**6.9**	**10.0**	**13.2**	**16.6**	**20.0**	**23.5**	**27.1**	**30.9**	**34.7**	**38.6**	**42.7**	**46.8**	**51.1**	**55.4**	**59.9**	**64.5**	**69.2**	**74.1**	**94.0**	**99.1**	**104.3**	**109.5**	**114.9**	**120.4**	**126.0**	**131.8**	**-**
BS LIABILITIES																													
Loan current debt	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	10.0	8.8	7.6	6.4	5.2	3.9	2.7	1.3	-0.0
Equity	-	3.8	6.9	10.0	13.2	16.6	20.0	23.5	27.1	30.9	34.7	38.6	42.7	46.8	51.1	55.4	59.9	64.5	69.2	74.1	84.0	90.2	96.6	103.1	109.7	116.5	123.4	130.4	-
LIABILITIES	**-**	**3.8**	**6.9**	**10.0**	**13.2**	**16.6**	**20.0**	**23.5**	**27.1**	**30.9**	**34.7**	**38.6**	**42.7**	**46.8**	**51.1**	**55.4**	**59.9**	**64.5**	**69.2**	**74.1**	**94.0**	**99.1**	**104.3**	**109.5**	**114.9**	**120.4**	**126.0**	**131.8**	**-0.0**

Table 2: Income statements (thousands of Euro)

INCOME STATEMENT	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Revenues	**-**	**0.5**	**0.5**	**0.4**	**0.3**	**0.2**	**0.2**	**0.1**	**-0.0**	**-0.1**	**-0.2**	**-0.3**	**-0.4**	**-0.5**	**-0.6**	**-0.7**	**-0.9**	**-1.0**	**-1.1**	**-1.2**	**-1.4**	**-1.4**	**-1.4**	**-1.4**	**-1.4**	**-1.4**	**-1.4**	**-1.4**	**-1.4**
New revenue: sale of energy	-	*1.8*	*1.7*	*1.7*	*1.6*	*1.5*	*1.4*	*1.4*	*1.3*	*1.2*	*1.1*	*1.0*	*0.9*	*0.8*	*0.7*	*0.6*	*0.5*	*0.4*	*0.3*	*0.1*	*0.0*	-	-	-	-	-	-	-	-
Lost rent from land property	-	*-1.3*	*-1.3*	*-1.3*	*-1.3*	*-1.3*	*-1.3*	*-1.3*	*-1.3*	*-1.3*	*-1.3*	*-1.3*	*-1.3*	*-1.3*	*-1.3*	*-1.3*	*-1.3*	*-1.4*	*-1.4*	*-1.4*	*-1.4*	*-1.4*	*-1.4*	*-1.4*	*-1.4*	*-1.4*	*-1.4*	*-1.4*	*-1.4*
(-) Operating costs	**-**	**6.8**	**7.1**	**7.4**	**7.7**	**8.1**	**8.4**	**8.7**	**9.1**	**9.4**	**9.8**	**10.1**	**10.5**	**10.9**	**11.3**	**11.7**	**12.1**	**12.5**	**12.9**	**13.3**	**13.8**	**20.3**	**20.6**	**20.9**	**21.2**	**21.5**	**21.9**	**22.2**	**19.6**
(-) Lease annual payment	-	*-6.3*	*-6.3*	*-6.3*	*-6.3*	*-6.3*	*-6.3*	*-6.3*	*-6.3*	*-6.3*	*-6.3*	*-6.3*	*-6.3*	*-6.3*	*-6.3*	*-6.3*	*-6.3*	*-6.3*	*-6.3*	*-6.3*	*-6.3*	-	-	-	-	-	-	-	-
(-) O&M cost	-	*-2.7*	*-2.7*	*-2.7*	*-2.7*	*-2.7*	*-2.7*	*-2.7*	*-2.8*	*-2.8*	*-2.8*	*-2.8*	*-2.8*	*-2.8*	*-2.8*	*-2.8*	*-2.9*	*-2.9*	*-2.9*	*-2.9*	*-2.9*	*-2.9*	*-2.9*	*-3.0*	*-3.0*	*-3.0*	*-3.0*	*-3.0*	*-3.0*
(-) Disposal costs	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	*-2.9*
(-) Cost saving: self consumption of energy	-	*15.8*	*16.1*	*16.4*	*16.7*	*17.0*	*17.4*	*17.7*	*18.1*	*18.5*	*18.8*	*19.2*	*19.6*	*20.0*	*20.4*	*20.8*	*21.2*	*21.6*	*22.1*	*22.5*	*22.9*	*23.3*	*23.6*	*23.9*	*24.2*	*24.5*	*24.9*	*25.2*	*25.5*
EBITDA	**-**	**7.4**	**7.6**	**7.8**	**8.1**	**8.3**	**8.6**	**8.8**	**9.1**	**9.3**	**9.6**	**9.8**	**10.1**	**10.4**	**10.6**	**10.9**	**11.2**	**11.5**	**11.8**	**12.1**	**12.4**	**18.9**	**19.2**	**19.5**	**19.8**	**20.1**	**20.4**	**20.7**	**18.2**
(-) Depreciation	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-3.1	-3.1	-3.1	-3.1	-3.1	-3.1	-3.1	-3.1
EBIT	**-**	**7.4**	**7.6**	**7.8**	**8.1**	**8.3**	**8.6**	**8.8**	**9.1**	**9.3**	**9.6**	**9.8**	**10.1**	**10.4**	**10.6**	**10.9**	**11.2**	**11.5**	**11.8**	**12.1**	**12.4**	**15.8**	**16.1**	**16.4**	**16.7**	**17.0**	**17.3**	**17.6**	**15.1**
Interest income	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(-) interest expenses	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-0.2	-0.2	-0.2	-0.1	-0.1	-0.1	-0.1	-0.0
EBT	**-**	**7.4**	**7.6**	**7.8**	**8.1**	**8.3**	**8.6**	**8.8**	**9.1**	**9.3**	**9.6**	**9.8**	**10.1**	**10.4**	**10.6**	**10.9**	**11.2**	**11.5**	**11.8**	**12.1**	**12.4**	**15.6**	**15.9**	**16.3**	**16.6**	**16.9**	**17.2**	**17.6**	**15.0**
(-) Taxes	-	-1.5	-1.5	-1.6	-1.6	-1.7	-1.7	-1.8	-1.8	-1.9	-1.9	-2.0	-2.0	-2.1	-2.1	-2.2	-2.2	-2.3	-2.4	-2.4	-2.5	-3.1	-3.2	-3.3	-3.3	-3.4	-3.4	-3.5	-3.0
NET INCOME	**-**	**5.9**	**6.1**	**6.3**	**6.5**	**6.6**	**6.8**	**7.0**	**7.2**	**7.4**	**7.7**	**7.9**	**8.1**	**8.3**	**8.5**	**8.7**	**9.0**	**9.2**	**9.4**	**9.7**	**9.9**	**12.5**	**12.7**	**13.0**	**13.3**	**13.5**	**13.8**	**14.1**	**12.0**

Table 3: Cash flow statements (thousands of Euro)

CASH FLOW	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
(+) CFO	-	4.1	6.1	6.3	6.5	6.7	6.9	7.1	7.3	7.5	7.7	8.0	8.2	8.4	8.6	8.8	9.1	9.3	9.6	9.8	-15.0	15.8	16.0	16.3	16.5	16.7	17.0	17.2	15.2
(-) CFD	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	10.0	-1.4	-1.4	-1.4	-1.4	-1.4	-1.4	-1.4	-1.4
FCFE (Free Cash Flow for Equity)	**-**	**4.1**	**6.1**	**6.3**	**6.5**	**6.7**	**6.9**	**7.1**	**7.3**	**7.5**	**7.7**	**8.0**	**8.2**	**8.4**	**8.6**	**8.8**	**9.1**	**9.3**	**9.6**	**9.8**	**-5.0**	**14.5**	**14.7**	**14.9**	**15.1**	**15.4**	**15.6**	**15.9**	**13.8**
(-) CFE	-	-2.1	-3.0	-3.1	-3.2	-3.3	-3.4	-3.5	-3.6	-3.7	-3.8	-3.9	-4.0	-4.1	-4.3	-4.4	-4.5	-4.6	-4.7	-4.8	-	-6.2	-6.4	-6.5	-6.6	-6.8	-6.9	-7.0	-142.5
(-) CFL	**-**	**2.1**	**3.1**	**3.2**	**3.3**	**3.4**	**3.5**	**3.6**	**3.7**	**3.8**	**3.9**	**4.0**	**4.1**	**4.2**	**4.4**	**4.5**	**4.6**	**4.7**	**4.8**	**5.0**	**-5.0**	**8.2**	**8.3**	**8.4**	**8.5**	**8.6**	**8.7**	**8.8**	**-128.6**

Note: Notwithstanding the existence and uniqueness of NPV and NPV^e, neither the IRR of the project cash-flow stream nor the IRR of the equity cash-flow stream exists.

Impact of financing and payout policy on the economic profitability of solar photovoltaic plants

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Impact of financing and payout policy on the economic profitability of solar photovoltaic plants

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ABSTRACT

This paper introduces an innovative comprehensive evaluation model for appraising an investment in a solar photovoltaic plant which encompasses both operational and financial management. We illustrate the intricate network of logical relations among technical (estimated) variables and financial (decision) variables and show that establishing transparent links between the former and the latter enhances the accuracy and soundness of the model. The results indicate that understanding the conceptual and formal relations of operating variables and financial decisions is necessary for correctly measuring shareholder value creation and making rational decisions, even for those projects (such as solar energy projects) where the operating, technical component is of paramount importance. We show how a firm's decision of replacing conventional energy with solar energy may be affected by managerial decisions regarding the firm's payout/retention policy and its financing policy to support the project. The model discloses insights on how to fine-tune the financing and distribution decisions in order to maximize the value creation for shareholders. We apply the model to a real-life photovoltaic project to be located in the province of Modena, in Northeast Italy, and quantify the effect of financial decisions on the project's net present value, showing that the financing and distribution policies may amplify or shrink the impact of changes in other inputs and may even revert an otherwise unprofitable project into a value-creating one. Finally, we allow operational variables as well as financial variables to change in order to measure their importance via the application of the Clean Finite Change Sensitivity Indices (Magni et al., 2020).

1. Introduction

Sustainabile operations are becoming a major trend in the manufacturing system and the fourth industrial revolution offers innovations which potentially could accelerate a green economic development, also because of the technological advancement in the fields of decentralised energy production and storage of electrical energy (Bai and Sarkis 2017; Wichmann et al. 2019; Bai et al., 2020). Photovoltaic (PV) technologies have been playing a central role in the development of a worldwide sustainable energy system, with their recent remarkable performance enhancement and cost reduction, transforming solar energy in electrical energy and combatting climate change and environmental pollution (Lupangu and Bansal 2017; Sinke 2019; Lei et al., 2019; Kang et al., 2020; Ezbakhe and Perez-Foguet, 2021), also supporting electrification opportunities in less developed and developing countries, isolated communities, and rural areas (Henao et al., 2012; Ferrer-Martí et al., 2013; Yu, 2017). In spite of its environmental benefits, the adoption of solar energy by the industrial, commercial, and residential sectors is strongly affected by economic considerations (e.g., Dong et al., 2017; Cucchiella et al., 2018; Pham et al., 2019). The mapping which links the key performance drivers and the investment's economic profitability requires a deep understanding of the intricate network of relations among technical aspects, accounting magnitudes, forecasting of financial data, and assumptions on financial decisions, which makes the project's evaluation particularly complex. It is then important to provide decision-aiding tools capable of measuring the project's economic profitability, taking into account uncertainty and providing insights on possible managerial actions that may affect the decision to adopt solar energy.

Several studies in the photovoltaic discipline have recently

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investigated technical, economic and institutional challenges to turn potential into reality (Welling 2016; Lupangu and Bansal 2017; Lei et al., 2019; Gorjian et al., 2019). From a managerial perspective, Bhattacharya et al. (2020) propose a risk management tool for solar energy producers, investigating both natural hedges embedded in cash flows and cross hedging strategies with temperature-based weather derivatives; Ferrer-Martí et al. (2013) and Billionnet et al. (2016) study the optimal design of a hybrid wind–photovoltaic system made of photovoltaic panels, wind turbines and battery elements for serving a given demand and minimizing the cost, and, analogously, Li et al. (2017) optimize the sizing of solar and wind generating units of hybrid systems aiming to minimize the levelized energy cost. Jufri et al. (2019) recently introduced a detection system for monitoring the abnormal conditions in the photovoltaic plants and maintaining their productivity; Mauritzen (2020) studies quality differences in terms of production degradation over time in photovoltaic panels produced by different manufacturers, supported by the theory of asymmetric information; Moret et al. (2020) provide a robust optimization framework for decision support under uncertainty in energy models including photovoltaic systems. From a financial perspective, Abdallah et al. (2013) present an economic model for evaluating the option of installing small-scale photovoltaic plants on facility rooftops, and Büyüközkan and Güleryüz (2016) introduce a multi-criteria decision making tool for selecting the most appropriate renewable energy resources (including the solar one), from an investor-focused point of view, by considering evaluation criteria in the technical, economic, political, social, and environmental areas.

Despite the substantial amount of contributions studying the economic consequences of technical features of solar PV plants, somewhat neglected in the literature is the role of the interaction between financial decisions and operational estimates in increasing or decreasing the firm's value and affecting the accept/reject decision. In particular, for a given set of technical inputs and assumptions, the role of financial decisions (financing and payout policies) may well turn a wealth-destroying project into a wealth-creating one or vice versa. In this respect, previous studies on the financial dimension of photovoltaic projects narrow the scope of application, in the sense that

(i) the financial policy is fixed and is not considered a decision variable capable of increasing the economic value created (e.g., Talavera et al. 2010; Talavera et al., 2015; Farias-Rocha et al., 2019);
(ii) the fundraising and the combination of different sources of financing is mainly aimed at covering the capital expenditures, not at maximizing the net present value for equityholders (e.g., Bolinger 2009; Coughlin and Cory 2009);
(iii) the financial models used for net-present-value maximization (or for other kinds of comparison among projects) do not consider the option to finance the project internally with liquid assets and disregard the role of the payout policy and the reinvestment of undistributed cash or possible financial needs occurring in the following years (e.g., Gu et al., 2018; Talavera et al., 2019; Jimenez-Castillo et al., 2020; Feldman et al., 2021), which are relevant aspects for the project's economic worthiness.

Building upon Magni (2020), we propose a detailed logical framework for modeling investment decisions in solar PV systems and capturing the effect of the financial variables on the project's economic profitability. Unlike the contributions in the extant literature, we explain why and how, for a given set of technical inputs, the decisions on the financing mix selected for raising the necessary funds and the decisions on the amount of cash distributed to shareholders as opposed to cash retained and reinvested in the firm may affect the decision to switch to solar energy. Also, this model explicitly acknowledges the distinction between *estimation variables* and *decision variables* on one hand and between *operating variables* and *financial variables* on the other hand, a distinction which is not made in the models currently available in engineering economics or operations research and is totally neglected in practice by modelers, who focus on the operational side alone and handle financial decisions implicitly via the determination of the weighted average cost of capital, assumed to be constant and based on a given pre-determined target leverage ratio and a full payout policy. The estimation variables necessitate some estimation process to be determined (e.g., operating and maintenance costs, disposal costs, interest rate on debt financing) while the decision variables are under the managers' control (e.g., timing and size of distributions to shareholders, recourse to debt borrowing or to cash withdrawals for covering the financial needs). The operating variables express the factors which have a direct impact on the firm's costs and revenues as a result of the adoption of solar energy (e.g., solar panel efficiency, avoided electric bill, energy prices, amount of self-consumption, credit terms for energy sales to the grid). The financial variables regard the factors which affect the mix of financing sources, the cash flow raised from the capital providers, and the cash flow distributed to debtholders and shareholders as opposed to the cash flow retained in the firm (e.g., interest rate on debt, interest rate on liquid assets, risk-adjusted cost of capital, payout ratio, retention ratio). This paper precisely shows that, for a given selected set of assumptions on the operating variables, the firm's decisions on the payout policy (i.e., the cash distributed to the firm's shareholders) and the financing mix may have a significant role in adding or subtracting value and even in turning an otherwise unprofitable project into a value-creating one (or vice versa) and that the impact of such decisions may be larger or smaller depending on the value of the other input factors.

Our paper also makes use of sensitivity analysis to assess the contribution of the operational and financial variables to the equityholders' net present value (NPV). We consider changes in all the (operational and financial) inputs and measure how much of the NPV variation is caused by the change of a given input. To accomplish this task, we employ the recently developed Clean FCSI (Magni et al., 2020), based on Borgonovo (2010) FCSI, and detect the drivers which are more likely to cause a change in the decision (from non-investment to investment, or vice versa).[1] Sensitivity analysis will also be of help to analysts for calibrating the model: If the contribution to value of some parameters is small, then there is no need of modeling those inputs in more detail; in contrast, if some parameters contribute significantly to value creation, then the analyst may consider a further development of the model for gaining deeper insights. Clean FCSI will also be of help to highlight the importance of correctly modeling the project, making analysts aware of the impact of the estimation process and the embedded decision variables on the accept/reject decision.

The results obtained suggest that some time and effort should be devoted by the firm's management to model the distribution policy and the borrowing policy explicitly and measure its effects on the project's value. In such a way, the firm may calibrate a suitable financing-and-distribution policy which maximizes shareholder value creation. These findings confirm that the role of the financial decisions in an industrial project deserves more attention than it usually arouses in traditional financial modeling (see Tham and Vélez-Pareja 2004 for an exception).

The logical scheme and the financial model here introduced can be applied whatever the operational inputs and is easily generalized upon the modeler's needs and experience. The model is currently applied in practice and the theoretical part of the paper is accompanied by a real-life application regarding a ground-mounted stand-alone solar PV plant to be installed in the province of Modena, Emilia-Romagna, Italy, by GRAF SpA, an Italian manufacturing company.

The remainder of the paper is structured as follows: In Section 2, we

[1] Some sensitivity analysis, applied to the internal rate of return (IRR) of PV solar plants, may be found in Talavera et al. (2010), who assume the financial policy as given and do not quantify the contributions of each input onto the IRR variation.

present the model setting, breaking down the input factors into estimation variables and decision variables, and introduce the notions of operating income, operating cash flow, and free cash flow to equity (FCFE). In Section 3, we show the link between the FCFE and the associated financing and distribution decisions made by the firm. Section 4 illustrates how to carve out the project's cash flow from the estimation variables and how each year's decisions affect the accounting and financial magnitudes of the next year. In Section 5, we operationalize the logical structure illustrated in the previous sections by showing, for a solar PV plant, how to pass from inputs to cash flows. Section 6 makes use of the estimated cash flows and the net-present-value (NPV) approach to estimate the shareholder value created by the project and to make an economically rational decision. In Section 7, we apply the model and the evaluation methodology to a real-life solar PV plant. Section 8 carries out a scenario analysis for computing the project's economic profitability resulting from different financing and distribution decisions. Section 9 quantifies the individual impacts and the interaction effect of financing policy and distribution policy on shareholder value creation for a given set of operational inputs, while Section 10 considers changes of operational inputs as well as financial ones and assesses the effect on the net-present-value variation via the Clean FCSI technique. Some remarks conclude the paper (Table 0 describes the abbreviations and the symbols used in this paper).

2. Operating cash flow and free cash flow to equity

The accounting-and-finance model we propose is based on a comprehensive economic evaluation of the option of switching to solar energy for a firm currently importing energy from electric grid. The framework is based on a twofold classification of the variables affecting benefits and costs. On one hand, we distinguish *estimation inputs* and *decision inputs*; on the other hand, we differentiate the *operating inputs* from the *financial inputs*:

(i) estimation inputs are stochastic variables whose representative values (e.g., mean values, most probable values) require an estimation process involving expert knowledge
(ii) decision inputs deal with decisions which must be made explicitly in order to build the financial model of the project
(iii) operating inputs have to do with the firm's operating activities and the related change in accounting and financial magnitudes under the assumption of project undertaking
(iv) financial inputs have to do with fund raising and distribution of cash to capital providers, with the interest rates (on debt and on reinvestment of cash), and with the minimum attractive rate of return required by the investors for undertaking the project.

Owing to this taxonomy, the estimation variables may be operating (e.g., solar degradation panel rate, operating and maintenance costs, annual energy consumption, energy prices, etc.) or financial (e.g., interest rate on debt, interest rate on retained cash, required return on operating assets, etc.). Likewise, decision variables may be operating or financial. The operating decisions have to do with technical aspects of the project (e.g., decisions on the amount of operating and maintenance costs) or with economic aspects such as the management of the net operating working capital and the operating cycle; the financial decisions deal with

– the *financing policies*, which are decisions on the financing mix to cover the financial deficits. The latter may be covered with debt capital, equity capital or internal financing, defined as the recourse to existing liquid assets such as cash or cash equivalents (e.g., cash withdrawals from bank accounts or sales of marketable securities)
– the *distribution* (or *payout*) *policies*, which are decisions on the amount of distribution to shareholders of cash generated by the

Table 0
Symbols and abbreviations.

Abbreviation	Description
CapEx	capital expenditures (purchase price of plant at the expiration date)
CFD	cash flow to debt
CFE	cash flow to equity
EBIT	earnings before interest and taxes
EBITDA	earnings before interest, taxes, depreciation, and amortization
EBT	earnings before taxes
FCFE	free cash flow to equity
NFA	net fixed assets
NOPAT	net operating profit after taxes
NOWC	net operating working capital
NPV	net present value
NPV^d	debt NPV
NPV^e	equity NPV
NPV^l	non-operating NPV
NPV^o	operating NPV
CFL	cash flow from/to liquid assets
OpC	operating costs
OCF	cash flow from operations
O&M	operations and maintenance and insurance (% of plant's total cost)
PV	photovoltaic
ProdLoss	productivity loss in case of O&M = 0%
Rev	sales revenues
SuggO&M	technical suggested O&M and insurance (% of plant's total cost)
Symbol	**Description**
α	payout ratio
Δ	variation, change
τ	tax rate
C_t	capital
C_t^d	debt
C_t^e	equity
C_t^l	liquid assets
C_t^o	operating assets
D	debt financing
d_m	first of year of CFE distribution
D_m	loan tenure
Dep_t	depreciation and amortization expenses
E	equity financing
g_c	growth rate for costs
g_p	growth rate of energy price
g_Q	solar panel degradation rate
H	disposal costs
i^d	interest rate on debt
i^l	interest rate on liquid assets
I_t	income
I_t^d	interest expense
I_t^e	net income
I_t^l	interest income
I_t^o	operating income
L	internal financing (cash withdrawal)
m	lease expiration date
n	project length (useful life of PV plant)
P	lease payment
p_p	energy purchase price
p_s	energy selling price
$Q_{\max}$	annual unit production (first year)
Q	annual energy consumption
R	opportunity costs (e.g., foregone rents)
r^d	required return on debt
r^e	required return on equity
r^l	required return on liquid assets
r^o	required return on operating assets
T_t	income taxes
V_0	economic value
V_0^d	economic value of debt
V_0^e	economic value of equity
V_0^l	economic value of liquid assets
V_0^o	economic value of operations

project and decisions on the amount of cash retained in the firm and reinvested in the liquid assets.

We assume that the values of the operating variables are given and focus on the financial decisions regarding the coverage of financial deficits and the distribution of available cash to the firm's equityholders, which we call the *embedded decisions*, since the accounting-and-finance model describing the project cannot be completed without their determination. In order to understand the role of the embedded decisions on the output, we illustrate the model and then, in Section 6, clarify how to evaluate the project and make an accept-reject decision. Following, we describe the setting of the decision process and some fundamental accounting and financial magnitudes alongside the logical connections between the operating variables and financial variables on one hand, and the estimation variables and decision variables on the other hand.

The model starts from the input variables, which are used to build three pro forma statements for each one of the n+1 dates (0 to n): Statement of capitals (or balance sheet), statement of incomes, statement of cash flows. The first one collects the capital invested and raised by the firm for undertaking the project, the second one reports the incomes, and the third one reports the cash flows generated by the project and distributed to the capital providers. Letting n be the duration of the solar PV plant, a total of $3(n+1)$ statements must be built.

To draw up the statements, the analyst should first focus on the operating components. Let Rev_t be the incremental revenues derived from the sale of excess energy, and OpC_t be the incremental operational costs (O&M, insurance costs, opportunity costs such as lost rents, etc.) brought about by the plant. Let Dep_t be the depreciation charge of the solar PV plant. The pre-tax operating income, also called *earnings before interest and taxes* (EBIT), is determined as

$$\mathrm{EBIT}_t = \mathrm{Rev}_t - \mathrm{OpC}_t - \mathrm{Dep}_t. \tag{1}$$

Subtracting the income taxes, T_t, one finds the after-tax operating income, denoted as I_t^o:

$$I_t^o = \mathrm{EBIT}_t - \mathrm{T}_t \qquad t = 0, 1, \ldots, n \tag{2}$$

where T is obtained as the product of the marginal corporate tax, τ, on the earnings before taxes (EBT):

$$\mathrm{T}_t = \tau\overbrace{(\mathrm{Rev}_t - \mathrm{OpC}_t - \mathrm{Dep}_t + I_t^l - I_t^d)}^{\mathrm{EBT}} \tag{3}$$

with I_t^l and I_t^d denoting, respectively, the interest income on liquid assets (cash and cash equivalents, marketable securities, other financial assets) and the interest expense on debt, obtained as

$$I_t^l = i^l \cdot C_{t-1}^l \tag{4}$$

$$I_t^d = i^d \cdot C_{t-1}^d \tag{5}$$

where

i^l = interest rate of liquid asset
i^d = interest rate of debt
C_{t-1}^l = balance of liquid assets at time $t-1$
C_{t-1}^d = debt outstanding at time $t-1$.

Once estimated the after-tax operating income,[2] the analyst must estimate the operating cash flow, that is, the cash flow generated (or absorbed, if negative) by the project's operations. To this end, it suffices to subtract the change in the operating capital invested in the project from the after-tax operating income. For doing so, the analyst has to add the depreciation charges (Dep_t) which are the opposite of fixed assets' variation, and subtract the change in net operating working capital (NOWC).[3] Letting C_t^o denote the capital invested in the operations at the beginning of period $[t, t+1]$, the operating cash flow is

$$\mathrm{OCF}_t = I_t^o - \Delta C_t^o \tag{7}$$

$$= I_t^o + \mathrm{Dep}_t - \Delta\mathrm{NOWC}_t \tag{8}$$

where Δ denotes variation, so that $\Delta C_t^o = C_t^o - C_{t-1}^o$ (with $C_{-1}^o = 0$).

The OCF represents cash available for distribution to the capital providers (shareholders and debtholders). Part of it is used to service the debt and the residual amount is the so-called *Free Cash Flow to Equity* (FCFE). Mathematically,

$$\mathrm{FCFE}_t = \mathrm{OCF}_t - \mathrm{CFD}_t \tag{9}$$

where CFD_t denotes the cash flow paid to debtholders.[4] When FCFE is positive, it indicates the maximum amount of cash that can be distributed to shareholders without making recourse to additional debt or to cash withdrawals from the firm's existing liquid assets; when it is negative, it indicates that the OCF provided by the operations is not sufficient to service the debt and represents the maximum amount that can be contributed by the shareholders to cover the financial shortage. In other words, FCFE is a financial surplus potentially distributable to shareholders if it is positive, whereas it expresses a financial deficit potentially contributable by shareholders if it is negative.

3. FCFE and the embedded decisions

The FCFE is the hub of the matter. It is the financial variable which triggers the financing and distribution decisions embedded in the model. Specifically, the firm's analysts must determine, for each period, how a financial deficit (FCFE<0) will be covered (financing policy) and how a financial surplus (FCFE>0) should be employed (payout/retention policy). Modeling such decisions explicitly for each year is important because, as we now see, a decision made in one year affects next year's after-tax cash flows and, hence, determines the project's overall economic profitability.[5] Furthermore, the explicit account of the embedded decisions enables the firm's analysts to study the interrelations of payout

[2] If one subtracts income taxes from EBT one gets the after-tax earnings, also known as *net income*, which is the profit accrued to equityholders:

$$I_t^e = \mathrm{EBT}_t - \mathrm{T}_t. \tag{6}$$

[3] In a solar PV plant, NOWC is represented by the accounts receivable generated by the sale of excess energy and the accounts payable generated by the purchase of energy from the grid whenever the plant does not meet the firm's electricity needs. Usually, the firm has no degree of freedom on the working capital because, in general, the payment conditions to the service operator are established by the operator. Also, the firm has little bargaining power regarding the credit terms relative to the sale of excess energy. (For the role of working capital in selecting an appropriate measure of value creation, see Magni and Marchioni 2020.)

[4] CFD is equal to the sum of interest expenses and principal repayments: $\mathrm{CFD}_t = \overbrace{I_t^d}^{\text{interest expenses}} + \overbrace{(-\Delta C_t^d)}^{\text{principal repayments}}$ with $\Delta C_t^d = C_t^d - C_{t-1}^d$. The residual debt C_t^d can be calculated either as $C_{t-1}^d + \Delta C_t^d$ or as $C_{t-1}^d(1+i_t^d) - F_t^d$, depending on whether the principal repayments, $-\Delta C_t^d$, are given or the cash payments, F_t^d, are given. Whenever $F_t^d < 0$, it represents a cash flow contributed by the creditors.

[5] Traditional modeling often neglects internal financing and usually assumes (implicitly or explicitly) that 100% of a financial surplus is distributed to shareholders and 100% of a financial deficit is covered by equity or debt. In practice, firms often use cash withdrawals from existing liquid assets (internal financing) to finance the installation of solar PV projects and do not distribute all the cash available for distribution but reinvest it, wholly or partially, into liquid assets. To abide by realistic assumptions is important to avoid over- or under-estimation of the project's economic profitability.

policy and financing policy and their impact on value creation, which helps find an optimal financial policy which maximizes shareholder wealth.

In general, once the OCF is estimated and the cash flow to debt is subtracted, two situations may occur for each date:

(i) $\mathrm{FCFE}_t > 0$: a financial surplus occurs (cash may be distributed); a decision on distribution of cash flow to equityholders is required which also determines automatically the amount of internal reinvestment (retained cash)
(ii) $\mathrm{FCFE}_t < 0$: a financial deficit occurs (cash must be contributed); a decision on contribution of cash flow from equityholders is required which also determines automatically the amount of internal financing (cash withdrawal).[6]

Let CFE_t denote the cash flow actually distributed to equityholders when $\mathrm{FCFE}_t > 0$ or the cash flow actually contributed by shareholders when $\mathrm{FCFE}_t < 0$; a positive CFE indicates that cash is distributed to equityholders and a negative CFE signals that cash is contributed by equityholders.

A decision on the CFE is a payout/financing decision. Such a decision is a decision on the amount of cash withdrawn from the firm's liquid assets to cover the financial deficit (whenever $\mathrm{FCFE}_t < 0$) or the amount of cash retained in the firm and invested in liquid assets (whenever $\mathrm{FCFE}_t > 0$). The decisions on distribution/retention and on the financing mix must be explicitly modeled *in each year* in order to get the estimation of the next year's incomes and cash flows. Indeed, next year's operating income and cash flow depend on the amount of next year's taxes and the latter is affected by the next year's interest income, which in turn depends on this year's beginning-of-period balance of liquid assets. For example, if $\mathrm{OCF}_t = 100$ and $\mathrm{CFD}_t = 40$, the cash available for distribution is $\mathrm{FCFE}_t = 60$. Suppose the firm decides to distribute to shareholders 70% of FCFE and suppose the beginning-of-period balance of liquid assets is $C^l_{t-1} = 500$ and the interest rate on liquid assets is $i^l_t = 1\%$. Then, the cash flow distributed to the firm's shareholders is $\mathrm{CFE}_t = 70\% \cdot 60 = 42$, which means that the retained cash is $60 - 42 = 18$. Only now it is possible to determine the time-t balance of liquid assets by summing the interest income and the retained cash: $C^l_t = 500 + 1\% \cdot 500 + 18 = 523$, which determines the time-t+1 interest income and, therefore, the income taxes and, hence, the time t+1 (after-tax) operating income and cash flow. Vice versa, if $\mathrm{OCF}_t = 100$ and $\mathrm{CFD}_t = 120$, then $\mathrm{FCFE}_t = -20$, which represents a financial deficit. Suppose the firm decides to cover 70% of this financial shortage with equity. The cash flow contributed by the equityholders is $\mathrm{CFE}_t = 70\% \cdot (-20) = -14$, which represents an outlay for shareholders. The residual amount, $20 - 14 = 6$ is financed internally, via cash withdrawal from liquid assets. Hence, the time-t balance of liquid assets is $C^l_t = 500 + 1\% \cdot 500 - 6 = 499$, which in turn impacts the (after-tax) operating income and cash flow in time $t + 1$.

In general, depending on the situation, the balance of liquid asset is set, respectively, as

(i) $C^l_t = C^l_{t-1} + I^l_t +$ retained cash (internal reinvestment) if $\mathrm{FCFE}_t > 0$
(ii) $C^l_t = C^l_{t-1} + I^l_t -$ cash withdrawal (internal financing) if $\mathrm{FCFE}_t < 0$

The two above equations may be compressed into a single recursive equation:

$$C^l_t = C^l_{t-1} + I^l_t - \mathrm{CFL}_t \qquad t = 0, 1, \ldots, n, \qquad C^l_{-1} = 0 \tag{10}$$

where the CFL denotes the cash flow withdrawn from the liquid assets (if $\mathrm{CFL}_t > 0$) or the undistributed cash which is reinvested in the firm's liquid assets (if $\mathrm{CFL}_t < 0$). As noted, the CFL is automatically determined by the decisions on equity contribution or distribution:

$$\mathrm{CFL}_t = \mathrm{CFE}_t - \mathrm{FCFE}_t \tag{11}$$

so that the balance of liquid assets is essentially affected by CFE and FCFE as follows:

$$C^l_t = C^l_{t-1} + I^l_t = C^l_{t-1} + (\mathrm{FCFE}_t - \mathrm{CFE}_t).$$

4. The logical loop

The firm's analysts evaluating an investment opportunity should build a model which computes the streams of OCF, CFL, and CFD. These cash-flow streams will be used for the calculation of the economic value created, as will be shown in section 6.

The project's cash-flows are dynamically interconnected via a logical loop such that the OCF of the current year, OCF_t, affects the cash available for distribution, FCFE_t, which affects the cash flow from liquid assets, CFL_t, which in turn affects the balance of liquid assets, C^l_t, which affects the next year's interest income, I^l_{t+1} and, in turn, the amount of taxes, T_{t+1} and, hence, the operating income, $I^o_{t+1} = \mathrm{EBIT}_{t+1} - \mathrm{T}_{t+1}$, which in turn affects next year's operating cash flow OCF_{t+1}. The logical loop that needs to be accounted for in the model is then as follows:

$$\begin{aligned} \mathrm{OCF}_t \quad & \Rightarrow \mathrm{FCFE}_t \\ & \Rightarrow \overbrace{\mathrm{CFE}_t}^{\text{payout/financing decision}} \\ & \Rightarrow \mathrm{CFL}_t \\ & \Rightarrow C^l_t \\ & \Rightarrow I^l_{t+1} \\ & \Rightarrow \mathrm{T}_{t+1} \\ & \Rightarrow I^o_{t+1} \Rightarrow \mathrm{OCF}_{t+1} \end{aligned} \tag{12}$$

for $t = 0, 1, 2, \ldots, n - 1$. Analytically, using (2)-(8), the loop linking OCF_t and OCF_{t+1}, mediated by the embedded decision about CFE_t, may be expressed as follows:

[6] Since the FCFE may well be positive or negative, the FCFE is a Free Cash Flow *to* Equity in the former case and a Free Cash Flow *from* Equity in the latter case. In other words, the firm might be said to be *free* to distribute FCFE to shareholders in the former case and to be *free* to ask for equity contribution from shareholders in the latter case.

$$\begin{aligned}
\mathrm{OCF}_{t+1} &= I^o_{t+1} - \Delta C^o_{t+1} \\
&= \mathrm{EBIT}_{t+1} - \mathrm{T}_{t+1} - \Delta C^o_{t+1} \\
&= \mathrm{EBIT}_{t+1} - \tau(\mathrm{EBIT}_{t+1} + I^l_{t+1} - I^d_{t+1}) - \Delta C^o_{t+1} \\
&= \mathrm{EBIT}_{t+1}(1-\tau) + \tau I^d_{t+1} - \tau I^l_{t+1} - \Delta C^o_{t+1} \\
&= \mathrm{EBIT}_{t+1}(1-\tau) + \tau I^d_{t+1} - \tau i^l_{t+1} C^l_t - \Delta C^o_{t+1} \\
&= \mathrm{EBIT}_{t+1}(1-\tau) + \tau I^d_{t+1} - \tau i^l_{t+1}\left(C^l_{t-1}(1+i^l_t) - \mathrm{CFL}_t\right) - \Delta C^o_{t+1} \\
&= \mathrm{EBIT}_{t+1}(1-\tau) + \tau I^d_{t+1} - \tau i^l_{t+1}\left(C^l_{t-1}(1+i^l_t) + \mathrm{FCFE}_t - \overbrace{\mathrm{CFE}_t}^{\text{payout/financing decision}}\right) - \Delta C^o_{t+1} \\
&= \mathrm{EBIT}_{t+1}(1-\tau) + \tau I^d_{t+1} - \tau i^l_{t+1}\left(C^l_{t-1}(1+i^l_t) + \mathrm{OCF}_t - \mathrm{CFD}_t - \mathrm{CFE}_t\right) - \Delta C^o_{t+1}
\end{aligned} \tag{13}$$

for $t = 0, 1, \ldots, n-1$.[7]

At time n (terminal date), the project is over and the entire available cash is distributed to equityholders, which is equal to the sum of the last FCFE and the terminal balance of liquid assets (i.e., net balance derived from the cash previously retained and withdrawn, with accumulated interest incomes):

$$\mathrm{CFE}_n = \overbrace{C^l_{n-1} + I^l_n}^{\text{terminal balance of liquid assets}} + \overbrace{\mathrm{FCFE}_n}^{\mathrm{OCF}_n - \mathrm{CFD}_n}. \tag{15}$$

It is worth noting that the CFE at time n is the result of decisions made at every date $t = 0, 1, \ldots, n-1$. These decisions affect the balance of liquid assets at every date, as well as the magnitude of the equity book value at every date. The final liquidation CFE is nothing but the total amount of cash available to the firm, which derives from the liquid assets and from the operations of the last period, net of the debt service of the last period.

The firm's analysts should calculate the balances of all the capitals involved (operating assets, liquid assets, debt, and equity), all the incomes (operating income, interest on liquid assets and on debt, net income), all the cash flows (OCF, CFL, CFD, and CFE). Hence, they should collect them in three pro forma statements for each time t: The statement of capitals (or balance sheet), the statement of incomes and the statement of cash flows. The internal consistency of the model must be certified by the following three balancing equations:

$$\text{Statement of capitals} \quad \overbrace{C^o_t + C^l_t}^{\text{Capital invested in the project}} = \overbrace{C^d_t + C^e_t}^{\text{Capital raised from capital providers}} \tag{16}$$

$$\text{Statement of incomes} \quad \overbrace{I^o_t + I^l_t}^{\text{Income generated by the project}} = \overbrace{I^d_t + I^e_t}^{\text{Income accrued to capital providers}} \tag{17}$$

$$\text{Statement of cash flows} \quad \overbrace{\mathrm{OCF}_t + \mathrm{CFL}_t}^{\text{Cash flow contributed by the project}} = \overbrace{\mathrm{CFD}_t + \mathrm{CFE}_t}^{\text{Cash flow distributed to capital providers}} \tag{18}$$

(see also Magni 2020, Ch. 2).

The logical steps required to determine all the project's cash flows and build the pro forma statements may be summarized as follows:

1. Use the operating inputs to estimate the EBIT (see eq. (1))
2. Determine the interest income via eq. (4) and the interest on debt via eq. (5)
3. Subtract the income taxes (see eq. (3)) from EBIT to get the after-tax operating income via eq. (2)
4. Add depreciation charges and subtract the change in NOWC to get the OCF (see eqs. (7) and (8))
5. Subtract the CFD to get the FCFE (see eq. (9))
6. If the FCFE is positive, make a decision on how to split the available cash between distribution to shareholders and cash retention in the firm. If the FCFE is negative, make a decision on how to split the financial deficit between equity contribution and internal financing.[8] This decision determines the CFE
7. Calculate the CFL via eq. (11) and determine the balance of liquid assets via eq. (10)
8. Repeat the steps above for $t = 0, 1, 2, \ldots, n-1$. (For $t = n$, step 6 is replaced by the calculation of CFE_n via eq. (15).)

Fig. 1 provides a graphical representation of the logical loop, from start of the system ($t = 0$) to the end of the system ($t = n$). The logical loop is reframed as an influence diagram in Fig. 2. It should be clear now that the *periodic* choice of the financing mix (equity, debt, or internal financing) and/or the choice of the amount distributed and reinvested in liquid assets play a functional role, and that debt and liquid assets should be separated for a better description of real-life situations. The precise identification of the steps is also a necessary requisite to complete the balance sheet. Indeed, the *project's cash flow cannot be correctly calculated if the amount of liquid assets is not calculated*, unless highly restrictive assumptions are added. This is a subtle point, which is mostly neglected in practice and in the relevant literature. Our logical framework enables one to dismiss simplifying and restrictive assumptions which may result in over- or under-estimation of the economic value created by the project. In particular, the extant literature on financial modeling of industrial projects rests on simplifying assumptions. For example, the so-called textbook WACC method (widely employed in practice) is based on two assumptions (i) the interest rate on net debt (i.e., debt minus liquid assets) is equal to the required return on net debt, (ii) the firm

[7] The logical loop may also be described starting and ending with any one of the involved variables. For example, starting and ending with the balance of liquid assets:

$$\begin{aligned}
C^l_t \;&\Rightarrow I^l_{t+1} \\
&\Rightarrow \mathrm{T}_{t+1} \\
&\Rightarrow I^o_{t+1} \\
&\Rightarrow \mathrm{OCF}_{t+1} \\
&\Rightarrow \mathrm{FCFE}_{t+1} \\
&\Rightarrow \overbrace{\mathrm{CFE}_{t+1}}^{\text{payout/financing decision}} \\
&\Rightarrow \mathrm{CFL}_{t+1} \Rightarrow C^l_{t+1}
\end{aligned} \tag{14}$$

[8] Or, possibly, a new increase of debt.

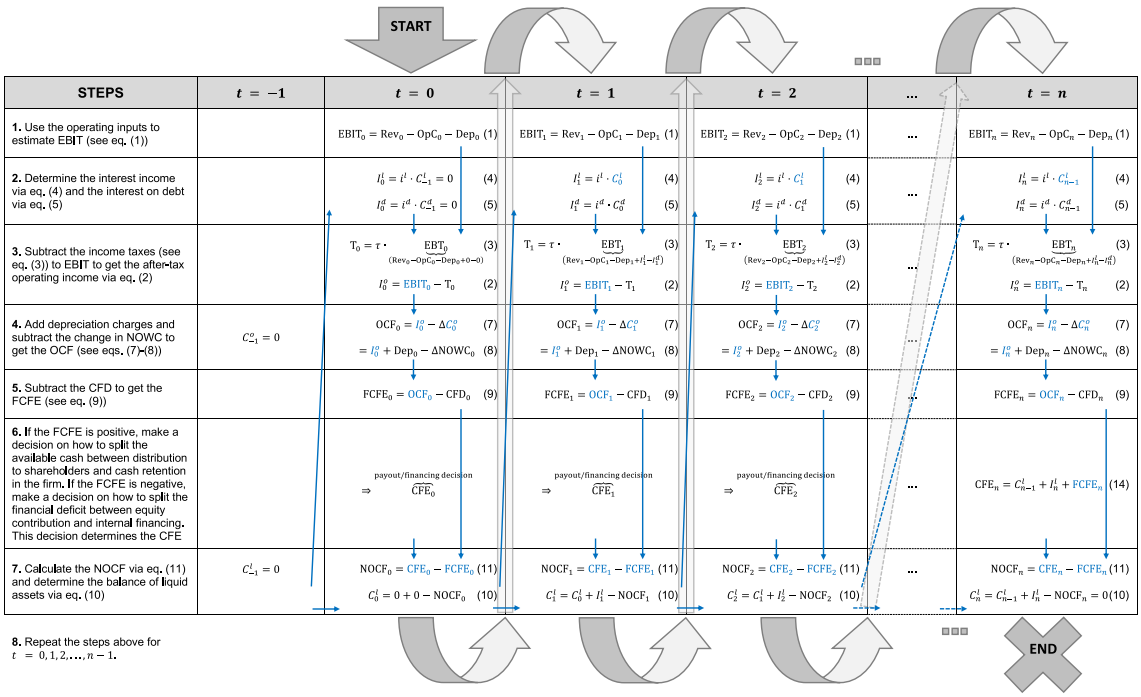

Fig. 1. The logical loop for calculating the project's cash flows.

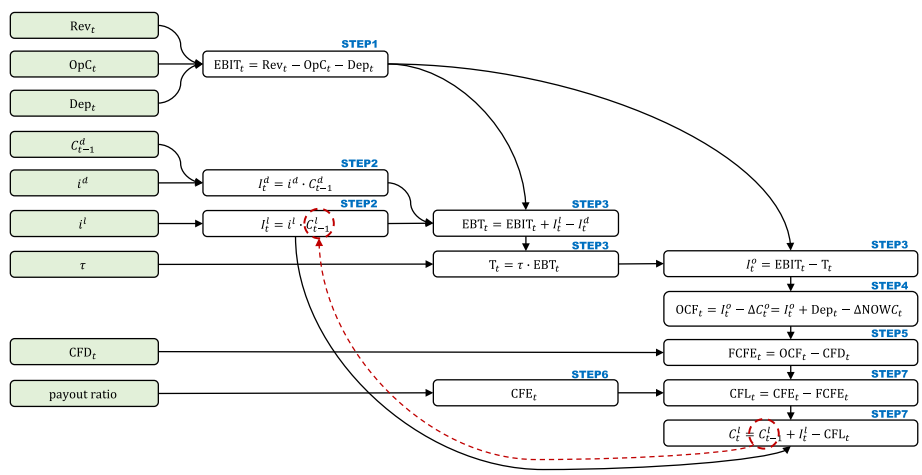

Fig. 2. Influence diagram of the logical loop represented in Fig. 1. The dashed line refers to time $t-1$: the balance of liquid assets at time $t-1$ determines the interest income in t via application of the interest on liquid assets, $I_t^l = i_t^l C_{t-1}^l$.

periodically adjusts the project's debt so as to maintain a target, pre-specified leverage ratio in market values (see Magni 2020, p. 350). This simplifies the financial model, because explicit forecasts of the financing policy are not needed to determine shareholder value creation (it suffices to discount Free Cash Flows at the after-tax WACC, calculated using assumption (ii)). However, these assumptions are not met in real-life applications of corporate projects, let alone in industrial projects.[9] Our model relaxes (i) and (ii) and takes explicit account of both the financing and payout policy. Another common model is the FCFE model, according to which the FCFEs are discounted at the cost of equity. This valuation scheme is correct only insofar as the undistributed cash is reinvested at the cost of equity capital and the arising financial deficits are financed at an interest rate equal to the cost of equity (this is, in essence, Miller and Modigliani, 1961, dividend irrelevance theorem). Under these assumptions, this valuation scheme enables the modeler to sidestep the impact of reinvested cash on the project's NPV. However, for capital asset projects, the undistributed cash is hardly ever invested at the cost of equity, and the financial deficits are not financed at the cost of equity, which implies that the adoption of such assumptions lead to over- or under-estimation (see also Magni 2020, pp. 344–5, for explanations). We relax these assumptions and cope with a more general setting in order to abide by more realistic assumptions about reinvestment of undistributed cash and about financing of financial deficits arising during the project's life, so our model does not suffer from the limitations of current models.

5. Feeding the model: from inputs to cash flows

In this section, we show how to plug the input factors in the financial model described above, making some assumptions on (the estimation variables and) the embedded decisions.

Consider a firm currently importing energy from electric grid, which is

[9] Not even at a firm level is debt rebalancing a common practice: Block (2011) surveyed 255 top-ranking financial officers; only 20.3% of them use debt rebalancing.

Table 1
Inputs for a solar PV plant.

Input	Symbol	Unit of measure	Type	Nature	Managerial area
Useful life of PV plant	n	years	Estimation	Operating	Project
Annual unit production (first year)	$Q_{\max}$	kWh/kWp/year	Estimation	Operating	Project
Solar panel degradation rate	g_Q	%	Estimation	Operating	Project
Disposal costs	H	€	Estimation	Operating	Project
Opportunity costs (e.g., foregone rents)	R	€/year	Estimation	Operating	Project
Growth rate for costs	g_c	%	Estimation	Operating	Project
Productivity loss in case of O&M = 0%	ProdLoss	%	Estimation	Operating	Project
Technical suggested O&M and insurance (% of plant's total cost)	SuggO&M	%	Estimation	Operating	Project
Lease expiration date	m	years	Estimation	Operating	Project
Lease payment	P	€/year	Estimation	Operating	Project
Purchase price of plant (at the expiration date)	CapEx	€	Estimation	Operating	Project
Annual energy consumption	q	kWh/year	Estimation	Operating	Company
Tax rate	τ	%	Estimation	Operating	Company
Energy purchase price	p_p	€/kWh	Estimation	Operating	Energy Market
Energy selling price	p_s	€/kWh	Estimation	Operating	Energy Market
Growth rate of energy price	g_p	%	Estimation	Operating	Energy Market
Required return on operating assets	r^o	%	Estimation	Financial	Capital market
Required return on liquid assets	r^l	%	Estimation	Financial	Capital market
Required return on debt	r^d	%	Estimation	Financial	Capital market
Interest rate on liquid assets	i^l	%	Estimation	Financial	Distribution
Interest rate on debt	i^d	%	Estimation	Financial	Financing
Loan tenure	D_m	years	Estimation	Financial	Financing
O&M and insurance (% of plant's total cost)	O&M	%	Decision	Operating	Project
First of year of CFE distribution	d_m	years	Decision	Financial	Distribution
Payout Ratio	α	%	Decision	Financial	Distribution
Equity financing	E	%	Decision	Financial	Financing
Internal financing (cash withdrawal)	L	%	Decision	Financial	Financing
Debt financing	D	%	Decision	Financial	Financing

offered the opportunity of switching to solar energy. Suppose the solar PV plant will be installed on a land property owned by the company and currently rented. With retail energy, the firm periodically pays a utility bill and receives a rental income from the rent of the land. If the solar PV plant is installed, the firm will stipulate a leasing contract whereby lease payments will be made periodically.[10] The plant will also require operating and maintenance costs (O&M) as well as insurance costs. After several years, at the expiration date, the lessee will pay a lump sum to acquire the plant, which may be financed with debt, equity, or internal financing (i.e., cash withdrawal from the firm's existing liquid assets). Once acquired the property of the plant, the solar PV system will continue to generate electric power for some years. At the end of its useful life, the plant will be removed, and the firm will incur disposal costs.

In terms of benefits and costs, if the retail system is replaced by the PV plant, the cash flows will increase as a result of the cost savings (avoided utility bill), but they will also decrease as a result of the operating and maintenance costs and the lost rental income (the solar panels will be ground-mounted). The two conflicting effects will determine a (positive or negative) change in the firm's EBIT and, hence, in the project's operating cash flow. At the expiration date, the firm will sustain a further expenditure for acquiring the property of the plant, which will bring about further benefits consisting of the ceased lease payments.

In Table 1, the input variables are reported (accompanied by their symbols and units of measure) with the specification of their nature (estimation, decision, operational, financial) and, in addition, with the managerial area with which they are associated.

The quantity of energy consumed for the firm's operations is estimated to be constant through time and equal to q; the current purchase price of energy is p_p, growing at a constant rate g_p per year. The utility bill is paid to the Energy Service Provider in the same year in which energy is consumed. The firm stipulates a lease contract with the following economic conditions: The lease payment, equal to P, is made periodically until the expiration date m; at time m, the firm acquires the property of the plant by paying a lump sum equal to CapEx (capital expenditure), and the solar PV system will keep on producing electric power for some years, until time n. From an accounting perspective, CapEx is a fixed asset, which is assumed to be depreciated evenly from $t = m + 1$ until $t = n$, so that the depreciation charge is $\text{Dep} = \text{CapEx}/(n - m)$. The PV plant is installed at $t = 0$ in a field owned by the firm, which is currently rented at a rent equal to R growing at the constant annual rate g_c. The latter represents an opportunity cost for the firm (a foregone income).

Starting from the first period, the PV plant requires operating, maintenance and insurance costs, expressed as a percentage of the total cost of the plant, which is the product between its nameplate capacity (in kWp) and its unit cost (per kWp). Technical experts determine a suggested level of these (percentage) costs for the first year in order to maximize the energy production, which we denote as SuggO&M. We denote as O&M the actual (percentage) expenses established by the management, which may be equal to or smaller than the suggested ones (i.e., O&M $\leq$ SuggO&M); both are assumed to grow at the constant annual rate g_c.

The solar panel degradation rate is g_Q. If O&M = SuggO&M, the PV system will produce $Q_{\max}$ units of energy in the first year, which decrease every year at the rate g_Q; if O&M = 0 (i.e., the company is not willing to spend for operating and maintenance costs), the energy

[10] A lease contract is an operating variable if lease payments are treated as operating expenses and the asset is not reported in the balance sheet during the lease term; it is a financial variable if it is treated like a loan, in which case accounting effects are shown on the balance sheets. In Italy, these kinds of costs are regarded operational costs, so they are not recorded in the balance sheet.

production suffers from a percentage loss due to lack of maintenance, denoted as ProdLoss. Furthermore, technical experts expect that the actual energy production in each period t, denoted as Q_t, will be proportional to the established level of O&M costs as compared to the suggested level. Specifically,

$$Q_t = Q_{\max}(1-g_Q)^{t-1}\cdot\left(1-\max\left(\text{ProdLoss}\cdot\frac{\text{SuggO\&M}-\text{O\&M}}{\text{SuggO\&M}},0\right)\right).$$

If the energy produced by the plant, Q_t, is higher than the energy consumed by the firm, the firm sells the differential quantity to the Energy Service Operator at the energy selling price p_s, growing at a constant rate g_p per year; the grid operator will pay the firm in the following year (this gives rise to accounts receivable). We assume that, at time $t = n$, the energy sold is paid immediately.

As a result, if the annual produced quantity is lower than the consumed energy in year t, that is, $Q_t < q$, energy costs savings arise equal to $Q_t \cdot p_p(1+g_p)^{t-1}$. If, instead, the produced quantity is higher than the consumed one, that is, $Q_t > q$, two benefits arise:

(i) energy costs savings arise equal to $q \cdot p_p(1+g_p)^{t-1}$
(ii) energy sales revenues equal to $(Q_t - q)\cdot p_s(1+g_p)^{t-1}$ are generated, which determine the presence of NOWC (i.e., accounts receivable).

Overall, the effect of the energy sales revenues and energy costs savings on the operating income can be summarized with the expression $\min(q, Q_t)\cdot p_p(1+g_p)^{t-1} + \max(0, Q_t - q)\cdot p_s(1+g_p)^{t-1}$ and the operating working capital can be represented with the formula $\text{NOWC}_t = \max(0, Q_t - q)\cdot p_s(1+g_p)^{t-1}$ and $\text{NOWC}_n = 0$. (See also Magni and Marchioni 2019).

At time n, disposal costs for removing the plant should be supported by the firm, whose current estimation for $t = 1$ is equal to H, expected to grow at the annual rate g_c in the time interval from 1 to n. Therefore, the expected disposal costs sustained at time n are equal to $H(1+g_c)^{n-1}$.

To sum up, conceptually

(i) the firm-without-the-project pays the utility bills and receives the rent for the land (for the whole period);
(ii) the firm-with-the-project sustains the lease payments (until $t = m$), the operating and maintenance costs (until $t = n$), the lump sum (in $t = m$), and the disposal costs (in $t = n$), and receives cash payments for the energy sold to the Energy Service Operator.

The project is, by definition, the difference between the firm-with-the-project and the firm-without-the project. Therefore, the pre-tax operating income is formally represented by

$$\text{EBIT}_t = \begin{cases} Z - P & \text{for } 1 \le t \le m \\ Z - \text{Dep} & \text{for } m+1 \le t \le n-1 \\ Z - \text{Dep} - H(1+g_c)^{t-1} & \text{for } t = n \end{cases} \tag{19}$$

where

$$Z = \min(q, Q_t)\cdot p_p(1+g_p)^{t-1} + \max(0, Q_t - q)\cdot p_s(1+g_p)^{t-1} - R\cdot(1+g_c)^{t-1} - \text{O\&M}\cdot(1+g_c)^{t-1}.$$

The project's operating assets, C_t^o, are represented by net operating working capital, NOWC_t, and, from time m on, by fixed assets, net of depreciation, NFA_t:

$$C_t^o = \begin{cases} \overbrace{\max(0, Q_t - q)\cdot p_s(1+g_p)^{t-1}}^{\text{NOWC}_t} & \text{for } 1 \le t \le m-1 \\ \overbrace{\max(0, Q_t - q)\cdot p_s(1+g_p)^{t-1}}^{\text{NOWC}_t} + \overbrace{\text{CapEx} - \text{Dep}\cdot(t-m)}^{\text{NFA}_t} & \text{for } m \le t \le n-1 \\ 0 & \text{for } t = n. \end{cases} \tag{20}$$

Using (2) and (7)-(8), one gets the OCF in each period. The OCF may be positive or negative (or zero). We assume that, whenever OCF is negative, all the financial needs will be covered by internal financing (cash withdrawal) except at time m, where CapEx is financed

(i) by equity capital with a proportion equal to $E \le 1$
(ii) by cash withdrawals from liquid assets (internal financing) with a proportion of $L \le 1$
(iii) by a loan contract of tenure $D_m = n - m$ with a proportion of $D = 1 - (E + L) \le 1$.

Whenever OCF is positive, FCFE is calculated subtracting the CFD associated to the loan stipulated at time m (see eq. (9)). If FCFE is negative, the financial needs will be covered with internal financing. If FCFE is positive, a decision on payout/retention is required, as seen in the previous sections. Let d_m be the first date at which some CFE is distributed; we assume that the firm will distribute a proportion α of the smaller between the net income and the FCFE, provided that they are both positive, that is

$$\text{CFE}_t = \begin{cases} 0 & \text{for } t = 1, 2, \ldots, d_m - 1 \\ \alpha\cdot\max[0, \min(I_t^e, \text{FCFE}_t)] & \text{for } t = d_m, d_m+1, \ldots, m-1, m+1, \ldots, n-1 \\ -E\cdot\text{CapEx} & \text{for } t = m. \end{cases} \tag{21}$$

The decision on CFE is also a decision on the amount of cash retained by the firm. The latter is equal to the FCFE minus the CFE (see eq. (11)). Finally, the project closes at time $t = n$ and we recall that CFE at $t = n$ is not a decision variable, since the available cash resulting from the retention decisions of the previous periods is entirely distributed to shareholders according to (15).

These nontrivial conceptual and formal relationships among estimation and decision variables and the impact on incomes and cash flows testify to the complexity of the financial modeling and suggest that the analyst should build a transparent model, where the embedded decisions are explicitly considered. Failing to do so would invalidate the determination of the financial magnitudes and even the internal consistency of the model.

Once this accounting-and-finance model of the project is built, all the cash flow streams associated with the project will be available. Using these cash-flow streams, the project is evaluated and the decision on whether undertaking the project or not will be made. In the next section, we illustrate the appraising process.

6. Shareholder value creation

In the previous sections, we have shown the first part of the financial model, consisting in drawing up three pro forma statements for the capitals, the incomes, and the cash flows. The second part of the financial modeling has to do with the evaluation of the project on the

basis of those statements, taking the point of view of the firm's shareholders.

Since the manager's mandate is to increase the wealth of the firm's shareholders, once the three pro forma statements have been built, the analyst must evaluate the shareholder value created by the project. As known, evaluation depends on the (opportunity) cost of capital: The economic (or market) value of any cash-flow stream is obtained by discounting its cash flows at the expected rate of return r on an equivalent-risk asset traded in the (assumed efficient) capital market: $V_0 = \sum_{t=1}^{n} F_t(1+r)^{-t}$. It represents the price that the cash-flow stream would have it were traded in the market.

Let r^o, r^l, r^d be, respectively, the required return on OCFs, the required return on CFLs, and the required return on CFDs, as estimated by the analyst.[11] Then, the economic values of the OCF stream, CFL stream and CFD stream are, respectively,

$$\begin{aligned} V_0^o &= \sum_{t=1}^{n} \frac{\mathrm{OCF}_t}{(1+r^o)^t} \\ V_0^l &= \sum_{t=1}^{n} \frac{\mathrm{CFL}_t}{(1+r^l)^t} \\ V_0^d &= \sum_{t=1}^{n} \frac{\mathrm{CFD}_t}{(1+r^d)^t}. \end{aligned} \tag{22}$$

Subtracting the respective initial capital, one gets the net present value (NPV) of the three areas:

$$\begin{aligned} \mathrm{NPV}^o &= V_0^o - C_0^o \quad \text{operating NPV} \\ \mathrm{NPV}^l &= V_0^l - C_0^l \quad \text{non–operating NPV} \\ \mathrm{NPV}^d &= V_0^d - C_0^d \quad \text{debt NPV.} \end{aligned} \tag{23}$$

The first one is the economic value generated by the operations (specifically, the production and consumption of energy, the maintenance of the plant, and the sale of excess energy to the grid operator); the second one is the economic value jointly generated by the internal financing and the reinvestment in liquid assets of the retained cash. The sum of NPV^o and NPV^l is the project's NPV, that is, the economic value created by the project as a result of the operations and the management of the non-operating cash flows (CFLs). The third one is the part of the project's NPV which is grasped by debtholders. (All of these NPVs may be either positive or negative or zero.) The residual amount obtained by subtracting the debt NPV from the project's NPV is the equity NPV, that is, the economic value created by the project and accrued to shareholders, after honoring the cash flows to debtholders:

$$\mathrm{NPV}^e = \overbrace{\mathrm{NPV}^o + \mathrm{NPV}^l}^{\text{project's NPV}} - \mathrm{NPV}^d \tag{24}$$

or, which is the same,

$$\mathrm{NPV}^e = \overbrace{(V_0^o + V_0^l - V_0^d)}^{\text{market value of equity}} - \overbrace{(C_0^o + C_0^l - C_0^d)}^{\text{initial equity investment}} = V_0^e - C_0^e. \tag{25}$$

NPV decision criterion. A project is worth undertaking if and only if it creates value for its equityholders, that is, $\mathrm{NPV}^e > 0$.

Owing to (24), shareholder value may be broken down to three components: The operating assets, the liquid assets, and the debt. Equityholders may then benefit not just from a value-creating operating activity (i.e., $\mathrm{NPV}^o > 0$), but also from an efficient management of liquid assets (i.e., $\mathrm{NPV}^l > 0$) and from the ability of borrowing at a rate i^d which is lower than the cost of debt, r^d, that is, the equilibrium rate prevailing in the capital markets (i.e., $\mathrm{NPV}^d < 0$). At the same time, the equity NPV may be positive even if the operating NPV is negative, as long as the management of the financial variables is efficient (resulting in $\mathrm{NPV}^l > 0$ and $\mathrm{NPV}^d < 0$).

If one assumes $i^d = r^d$, then the market value of debt coincides with the nominal value of debt, that is, $V_0^d = C_0^d$, which means $\mathrm{NPV}^d = 0$. In this case, the equity NPV is the sum of operating NPV and non-operating NPV:

$$\mathrm{NPV}^e = \mathrm{NPV}^o + \mathrm{NPV}^l. \tag{26}$$

If, in addition, $i^l = r^l$, then $V_0^l = C_0^l$ and $\mathrm{NPV}^l = 0$ so that the equity NPV is equal to the operating NPV:

$$\mathrm{NPV}^e = \mathrm{NPV}^o. \tag{27}$$

In the next section, we propose the application of this model to a firm facing the opportunity of switching from retail energy to solar energy.

7. The GRAF project: base scenario

In this section, we analyse value creation for a real-life, ground-mounted, stand-alone solar PV plant, recently offered by an Italian solar PV installer company, GRAF SpA, to a small-sized firm which we refer to as Alpha for confidential reasons. The specific assumptions about estimation and decision variables of the PV project (henceforth denoted as the GRAF project), reported in Table 2, have been directly provided by Alpha and by GRAF, where one of this paper's authors works as an engineer and a project analyst.

More precisely, the annual unit production $Q_{\max} = 1{,}080.00$ kWh/kWp/year has been determined collaboratively by the two companies, starting from several underlying factors:

(i) Alpha identified the place of installation in a proprietary land in the province of Modena, Emilia-Romagna (Italy), and defined the plant size according to the available space for panels, the acceptable investment size, and its energy demand (i.e., the forecast of its annual energy consumption).
(ii) The engineers of GRAF designed the installation features and estimated the expected annual unit production, according to the technical characteristics of the solar panels (as declared by the panels' manufacturer). In this estimation process, GRAF considered several parameters including the in-plane irradiation and the typical meteorological year at the geographical location, the panels' degradation rate, the system losses and the productivity loss in case of lack of maintenance.[12]

Furthermore, Alpha has provided relevant information on operational variables such as the lost rent from land property (opportunity cost), the annual energy consumption, the price paid for energy purchase, the tax rate, and its willingness to stipulate a 20-year lease contract, and has informed us about its intentions regarding timing and size of the financing of possible financial deficits and the distribution of the inflows generated by the project.

[11] Required returns are usually estimated by summing the risk-free rate to a risk premium compensating for risk. This is established by the market, possibly integrating it with subjective considerations (see Damodaran 1999, 2006, Berk and DeMarzo 2014, Titman and Martin 2016, Magni 2020, Sect. 5. See also Boudreaux et al., 2011, Bora and Vanek 2017 for the use of build-up models).

[12] Several tools are available for the estimation of the annual unit production $Q_{\max}$. An example is the Photovoltaic Geographical Information System (PVGIS), the science and knowledge service offered by the European Commission, available at https://ec.europa.eu/jrc/en/pvgis which provides free and open access to solar irradiation data and to Typical Meteorological Year data. The result $Q_{\max} = 1{,}080.00$ kWh/kWp/year is consistent with the PVGIS application using the following parameters: Latitude/Longitude: 44.646, 10.926 (the city center of Modena); Database used: PVGIS-SARAH; PV technology: Crystalline silicon; PV installed: 1 kWp; System loss: 17.75%; Slope angle: 0°; Azimuth angle: 0°; Mounting position: Free-standing.

Table 2
Assumptions.

Input	Assumption
ESTIMATED VARIABLES	
Useful life of PV plant	$n = 25$ years
Annual unit production (first year)	$Q_{\max} = 1{,}080.00$ kWh/kWp/year
Solar panel degradation rate	$g_Q = 0.90\%$
Disposal costs	$H = 5000$ €
Lost rent from land property	$R = 3{,}000$ €/year
Growth rate for costs	$g_c = 1.25\%$
Productivity loss in case of O&M = 0%	ProdLoss = 15%
Technical suggested O&M and insurance	SuggO&M = 4%
Lease expiration date	$m = 20$ years
Lease payment	$P = 6{,}268.45$ €/year
Purchase price of plant (at the expiration date)	CapEx = 25,000€
Required return on operating assets	$r^o = 6\%$
Required return on liquid assets	$r^l = 3\%$
Required return on debt	$r^d = 3\%$
Annual energy consumption	$q = 30{,}000$ kWh/year
Tax rate	$\tau = 27.9\%$
Energy purchase price	$p_p = 0.160$€/kWh
Energy selling price	$p_s = 0.130$€/kWh
Growth rate of energy price	$g_p = 1.25\%$
Interest rate on liquid assets	$i^l = 0.50\%$
Interest rate on debt	$i^d = 4\%$
Loan tenure (dependent variable)	$D_m = n - m = 5$ years
DECISION VARIABLES	
O&M and insurance	O&M = 3.50%
First year of CFE distribution	$d_m = 15$th year
Payout Ratio	$\alpha = 50.0\%$
Equity financing	$E = 25\%$
Internal financing (cash withdrawal)	$L = 25\%$
Debt financing (loan with level payments)	$D = 1 - (E + L) = 50\%$

In the following, we build the financial model of this real-life project and analyse value-creation for equityholders, with a special interest in the impact of different ways for financing the project combined with different distribution policies.

From the input data, the statements of capitals (balance sheets), the income statements, and cash flow statements are drawn in the way described in the previous sections. We report them in Fig. 3.[13] The three pro forma statements are logically interconnected in the non-trivial logical loop described in (14) and formalized in (13), owing to the embedded decisions: The decisions on financing and cash flow distribution affect the amount of liquid assets; this in turn affects next-period interest on liquid assets, which in turn affects next-period operating income and, therefore, the next-period OCF.

As described in the first part of this paper, the model logically chains estimated data and decisions regarding the proportions of distribution and retention and the proportions of equity financing and internal financing. For example, to build the balance of liquid assets at $t = 24$, C^l_{24}, we consider the balance of liquid assets at time $t = 23$, which is $C^l_{23} = 2{,}390.66$. This amount increases by the interest income $I^l_{24} = i^l \cdot C^l_{23} = 0.5\% \cdot 2{,}390.66 = 11.95$, and by the retained cash (i.e., the amount not distributed to the equityholders) at time $t = 24$. The latter is obtained via eq. (11) as

$$\begin{aligned}\text{Retained cash} &= -\text{CFL}_{24}\\ &= \text{FCFE}_{24} - \text{CFE}_{24}\\ &= \text{FCFE}_{24} - \alpha \cdot \max[0, \min(I^e_{24}, \text{FCFE}_{24})]\\ &= 3{,}279.58 - 50\% \cdot \max[0, \min(869.72, 3{,}279.58)]\\ &= 3{,}279.58 - 50\% \cdot 869.72 = 2{,}844.72.\end{aligned}$$

As a result, using (10), the balance of liquid assets at time $t = 24$ as

$$C^l_{24} = C^l_{23} + I^l_{24} - \text{CFL}_{24} = 2{,}390.66 + 11.95 + 2{,}844.72 = 5{,}247.33.$$[14]

This amount enables calculating the terminal CFE at time $t = 25$ via eq. (15) as $\text{FCFE}_{25} + C^l_{24} + I^l_{25} = 6{,}849.34 + 5{,}247.33 + 26.24 = 12{,}122.91$, which entirely liquidates the investment project.[15]

The pro forma financial statements in Fig. 3 represent the changes in the pro forma financial statements of the firm as a result of switching to solar energy. For example, the revenues express the increase in the firm's revenues, the operating costs express the increase in the firm's operating costs (note that, from year 21 to year 24, a decrease in the firm's operating costs occurs, since the cost savings due to avoided bills outweigh the plant's operating and maintenance costs). The last line of the cash-flow statement highlights the project's non-operating cash flows. In year 1, a decrease of cash occurs in order to cover the financial deficit ($\text{CFL}_1 > 0$). From year 2 to year 19 part of FCFE generated by the project is retained in the firm ($\text{CFL}_t < 0$). In year 20, cash is withdrawn again from liquid assets ($\text{CFL}_{20} > 0$) to partially finance the purchase of the solar PV plant. From year 21 to year 24, part of the FCFE is retained ($\text{CFL}_t > 0$) and, in year 25, cash is distributed to capital providers ($\text{CFL}_{25} > 0$).

The expected (equity) NPV is the model output. With the assumptions made, it is slightly positive: $\text{NPV}^e = 32.84 > 0$. A better understanding of this result is presented in the following table, obtained from (24):

Equity NPV decomposition	
+ NPV of operating assets	$+ \text{NPV}^o = -1{,}188.91$
+ NPV of liquid assets	$+ \text{NPV}^l = 1{,}420.57$
− NPV of debt	$- \text{NPV}^d = -198.81$
= NPV of equity	$= \text{NPV}^e = 32.84$

The NPV of the OCFs is negative and tends to destroy value. However, it would be unwise to recommend rejection on the basis of this operating NPV alone. The NPV of the CFLs (i.e., the cash withdrawals and the reinvestment in financial assets) creates more value than the operating assets destroy: The way the firm will manage the financial policy is able to compensate and turn an otherwise unprofitable project into a profitable one. The NPV of the project is then $\text{NPV}^o + \text{NPV}^l = -1{,}188.91 + 1{,}420.57 = 231.66$. Part of the project's value created is captured by debtholders; specifically, equityholders lose this part of the value created at the expense of the debtholders, but this loss is tiny, due to the limited scale of the debt. As a result, the assumptions made are such that the financial decisions more than compensate, albeit slighlty, the negative performance of the operations. This case testifies to the importance of the financial variables and, in particular, of the embedded decisions, in creating value.

The next section shows that the value created by this project may be changed (increased or decreased) by changing choices of financing and distribution.

8. The GRAF project: scenario analysis

The impact of the financing-and-distribution decisions on the value of the GRAF project may be best appreciated by showing the effect of changes in the input data on the equity NPV. Table 3 and Figs. 4–5 present 8 scenarios for the GRAF project with different assumptions on how financial deficits are covered and how financial surplus are employed. In particular, we have considered different proportions of

[13] The debt is reimbursed with level payments and the plant's total cost is 96,600 euro, obtained as the product of the plant's nameplate capacity (92 kWp) and its unit cost (1,050 euro per kWp).

[14] Only after this computation is done, the balance sheet at time $t = 24$ may be completed by calculating the equity capital:
$C^e_{24} = C^o_{24} + C^l_{24} - C^d_{24} = 13{,}510.01 + 5{,}247.33 - 2{,}699.85 = 16{,}057.50$.

[15] The terminal CFE may equivalently be obtained as $C^e_{24} + I^e_{25} = 16{,}057.50 - 3{,}934.59 = 12{,}122.91$, confirming the logical consistency of (this part of) the model.

BALANCE SHEET	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
BS ASSETS																										
Operating Assets	-	8,775	8,769	8,763	8,756	8,749	8,741	8,733	8,724	8,715	8,705	8,695	8,684	8,673	8,661	8,648	8,635	8,622	8,608	8,593	33,578	28,562	23,545	18,528	13,510	-
Accounts receivable from grid operator	-	*8,775*	*8,769*	*8,763*	*8,756*	*8,749*	*8,741*	*8,733*	*8,724*	*8,715*	*8,705*	*8,695*	*8,684*	*8,673*	*8,661*	*8,648*	*8,635*	*8,622*	*8,608*	*8,593*	*8,578*	*8,562*	*8,545*	*8,528*	*8,510*	-
Net fixed assets	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	*25,000*	*20,000*	*15,000*	*10,000*	*5,000*	-
Liquid assets	-	-8,108	-7,482	-6,873	-6,281	-5,707	-5,150	-4,612	-4,093	-3,593	-3,115	-2,657	-2,221	-1,807	-1,417	-1,227	-1,050	-885	-733	-595	-6,610	-3,541	-540	2,391	5,247	-
ASSETS	-	667	1,286	1,889	2,474	3,042	3,591	4,121	4,631	5,121	5,590	6,038	6,463	6,865	7,244	7,421	7,585	7,737	7,874	7,998	26,967	25,021	23,005	20,918	18,757	-
BS LIABILITIES																										
Debt	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	12,500	10,192	7,792	5,296	2,700	-
Equity	-	667	1,286	1,889	2,474	3,042	3,591	4,121	4,631	5,121	5,590	6,038	6,463	6,865	7,244	7,421	7,585	7,737	7,874	7,998	14,467	14,828	15,213	15,623	16,057	-
LIABILITIES	-	667	1,286	1,889	2,474	3,042	3,591	4,121	4,631	5,121	5,590	6,038	6,463	6,865	7,244	7,421	7,585	7,737	7,874	7,998	26,967	25,021	23,005	20,918	18,757	-

INCOME STATEMENT	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Revenues	-	5,775	5,731	5,687	5,642	5,596	5,549	5,500	5,451	5,401	5,350	5,298	5,245	5,190	5,135	5,078	5,021	4,962	4,902	4,841	4,779	4,715	4,651	4,585	4,518	4,449
New revenue: sale of energy	-	*8,775*	*8,769*	*8,763*	*8,756*	*8,749*	*8,741*	*8,733*	*8,724*	*8,715*	*8,705*	*8,695*	*8,684*	*8,673*	*8,661*	*8,648*	*8,635*	*8,622*	*8,608*	*8,593*	*8,578*	*8,562*	*8,545*	*8,528*	*8,510*	*8,492*
Lost rent from land property	-	*-3,000*	*-3,038*	*-3,075*	*-3,114*	*-3,153*	*-3,192*	*-3,232*	*-3,273*	*-3,313*	*-3,355*	*-3,397*	*-3,439*	*-3,482*	*-3,526*	*-3,570*	*-3,614*	*-3,660*	*-3,705*	*-3,752*	*-3,799*	*-3,846*	*-3,894*	*-3,943*	*-3,992*	*-4,042*
(-) Operating costs	-	-4,849	-4,832	-4,814	-4,796	-4,777	-4,759	-4,740	-4,721	-4,701	-4,682	-4,662	-4,642	-4,621	-4,601	-4,580	-4,559	-4,537	-4,516	-4,494	-4,472	1,819	1,842	1,865	1,888	-4,825
(-) Lease annual payment	-	*-6,268*	*-6,268*	*-6,268*	*-6,268*	*-6,268*	*-6,268*	*-6,268*	*-6,268*	*-6,268*	*-6,268*	*-6,268*	*-6,268*	*-6,268*	*-6,268*	*-6,268*	*-6,268*	*-6,268*	*-6,268*	*-6,268*	*-6,268*	-	-	-	-	-
(-) O&M cost	-	*-3,381*	*-3,423*	*-3,466*	*-3,509*	*-3,553*	*-3,598*	*-3,643*	*-3,688*	*-3,734*	*-3,781*	*-3,828*	*-3,876*	*-3,925*	*-3,974*	*-4,023*	*-4,074*	*-4,124*	*-4,176*	*-4,228*	*-4,281*	*-4,335*	*-4,389*	*-4,444*	*-4,499*	*-4,555*
(-) Disposal costs	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	*-6,737*
(-) Cost saving: self consumption of energy	-	*4,800*	*4,860*	*4,921*	*4,982*	*5,045*	*5,108*	*5,171*	*5,236*	*5,302*	*5,368*	*5,435*	*5,503*	*5,572*	*5,641*	*5,712*	*5,783*	*5,855*	*5,929*	*6,003*	*6,078*	*6,154*	*6,231*	*6,309*	*6,387*	*6,467*
EBITDA	-	925	900	873	846	819	790	761	731	700	668	636	603	569	534	499	462	425	386	347	307	6,535	6,493	6,450	6,406	-375
(-) Depreciation	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-5,000	-5,000	-5,000	-5,000	-5,000
EBIT	-	925	900	873	846	819	790	761	731	700	668	636	603	569	534	499	462	425	386	347	307	1,535	1,493	1,450	1,406	-5,375
Interest income	-	-	-41	-37	-34	-31	-29	-26	-23	-20	-18	-16	-13	-11	-9	-7	-6	-5	-4	-4	-3	-33	-18	-3	12	26
(-) interest expenses	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-500	-408	-312	-212	-108
EBT	-	925	859	836	812	787	762	735	708	680	651	621	590	558	525	491	456	419	382	344	304	1,002	1,067	1,136	1,206	-5,457
(-) Taxes	-	-258	-240	-233	-227	-220	-212	-205	-197	-190	-181	-173	-165	-156	-147	-137	-127	-117	-107	-96	-85	-279	-298	-317	-337	1,523
NET INCOME	-	667	619	603	585	568	549	530	510	490	469	447	425	402	379	354	329	302	275	248	219	722	770	819	870	-3,935

EBIT	-	925	900	873	846	819	790	761	731	700	668	636	603	569	534	499	462	425	386	347	307	1,535	1,493	1,450	1,406	-5,375
(-) Taxes	-	-258	-240	-233	-227	-220	-212	-205	-197	-190	-181	-173	-165	-156	-147	-137	-127	-117	-107	-96	-85	-279	-298	-317	-337	1,523
OPERATING INCOME	-	667	660	640	620	599	578	556	533	510	487	463	438	413	388	361	335	308	280	251	222	1,255	1,195	1,133	1,070	-3,853

CASH FLOW	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
(+) OCF	-	-8,108	666	646	627	606	585	564	542	520	497	473	449	425	400	374	348	321	294	266	-24,762	6,271	6,212	6,150	6,087	9,657
(-) CFD	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	12,500	-2,808	-2,808	-2,808	-2,808	-2,808
FCFE (Free Cash Flow for Equity)	-	-8,108	666	646	627	606	585	564	542	520	497	473	449	425	400	374	348	321	294	266	-12,262	3,463	3,404	3,342	3,280	6,849
(-) CFE	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-177	-164	-151	-138	-124	6,250	-361	-385	-409	-435	-12,123
(-) CFL	-	-8,108	666	646	627	606	585	564	542	520	497	473	449	425	400	197	183	170	156	142	-6,012	3,102	3,019	2,933	2,845	-5,274
CFL		8,108	-666	-646	-627	-606	-585	-564	-542	-520	-497	-473	-449	-425	-400	-197	-183	-170	-156	-142	6,012	-3,102	-3,019	-2,933	-2,845	5,274
interpretation		*cash withdrawal*	*retained cash*	*retained cash*	*retained cash*	*retained cash*	*retained cash*	*retained cash*	*retained cash*	*retained cash*	*retained cash*	*retained cash*	*retained cash*	*retained cash*	*retained cash*	*retained cash*	*retained cash*	*retained cash*	*retained cash*	*retained cash*	*cash withdrawal*	*retained cash*	*retained cash*	*retained cash*	*retained cash*	*cash withdrawal*

Fig. 3. Balance Sheets, Income Statements, Cash-flow Statements of the GRAF project.

Table 3
Scenario analysis for the decision inputs.

SCENARIO	1st year of CFE distribution (d_m)	Payout Ratio (α)	Equity financing (E)	Internal financing (L)	Debt financing (D)	Equity NPV (NPV^e)
1	25	0%	100%	0%	0%	−772.69
2	25	0%	75%	0%	25%	−642.60
3	20	25%	50%	25%	25%	−202.75
4	15	50%	25%	25%	50%	32.84
5	10	50%	0%	50%	50%	651.21
6	5	50%	0%	75%	25%	1331.60
7	1	75%	0%	75%	25%	2215.90
8	1	100%	0%	100%	0%	3041.44

financing sources for the purchase of the solar PV plant, different payout ratios and different years as first year of cash distribution. Scenario 4 is the base scenario described in Section 7. In this scenario, the financial policy offsets the negative performance of the operations. In scenario 1 to 3, where equity financing for the purchase of the PV plant is predominant, the payout ratio is low, and the firm distributes the available cash late, the financial decisions do not compensate the bad performance of the operations. However, by (i) increasing the payout ratio, (ii) anticipating the cash distributions, (iii) reducing the equity financing, and (iv) increasing the internal financing, the equity NPV is greatly increased. The variance between scenario 1 and scenario 8 is significant, as illustrated in Fig. 5: The equity NPV turns from a negative −772.69€ (scenario 1) to a positive 3,041.44€ (scenario 8) representing an increase of 3,814.13€.

It is easy to see that the choice of internal financing for purchasing the solar PV plant at time 20 (expiration date of the lease contract) contributes positively to create value. To see it, consider scenario 8, where 100% internal financing is assumed and the resulting equity NPV is 3,041.44. With distribution policy unvaried (i.e., $d_m = 1$ and $\alpha = 100\%$) if one assumes 100% equity financing or 100% debt financing or a mix of equity and debt, then the equity NPV will be much smaller than 3,041.44. Following are some results for different mixes of financing sources in scenario 8:

100% internal financing (scenario 8)	$\text{NPV}^e = 3041.44$
100% equity, 0% debt	$\text{NPV}^e = 1410.84$
80% equity, 20% debt	$\text{NPV}^e = 1499.57$
60% equity, 40% debt	$\text{NPV}^e = 1588.29$
50% equity, 50% debt	$\text{NPV}^e = 1632.65$
40% equity, 60% debt	$\text{NPV}^e = 1677.02$
20% equity, 80% debt	$\text{NPV}^e = 1765.74$
0% equity, 100% debt	$\text{NPV}^e = 1865.36$

We mention that the equity NPV increases as the equity financing decreases and debt financing increases until the maximum value of 1,865.36 is achieved with 100% debt, which is still much lower than the equity NPV resulting from the original scenario 8 with 100% of internal financing.

It is worth noting that the impact of the financing-and-distribution decisions on shareholder value creation is sensitive to other input factors, which may amplify or shrink their effect. Next, we analyse a change

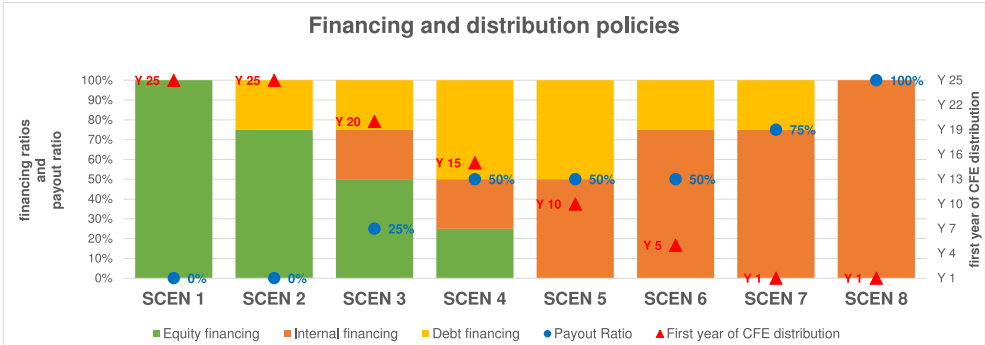

Fig. 4. Scenario analysis: Financing and distribution policies (base scenario = 4). The financial policy in scenario 8 consists of 100% internal financing (i.e., no recourse to debt nor equity), full payout policy, and year 1 as the first year of distribution.

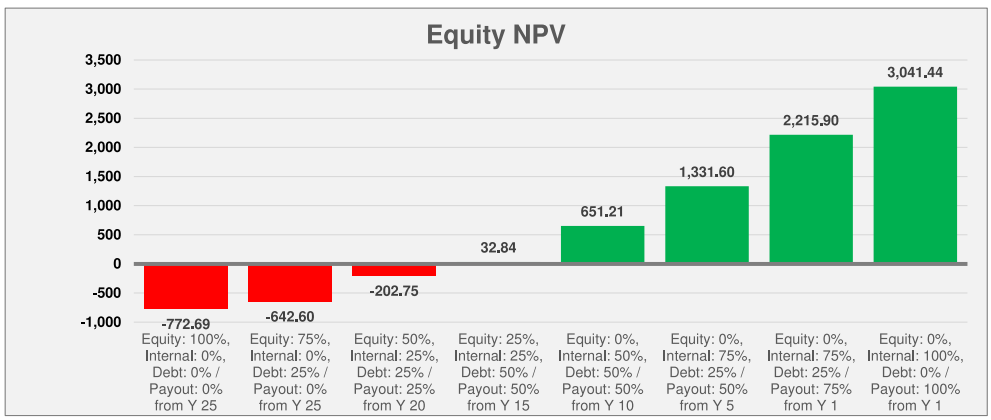

Fig. 5. Scenario analysis: Equity NPV for the 8 different financing and distribution policies.

in three different parameters:

- the interest rate on liquid asset (financial parameter), which triggers a shrinking effect on value,
- the annual energy consumption (operational parameter), which triggers a magnifying effect on value, and
- the annual unit production (operational parameter), which triggers a magnifying effect on value.

This allows us to verify whether the preference for Scenario 8 is robust under several changes in the assumptions.

Table 4 shows that by increasing i^l, other things unvaried, the effect of financial decisions on equity NPV is diminished, as testified by the max-min deviation (last line). Note that, given a scenario, the higher the interest rate, the lower the equity NPV. This is because, in most periods, the balance of liquid assets is negative (indicating that the project entails a reduction in the firm's liquid assets), so i^l represents a foregone rate of return on those liquid assets. The analysis in Table 4 confirms Scenario 8 as the best scenario, regardless of the level of the interest rate i^l, that is, creating the highest value or destroying the lowest value for equityholders. Also, the accept/reject decision itself may be different under different financial policies.

Consider now a change in the annual energy consumption. Table 5 shows that, by increasing the annual energy consumption (q), other things unvaried, the effect of financial decisions on NPV is augmented, as measured via the max-min deviation. Note that, for any given scenario, a higher energy consumption generates a higher NPV. The reason is that higher q implies higher cost savings, higher net incomes and, therefore, higher CFE, which results in higher NPVs. This effect is amplified by the financing and distribution decisions. Again, Scenario 8 is confirmed as the best scenario, regardless of the level of the energy consumption q. Further, note that, at a given level of $q = 25{,}000$ and $q = 30{,}000$, the accept/reject decision may be reverted, depending on the financial policies.

Finally, we consider a change in the first-year unit production, Q_{max} (which in turn affects the next years' production). From Section 7, we

Table 4
NPV deviations under different assumptions of interest rate on liquid assets.

Interest rate (i^l)	0.5%	1.5%	2%	2.5%	3%	3.5%
SCENARIO 1	−772.69	−815.11	−867.17	−942.69	−1044.07	−1173.90
SCENARIO 2	−642.60	−741.43	−821.99	−926.22	−1056.50	−1215.46
SCENARIO 3	−202.75	−444.29	−597.63	−775.54	−980.38	−1214.72
SCENARIO 4	32.84	−301.90	−501.94	−726.19	−976.64	−1255.42
SCENARIO 5	651.21	109.55	−193.55	−520.05	−871.47	−1249.41
SCENARIO 6	1331.60	572.24	163.38	−266.20	−717.48	−1191.51
SCENARIO 7	2215.90	1131.89	575.30	8.72	−568.05	−1155.23
SCENARIO 8	3041.44	1664.88	975.66	285.83	−404.62	−1095.70
Max-Min deviation	3814.13	2479.99	1842.84	1228.52	651.88	159.72

Table 5
NPV deviations under different assumptions of annual energy consumption.

Energy consumption (q)	20,000	25,000	30,000	40,000	50,000	60,000
SCENARIO 1	−3110.53	−1941.61	−772.69	1565.16	3903.00	6240.84
SCENARIO 2	−2980.45	−1811.53	−642.60	1695.24	4033.08	6370.93
SCENARIO 3	−2550.31	−1376.53	−202.75	2144.80	4492.36	6839.92
SCENARIO 4	−2400.37	−1184.89	32.84	2468.31	4903.78	7338.79
SCENARIO 5	−1909.79	−630.48	651.21	3214.60	5777.99	8341.39
SCENARIO 6	−1407.68	−39.35	1331.60	4073.49	6815.39	9557.28
SCENARIO 7	−916.45	647.42	2215.90	5352.85	8489.80	11,626.75
SCENARIO 8	−349.03	1342.88	3041.44	6438.58	9835.71	13,232.84
Max-Min deviation	2761.50	3284.48	3814.13	4873.42	5932.71	6991.99

Table 6
NPV deviations under different assumptions of annual unit production (Q_{max}).

Annual unit production (Q_{max})	980	1030	1080	1130	1180	1230
SCENARIO 1	−7407.50	−4090.09	−772.69	2544.72	5862.13	9179.54
SCENARIO 2	−7277.42	−3960.01	−642.60	2674.80	5992.21	9309.62
SCENARIO 3	−6866.91	−3535.57	−202.75	3130.07	6462.89	9795.71
SCENARIO 4	−6736.83	−3401.20	32.84	3507.25	6976.39	10,445.53
SCENARIO 5	−6326.32	−2989.65	651.21	4350.10	8035.14	11,717.18
SCENARIO 6	−6045.89	−2631.49	1331.60	5353.81	9372.96	13,383.77
SCENARIO 7	−6045.89	−2430.18	2215.90	6952.22	11,688.54	16,424.86
SCENARIO 8	−5760.78	−2035.54	3041.44	8239.56	13,437.68	18,635.80
Max-Min deviation	1646.72	2054.55	3814.13	5694.84	7575.55	9456.26

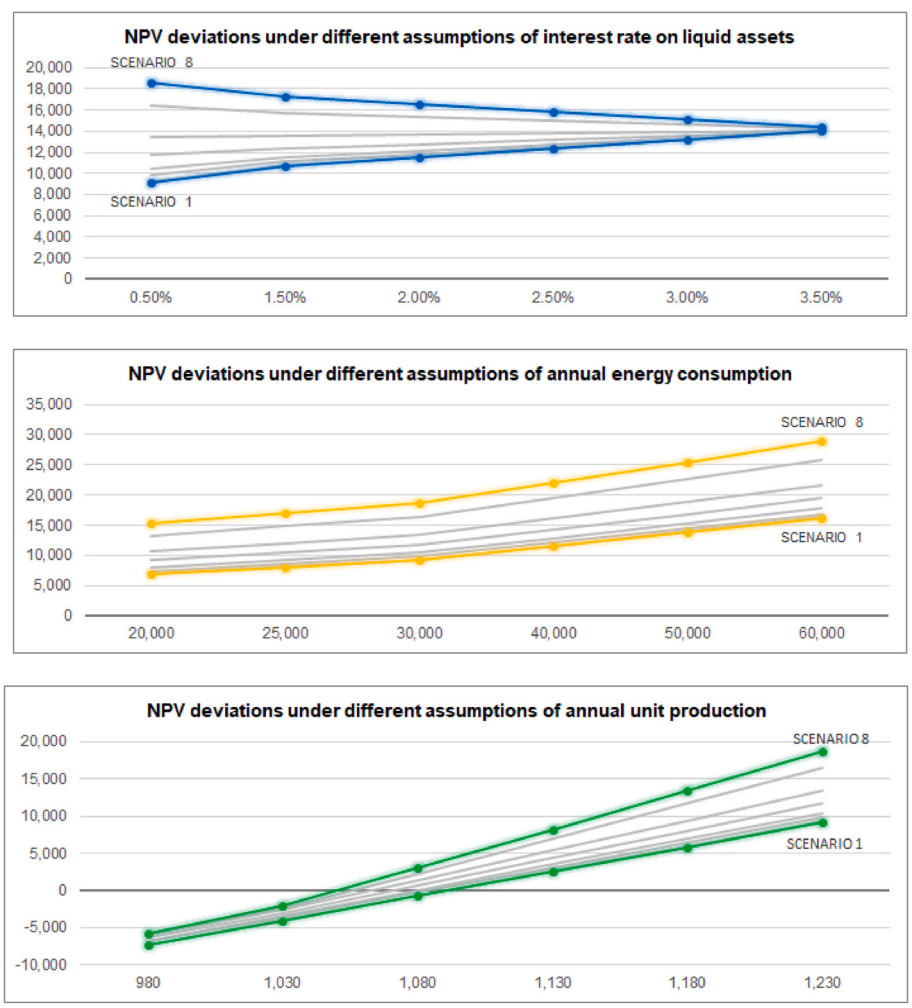

Fig. 6. NPV deviations under different assumptions of interest rate on liquid asset, annual energy consumption, and annual unit production associated with different financing/payout policies (scenarios 1 to 8).

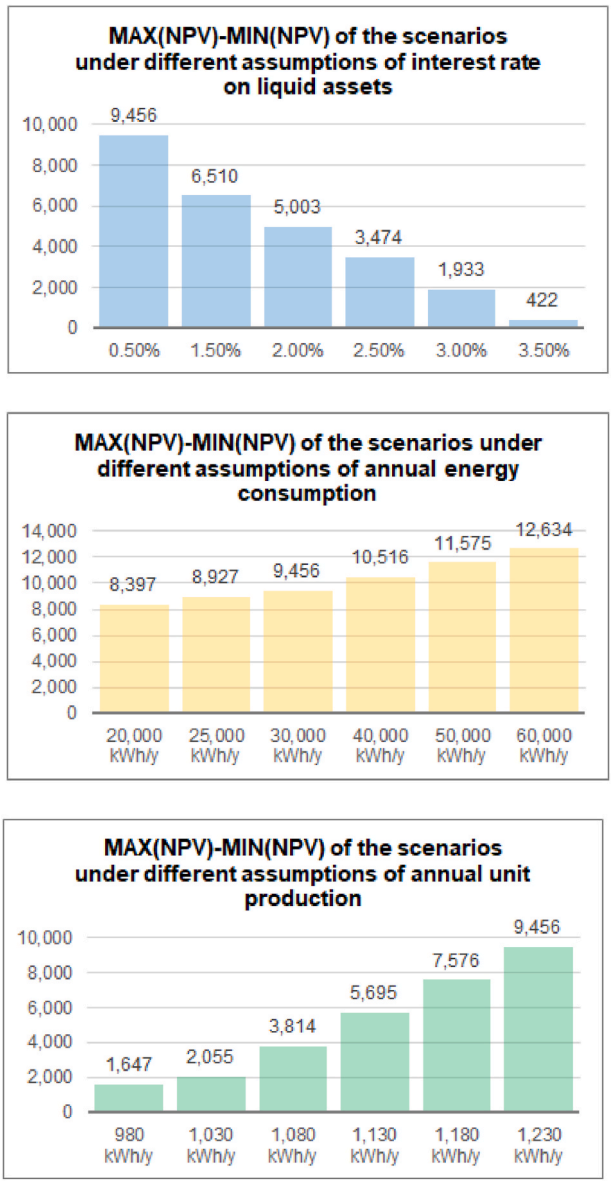

Fig. 7. Maximum and minimum net present values for different values of i^l, q, and Q_{max}.

recall that Q_{max} is the result of a joint estimation process of Alpha and GRAF, and depends on several underlying drivers, some of which are certain and constant through the plant's life such as the geographical location, panel orientation, and inclination, while other are uncertain and estimated such as the external condition (weather condition and ambient temperature) and the plant's technical efficiency (panels' degradation rate and productivity loss in case of no maintenance). In this sense, the annual unit production Q_{max} incorporates and summarizes several sources of operational uncertainty.

Table 6 shows that, other things unvaried, the effect of financial decisions on NPV is magnified for increasing values of Q_{max}. In particular, for any given scenario, higher production means higher NPV for equityholders, because more energy implies more sales revenues and, therefore, higher CFE, which results in higher value creation. This effect is amplified by the financing and distribution decisions, and the greater Q_{max} the greater the impact of the financing policy on the NPV. (At a level of 1,080, different financial scenarios provide different decisions).

As a result of the analysis presented in Tables 3–6, different financial decisions lead to different NPVs for a given set of estimation variables, and different sets of estimation variables make those decisions more or less impactful. Figs. 6 and 7 summarise the scenario analysis illustrated above (interest rates in the top charts, energy consumption in the middle charts, annual unit production in the bottom charts). The dominance of the financial policy in Scenario 8 over the others is always evident.

Furthermore, simultaneous changes in more than one input have different effects under different financial policies. For example, consider the effect of a simultaneous change in annual energy production Q_{max} and interest rate i^l. Tables 7 and 8 describe the effects of different values of the pair (Q_{max}, i^l) on the equity NPV for scenario 1 and scenario 8, other things unvaried. Though Scenario 8 dominates over Scenario 1 for any pair (Q_{max}, i^l), the differences in the two scenarios may be greater or smaller depending on the value of the pair (Q_{max}, i^l). For example, when (Q_{max}, i^l) = (980, 3.5%) the effect of the financing/payout policy is minimum and the financing/payout policy in scenario 8 is only slightly better than scenario 1 (−11,302.06 − (−11,305.44) = 3.39), with scenario 8 being incapable of making the project economically profitable.

In contrast, when (Q_{max}, i^l) = (1,230, 0.5%) the effect of the financing/payout policy is maximum, with a sharp increase in value creation changing the financing/payout policy from scenario 1 to scenario 8: 18,635.80–9,179.54 = 9,456.26. In some cases, the financing/payout policy even reverts the sign of the project's economic profitability; for example, when (Q_{max}, i^l) = (1,080, 0.5%), by changing the financing/payout policy from scenario 1 to scenario 8, the financial efficiency turns from negative (NPV^e = −772.69) to positive (NPV^e = 3,041.44) with a sharp increase of 3,814.13 (see Table 9).

In conclusion, this section has proved that the economic value created by the GRAF project strongly depends on financing and payout policies, which can turn the project from unprofitable to profitable (or viceversa); we have tested the robustness of Scenario 8, which is confirmed as the best scenario under all tests we have conducted. As a result, Alpha has been recommended to use 100% internal financing, to follow a full payout policy, and to distribute cash starting from year 1 in order to maximize the wealth of Alpha's equityholders.

In the proposed model, the impact of the financing and dividend

Table 7
NPV for different values of Q_{max} and i^l - Scenario 1.

(Q_{max}, i^l)	0.50%	1.50%	2.00%	2.50%	3.00%	3.50%
980	−7407.50	−8495.56	−9112.91	−9784.19	−10,513.55	−11,305.44
1030	−4090.09	−4655.33	−4990.04	−5363.44	−5778.81	−6239.67
1080	−772.69	−815.11	−867.17	−942.69	−1044.07	−1173.90
1130	2544.72	3025.12	3255.69	3478.06	3690.68	3891.87
1180	5862.13	6865.34	7378.56	7898.81	8425.42	8957.64
1230	9179.54	10,705.56	11,501.43	12,319.56	13,160.16	14,023.40

Table 8
NPV for different values of Q_{max} and i^l - Scenario 8.

(Q_{max}, i^l)	0.50%	1.50%	2.00%	2.50%	3.00%	3.50%
980	−5760.78	−7384.26	−8272.72	−9219.00	−10,227.28	−11,302.06
1030	−2035.54	−3375.76	−4060.87	−4761.41	−5482.15	−6235.39
1080	3041.44	1664.88	975.66	285.83	−404.62	−1095.70
1130	8239.56	6848.53	6152.06	5454.96	4757.22	4058.84
1180	13,437.68	12,032.19	11,328.46	10,624.09	9919.06	9213.38
1230	18,635.80	17,215.84	16,504.86	15,793.22	15,080.90	14,367.92

Table 9
NPV deviations for different values of Q_{max} and i^l - Scenario 1 vs Scenario 8.

(Q_{max}, i^l)	0.50%	1.50%	2.00%	2.50%	3.00%	3.50%
980	1646.72	1111.30	840.19	565.19	286.27	3.39
1030	2054.55	1279.58	929.17	602.03	296.66	4.29
1080	3814.13	2479.99	1842.84	1228.52	639.44	78.21
1130	5694.84	3823.42	2896.37	1976.90	1066.54	166.98
1180	7575.55	5166.85	3949.90	2725.28	1493.64	255.75
1230	9456.26	6510.28	5003.43	3473.66	1920.74	344.52

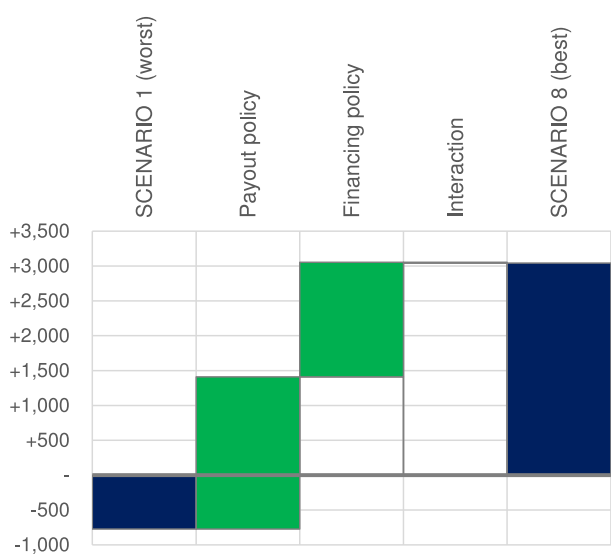

Fig. 8. Contribution of payout policy and financing policy to value creation.

policies on the project's economic profitability is investigated by evaluating the objective function over (a broad grid of) the decision variables domain. From it, one can easily see which types of policy can increase the NPV. This analysis mimics an optimization process, and the NPV-maximizing policy occurs at a corner solution. However, for an even more refined analysis, one might take account of more subtle relations (e.g., the interest rate on debt may increase as the debt/equity ratio increases, the interest rate on liquid assets may depend on the amount of liquid assets, both the payout ratio and the first year of distribution may be functions of net income and FCFE, etc.). Future researches may then be conducted to model the decision variables in more depth and perform a rigorous optimization over the whole domain to find the optimal financial policy.

9. Contribution of financing and distribution to value creation

In the light of what we have seen in the previous sections, it should be clear that the decision variables may play a significant role in increasing or decreasing the attractiveness of the solar PV project. One may wonder whether the contribution to value creation of the financing policy is smaller or greater than the contribution of the payout policy. Given the complexity of the relations between the estimation variables and the decision variables, and even among variables within the same group, there is no general answer. However, for each situation, one can find the contribution to the change in the equity NPV of the distribution policy as opposed to the financing policy with the following simple technique. Let f be the group of financing variables (equity, liquid assets, debt) and d be the group of distribution variables (first year of distribution, payout ratio):

$$f = (E, L, D), \qquad d = (d_m, \alpha).$$

We focus on the two extreme scenarios presented in Table 3: Scenario 1 is the worst case with value destruction equal to − 772.69 euro and scenario 8 is the best one with value creation equal to 3,041.44 euro. The pair (f, d) represents a pair of macro-inputs: the vector (f^1, d^1) = (100%, 0%, 0%, 25, 0%) describes the assumptions in scenario 1 and (f^8, d^8) = (0%, 100%, 0%, 1, 100%) describes the assumptions in scenario 8. We denote as $h(f, d)$ the equity NPV obtained by setting the financing and payout macro-inputs as (f, d). The increase in NPV from the worst case (scenario 1) to the best case (scenario 8) is

$$\Delta\text{NPV}^e = \text{NPV}^{e,8} - \text{NPV}^{e,1} = h(f^8, d^8) - h(f^1, d^1). \tag{28}$$

The individual contribution of the financing variables, denoted as ΔNPV^e_f, may be obtained by calculating the change that the NPV would have if f changed from f^1 to f^8 while leaving the values of the distribution group unvaried at the worst case d^1:

$$\Delta\text{NPV}^e_f = h(f^8, d^1) - h(f^1, d^1). \tag{29}$$

Analogously, the individual contribution of the distribution variables, denoted as ΔNPV^e_d, may be obtained by calculating the change that the NPV would have if d changed from d^1 to d^8 while leaving the values of the financing group unvaried at the worst case f^1:

$$\Delta\text{NPV}^e_d = h(f^1, d^8) - h(f^1, d^1). \tag{30}$$

The difference between the overall equity-NPV increase (ΔNPV^e), and the individual contributions of the two groups represents the interaction effect between financing and payout policy:

$$\Delta\text{NPV}^e_{f,d} = \Delta\text{NPV}^e - (\Delta\text{NPV}^e_f + \Delta\text{NPV}^e_d) \tag{31}$$

(see Saltelli et al., 2004, Borgonovo, 2010, 2017 and Borgonovo et al., 2010 on measures of individual contributions and interaction effects).

In such a way, the change in the equity NPV from the worst scenario to the best scenario may be apportioned to the financing variables, to the distribution variables, and to a possible interaction effect between the two groups. Fig. 8 depicts the results for the project at hand. As can be gleaned from inspection of the figure, the effect of the financing group from eq. (29) amounts to $\text{NPV}^e_f = 869.36 - (-772.69) = 1{,}642.04$€, which represents the 43.1% of the NPV increase; the effect of the distribution group from eq. (30) amounts to $\text{NPV}^e_d = 1{,}410.84 - (-772.69) = 2{,}183.53$€, meaning that the 57.2% of the NPV increase from the worst to the best scenario is explained by the payout policy. The interaction effect from eq. (31) is

Input	pessimistic base case	optimistic base case	clean total order	weight	RANK
Useful life of PV plant	24	26	1,729.73	8.00%	**6**
Annual unit production (first year)	1,060.0	1,100.0	3,182.65	14.72%	**1**
Solar panel degradation rate	0.95%	0.85%	867.99	4.01%	**12**
O&M and insurance	3.70%	3.30%	1,698.94	7.86%	**7**
Disposal costs	5,500.0	4,500.0	231.81	1.07%	**16**
Lost rent from land property	3,100.0	2,900.0	1,556.06	7.20%	**8**
Growth rate for costs	1.30%	1.20%	541.08	2.50%	**13**
Annual energy consumption	25,000	35,000	2,720.27	12.58%	**2**
Tax rate	28.00%	27.70%	7.99	0.04%	**17**
Energy purchase price	0.158	0.162	898.57	4.16%	**11**
Energy selling price	0.128	0.132	1,777.47	8.22%	**5**
Growth rate of energy price	1.20%	1.30%	926.36	4.28%	**10**
First year of CFE distribution	24	1	1,782.91	8.25%	**4**
Payout Ratio	0%	100%	1,813.33	8.39%	**3**
Interest rate on liquid assets	0.70%	0.30%	427.45	1.98%	**14**
Equity financing	100%	0%	384.76	1.78%	**15**
Internal financing (cash withdrawal)	0%	100%	1,076.26	4.98%	**9**
TOTAL (Δ NPV)			**21,623.62**		

Fig. 9. Clean FCSIs and ranks of the input parameters for the GRAF project. The importance of the payout ratio and the year of first distribution are significant (ranks 3 and 4, respectively), just after the annual unit production and energy consumption.

Fig. 10. Breakdown of ΔNPV^e from the pessimistic base scenario to the optimistic base scenario.

negligible since $NPV^e_{f,d} = 3{,}814.13 - (1{,}642.04 + 2{,}183.53) = -11.44€$.

10. Finite change sensitivity indices for investment appraisal

In section 8, we have considered deviations in the NPV due to changes in the financial policy for a given set of operating inputs. We have also considered the deviations of the NPV due to potential deviations of three estimation variables, a financial one (interest rate on liquid assets) and two operating ones (annual unit production and annual energy consumption), for different financial policies. However, in that section, we have used a one-at-a-time method, whereby only one parameter at a time is changed, and a two-at-a-time method, with simultaneous change of two parameters (see Tables 7–9). In this section, we do not only consider the simultaneous change of all the input parameters, but also quantify the effect of each one on the equityholders' NPV. To accomplish this task, we apply the Clean Finite Change Sensitivity Indices (FCSIs) technique, introduced in Magni et al. (2020, section 4), who provide a refinement of Borgonovo (2010) FCSI. The Clean FCSI is a risk management tool allowing for the exact decomposition of a finite functional variation (in our case, the change in the equityholders' NPV) in terms of the finite input changes (in our case, the variation in the estimated and decision inputs).

More precisely, the Clean Total FCSI of a parameter measures its total effect on the change in the model output, cosidering both its individual contribution and its interaction effect with the other parameters. Furthermore, its normalized version, obtained by dividing the Clean Total FCSI by the output variation, represents the relative weight of the parameter for the functional variation, such that the sum of all the normalized Clean Total FCSIs exactly explains 100% of the output change (we crossrefer the reader to Magni et al., 2020 for details of this technique).

The sign of the normalized Clean Total FCSI signals the concordance, if positive (discordance, if negative) of the parameter's total effect with the functional variation, and its absolute value measures the magnitude

of the effect, such that higher absolute values mean higher influence on the output change. The higher the parameter's influence, the higher its rank.

We apply the Clean FCSI technique to the equity NPV (see Fig. 9). As for the operational variables, we consider an interval of values around the base case, which is regarded by Alpha's and GRAF's analysts as the most plausible interval of values. We call them *pessimistic base case* and *optimistic base case*. For example, the annual unit production (in the first year) is equal to 1,080 KWh/KWp/y in the base case. However, the most plausible value may span from a more pessimistic estimate of 1,060 KWh/KWp/y (pessimistic base case) to a more optimistic estimate of 1,100 KWh/KWp/y (optimistic base case). The optimistic base case is associated with a full payout policy from year 1 and 100% internal financing; the pessimistic base case is associated with no distribution until the end of the project's life and 100% equity financing.

Fig. 9 reports the sensitivity-analysis results on the equity NPV. Shareholder value created varies from value destruction in the pessimistic base scenario (with $NPV^e = -7{,}747.66€$) to value creation in the optimistic base scenario (with $NPV^e = 13{,}875.96€$), with an increase of $\Delta NPV^e = 21{,}623.62€$. The Clean FCSIs help explain why this dramatic change occurs. Column 4 of Fig. 9 collects the Clean FCSIs, column 5 reports its normalized version, that is, the weight of each input in explaining the performance variation, and column 6 shows the parameters' resulting ranks. (See Magni et al., 2020 for further details on FCSI.) All the Clean FCSIs are positive, meaning that each parameter has a monotonically increasing effect on the equity NPV. It is worth noting that the most important driver of change is the annual unit production (rank 1), followed by the annual energy consumption (rank 2), which might call for some further estimation analysis. The payout ratio and the first year of CFE distribution are next in importance (ranks 3 and 4), meaning that financial decisions have a substantial impact on the overall result of the GRAF project. The FCSIs also make it clear that the most important financial decisions are the payout decisions rather than the financing decisions, which have a much smaller rank, which suggests that Alpha should immediately pay out to its shareholders all the cash flow generated by the project, with no cash retention.

Fig. 10 shows the decomposition of NPV variation from the pessimistic base case to the optimistic base case, with green bars representing the Clean FCSI of each parameter.

11. Concluding remarks

Since solar energy undeniably contributes to a sustainable economy, the decision of adopting a solar energy system by firms is important to achieve a substantial cumulative effect in the environment. However, firms' decisions are mostly motivated by the economic profitability of a project and by the value created for the firm's shareholders. Building upon Magni (2020), we present an analytical tool increasing the analysts' and managers' awareness of the importance of modeling the financial variables associated with an industrial project.

We show that, while operating variables (energy prices, O&M costs, solar panel degradation rate, etc.) are important, financial variables may have a substantial impact on the value created as well. In particular, the embedded decisions are of special importance: They deal with the amount of cash distribution to shareholders, the retained cash, the proportion of equity financing and debt financing as opposed to cash withdrawals from liquid assets. Our model takes these variables into explicit consideration and measures their impact on the firm's pro forma financial statements and, hence, on the shareholder value created. The model may be helpful in real-life applications, especially considering that, in practice, a substantial amount of solar PV plants is financed by firms with internal financing, with no recourse to equity issuance (and sometimes not even to debt financing). We have applied the model to a real-life project of a solar photovoltaic plant to be installed in Northeast Italy.

We apportion the overall value created according to the various sources of value, namely, the operating activities (operating NPV), the liquid assets (non-operating NPV), and the debt borrowing (debt NPV). We show that the non-operating NPV may play a role in creating value, and may even turn an otherwise unprofitable project into a profitable one.

As a result, this paper's findings suggest that, while the technical inputs describing the functioning of the plant are of paramount importance, the financial variables and the embedded decisions should not be disregarded, for their impact on the project's attractiveness may be non-negligible, as resulting from our scenario analysis and sensitivity analysis. Armed with an appropriate model including both estimation variables and decision variables, the firm's analysts may fine-tune the financial decisions for a given set of expected value of estimated inputs and optimize the NPV for the firm's shareholders. Future researches may address the problem of finding, for any given set of estimation variables, the optimal financing and payout policy, making use of rigorous optimization techniques.

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The Attribution Matrix and the joint use of Finite Change Sensitivity Index and Residual Income for value-based performance measurement

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The Attribution Matrix and the joint use of Finite Change Sensitivity Index and Residual Income for value-based performance measurement

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Abstract. We present a model of performance measurement and attribution for delegated investments. We introduce an innovative two-dimensional approach that, on one hand, detects the (manager and client) *decision effects*, measuring the impact on the overall investment performance of the choices made by manager and client in a given period and, on the other hand, detects the (manager and client) *period effects*, measuring the impact on the period investment performance generated by the entire set of decisions made by manager and client.

To accomplish the task, we employ two techniques of analysis and a three-step procedure: we first use the Finite Change Sensitivity Index (FCSI) for measuring the decision effects (step 1); then, we use the Residual Income (RI) approach for measuring the period effects (step 2); finally, we combine the two attribution dimensions into an Attribution Matrix (AM) (step 3) which contains the attribution values. An attribution value provides the amount of value added generated in a given period by the decisions made by the manager or by the investor in (the same or) another period. We apply the AM to an Italian fund, "Anima Italia A", between 2013 and 2020.

Keywords. Value added, performance measurement, attribution matrix, sensitivity analysis, FCSI, residual income, manager effect, client effect.

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1 Introduction

The aim of this paper is to provide an innovative tool of performance measurement and attribution for delegated investments involving a principal that delegates the execution of an economic activity to an agent. The methodology we propose suits several situations. In particular, we analyze the setting where an investor/client endows a fund manager a monetary amount and makes periodic capital calls or additional injections in the fund, while the fund manager makes decisions on asset selection and allocation. However, the results obtained may be applied to any situation where a principal retains the right of periodically withdrawing funds or injecting additional funds into the investment. For example, in decentralized organizations, the board of directors may delegate the investment policy to a business unit's managers while retaining the right of increasing or decreasing the amount of resources the managers may employ (see, for example, Magni 2015).

In such situations, it is important to measure the performance of the investment. A substantial amount of contributions have recently dealt with pros and cons of various metrics from several points of view, all of which taking into account the role of a benchmark return in assessing the investment's value added. Most of these measures are return-based, that is, expressed as relative measures of worth, such as the internal rate of return and the time-weighted rate of return (see Gray and Dewar 1971, Long and Nickels 1996, and, more recently, Feibel 2003, Bacon 2008, Magni 2013, 2014, Gredil, Griffiths, and Stucke 2014, Altshuler and Magni 2015, Jiang 2017, Cuthbert and Magni 2018). However, investment performance of a delegated portfolio depends on two sets of decisions: Investment decisions made by the manager and cash-flow decisions made by the client/investor, so it is of paramount importance to separate the effects of the managers's decisions from the effects of the investor's decisions onto the value added. Yet, the role of client's performance has been somewhat disregarded in the literature, notwithstanding the prosperous development of mathematical techniques for the optimization of portfolio allocation and selection (Jin and Yu Zhou 2008, Lim, Shanthikumar, and Watewai 2011, Low, Pachamanova, and Sim 2012, Wang and Yu Zhou 2020, Cerny 2020), and a long-standing tradition in academia has been mainly focused on the empirical measurement of managerial skills such as the ability to invest/disinvest in undervalued/overvalued securities (asset allocation and selection policy) or the capability to anticipate market behaviour and vary the levels of risk exposures in upward and downward markets accordingly (market timing) (Angelidis, Giamouridis, and Tessaromatis 2013, Spaulding 2014, Andreu Sánchez, Matallín-Sáez, and Sarto Marzal 2018, Crane and Crotty 2018, Elton and Gruber 2020, Bali et al. 2021. See also Banker, Chen, and Klumpes 2016 for the asymmetric ability of managers in buying and selling), as well as the ability to control expenses and transaction costs (Andreu, Serrano, and Vicente 2019, Galagedera et al. 2020). With a similar attitude, Levy (1968) segregated (more precisely, cleaned) the manager's results from the client's contribution and distribution decisions. Considering the client's perspective, strong empirical evidence has been found about the relation between past investment performance and investors' contribution-and-distribution decisions (Ippolito 1992, Chevalier and Ellison 1997, Del Guercio and Tkac 2002, Bollen 2007,

Goyal, and Wahal 2008, Goriaev, Nijman, and Werkel 2008). Another important strand in the literature empirically analyzes the relation between the client's flows and the subsequent realized returns (e.g., Warther 1995, Gruber 1996, Zheng 1999, Sapp and Tiwari 2004, Frazzini and Lamont 2008, Munoz, Vargas and Vicente 2014, Akbas et al. 2015, Jiang and Yuksel 2017). Some of these studies suggest the existence of the clients' selection ability for skilled fund managers (so called smart money effect), whereas some other researches contradict such an effect by finding that investors direct their money to funds with low future returns (so called "dumb money effect"). Furthermore, Rakowski (2010) analysed the effect of daily mutual fund flow volatility on fund performance, Jones and Martinez (2017) studied the impact on asset allocation decisions of the investors' expectations about the fund's future performance, and Kostovetsky and Warner (2015) found evidence on how past fund investment perfomance and past cash contributions and distributions predict managerial turnover.

Bagot and Armitage (2004) moved a step forward, from the analysis of managerial skills to the contributions to value creation, however still concentrating just on managerial performance. They noted that return-based methods of performance measurement and attribution, such as the time-weighted return (TWR), do not answer the question about 'What has the manager done for me, given my initial investment and the cash inflows and outflows by me along the way?', since these metrics assess the manager's skills but do not measure the manager's contribution to the investment's value added. Bagot and Armitage (2004) and Armitage and Bagot (2009) endorsed a value-based method to answer this question about managerial contribution, since multiperiod attribution analysis is easier using values than using returns. Furthermore, in a multiperiod relationship between a fund manager and a client, Heinkel and Stoughton (1994) studied the contracts and the client's retention policies that most motivate the manager to acquire valuable information. As for relative measures of worth, Magni (2014) detected the role of manager's decisions with the manager's AIRR, that is, an annual rate of return appraising the manager's skills, and Magni (2015) defined the manager's profitability index, which is invariant under changes in the cash flows and neutralizes the effect of the client's decisions.

Despite the considerable attention drawn on the appropriateness of a performance criterion and on the role of the manager in affecting the performance, financial models explicitly measuring the impact of the investor's decisions and the interaction between the two kinds of decisions are lacking. This paper aims to fill the gap. We elaborate on the question by Bagot and Armitage (2004) and further ask, 'What has the client done for himself, with his own decisions on the intermediate cash deposits and withdrawals into the investment portfolio, given the previous and future realized returns derived from the investment policies of the managers?'. And we accomplish this attribution assessment by merging two notions developed in different fields: the Finite Change Sensitivity Index, developed in the operational research literature, and the Residual Income notion, developed in business economics, accounting and finance.

To be more precise, our paper measures the *manager effect* and the *client effect*, respectively defined as the impact of the decisions made by the manager and by the investor on the value added by an actively managed investment as opposed to a passively-managed investment in

the benchmark over a pre-selected assessment interval $[0, n]$.

We identify, for each period and for each decision maker (manager and investor), the decisions and the group of decisions that have been the most influential ones; we also attribute a specific value for each decision made in every period and rank the decisions according to their impact on the investment's performance.

Since the decisions by the fund manager about selection and allocation of assets in a given period generate a well-determined holding period rate and the decisions of the investor gives rise to a cash flow (into or out of the investment), the problem of measuring the impact of decisions boils down to measuring the impact of the holding period rates and the intermediate cash flows on the investment's value added. Holding period rates and interim cash flows will then give rise to the set of input parameters of the model, the output being the investment's value added. The analysis is then refined so as to assess the impact of the decisions on each period performance. As a result, we propose a two-dimensional model which enables one to understand in which periods and by whom the most important (and less important) decisions have been made. The first dimension of the analysis addresses the problem of assessing the impact of the investment decisions and contribution-and-distribution decisions made in a given period onto the investment's value added in the assessment interval $[0, n]$. To accomplish this objective, the active investment derived from the decisions of manager and client is compared with a passive investment in a benchmark portfolio with no intermediate cash flows. We make use of a recently-conceived technique of sensitivity analysis, which apportions a discrete change in a model output to the discrete changes in the model inputs, the Finite Change Sensitivity Index (FCSI), introduced in Borgonovo (2010a, 2010b). We suitably supplement this technique with the fine-tuning of the FCSI procedure introduced in Magni et al. (2020) which allows a perfect (i.e., 100%) decomposition of the value added. In the following step, we address the second analytical dimension, namely, the determination of the effects of the decisions made in the assessment interval $[0, n]$ onto the value created in one single period. To accomplish this objective, we use the Residual Income (RI) approach, whereby the investment's period excess return is assessed (Peasnell 1981, 1982, Peccati 1989, Stewart 1991, Ohlson 1995, Arnold and Davies 2000, Young and O'Byrne 2001, Lundholm and O'Keefe 2001, O'Hanlon and Peasnell 2002, Martin, Petty and Rich 2003, Magni 2009). In the literature, residual income and delegated investments have often been studied jointly, especially in the field of incentive mechanisms for managers (e.g., Wallace 1997, Bromwich and Walker 1998, Georgieva 2015), but the separation between residual income generated by the manager and residual income generated by the investor in a principal/agent relationship has never been addressed in the literature and, consequently, the decomposition of residual income into manager effect and client effect is innovative.

Finally, combining the two attribution analyses we obtain an Attribution Matrix (AM) whose cells are the *attribution values*; an attribution value measures the value added in a period t by the decisions made by the manager or the investor in a (same or other) period. The sum of the elements of a row of the AM attributable to the manager is the *manager decision*

effect, whereas the sum of the elements of a row attributable to the client is the *client decision effect*. The sum of the manager decision effect and the client decision effect referred to the same period is the *joint decision effect*. Furthermore, the sum of the elements of a column is the *period effect*, which may be partitioned into *manager period effect* and *client period effect*.

We prove that the investment's value added is equal to the sum of the decision effects, which is also equal to the sum of the period effects. As a result, the model generates a twofold decomposition of the investment's value added in terms of decisions and in terms of periods.

This paper positions itself in the interfaces of operations research (OR) and finance. The connections between OR and finance have been recognized long since (e.g., Small 1956, Weingartner 1963, Adelson 1965, Hespos and Strassman 1965, Ignizio 1976, Ashford et al. 1988) and have been increasingly investigated by scholars in the last decades (e.g., Grubbström and Ashcroft 1991, Murthi et al. 1997, Meier et al. 2001, Gondzio and Kouwenberg 2001, Baesens et al. 2003, Steuer and Na 2003, Fabozzi et al. 2010, Thomas 2010, Zhao and Huchzermeier 2015, Trigeorgis and Tsekrekos 2018, Marchioni and Magni 2018). The relation between OR and finance is bidirectional, in the sense that finance provides the economic principles, the criteria, the methodologies, and a firm's main objectives. This helps operational managers to grasp the impact of their decisions on shareholders' wealth and make (economically) rational decisions. On the other hand, OR provides techniques and tools that may be applied to several finance problems. Indeed, finance is a privileged area of application of OR, to such an extent that "several thousand papers concerning the application of OR techniques to finance have appeared in academic journals" (Board et al. 2003, p. 12) and the role of OR techniques in finance "may only increase with time" (Board et al. 2003, p. 12). This paper is in line with this strand of literature and is innovative in both theory and application: as for theory, we blend two unrelated frameworks, rooted in OR and in finance (respectively, FCSI and RI), which have never been combined for attribution analysis purposes; in terms of application, the model is immediately applicable and adds information which is not available in the current models, since attribution analysis is traditionally concerned with measuring the so-called allocation effect (which measures the manager's decision to over- or underweight a particular market segment) and the selection effect (which measures the manager's ability to select securities that generate an above-normal return); as far as we can tell, the current literature does not answer the questions we pose in this paper: what is the impact of the (allocation and selection) manager's choices in a given period? What is the impact of the client's choices of drawdowns and injections? What is the influence on the value added in a given period of the set of manager's decisions and the set of client's decisions? And, given any two periods, what is the impact of the decisions made in a period by the manager and the client onto the value added in another period? (See Reilly and Brown 2002, Ch. 266, for the classical questions formulated and answered in investment analysis and portfolio management). Moreover, the perspectives and the methods adopted in traditional performance attribution analysis have to do with rates of return (whereas we focus on value added) and/or refer to one single period (whereas we cope with a multiperiod performance assessment) and/or assess the impact of manager's choices rather

than the impact of client's choices (whereas we assess both). Finally, we do not use the two techniques separately: we commingle them into a unified framework, which supplies a rigorous method for multiperiod analysis and for quantifying the roles of manager and client in both single-period and multiperiod terms. As such, this paper presents a relevant and innovative interface between OR and finance.

The remainder of the paper is structured as follows. Section 2 presents the setting and Section 3 defines the investment's value added and reframes it in terms of finite changes. Section 4 defines the tools used in the paper: Clean Finite Change Sensitivity Index (FCSI) and Residual Income (RI). Section 5 uses the Clean FCSI technique (step 1): we find the (manager and client) *decision effects*, that is, the impact of the investment decisions and the contribution-and-distribution decisions made in a given period onto the overall performance of an actively-managed investment. Section 6 uses the RI approach (step 2): we truncate the investment at various dates to find the *period effects*, that is, the impact of the investment decisions and contribution-and-distribution decisions made in the overall assessment interval onto the value created in a single period. Section 7 combines FCSI and RI approaches (step 3) to give rise to the Attribution Matrix (AM) which mingles the two dimensions (decision and period) and produces the attribution values. Section 8 illustrates the procedure with a numerical example for an eight-period investment. Some remarks conclude the paper and give a succinct summary of the three-step procedure. An application of the AM approach to a real-life Italian fund is illustrated in Appendix D.

2 Economic setting

We analyze an investment (a fund or a portfolio of assets), starting at time $t = 0$ and liquidating at time $t = n$, involving a client/investor who endows the fund manager a monetary amount for actively managing the investment. Cash flows into and out of the fund at time t are denoted as F_t, where $F_t < 0$ represents a net contribution into the fund (outflow for the investor) and $F_t > 0$ represents a distribution from the fund (inflow for the investor), with $t = 0, 1, 2, \ldots, n$. While the investor makes the periodic decisions on contributions and distributions, the fund manager makes periodic decisions on the selection and allocation of the amount that remains invested in the fund. These decisions affect the beginning-of-period capital invested (investor's decisions) and the single-period rate of return of the fund managed (manager's decisions). Consider period t (i.e., the interval $[t-1, t]$) and let E_t be the end-of-period portfolio value at time t (i.e., before cash movement) and B_t denote the beginning-of-period portfolio value at time t (i.e., after cash movement). The rate of return is calculated as

$$i_t = \frac{E_t}{B_{t-1}} - 1. \tag{1}$$

This relation says that the investment's holding period rate i_t represents the relative increase in the investment value. For example, if $B_{t-1} = 100$, E_t=110, then the increase in value is

$i_t = 110/100 - 1 = 10\%$. Therefore, one may also write

$$E_t = B_{t-1} \cdot (1 + i_t) \tag{2}$$

which says that the end-of-period value is equal to the beginning value marked up by the return rate i_t.

The beginning-of-period value at time t (i.e., at the beginning of period $t+1$) may be obtained from the end-of-period value by deducting the cash withdrawal from the fund or adding the capital injections into the fund. This may be expressed formally as

$$\begin{aligned} B_t &= E_t - F_t \\ B_t &= B_{t-1}(1 + i_t) - F_t. \end{aligned} \tag{3}$$

Completing the numerical example, if $F_t = 20$, the portfolio value at the beginning of period $t+1$ will be $B_t = 110 - 20 = 90$ or, equivalently, $100(1 + 10\%) - 20 = 90$.

The above relation formally describes the change in portfolio value caused by both the fund manager and the client/investor. More precisely, eq. (3) depends on both the manager's decisions, which affect i_t via the allocation and selection choices, and the client's decisions, which determine F_t via the contribution and distribution choices. The two effects are intertwined, since B_{t-1} is determined by past decisions of both manager and investor. This means that the manager's decisions and the investor's decisions interact in each period to determine the next-period investment value (see also Table 1).

Table 1: Breakdown of beginning-of-period investment value, B_t

decisions made by the manager and the investor in the interval $[0, t-1]$	$\Longrightarrow B_{t-1}$
decisions made by the manager in period t, i.e. in the interval $[t-1, t]$	$\Longrightarrow i_t$
decisions made by the investor at the end of period t, i.e. in date t	$\Longrightarrow F_t$

At time 0, the beginning-of-period value is $B_0 = -F_0 > 0$, and the ending value of the portfolio at the liquidation time n, denoted as E_n, is entirely distributed to the investor (i.e., $F_n = E_n$), so that the cash-flow stream for the investor is $(F_0, F_1, \ldots, E_n)$. We denote as $F = (F_1, F_2, \ldots, F_{n-1}) \in \mathbb{R}^{n-1}$ the vector collecting the intermediate cash flows, from $t = 1$ to $t = n-1$, while $i = (i_1, i_2, \ldots, i_n) \in \mathbb{R}^n$ denotes the vector collecting the single-period rates of return from $t = 1$ until the liquidation date $t = n$.

Focusing on the terminal date n, and using (1)-(3), one can express the net terminal value E_n as a function of the return rates and the cash flows prior to n, collected in vectors i and F, respectively:

$$E_n = E_n(i, F) = -\sum_{t=0}^{n-1} (1 + i_{t+1})(1 + i_{t+2}) \ldots (1 + i_n) \cdot F_t. \tag{4}$$

The above relation tells us that the portfolio's terminal value E_n is the result of the previous decisions made by both the manager (who affects i_t) and the client (who affects F_t), and it is

formally equal to the difference between the future value of the contributions and the future value of the distributions.

Consider now a benchmark index traded on the financial market and let $i^* = (i_1^*, i_2^*, \dots, i_n^*)$ be the vector collecting the benchmark single-period returns. The benchmark is used as a reference index, and the single-period benchmark returns i_t^* are used to capitalize, to a given point in time, the interim contributions and distributions as well as the portfolio's net terminal value. If the point in time is $t = 0$, the discounting process leads to the investor's Net Present Value (NPV):

$$\text{NPV} = \sum_{t=0}^{n} \frac{F_t}{(1+i_1^*)(1+i_2^*)\dots(1+i_t^*)}. \tag{5}$$

If the point in time is $t = n$, the compounding process leads to the investor's Net Future Value, also known as Value Added (VA):

$$\text{VA} = \sum_{t=0}^{n} (1+i_{t+1}^*)(1+i_{t+2}^*)\dots(1+i_n^*) \cdot F_t = \text{NPV} \cdot (1+i_1^*)(1+i_2^*)\dots(1+i_n^*). \tag{6}$$

Since we aim at measuring the economic value created by the investment *ex post*, we will focus on the latter. The investment creates value for the client if and only if the value added is positive: $\text{VA} > 0$.

3 Value added: Active vs. passive investment

For a given vector of benchmark rates $i^* = (i_1^*, i_2^*, \dots, i_n^*)$ and a given initial contribution F_0, (6) may be reframed in terms of i and F as follows:

$$\begin{aligned}
\text{VA} = f(i,F) &= \Big(\sum_{t=0}^{n-1} (1+i_{t+1}^*)(1+i_{t+2}^*)\dots(1+i_n^*) \cdot F_t\Big) + E_n = \\
&= \Big(\sum_{t=0}^{n-1} (1+i_{t+1}^*)(1+i_{t+2}^*)\dots(1+i_n^*) \cdot F_t\Big) + \Big(-\sum_{t=0}^{n-1} (1+i_{t+1})(1+i_{t+2})\dots(1+i_n) \cdot F_t\Big) \\
&= \sum_{t=0}^{n-1} \Big((1+i_{t+1}^*)(1+i_{t+2}^*)\dots(1+i_n^*) - (1+i_{t+1})(1+i_{t+2})\dots(1+i_n)\Big) \cdot F_t.
\end{aligned} \tag{7}$$

Abusing notation, we denote as $0 \in \mathbb{R}^{n-1}$ the null vector, whose components are all equal to zero. It is then worth noting that $f(i^*, 0)$ denotes the value added of a passive investment whereby an investor invests in the benchmark index and does not make any contribution nor distribution between $t = 1$ and $t = n$. Replacing i with i^* and F with 0 in (7) one finds that the value added by such a passive investment is zero (as expected):

$$f(i^*, 0) = \Big((1+i_1^*)(1+i_2^*)\dots(1+i_n^*) - (1+i_1^*)(1+i_2^*)\dots(1+i_n^*)\Big) F_0 = 0. \tag{8}$$

In other words, the passive investment is value neutral. This implies that the value added by the investment under consideration, VA, may be viewed as the result of switching from a passive investment where the single-period rate is i_t^* and the interim cash flows are zero to an active investment where the single-period rate is i_t and the interim cash flows are equal to F. Switching from $(i^*, 0)$ to (i, F) means to switch from a passive investment in the benchmark (with no interim contributions nor distributions) to an active investment where

(i) the fund manager selects assets and allocates the endowed amounts to the various assets

(ii) the client selects the time and the size of contributions and distributions.

As a result, the value added changes from $f(i^*, 0)$ to $f(i, F)$. Since $f(i^*, 0) = 0$, eq. (6) may be rewritten as

$$\text{VA} = \overbrace{f(i, F)}^{\substack{\text{value added} \\ \text{by the active investment}}} - \overbrace{f(i^*, 0)}^{\substack{\text{value added} \\ \text{by the passive investment}}}. \tag{9}$$

We now analyze and interpret (9) in some detail.

Given a generic initial outflow y_0 and a vector of benchmark returns i^*, consider an asset with a set of single-period rates $x = (x_1, x_2, \ldots, x_n)$ and a set of interim contributions and distributions $y = (y_1, y_2, \ldots, y_{n-1})$. The terminal asset value, denoted as $E_n(x, y)$, is

$$E_n(x, y) = -\sum_{t=0}^{n-1} (1 + x_{t+1})(1 + x_{t+2}) \ldots (1 + x_n) \cdot y_t \tag{10}$$

while the value added is

$$f(x, y) = \sum_{t=0}^{n-1} \Big((1 + i_{t+1}^*)(1 + i_{t+2}^*) \cdots (1 + i_n^*) - (1 + x_{t+1})(1 + x_{t+2}) \cdots (1 + x_n) \Big) \cdot y_t. \tag{11}$$

Therefore, (9) expresses the difference between the function f evaluated at the point $(x^1, y^1) = (i, F)$ and the same function evaluated at the point $(x^0, y^0) = (i^*, 0)$, assuming $y_0 = F_0$. Hence, (9) tells us that the economic value created by any investment is the *change* in the value added obtained by turning from a passive strategy to an active strategy, which shifts the value added from $f(i^*, 0)$ to $f(i, F)$.

From now on, we will use eq. (9), not eq. (6). The reason is that, analytically, eq. (9) is more useful for our ends, because it represents a finite change: The change of $f(x, y)$ when the independent variables shift from the point $(x^0, y^0) = (i^*, 0)$ to the point $(x^1, y^1) = (i, F)$. This fact enables us to apply a most recent technique of sensitivity analysis to $f(x, y)$ so as to measure the effects of the decisions made by the manager and the investor on VA (i.e., the manager decision effects and the client decision effects). In Section 4, we describe the technique, so-called Finite Change Sensitivity Index and, then, in Section 5, we show how to derive the (manager and client) decision effects.

4 The two tools: FCSI and RI

4.1 Finite Change Sensitivity Index (FCSI)

Sensitivity analysis (SA) is the study of how the variance of the output of a model (numerical or otherwise) can be apportioned to different input key parameters (Saltelli et al. 2004). As such, it aims at quantifying how much of an output change is attributed to a given parameter or a set of parameters. It is widely employed in finance and management (Huefner 1972), for instance in analyzing the value creation of industrial projects (Borgonovo and Peccati 2004, 2006; Borgonovo, Gatti, and Peccati 2010; Percoco and Borgonovo 2012; Marchioni and Magni 2018; Magni and Marchioni 2020), the composition of optimal financial portfolios (Luo, Seco and Wu 2015), and the effects of corporate debt (Donders, Jara and Wagner 2018; Délèze and Korkeamäki 2018). There exist several SA techniques defined in the literature (see Borgonovo and Plischke 2016; Pianosi et al. 2016; Saltelli et al. 2004, Saltelli et al. 2008 for reviews of SA methods). Among others, the Finite Change Sensitivity Indices (FCSIs) have been recently introduced in Borgonovo (2010a, 2010b) for analyzing the impact of a finite change in the model inputs on the model output and apportioning the influence of each input on the output change. Formally, let f be the objective function, which maps the vector of inputs (also called parameters, or key drivers) $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_p) \in \mathbb{R}^p$ onto the model output $f(\alpha) \in \mathbb{R}$. Let the inputs vary from $\alpha^0 = (\alpha_1^0, \ldots, \alpha_p^0)$, the so-called *base value*, to $\alpha^1 = \left(\alpha_1^1, \alpha_2^1, \ldots, \alpha_p^1\right)$, the *realized value*. The corresponding model outputs are $f(\alpha^0)$ and $f(\alpha^1)$, so that the output variation is

$$\Delta f = f(\alpha^1) - f(\alpha^0). \tag{12}$$

Let $(\alpha_j^1, \alpha_{(-j)}^0) = (\alpha_1^0, \alpha_2^0, \ldots, \alpha_{j-1}^0, \alpha_j^1, \alpha_{j+1}^0, \ldots, \alpha_p^0)$ be the vector consisting of all the inputs set at their base value α^0, except parameter α_j which is given the realized value α_j^1. Analogously, let $(\alpha_j^1, \alpha_k^1, \alpha_{(-j,k)}^0) = (\alpha_1^0, \alpha_2^0, \ldots, \alpha_{j-1}^0, \alpha_j^1, \alpha_{j+1}^0, \ldots, \alpha_{k-1}^0, \alpha_k^1, \alpha_{k+1}^0, \ldots, \alpha_p^0)$ be the input vector where α_j and α_k are set to the realized values, while the remaining $p-2$ parameters are set at their base value, and so forth for all s-tuples of inputs, $s = 1, 2, \ldots, p$.

Borgonovo (2010a, 2010b) defines two versions of FCSIs: First Order FCSI and Total Order FCSI. The First Order FCSI of parameter α_j measures the individual effect of α_j (Borgonovo 2010a) on the output change and is obtained as

$$\Delta_j^1 f = f(\alpha_j^1, \alpha_{(-j)}^0) - f(\alpha^0) \tag{13}$$

or, in normalized version, $\Phi_j^1 f = \frac{\Delta_j^1 f}{\Delta f}$. The Total Order FCSI quantifies the total effect of α_j, including both its individual contribution and its interactions with the other parameters. Before giving the definition of the Total Order FCSI, we need to understand the interaction effects. Let $\Delta_{j,k} f$ be the interaction between α_j and α_k, that is, the portion of $f(\alpha_j^1, \alpha_k^1, \alpha_{(-j,k)}^0) - f(\alpha^0)$

which is not explained by the individual effects $\Delta_j^1 f$ and $\Delta_k^1 f$. Specifically,

$$\overbrace{f(\alpha_j^1, \alpha_k^1, \alpha_{(-j,k)}^0) - f(\alpha^0)}^{\text{change in } f \text{ caused by } \alpha_j \text{ and } \alpha_k} = \overbrace{\Delta_j^1 f + \Delta_k^1 f}^{\text{individual contributions of } \alpha_j \text{ and } \alpha_k} + \overbrace{\Delta_{j,k} f}^{\text{interaction effect of } \alpha_j \text{ and } \alpha_k}$$

whence the interaction effect can be calculated as

$$\Delta_{j,k} f = f(\alpha_j^1, \alpha_k^1, \alpha_{(-j,k)}^0) - f(\alpha^0) - \Delta_j^1 f - \Delta_k^1 f.$$

Similarly, let $\Delta_{j,k,h} f$ be the interaction among the inputs α_j, α_k and α_h, which is the portion of $f(\alpha_j^1, \alpha_k^1, \alpha_h^1, \alpha_{(-j,k,h)}^0) - f(\alpha^0)$ not explained by the individual effects and by the interactions between any pair:

$$\overbrace{f(\alpha_j^1, \alpha_k^1, \alpha_h^1, \alpha_{(-j,k,h)}^0) - f(\alpha^0)}^{\text{change in } f \text{ caused by } \alpha_j,\ \alpha_k, \text{ and } \alpha_h} = \overbrace{\Delta_j^1 f + \Delta_k^1 f + \Delta_h^1 f}^{\text{individual contributions of } \alpha_j,\ \alpha_k, \text{ and } \alpha_h}$$
$$+ \overbrace{\Delta_{j,k} f + \Delta_{j,h} f + \Delta_{k,h} f}^{\text{pairwise interaction effect of } \alpha_j,\ \alpha_k, \text{ and } \alpha_h} + \overbrace{\Delta_{j,k,h} f}^{\text{threewise interaction effect of } \alpha_j,\ \alpha_k, \text{ and } \alpha_h} \tag{14}$$

whence

$$\Delta_{j,k,h} f = f(\alpha_j^1, \alpha_k^1, \alpha_h^1, \alpha_{(-j,k,h)}^0) - f(\alpha^0) - \Delta_j^1 f - \Delta_k^1 f - \Delta_h^1 f - \Delta_{j,k} f - \Delta_{j,h} f - \Delta_{k,h} f$$

(analogously for a s-tuple, with $s > 3$). Switching from α^0 to α^1, the output change is equal to the sum of all the individual effects and all the s-wise interactions, $s = 1, 2, \ldots, p$ between parameters:

$$\Delta f = \overbrace{\sum_{i=j}^{p} \Delta_j^1 f}^{\text{individual contributions}} + \underbrace{\overbrace{\sum_{j_1<j_2} \Delta_{j_1,j_2} f}^{\text{pairs}} + \overbrace{\sum_{j_1<j_2<j_3} \Delta_{j_1,j_2,j_3} f}^{\text{triplets}} + \cdots + \overbrace{\sum_{j_1<j_2\cdots<j_s} \Delta_{j_1,j_2,\ldots,j_s} f}^{s\text{-tuples}} + \ldots + \overbrace{\Delta_{j_1,j_2,\ldots,j_p} f}^{p\text{-tuple}}}_{\text{overall interaction effects}},$$

where $\sum_{j_1<j_2\cdots<j_s} \Delta_{j_1,j_2,\ldots,j_s} f$ is the sum of the interactions between s-tuples.

Borgonovo (2010a) defines the Total Order FCSI of α_j, denoted as $\Delta_j^{\mathcal{T}} f$, as the sum of First Order FCSI of α_j, $\Delta_j^1 f$, and the interaction effect of α_j, denoted as $\Delta_j^{\mathcal{I}} f$ and called Interaction FCSI, which is the sum of every interaction involving α_j:

$$\Delta_j^{\mathcal{I}} f = \sum_{\substack{j_1<j_2 \\ j\in\{j_1,j_2\}}} \Delta_{j_1,j_2} f + \ldots + \sum_{\substack{j_1<j_2\ldots<j_s \\ j\in\{j_1,j_2,\ldots,j_s\}}} \Delta_{j_1,j_2,\ldots,j_s} f + \ldots + \Delta_{j_1,j_2,\ldots,j_p} f.$$

Therefore,

$$\Delta_j^{\mathcal{T}} f = \Delta_j^1 f + \Delta_j^{\mathcal{I}} f = \Delta_j^1 f + \sum_{\substack{j_1<j_2 \\ j\in\{j_1,j_2\}}} \Delta_{j_1,j_2} f + \ldots + \sum_{\substack{j_1<j_2\cdots<j_s \\ j\in\{j_1,j_2,\ldots,j_s\}}} \Delta_{j_1,j_2,\ldots,j_s} f + \ldots + \Delta_{j_1,j_2,\ldots,j_p} f \quad (15)$$

and, in normalized version, $\Phi_j^{\mathcal{T}} f = \frac{\Delta_j^{\mathcal{T}} f}{\Delta f}$. Computationally, the calculation of the Interaction FCSIs (and, therefore, the Total Order FCSIs) may be extremely burdensome if the model does not contain a very small number of inputs.[1] However, Borgonovo (2010a, Proposition 1) provides a useful result for reducing the number of calculations:

$$\Delta_j^{\mathcal{T}} f = f(\alpha^1) - f(\alpha_j^0, \alpha_{(-j)}^1),\ \forall j = 1, 2, \ldots, p, \quad (16)$$

where $(\alpha_j^0, \alpha_{(-j)}^1)$ denotes the vector with each input equal to the realized value α^1, except for α_j which is set equal to α_j^0. This enables computing the total FCSI of α_j without calculating the Interaction FCSI of α_j.

Unfortunately, as Magni et al. (2020, p. 1948) have shown, the Total Order FCSI has an unpleasant feature: It does not provide a complete decomposition of the output change. That is,

$$\sum_{l=1}^{p} \Delta_l^{\mathcal{T}} f \neq \Delta f = f(\alpha^1) - f(\alpha^0) \quad \text{or, equivalently,} \quad \sum_{l=1}^{p} \Phi_l^{\mathcal{T}} f \neq 1.$$

In other words, the sum of Total FCSIs explains less (or more) than 100% of the output change.[2] Recently, Magni et al. (2020) introduced a duplication-clearing factor which eliminates the redundant, multiple interactions and allows a complete and exact decomposition of the output change. The *Clean* Interaction FCSI of α_j, here denoted as $\Delta_j^{I} f$, is defined as the product of the Interaction FCSI $\Delta_j^{\mathcal{I}} f$ and a suitable correction factor, defined as the ratio of the overall

[1] The number of individual contributions is p and the number of the interactions between parameters and groups of parameters is equal to $2^p - p - 1$.

[2] To understand why this happens, consider that, in the sum of the Interaction FCSIs, $\sum_{l=1}^{p} \Delta_l^{\mathcal{I}} f$, the pairwise interactions of α_j and α_k, appear twice (in $\Delta_j^{\mathcal{I}} f$ and in $\Delta_k^{\mathcal{I}} f$); the three-wise interactions of α_j, α_k, and α_h appear three times (in $\Delta_j^{\mathcal{I}} f$, in $\Delta_k^{\mathcal{I}} f$, and in $\Delta_h^{\mathcal{I}} f$); and so on for all the s-wise interactions, $s = 2, 3, \ldots, p$. This implies that the sum of Interaction FCSIs does not equate the overall interaction effects:

$$\underbrace{\sum_{l=1}^{p} \Delta_l^{\mathcal{I}} f}_{\text{sum of Interaction FCSIs}} \neq \underbrace{\overbrace{\sum_{j_1<j_2} \Delta_{j_1,j_2} f}^{\text{pairs}} + \overbrace{\sum_{j_1<j_2<j_3} \Delta_{j_1,j_2,j_3} f}^{\text{triplets}} + \cdots + \overbrace{\sum_{j_1<j_2\cdots<j_s} \Delta_{j_1,j_2,\ldots,j_s} f}^{s\text{-tuples}} + \ldots + \overbrace{\Delta_{j_1,j_2,\ldots,j_p} f}^{p\text{-tuple}}}_{\text{overall interaction effects}}$$

and, therefore, $\sum_{l=1}^{p} \Delta_l^{\mathcal{T}} f \neq \Delta f$ (see also footnote 4 and the counterexample in Magni et al. 2020, p. 1948).

interaction effects over the sum of Interaction FCSIs (Magni et al. 2020):

$$\Delta_j^I f = \Delta_j^{\mathcal{I}} f \cdot \frac{\overbrace{\sum_{j_1<j_2} \Delta_{j_1,j_2} f + \cdots + \sum_{j_1<j_2\cdots<j_s} \Delta_{j_1,j_2,\ldots,j_s} f + \cdots + \Delta_{j_1,j_2,\ldots,j_p} f}^{\text{overall interaction effects}}}{\underbrace{\sum_{l=1}^{p} \Delta_l^{\mathcal{I}} f}_{\text{sum of Interaction FCSIs}}}. \tag{17}$$

Considering that $\Delta_j^{\mathcal{I}} f = \Delta_j^{\mathcal{T}} f - \Delta_j^1 f$ and

$$\sum_{j_1<j_2} \Delta_{j_1,j_2} f + \cdots + \sum_{j_1<j_2\cdots<j_s} \Delta_{j_1,j_2,\ldots,j_s} f + \cdots + \Delta_{j_1,j_2,\ldots,j_p} f = \Delta f - \sum_{j=1}^{p} \Delta_j^1 f,$$

one may reframe (17) as

$$\Delta_j^I f = \frac{\Delta_j^{\mathcal{T}} f - \Delta_j^1 f}{\sum_{l=1}^{p} (\Delta_l^{\mathcal{T}} f - \Delta_l^1 f)} \cdot \big(\Delta f - \sum_{l=1}^{p} \Delta_l^1 f\big). \tag{18}$$

In other words, the Clean Interaction FCSI is computed by imputing a share of the overall true interaction effect ($\Delta f - \sum_{l=1}^{p} \Delta_l^1 f$) to parameter α_j. This share is obtained as the ratio of the Interaction FCSI of α_j and the sum of all Interaction FCSIs (Magni et al. 2020).

The Clean Total Order FCSI of parameter α_j, denoted as $\Delta_j^T f$, is defined as the sum of individual contribution and Clean Interaction FCSI of α_j (Magni et al. 2020):

$$\Delta_j^T f = \Delta_j^1 f + \Delta_j^I f \tag{19}$$

and, in normalized version, $\Phi_j^T f = \frac{\Delta_j^T f}{\Delta f}$. It is easy to see that the Clean Total FCSIs completely explain the output variation:

$$\sum_{l=1}^{p} \Delta_l^T f = \Delta f, \tag{20}$$

and, in normalized version, $\sum_{l=1}^{p} \Phi_l^T f = 1$.

The sign of a Clean Total FCSI, $\Delta_j^T f$, signals the directional effect of an input change onto the output change: A positive (negative) index signals that the change in the input has the effect of increasing (decreasing) the output. The absolute value of the Clean Total FCSI quantifies the magnitude of the effect; one may then rank the input factors according to their influence on the change in the objective function: Input α_j has higher rank than α_k if and only if $|\Delta_j^T f| > |\Delta_k^T f|$. We denote the rank of parameter α_j as R_j. The rank vector is $R = (R_1, R_2, \ldots, R_p)$.

4.2 Residual Income (RI)

The Residual Income (RI) expresses the value created by an investment in a given period $[m\text{–}1, m]$. It is variously named Economic Value Added, excess profit, economic profit, abnormal

return, etc. (see Magni 2009). Formally, it is defined as $\mathrm{RI}_t = B_{m-1}(i_m - i_m^*)$ and, as such, it measures the return over and above the normal return that would be generated by investing the same beginning-of-period capital B_{m-1} in the passive benchmark portfolio. It is well-known that the sum of an investment's capitalized RIs is equal to the investment's value added:

$$\sum_{m=1}^{n} \mathrm{RI}_m \cdot (1 + i_{m+1}^*) \cdots (1 + i_n^*) = \mathrm{VA}. \tag{21}$$

As such, the RI is a period measure that brings about a (perfect) decomposition of the investment's VA.

5 FCSI and the decision effects (step 1)

Consider the vector of input factors $\alpha = (x, y) = (x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_{n-1}) \in \mathbb{R}^{2n-1}$ where x_t denotes a rate of return and y_t denotes a cash flow. For a given initial outflow $y_0 < 0$ (investor's initial contribution) and a given vector of benchmark returns i^*, the asset allocation-and-selection policy followed by the manager in the various periods has the effect of shifting the rates from $x^0 = i^*$ to $x^1 = i$ and the client's decisions about contributions and distributions shift the cash flows from $y^0 = 0$ to $y^1 = F$. As already seen, the change from

$$\alpha^0 = (i^*, 0) = (i_1^*, i_2^*, \ldots, i_n^*, 0, 0, \ldots, 0)$$

to

$$\alpha^1 = (i, F) = (i_1, i_2, \ldots, i_n, F_1, F_2, \ldots, F_{n-1})$$

expresses the change from a passive investment policy to an active investment policy, which makes the value added change from $f(\alpha^0) = f(i^*, 0)$ to $f(\alpha^1) = f(i, F)$ (see Table 2). The output change is

$$f(\alpha^1) - f(\alpha^0) = f(i, F) - f(i^*, 0)$$

which is (9). Therefore, one may apply the Clean FCSI technique illustrated in Section 4 for decomposing VA in terms of period return rates and interim cash flows.

It is then possible to identify the investment choices made by the manager and the contributions/distributions decisions made by the client which have most affected the overall investment's performance. In particular, the value added may be considered as the sum of all the effects of the active selection and allocation choices made in the various periods and the contribution-and-distribution decisions, as opposed to the passive strategy consisting in investing in the benchmark portfolio with no contributions nor distributions.

The Clean Total FCSI, $\Delta_j^T f$, provides the amount of value added that is determined by the decision made in a period by the manager or the client. We call $\Delta_j^T f$ the *decision effect* of parameter α_j. It is worth noting that the piece of information provided by $\Delta_j^T f$ is not whether and how much the investment outperforms or underperforms the benchmark in a given period,

Table 2: Passive vs. active investment: Inputs, terminal value and value added

Inputs	**Passive**	**Active**
$\alpha = (x, y)$	α^0	α^1
$\alpha_1 = x_1$	i_1^*	i_1
$\alpha_2 = x_2$	i_2^*	i_2
$\alpha_3 = x_3$	i_3^*	i_3
$\vdots$	$\vdots$	$\vdots$
$\alpha_{n-1} = x_{n-1}$	i_{n-1}^*	i_{n-1}
$\alpha_n = x_n$	i_n^*	i_n
$\alpha_{n+1} = y_1$	0	F_1
$\alpha_{n+2} = y_2$	0	F_2
$\alpha_{n+3} = y_3$	0	F_3
$\vdots$	$\vdots$	$\vdots$
$\alpha_{2n-1} = y_{n-1}$	0	F_{n-1}
Terminal value and value added		
$E_n(\alpha) = E_n(x, y)$	$-\prod_{l=1}^{n}(1 + i_l^*) \cdot F_0$	$-\sum_{t=0}^{n-1} \prod_{l=t+1}^{n}(1 + i_l) \cdot F_t$
$f(\alpha) = f(x, y)$	$f(\alpha^0)$	$f(\alpha^1)$

but whether the decisions made by the manager or the client in a given period have contributed, overall, to outperform or underperform the passive benchmark investment in the time interval $[0, n]$ and how much of the value added is attributable to them. This piece of information necessarily takes account of the interactions with the decisions made in the other periods. Indeed, the manager's investment decisions made in period t determine i_t (which measures the relative period growth in the investment's value) and, therefore, affect the magnitude of the value added (not only in period t, but also) in the following periods $t + 1$, $t + 2$, ..., n. Analogously, the client's choices about contributions and distributions made by the client in period t determine F_t, which affects the beginning-of-period capital B_t, and, therefore, the magnitude of the value added in period $t + 1$, and also in the following periods $t + 2$, $t + 3$, ..., n. Overall, there are $p = 2n - 1$ decision effects attributable to the decisions of manager and client: The first n effects are attributable to the manager's decisions and are called *manager decision effects*, while the remaining $n - 1$ effects are attributable to the investor's decisions and are called *client decision effects*. Finally, we define the *joint decision effect* as the sum of the manager decision effect and the client decision effect related to the decisions made in the same period:

$$\text{joint decision effect in period } j = \overbrace{\Delta_j^T f}^{\text{manager decision effect}} + \overbrace{\Delta_{n+j}^T f}^{\text{client decision effect}} \quad \text{for } j = 1, \ldots, n \tag{22}$$

with $\Delta_{2n}^T f = 0$. The value added is equal to the sum of all the joint decision effects:

$$\mathrm{VA} = \sum_{j=1}^{n} (\Delta_j^T f + \Delta_{n+j}^T f) \qquad \text{for } j = 1, \ldots, n. \tag{23}$$

For summarizing the role of the two decision makers on value creation, we define the *manager effect* as the sum of the n manager decision effects, $\sum_{j=1}^{n} \Delta_j^T f$ (and, in normalized version, $\sum_{j=1}^{n} \Phi_j^T f$) and the *client effect* as the sum of the $n-1$ client decision effects, $\sum_{j=n+1}^{2n-1} \Delta_j^T f$ (and, in normalized version, $\sum_{j=n+1}^{2n-1} \Phi_j^T f$), such that the value added is equal to the addition of manager effect and client effect:

$$\mathrm{VA} = \overbrace{\sum_{j=1}^{n} \Delta_j^T f}^{\text{manager effect}} + \overbrace{\sum_{j=n+1}^{2n-1} \Delta_j^T f}^{\text{client effect}}. \tag{24}$$

6 RI and the period effects (step 2)

As seen, the Clean Total FCSI $\Delta_j^T f$ represents the decision effect and measures the global effect of a given *decision* onto VA. In this section, we want to capture the period effect, that is, the global effect of a given *period* onto VA. In order to do so, we assume that the investment is fully liquidated at the date m such that $0 \leq m \leq n$. This implies that the cash-flow stream of the investment truncated at time m is $F^{(m)} = (F_0, F_1, \ldots, F_{m-1}, E_m, 0, 0, \ldots, 0)$ where

$$E_m = E_m(x, y) = -\sum_{t=0}^{m-1} (1 + x_{t+1})(1 + x_{t+2}) \ldots (1 + x_m) \cdot y_t. \tag{25}$$

From (5), the NPV of such a truncated project, denoted as $\mathrm{NPV}^{(m)}$, is

$$\mathrm{NPV}^{(m)} = \sum_{t=0}^{m-1} \frac{F_t}{(1 + i_1^*)(1 + i_2^*) \ldots (1 + i_t^*)} + \frac{E_m}{(1 + i_1^*)(1 + i_2^*) \ldots (1 + i_m^*)}$$

for $m = 1, 2, \ldots, n$. We denote as $\mathrm{VA}^{(m)}$ the value added (at time $t = n$) by the project truncated at time $t = m$. Using (6),

$$\begin{aligned} \mathrm{VA}^{(m)} &= \mathrm{NPV}^{(m)} \cdot (1 + i_1^*)(1 + i_2^*) \ldots (1 + i_n^*) \\ &= \sum_{t=0}^{m-1} F_t \cdot (1 + i_{t+1}^*)(1 + i_{t+2}^*) \ldots (1 + i_n^*) + E_m \cdot (1 + i_{m+1}^*)(1 + i_{m+2}^*) \ldots (1 + i_n^*) \end{aligned} \tag{26}$$

with $\mathrm{VA}^{(0)} = 0$. Consider now two consecutive truncated projects: The difference $\mathrm{VA}^{(m)} - \mathrm{VA}^{(m-1)}$ represents that part of the investment's VA generated in period m (i.e., the interval $[m-1, m]$). We denote it as $\Delta^T f_m$ and call it *period effect*:

$$\Delta^T f_m = \mathrm{VA}^{(m)} - \mathrm{VA}^{(m-1)}; \tag{27}$$

its normalized version is denoted as $\Phi^T f_m = \frac{\Delta^T f_m}{\text{VA}}$.

The period effect is the effect on the value created in period m by the decisions made by the manager and the investor in the various periods .[3] It is easy to check that the sum of the period effects is exactly equal to VA:

$$\begin{aligned}\sum_{m=1}^{n} \Delta^T f_m &= \sum_{m=1}^{n} (\text{VA}^{(m)} - \text{VA}^{(m-1)}) \\ &= (\text{VA}^{(1)} - \text{VA}^{(0)}) + (\text{VA}^{(2)} - \text{VA}^{(1)}) + \ldots (\text{VA}^{(n-1)} - \text{VA}^{(n-2)}) + (\text{VA}^{(n)} - \text{VA}^{(n-1)}) \\ &= \text{VA}^{(n)} = f(i, F) = \text{VA}.\end{aligned}$$

Table 3: Period effect and Residual Income

Time	$F^{(m-1)}$	$F^{(m)}$	$\Delta F^{(m)}$
0	F_0	F_0	0
1	F_1	F_1	0
2	F_2	F_2	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$m-2$	F_{m-2}	F_{m-2}	0
$m-1$	E_{m-1}	F_{m-1}	$-B_{m-1}$
m	0	E_m	E_m
$m+1$	0	0	0
$m+2$	0	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	0	0	0

Using (26) and (27), one gets

$$\begin{aligned}\Delta^T f_m &= \Big((1+i^*_{m+1})\ldots(1+i^*_n)\Big) \cdot \Big(E_m - (E_{m-1} - F_{m-1})(1+i^*_m)\Big) \\ &= \Big((1+i^*_{m+1})\ldots(1+i^*_n)\Big) \cdot \Big(B_{m-1}(1+i_m) - B_{m-1}(1+i^*_m)\Big) \\ &= \Big((1+i^*_{m+1})\ldots(1+i^*_n)\Big) \cdot B_{m-1}(i_m - i^*_m) \\ &= \Big((1+i^*_{m+1})\ldots(1+i^*_n)\Big) \cdot \text{RI}_m. \qquad (28)\end{aligned}$$

Therefore, the period-m effect, $\Delta^T f_m$, is the value, at time n, of the RI of period m. Conceptually, eq. (28) may be best understood by considering two consecutive truncated projects. Continuing the investment from $m-1$ to m, and considering that $E_{m-1} - F_{m-1} = B_{m-1}$, the incremental cash-flow stream is $\Delta F^{(m)} = F^{(m)} - F^{(m-1)} = (0, 0, \ldots, -B_{m-1}, E_m, 0, 0, \ldots, 0)$ (see Table 3). This incremental cash-flow stream tells us that, by continuing the investment

[3]Notably, only the decisions made up to time m (may) have a nonzero impact on the VA generated in period m and following periods, whereas any decision made after time m has no effect whatsoever on period m and previous periods (see Remark 1).

from $m-1$ to m the investor gives up B_{m-1} at time $m-1$ but receives an additional E_m at time m. Whether the continuing value is greater or smaller of the truncation value depends on whether the manager's investment decisions are value-creating or not. Specifically, if B_{m-1} were invested in the benchmark, the end-of-period market value would be $B_{m-1}(1+i_m^*)$. The difference $E_m - B_{m-1}(1+i_m^*)$ is then the incremental value generated in period m; owing to (2), this is just the RI of period m: $E_m - B_{m-1}(1+i_m^*) = B_{m-1}(i_m - i_m^*) = \mathrm{RI}_t$. The value of this RI at time n is precisely the m-period effect, $\Delta^T f_m$. To sum up, we have used a RI approach to find the period effects.

In the first two steps of the model, we have introduced two dimensions of analysis and two vectors, $(\Delta_1^T f, \Delta_2^T f, \ldots, \Delta_p^T f) \in \mathbb{R}^p$ and $(\Delta^T f_1, \Delta^T f_2, \ldots, \Delta^T f_n) \in \mathbb{R}^n$, both accomplishing a perfect breakdown of the VA. In other words, we have generated two attribution groups, the group of the manager and client decision effects (the clean FCSIs) and the group of the period effects (the capitalized RIs), both of which perfectly decompose the investment's VA:

$$\overbrace{\sum_{j=1}^{p} \Delta_j^T f}^{\text{sum of Clean FCSI}} = \mathrm{VA} = \overbrace{\sum_{t=1}^{n} \Delta^T f_t}^{\text{sum of capitalized RIs}}. \tag{29}$$

In the next section, we combine the decision effects and the period effects and flesh out the contribution to VA of a given parameter α_j in a given period t.

In the next section, we combine the FCSI technique (which measures the decision effects) and the RI approach (which measures the period effects) to give rise to the *Attribution Matrix*, which contains $p \times n$ attribution values (which measure the contribution onto VA of a given parameter α_j in a given period t).

7 The Attribution Matrix (step 3)

In the previous sections, we have used FCSI to find the decision effects and RI to find the period effects. In this section, we combine the two techniques and give rise to Attribution Matrix (AM) which commingles decision effects and period effects. Let $\Delta_j^T \mathrm{VA}^{(m)}$ denote the total effect of α_j on $\mathrm{VA}^{(m)}$ (i.e., the total clean FCSI of $\mathrm{VA}^{(m)}$); it measures the global impact of parameter α_j on the value added in the interval $[0, m]$. Likewise, $\Delta_j^T \mathrm{VA}^{(m-1)}$ measures the global impact of parameter α_j on the value added in the interval $[0, m-1]$. Therefore, the difference

$$\Delta_j^T f_m = \Delta_j^T \mathrm{VA}^{(m)} - \Delta_j^T \mathrm{VA}^{(m-1)} \tag{30}$$

measures that part of VA which is generated in period m, that is in the interval $[m-1, m]$, by parameter α_j, $j = 1, 2, \ldots, p$. We call $\Delta_j^T f_m$ the *attribution value* of α_j in period m; it is the effect of α_j on the economic value generated in period m (i.e., between $m-1$ and m), with $1 \leq m \leq n$.

For any given decision made by the manager, represented by α_j, $j = 1, 2, \ldots, n$, the sum

of the attribution values amounts to the manager decision effect of α_j on VA; analogously, for any given decision made by the client, represented by α_j, $j = n+1, n+2, \ldots, 2n-1$, the sum of the attribution values amounts to the client decision effect of α_j on VA; formally,

$$\Delta_j^T f = \sum_{m=1}^{n} \Delta_j^T f_m \tag{31}$$

(see proof in Appendix A). Symmetrically, for any given period t, the sum of the attribution values is the period effect, that is, the contribution of period t to VA:

$$\sum_{j=1}^{p} \Delta_j^T f_t = \Delta^T f_t \tag{32}$$

(see proof in Appendix B). Owing to (29), the sum of the period effects coincides with the sum of the decision effects, therefore offering a twofold decomposition of the economic created value. To better appreciate it, we gather the attribution values in a $p \times n$ *Attribution Matrix* (AM) such the element (j, t) reports the attribution value $\Delta_j^T f_t$, which expresses the value added by parameter α_j in period t, with $j = 1, 2, \ldots p$ and $t = 1, 2, \ldots, n$. Table 4 reports the AM, which is ideally partitioned into two submatrices, one regarding the manager effects (rows $1, 2, \ldots, n$), the other one regarding the client effects (rows $n+1, n+2, \ldots, 2n-1$).

For instance, referring to our example in Section 8 where $p = 15$ and $n = 8$, $\Delta_3^T f_7$ represents the value added in period 7 by the investment decisions made by the manager in period 3 ($\alpha_3 = x_3$). Likewise, the attribution value $\Delta_{12}^T f_4$ represents the value added in period 4 by the contribution or distribution decision made by the investor in period 4 ($\alpha_{12} = y_4$).

For a given column t, summing by row one gets the contribution of all the decisions made by the manager and the investor in the assessment interval $[0, n]$ to the value created in period t (i.e., in the interval $[t-1, t]$) (period effect). For a given row $j = 1, 2, \ldots, n$, summing by column one gets the contribution to VA generated by the decisions made in period j by the manager (manager decision effect); likewise, for a given row $n + j$, summing by column one gets the contribution to VA generated by the decisions made at time j by the investor (client decision effect) (see Figure 1).

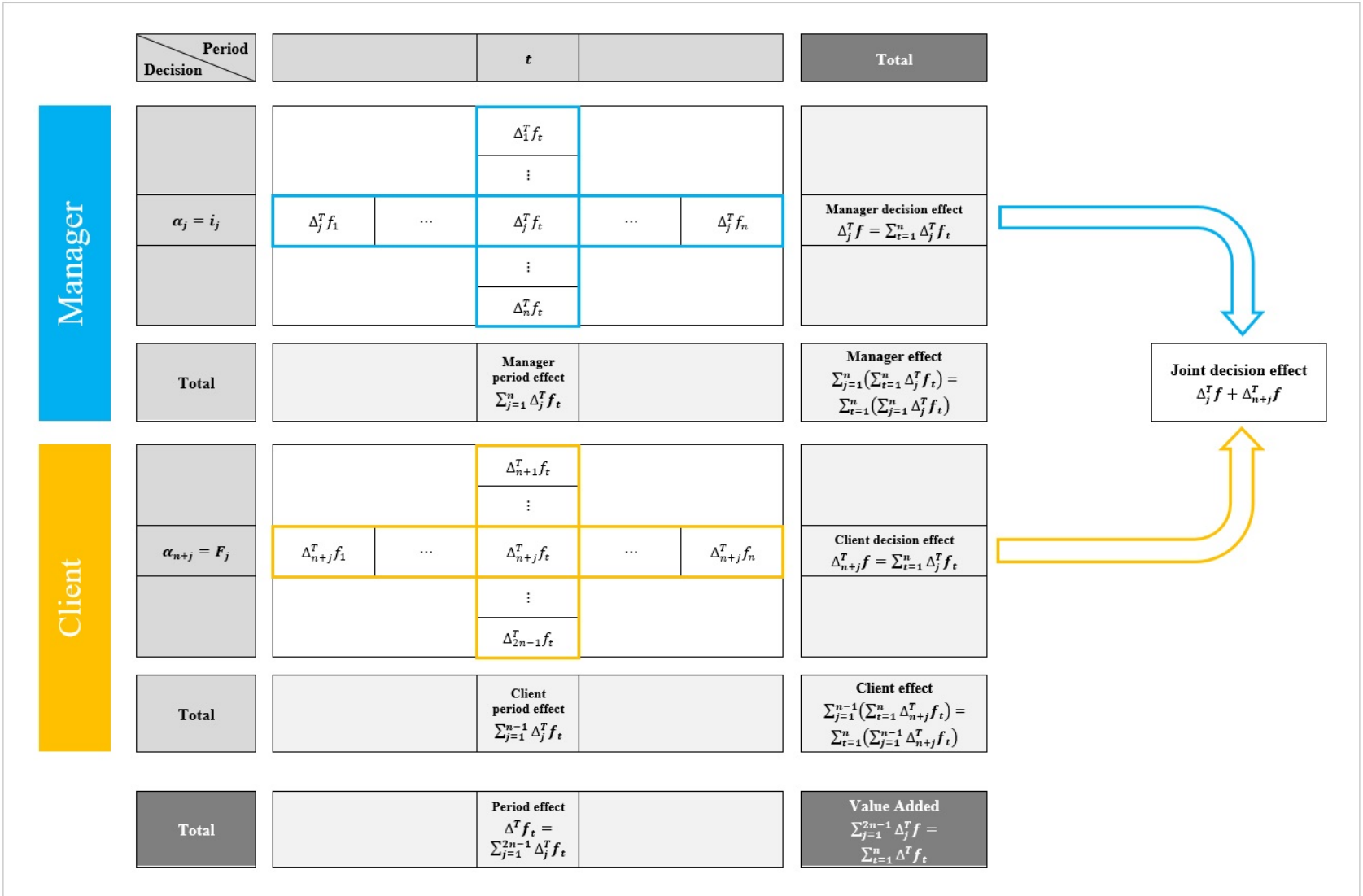

Figure 1: Attribution Matrix: Summary of decision effects, period effects, manager effect, and client effect.

Table 4: The Attribution Matrix

α	$\Delta_j^T f_1$	$\Delta_j^T f_2$	$\dots$	$\Delta_j^T f_n$	$\Delta_j^T f$
α_1	$\Delta_1^T f_1$	$\Delta_1^T f_2$	$\dots$	$\Delta_1^T f_n$	$\Delta_1^T f$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
α_j	$\Delta_j^T f_1$	$\Delta_j^T f_2$	$\dots$	$\Delta_j^T f_n$	$\Delta_j^T f$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
α_n	$\Delta_n^T f_1$	$\Delta_n^T f_2$	$\dots$	$\Delta_n^T f_n$	$\Delta_n^T f$
α_{n+1}	$\Delta_{n+1}^T f_1$	$\Delta_{n+1}^T f_2$	$\dots$	$\Delta_{n+1}^T f_n$	$\Delta_{n+1}^T f$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
α_{n+j}	$\Delta_{n+j}^T f_1$	$\Delta_{n+j}^T f_2$	$\dots$	$\Delta_{n+j}^T f_n$	$\Delta_{n+j}^T f$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
α_p	$\Delta_p^T f_1$	$\Delta_p^T f_2$	$\dots$	$\Delta_p^T f_n$	$\Delta_p^T f$
$\Delta^T f_t$	$\Delta^T f_1$	$\Delta^T f_2$	$\dots$	$\Delta^T f_p$	Δf

Summing by rows *and* by columns, one gets the VA:

$$(1 \quad 1 \quad \dots \quad 1) \cdot \begin{pmatrix} \Delta_1^T f_1 & \Delta_1^T f_2 & \dots & \Delta_1^T f_n \\ \Delta_2^T f_1 & \Delta_2^T f_2 & \dots & \Delta_j^T f_n \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \Delta_p^T f_1 & \Delta_p^T f_2 & \dots & \Delta_p^T f_n \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} = \text{VA} \tag{33}$$

or, equivalently, $\sum_{j=1}^{p} \sum_{t=1}^{n} \Delta_j^T f_t = \text{VA}$. In words, the sum of all elements of the AM amounts to the investment's value added (see bottom-right corner of Figure 1).

We can also define the normalized attribution values, $\Phi_j^T f_t$, as

$$\Phi_j^T f_t = \frac{\Delta_j^T f_t}{\text{VA}}. \tag{34}$$

We gather the normalized attribution values in a normalized AM in Table 5, where the sum $\sum_{t=1}^{n} \Phi_j^T f_t = \Phi_j^T f$ is the normalized decision effect and, analogously, the sum $\sum_{j=1}^{p} \Phi_j^T f_t = \Phi^T f_t$ is the normalized period effect.

It is trivial to derive the perfect decomposition of the value added in normalized terms:

$$\sum_{j=1}^{p} \Phi_j^T f = 100\% = \sum_{t=1}^{n} \Phi^T f_t$$

and

$$\sum_{j=1}^{p} \sum_{t=1}^{n} \Phi_j^T f_t = 100\%.$$

Remark 1. Inspecting (25) and (26), it should be clear that the return rates x_t of periods

Table 5: The normalized Attribution Matrix

α	$\Phi_j^T f_1$	$\Phi_j^T f_2$	$\dots$	$\Phi_j^T f_n$	$\Phi_j^T f$
α_1	$\Phi_1^T f_1$	$\Phi_1^T f_2$	$\dots$	$\Phi_1^T f_n$	$\Phi_1^T f$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
α_j	$\Phi_j^T f_1$	$\Phi_j^T f_2$	$\dots$	$\Phi_j^T f_n$	$\Phi_j^T f$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
α_n	$\Phi_n^T f_1$	$\Phi_n^T f_2$	$\dots$	$\Phi_n^T f_n$	$\Phi_n^T f$
α_{n+1}	$\Phi_{n+1}^T f_1$	$\Phi_{n+1}^T f_2$	$\dots$	$\Phi_{n+1}^T f_n$	$\Phi_{n+1}^T f$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
α_{n+j}	$\Phi_{n+j}^T f_1$	$\Phi_{n+j}^T f_2$	$\dots$	$\Phi_{n+j}^T f_n$	$\Phi_{n+j}^T f$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
α_p	$\Phi_p^T f_1$	$\Phi_p^T f_2$	$\dots$	$\Phi_p^T f_n$	$\Phi_p^T f$
$\Phi^T f_t$	$\Phi^T f_1$	$\Phi^T f_2$	$\dots$	$\Phi^T f_n$	100.00%

$t > m$ and the cash flows y_t of periods $t \geq m$ do not affect $E_m(x, y)$ and $\mathrm{VA}^{(m)}(x, y)$, because these decisions only intervene after the liquidation date m. Therefore,

$$\Delta_j^T \mathrm{VA}^{(m)} = 0 \quad \text{for the inputs } \alpha_j = x_t \text{ with } t > m \text{ or } \alpha_j = y_t \text{ with } t \geq m. \tag{35}$$

From eq. (30), this implies that the attribution value of α_j is null on the periods preceding the decision, that is,

$$\Delta_j^T f_m = 0 \text{ for the inputs } \alpha_j = x_t \text{ with } t > m \text{ or } \alpha_j = y_t \text{ with } t \geq m. \tag{36}$$

Remark 2. It is worth noting that, for any $k \in \{1, 2, \dots, n\}$, if $i_k = i_k^*$, then both row k and column k of the AM are zero vectors. Formally,

- row k: $\Delta_k^T f_1 = \Delta_k^T f_2 = \dots = \Delta_k^T f_n = 0$ (α_k has no impact on any period)
- column k: $\Delta_1^T f_k = \Delta_2^T f_k = \dots = \Delta_p^T f_k = 0$ (no decision has any impact on period k).

(see proof in Appendix C).

Remark 3. An interesting feature of the normalized AM is that it is invariant under changes in the evaluation date. Specifically, if the NPV is selected as the model output, then the AM associated with NPV is equal to the AM associated with VA, premultiplied by the discounting factor $\prod_{t=1}^n (1 + i_t^*)^{-1}$. The normalized AM found by such a matrix is equal to the normalized AM associated with VA described in Table 5. In other words, whether one refers value creation at time $t = 0$ (NPV) or at time $t = n$ (VA) or at any other date t, $0 < t < n$, the normalized attribution values do not change.

Finally, we summarize the contribution of the two decision makers on the value created in a period by defining the *manager period effect* and the *client period effect*: For any fixed

period m, the manager period effect is the sum of the n attribution values attributable to the manager, $\sum_{j=1}^{n} \Delta_j^T f_m$ (and, in normalized version, $\sum_{j=1}^{n} \Phi_j^T f_m$); likewise, for any fixed period m, the client period effect is the sum of the $n-1$ attribution values attributable to the client, $\sum_{j=n+1}^{2n-1} \Delta_j^T f_m$ (and, in normalized version, $\sum_{j=n+1}^{2n-1} \Phi_j^T f_m$). The period effect is equal to the sum of the manager period effect and the client period effect:

$$\overbrace{\Delta^T f_m}^{\text{period effect}} = \overbrace{\sum_{j=1}^{n} \Delta_j^T f_m}^{\text{manager period effect}} + \overbrace{\sum_{j=n+1}^{2n-1} \Delta_j^T f_m}^{\text{client period effect}}. \tag{37}$$

8 Worked example

In this section, we consider an investment management agreement whereby an investor endows a fund manager the capital amount $B_0 = -F_0 = 100$ (in thousands). The investment lasts $n = 8$ periods. The input data are described in Table 6: The first column describes the $15(= 2 \cdot 8 - 1)$ variables of the model, distinguishing the rates (x_t) from the cash flows (y_t) (and, therefore, the manager's decisions from the investor's decisions). The second column expresses the benchmark (i.e., base) case and the third column describes the realized case.

Table 7 describes the beginning-of-period and end-of-period values of both passive investment and active investment, as well as the returns and the cash flows. From (8), the value added of the passive investment is 0 (as expected), and, from (7), the value added of the active investment is $f(i, F) = 2.466$ (see last row of the table).

Therefore, from (9), the increase in value added from the passive (value-neutral) investment policy to the active investment policy is $\text{VA} = f(i, F) - f(i^*, 0) = 2.466 - 0 = 2.466 > 0$, meaning that the active investment creates value.

Table 6: Worked example: Inputs

α	$\alpha^0 = (i^*, 0)$	$\alpha^1 = (i, F)$
$\alpha_1 = x_1$	$i_1^* = 3\%$	$i_1 = 4\%$
$\alpha_2 = x_2$	$i_2^* = 4\%$	$i_2 = 5\%$
$\alpha_3 = x_3$	$i_3^* = 3\%$	$i_3 = 2\%$
$\alpha_4 = x_4$	$i_4^* = 6\%$	$i_4 = 4\%$
$\alpha_5 = x_5$	$i_5^* = 1\%$	$i_5 = 3\%$
$\alpha_6 = x_6$	$i_6^* = 2\%$	$i_6 = 3\%$
$\alpha_7 = x_7$	$i_7^* = 2\%$	$i_7 = 5\%$
$\alpha_8 = x_8$	$i_8^* = 5\%$	$i_8 = 4\%$
$\alpha_9 = y_1$	0.00	30.00
$\alpha_{10} = y_2$	0.00	−20.00
$\alpha_{11} = y_3$	0.00	40.00
$\alpha_{12} = y_4$	0.00	10.00
$\alpha_{13} = y_5$	0.00	−30.00
$\alpha_{14} = y_6$	0.00	60.00
$\alpha_{15} = y_7$	0.00	20.00

Table 7: Passive vs. active investment: Cash flows, market values, and value added

	Passive Investment $(i^*, 0)$					**Active Investment** (i, F)					
t	Beginning value $(t-1)$	Rate of return	Ending value (t)	Cash flow	Beginning value (t)	Beginning value $(t-1)$	Rate of return	Ending value (t)	Cash flow	Beginning value (t)	t
0				−100.00	100.00				−100.00	100.00	0
1	100.00	3.00%	103.00	0.00	103.00	100.00	4.00%	104.00	30.00	74.00	1
2	103.00	4.00%	107.12	0.00	107.12	74.00	5.00%	77.70	−20.00	97.70	2
3	107.12	3.00%	110.33	0.00	110.33	97.70	2.00%	99.65	40.00	59.65	3
4	110.33	6.00%	116.95	0.00	116.95	59.65	4.00%	62.04	10.00	52.04	4
5	116.95	1.00%	118.12	0.00	118.12	52.04	3.00%	53.60	−30.00	83.60	5
6	118.12	2.00%	120.49	0.00	120.49	83.60	3.00%	86.11	60.00	26.11	6
7	120.49	2.00%	122.90	0.00	122.90	26.11	5.00%	27.41	20.00	7.41	7
8	122.90	5.00%	129.04	129.04		7.41	4.00%	7.71	7.71		8
$f(x, y)$				0.000					2.466		$f(x, y)$

Step 1 (Split the value added into 15 decision effects) Using Clean FCSIs, we now decompose the value added in terms of the influences of active investment choices and contribution/distribution decisions made in the various periods, by evaluating the effect on $f(\alpha)$ when the input vector is changed from the benchmark vector $\alpha^0 = (i^*, 0)$ to the active-investment vector $\alpha^1 = (i, F)$.

Table 8: Decomposition of the value added: Decision effects $(\Delta_j^T f)$

$\alpha = (x, y)$	$\Delta_j^1 f$	$\Delta_j^I f$	$\Delta_j^T f$	$\Phi_j^T f$	R_j
Manager decision effects					
$\alpha_1 = x_1$	1.253	0.019	1.272	51.57%	4
$\alpha_2 = x_2$	1.241	−0.167	1.074	43.56%	7
$\alpha_3 = x_3$	−1.253	0.038	−1.215	−49.26%	5
$\alpha_4 = x_4$	−2.435	0.529	−1.905	−77.27%	2
$\alpha_5 = x_5$	2.555	−0.696	1.859	75.39%	3
$\alpha_6 = x_6$	1.265	−0.177	1.088	44.12%	6
$\alpha_7 = x_7$	3.795	−1.499	2.296	93.13%	1
$\alpha_8 = x_8$	−1.229	0.581	−0.648	−26.29%	9
Client decision effects					
$\alpha_9 = y_1$	0.000	−0.567	−0.567	−22.99%	11
$\alpha_{10} = y_2$	0.000	0.244	0.244	9.91%	14
$\alpha_{11} = y_3$	0.000	−0.710	−0.710	−28.79%	8
$\alpha_{12} = y_4$	0.000	−0.277	−0.277	−11.25%	13
$\alpha_{13} = y_5$	0.000	0.488	0.488	19.79%	12
$\alpha_{14} = y_6$	0.000	−0.634	−0.634	−25.70%	10
$\alpha_{15} = y_7$	0.000	0.101	0.101	4.08%	15
Total	5.193	−2.727	2.466	100%	

Table 8 collects the results of the sensitivity analysis: Column 1 presents the input parameters, column 2 supplies the individual contributions of α_j, calculated as in (13); column 3 reports the Clean Interaction FCSI, which is computed as in (18); column 4 (in gray) shows the Clean Total Order FCSI as defined in (19). They represent the manager decision effects (the first eight effects, whose sum is the manager effect) and the client decision effects (the following

seven effects, whose sum is the client effect). As expected, they exactly decompose the value added, with $\sum_{j=1}^{15} \Delta_j^T f = f(i, F) - f(i^*, 0) = 2.466 = \text{VA}$.[4] Column 5 reports the normalized decision effects $\Phi_j^T f$ and, finally, column 6 shows their ranking.

The most influential parameter on VA is the return rate in period $t = 7$, $\alpha_7 = x_7$, with $\Delta_7^T f = 2.296$, signifying that the investment decisions made by the manager in period 7, realizing the return rate $i_7 = 5\%$ (greater than the benchmark index of the same period $i_7^* = 2\%$), have overall contributed positively to the active-investment performance and have had the greatest impact on VA.

Table 9: Individual contribution of the decisions made by the manager in period 7

$\alpha = (x, y)$	(α^0)	$(\alpha_7^0, \alpha_{(-7)}^0)$
$\alpha_1 = x_1$	$i_1^* = 3\%$	$i_1^* = 3\%$
$\alpha_2 = x_2$	$i_2^* = 4\%$	$i_2^* = 4\%$
$\alpha_3 = x_3$	$i_3^* = 3\%$	$i_3^* = 3\%$
$\alpha_4 = x_4$	$i_4^* = 6\%$	$i_4^* = 6\%$
$\alpha_5 = x_5$	$i_5^* = 1\%$	$i_5^* = 1\%$
$\alpha_6 = x_6$	$i_6^* = 2\%$	$i_6^* = 2\%$
$\alpha_7 = x_7$	$\boldsymbol{i_7^* = 2\%}$	$\boldsymbol{i_7 = 5\%}$
$\alpha_8 = x_8$	$i_8^* = 5\%$	$i_8^* = 5\%$
$\alpha_9 = y_1$	0	0
$\alpha_{10} = y_2$	0	0
$\alpha_{11} = y_3$	0	0
$\alpha_{12} = y_4$	0	0
$\alpha_{13} = y_5$	0	0
$\alpha_{14} = y_6$	0	0
$\alpha_{15} = y_7$	0	0

For the sake of interpretability, it is worth noting that the *individual* contribution of $\alpha_7 = x_7$ to the value added is obtained with the following argument: Suppose the client invests passively in the benchmark index from time $t = 0$ to time $t = 6$, then switches to the fund manager's active investment at time $t = 6$ and then switches back to the benchmark index at time $t = 7$, without intermediate contributions and distributions. This means that α shifts from α^0 to $(\alpha_7^1, \alpha_{(-7)}^0)$ (i.e., all parameters are unvaried at their base value while $\alpha_7 = x_7$ is changed from $\alpha_7^0 = x_7^0 = 2\%$ to $\alpha_7^1 = x_7^1 = 5\%$. From (11), and considering that $y_t = 0$ for $t = 1, 2, \ldots, 7$, the switching strategy leads to

$$\begin{aligned} f(\alpha_7^1, \alpha_{(-7)}^0) &= f(0.03, 0.04, 0.03, 0.06, 0.01, 0.02, \mathbf{0.05}, 0.05, 0, 0, 0, 0, 0, 0, 0) \\ &= -100\Big((1.03)(1.04)(1.03)(1.06)(1.01)(1.02)(1.02)(1.05) + \\ &\qquad - (1.03)(1.04)(1.03)(1.06)(1.01)(1.02)(\mathbf{1.05})(1.05)\Big) = 3.795 \end{aligned}$$

[4]It can be shown that the standard Total FCSIs (Borgonovo 2010a, 2010b) do not accomplish a perfect decomposition. Specifically, in this case, $\sum_{j=1}^{15} \Delta_j^{\mathcal{T}} f = -0.230 \neq 2.466 = \text{VA}$.

and the no-switching strategy is the passive investment which, owing to (8), leads to

$$\begin{aligned} f(\alpha^0) &= f(0.03, 0.04, 0.03, 0.06, 0.01, 0.02, \mathbf{0.02}, 0.05, 0, 0, 0, 0, 0, 0, 0) = \\ &= -100\Big((1.03)(1.04)(1.03)(1.06)(1.01)(1.02)(1.02)(1.05)+ \\ &\qquad - (1.03)(1.04)(1.03)(1.06)(1.01)(1.02)(\mathbf{1.02})(1.05)\Big) = 0. \end{aligned}$$

The difference

$$\Delta_7^1 f = f(\alpha_7^1, \alpha_{(-7)}^0) - f(\alpha^0) = 3.795 - 0 = 3.795$$

represents the individual contribution of $\alpha_7 = x_7$, calculated as in (13), that is the impact of the investment decisions made by the manager in period $t = 7$ on the value added, taken in isolation from the other inputs. The interaction effect is calculated as in eq. (18): $\Delta_7^I f = -1.499$. That is, the interaction shows a partial compensating effect. Overall, the manager contribution (i.e., the contribution to VA of the investment policy made by the manager) in period 7 is $\Delta_7^T f = 3.795 - 1.499 = 2.296$.

In terms of weight, the manager's contribution in period 7 explains VA almost entirely ($\Phi_7^T f = 93.13\%$). However, this does not mean that the impact of the decisions made in the other periods is small, because some of the parameters have had a strong *positive* impact and some other parameters have had a strong *negative* impact. For example, $\alpha_4 = x_4$ is the second most influential input and it contributes negatively ($i_4 = 4\%$ is lower than the passive index $i_4^* = 6\%$) with $\Delta_4^T f = -1.905$, corresponding to $\Phi_4^T f = -77.27\%$, which means that the manager has destroyed much value in that period. However, in the following period, the manager's decisions have created value ($i_5 = 3\% > 1\% = i_5^*$): The total contribution of $\alpha_5 = x_5$ is $\Delta_5^T f = 1.859$ which corresponds to $\Phi_5^T f = 75.39\%$ of VA, implying that this is the third most influential parameter and that it has almost entirely offset the poor performance of period 4.

At the opposite side of the parameters' ranking, the least influential input in the whole set is the client contribution in period 7, $y_7 = \alpha_{15}$. This means that the client's decision of withdrawing 20 from the investment at $t = 7$ is the lowest-impact decision. The contribution of y_7 is $\Delta_{15}^T f = 0.101$, corresponding to a 4.08% of the value added. The penultimate rank and the third-last rank are also determined by client's decisions, namely, $y_2 = \alpha_{10}$ and $y_4 = \alpha_{12}$, with $\Delta_{10}^T f = 0.244$ and $\Delta_{12}^T f = -0.277$.

As anticipated, for any fixed period t, a joint decision effect is obtained as the sum of the manager decision effect and the client decision effect of period t. Table 10 reports the joint decision effects as defined in eq. (22). The highest positive effect is in period $t = 7$, equal to 2.397, meaning that the decisions made in period 7 by manager and client jointly generate 2.397; the highest negative effect is in period $t = 4$ and amounts to -2.182.

The manager effect, determined by the group of parameters $x = \{x_1, x_2, \ldots, x_8\}$ and computed as $\sum_{j=1}^{8} \Delta_j^T f = 3.821$, is considerably more impactful than the client effect, determined by the group of parameters $y = \{y_1, y_2, \ldots, y_7\}$ and calculated as $\sum_{j=9}^{15} \Delta_j^T f = -1.355$. Moreover, the former is positive while the latter is negative. Therefore, the manager has, overall, performed positively and created value, thereby offsetting the value destruction caused by the

Table 10: Decision effects as the sum of manager decision effects and client decision effects

Effect	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t=7$	$t=8$	Sum
Manager decision effect	1.272	1.074	−1.215	−1.905	1.859	1.088	2.296	−0.648	3.821
Client decision effect	−0.567	0.244	−0.710	−0.277	0.488	−0.634	0.101	0	−1.355
Joint decision effect	0.705	1.318	−1.925	−2.182	2.347	0.454	2.397	−0.648	2.466

investor's decisions regarding interim contributions and distributions.

Remark 4. The decisions of contributions and distributions of the investor, taken in isolation, have no effect on the value added: $\Delta_j^1 f = 0$ for all $j = 9, 10, \ldots, 15$ (see column 2 of Table 8). Indeed, if in a given period the investor funds are invested at a rate of return equal to the benchmark return, the amount of money which is deposited or withdrawn at the beginning of that period will neither increase the value added nor decrease it (the decisions will be neutral). The effects of deposits and withdrawals are indirect, mediated by the manager's performance. In other words, it is the interaction between rates (affected by manager's decisions) and cash flows (determined by the investor) that activates a nonzero effect of the cash flows on the investment's performance (see columns 3 and 4 of Table 8). Specifically, if the investor deposits (withdraws) money at the beginning of a value-creating period (i.e., $i_t > i_t^*$), then the investor's decision will amplify (reduce) the good manager's performance; if, instead, the investor deposits (withdraws) money at the beginning of a value-destroying period (i.e., $i_t < i_t^*$), then the investor's decisions will amplify (reduce) the bad manager's performance.[5] If the investor deposits (withdraws) money at the beginning of a value-neutral period (i.e., $i_t = i_t^*$), the investor's decisions has no effect on the same period but will have effects on the following periods (as long as the holding rate differs from the benchmark return).

Step 2 (Split the Value Added into 8 period effects) Table 11 reports the truncated investments at time m, with $1 \leq m \leq 8$ and the resultant period effects, which are highlighted in the last row. The upper side of the table collects the cash flows

$$F^{(m)} = (F_0, F_1, \ldots, F_{m-1}, E_m, 0, \ldots, 0),$$

whereas the lower part shows the value added $\mathrm{VA}^{(m)}(i, F)$ and the period effects $\Delta^T f_m(i, F)$ (numbers are rounded).

Step 3 (Build the Attribution Matrix) We now consider the AM, presented in Table 12. Inspecting the AM, it is clear that a decision made in a given period has no effect on previous periods, so the resulting attribution value is zero. For example, $\Delta_5^T f_2 = 0$ means that the decisions made by the manager in period 5 has no effect on the value added in period 2. Also, focusing on the manager's attribution value, $\Delta_j^T f_t$ with $1 \leq j \leq 8$, it is worth noting that $\Delta_j^T f_j$ is (i) positive if the manager's decisions in period j are such that fund's holding period rate i_j exceeds the benchmark return i_j^*, (ii) negative if the manager's decisions in period j are such

[5]This is because a higher (smaller) scale of the investment amplify (reduce) the (good or bad) performance.

Table 11: Decomposition of the value added: Period effects ($\Delta^T f_m$)

	Truncation dates (m)							
Cash-flow dates (t)	$F^{(1)}$	$F^{(2)}$	$F^{(3)}$	$F^{(4)}$	$F^{(5)}$	$F^{(6)}$	$F^{(7)}$	$F^{(8)}$
$t=0$	− 100.00	− 100.00	− 100.00	− 100.00	− 100.00	− 100.00	− 100.00	− 100.00
$t=1$	104.00	30.00	30.00	30.00	30.00	30.00	30.00	30.00
$t=2$	0.00	77.70	− 20.00	− 20.00	− 20.00	− 20.00	− 20.00	− 20.00
$t=3$	0.00	0.00	99.65	40.00	40.00	40.00	40.00	40.00
$t=4$	0.00	0.00	0.00	62.04	10.00	10.00	10.00	10.00
$t=5$	0.00	0.00	0.00	0.00	53.60	− 30.00	− 30.00	− 30.00
$t=6$	0.00	0.00	0.00	0.00	0.00	86.11	60.00	60.00
$t=7$	0.00	0.00	0.00	0.00	0.00	0.00	27.41	20.00
$t=8$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	7.71
VA and period effect	$m=1$	$m=2$	$m=3$	$m=4$	$m=5$	$m=6$	$m=7$	$m=8$
$\mathrm{VA}^{(m)}(i,F)$	1.253	2.144	1.002	−0.315	0.822	1.718	2.540	2.466
$\Delta^T f_m(i,F)$	1.253	0.891	−1.143	−1.316	1.137	0.895	0.822	−0.074

[a] The element (t,m) of this matrix represents the cash flow at time t of the investment truncated at time m.

that fund's holding period rate i_j falls short the benchmark return i_j^*. For example, in period 6, the manager's decisions give rise to a positive performance (since $3\% = i_6 > i_6^* = 2\%$) and the impact of the manager's decisions is $\Delta_6^T f_6 = 1.078$.

Table 12: Attribution matrix for the 8-period investment

$\alpha=(x,y)$	$\Delta_j^T f_1$	$\Delta_j^T f_2$	$\Delta_j^T f_3$	$\Delta_j^T f_4$	$\Delta_j^T f_5$	$\Delta_j^T f_6$	$\Delta_j^T f_7$	$\Delta_j^T f_8$	$\Delta_j^T f$
$\alpha_1=x_1$	1.253	0.006	−0.006	−0.012	0.012	0.006	0.019	−0.006	1.272
$\alpha_2=x_2$	0	1.066	−0.006	−0.006	0.003	0.007	0.015	−0.005	1.074
$\alpha_3=x_3$	0	0	−1.197	0.010	−0.009	−0.006	−0.018	0.006	−1.215
$\alpha_4=x_4$	0	0	0	−1.878	0.003	−0.014	−0.026	0.010	−1.905
$\alpha_5=x_5$	0	0	0	0	1.829	0.015	0.025	−0.010	1.859
$\alpha_6=x_6$	0	0	0	0	0	1.078	0.015	−0.006	1.088
$\alpha_7=x_7$	0	0	0	0	0	0	2.310	−0.014	2.296
$\alpha_8=x_8$	0	0	0	0	0	0	0	−0.648	−0.648
$\alpha_9=y_1$	0	−0.181	0.184	0.353	−0.364	−0.186	−0.556	0.182	−0.567
$\alpha_{10}=y_2$	0	0	−0.118	−0.223	0.228	0.120	0.354	−0.116	0.244
$\alpha_{11}=y_3$	0	0	0	0.439	−0.453	−0.232	−0.692	0.227	−0.710
$\alpha_{12}=y_4$	0	0	0	0	−0.112	−0.054	−0.165	0.054	−0.277
$\alpha_{13}=y_5$	0	0	0	0	0	0.162	0.484	−0.159	0.488
$\alpha_{14}=y_6$	0	0	0	0	0	0	−0.944	0.311	−0.634
$\alpha_{15}=y_7$	0	0	0	0	0	0	0	0.101	0.101
$\Delta^T f_t$	1.253	0.891	−1.143	−1.316	1.137	0.895	0.822	−0.074	2.466

As for the investor's attribution values, $\Delta_j^T f_t$, with $1 \le t \le 8$ and $9 \le j < t+8$, the attribution values $\Delta_j^T f_t$ are

- positive if (i) the fund's holding period rate i_t exceeds the benchmark return i_t^* and the investor contributes cash to the fund at time $j-8$, or (ii) the fund's holding period rate i_t falls short of the benchmark return i_t^* and the investor withdraws cash from the fund at time $j-8$
- negative if (i) the fund's holding period rate i_t falls short of the benchmark return i_t^* and the investor contributes cash to the fund at time $j-8$, or (ii) the fund's holding period

Table 13: Normalized attribution matrix for the 8-period investment

$\alpha = (x, y)$	$\Phi_j^T f_1$	$\Phi_j^T f_2$	$\Phi_j^T f_3$	$\Phi_j^T f_4$	$\Phi_j^T f_5$	$\Phi_j^T f_6$	$\Phi_j^T f_7$	$\Phi_j^T f_8$	$\Phi_j^T f$
$\alpha_1 = x_1$	50.81%	0.24%	−0.25%	−0.48%	0.49%	0.25%	0.75%	−0.25%	51.57%
$\alpha_2 = x_2$	0.00%	43.23%	−0.23%	−0.25%	0.14%	0.27%	0.62%	−0.22%	43.56%
$\alpha_3 = x_3$	0.00%	0.00%	−48.56%	0.41%	−0.38%	−0.26%	−0.71%	0.24%	−49.26%
$\alpha_4 = x_4$	0.00%	0.00%	0.00%	−76.16%	0.11%	−0.55%	−1.07%	0.41%	−77.27%
$\alpha_5 = x_5$	0.00%	0.00%	0.00%	0.00%	74.17%	0.60%	1.03%	−0.41%	75.39%
$\alpha_6 = x_6$	0.00%	0.00%	0.00%	0.00%	0.00%	43.72%	0.62%	−0.22%	44.12%
$\alpha_7 = x_7$	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	93.68%	−0.55%	93.13%
$\alpha_8 = x_8$	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	−26.29%	−26.29%
$\alpha_9 = y_1$	0.00%	−7.33%	7.47%	14.31%	−14.75%	−7.56%	−22.53%	7.39%	−22.99%
$\alpha_{10} = y_2$	0.00%	0.00%	−4.78%	−9.03%	9.23%	4.86%	14.36%	−4.72%	9.91%
$\alpha_{11} = y_3$	0.00%	0.00%	0.00%	17.82%	−18.36%	−9.40%	−28.05%	9.20%	−28.79%
$\alpha_{12} = y_4$	0.00%	0.00%	0.00%	0.00%	−4.54%	−2.21%	−6.69%	2.18%	−11.25%
$\alpha_{13} = y_5$	0.00%	0.00%	0.00%	0.00%	0.00%	6.59%	19.64%	−6.45%	19.79%
$\alpha_{14} = y_6$	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	−38.29%	12.60%	−25.70%
$\alpha_{15} = y_7$	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	4.08%	4.08%
$\Phi^T f_t$	50.81%	36.15%	−46.34%	−53.38%	46.11%	36.31%	33.35%	−3.01%	100.00%

rate i_t exceeds the benchmark return i_t^* and the investor withdraws cash from the fund at time $j - 8$

(if $j \geq t+8$, then $\Delta_j^T f_t = 0$). This means that the effect of the client's decisions on a given period depends on whether the client contributes (withdraws) cash into the fund at the beginning of a value-creating period, so determining a positive (negative) effect, or the client contributes (withdraws) cash into the fund at the beginning of a value-destroying period, so determining a negative (positive) effect (see also Remark 4). For example, $\Delta_9^T f_2 = -0.181 < 0$ because, at time $t = 1$ (i.e. $9 - 8$), the investor withdraws 30 ($F_1 = \alpha_9 = 30$) from the investment and period $t = 2$ is a value-creating period ($5\% = i_2 > i_2^* = 4\%$); therefore, reducing the investment scale has a negative impact. However, the same decision has a positive effect in period $t = 3$ ($\Delta_9^T f_3 = 0.184 > 0$), because period 3 is a value-destroying one ($2\% = i_3 < i_3^* = 3\%$); in other words, the reduction of the investment scale at the end of period 1 partially offsets the negative performance of period 3. And so on for the following periods. Overall, in the assessment interval $[0, 8]$, the drawdown decision made by the investor at time $t = 1$ has a net negative effect, equal to $\Delta_9^T f = -0.567$. Consider now the investor's decision of contributing 20 at time $t = 2$ ($F_2 = \alpha_{10} = -20$). The impact of such a decision in period 3 is negative ($\Delta_{10}^T f_3 = -0.118 < 0$) because that period is a value-destroying period ($2\% = i_3 < i_3^* = 3\%$); therefore, augmenting the investment scale is not a good decision. A negative impact of that decision on the following period $t = 4$ occurs as well ($\Delta_{10}^T f_4 = -0.223 < 0$) for the same reason. However, in period $t = 5$, that contribution has a positive effect ($\Delta_{10}^T f_5 = 0.228 > 0$), so the decision of increasing the scale at the end of period 2 has a positive effect after three periods. Overall, the net effect of the investor's decision made at time $t = 2$ is positive: $\Delta_{10}^T f = 0.224 > 0$.

Given a row, summing by columns, one gets the overall effect of a decision made by the manager or the client (decision effect); given a column, summing by rows one gets the overall effect onto a single period of the decisions made by the manager and the investor in all periods (period effect). For example, the overall effect of the decisions made by the manager in period

$t = 4$ is $\Delta_4^T f = -1.905$ and the overall effect of all the decisions made by manager and investors in the various periods onto period $t = 7$ is $\Delta^T f_7 = 0.822$. Table 13 reports the normalized AM, obtained by dividing each cell of the AM by the investment value added VA = 2.466. As previously noted, the highest normalized decision effect is $\Phi_7^T f = 93.13\%$. Accordingly, the managerial decision in period 7 is the most relevant one for the overall investment performance. Inspecting the normalized AM, we understand that most of that value is generated in period 7 ($\Phi_7^T f_7 = 93.68\%$), whereas the effect of the same decision on period 8 is negligible. The period which is most impacted by all the decisions is period 4, responsible for a normalized value destruction equal to $\Phi^T f_4 = -53.38\%$. Period 3, 4, and 8 are value-destroying periods, whereas the other periods are value-creating. The normalized AM also shows that the impact of the manager's decisions is mostly concentrated on the same period where the decisions are made; conversely, the impact of the investor's decision in one period may have a greater impact in some later period. This depends on the magnitude of the excess return $i_t - i_t^*$ which depends on the manager's decisions.[6] Finally, we remind that the normalized AM is the same if one disaggregates the NPV instead of the VA.

Finally, we compute the manager period effects and client period effects and report their absolute and normalized values in a concise AM (see Tables 14 and 15). The manager period effect is higher (in absolute terms) than the client period effect for every $t = 1, 2, \ldots, 8$, suggesting that the manager's decisions are considerably more impactful than the client's decisions. The highest impacts are in period $t = 7$, where the (positive) manager effect is $\sum_{j=1}^{n} \Delta_j^T f_7 = 2.340$ (corresponding to about 95% of the overall value added) and the (negative) client effect is $\sum_{j=n+1}^{2n-1} \Delta_j^T f_7 = -1.518$ (corresponding to about 62% of the overall value added, with opposite sign). Table 16 summarizes the two-dimensional decomposition (decision vs. period) of manager effect and client effect. (Figure 4 in Appendix D summarizes the various effects.)

Table 14: Concise Attribution Matrix

Effect	$\Delta_j^T f_1$	$\Delta_j^T f_2$	$\Delta_j^T f_3$	$\Delta_j^T f_4$	$\Delta_j^T f_5$	$\Delta_j^T f_6$	$\Delta_j^T f_7$	$\Delta_j^T f_8$	$\Delta_j^T f$
Manager period effect	1.253	1.072	−1.209	−1.886	1.838	1.086	2.340	−0.673	3.821
Client period effect	0	−0.181	0.066	0.570	−0.701	−0.190	−1.518	0.599	−1.355
$\Delta^T f_t$	1.253	0.891	−1.143	−1.316	1.137	0.895	0.822	−0.074	2.466

Table 15: Normalized concise Attribution Matrix

Effect	$\Phi_j^T f_1$	$\Phi_j^T f_2$	$\Phi_j^T f_3$	$\Phi_j^T f_4$	$\Phi_j^T f_5$	$\Phi_j^T f_6$	$\Phi_j^T f_7$	$\Phi_j^T f_8$	$\Phi_j^T f$
Manager period effect	50.81%	43.48%	−49.03%	−76.48%	74.53%	44.03%	94.91%	−27.29%	154.95%
Client period effect	0%	−7.33%	2.69%	23.10%	−28.42%	−7.72%	−61.56%	24.29%	−54.95%
$\Phi^T f_t$	50.81%	36.15%	−46.34%	−53.38%	46.11%	36.31%	33.35%	−3.01%	100.00%

[6] We remind that the investor's effects depend on the manager's decisions; the individual contribution of an injection/withdrawal onto VA is zero.

Table 16: Twofold decomposition of manager effect and client effect

Effect	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t=7$	$t=8$	Manager effect
Manager period effect	1.253	1.072	−1.209	−1.886	1.838	1.086	2.340	−0.673	3.821
Manager decision effect	1.272	1.074	−1.215	−1.905	1.859	1.088	2.296	−0.648	3.821
									Client effect
Client period effect	0	−0.181	0.066	0.570	−0.701	−0.190	−1.518	0.599	−1.355
Client decision effect	−0.567	0.244	−0.710	−0.277	0.488	−0.634	0.101	0	−1.355

9 Concluding remarks

Performance of an investment in a given span of time depends on the decisions made in each period by two decision makers, the manager and the investor/client. We blend two techniques: a recent technique of sensitivity analysis, borrowed from OR (the Finite Change Sensitivity Index, introduced in Borgonovo 2010a, 2010b and refined in Magni et al. 2020) and a non-recent technique of economic analysis, borrowed from accounting and finance (the Residual Income. See Magni 2009 for a historical review of its origins and uses). Precisely, we first use FCSI for finding the (manager and client) decision effects, that is, the contributions to the overall investment's performance of the decision made by manager or investor in a period (summing the manager decision effects one gets the *manager effect* and summing the client decision effects one gets the *client effect*). Then, we use the Residual Income approach for finding the contribution to the period investment's performance of all the decisions made by either decision maker over the investment lifespan (period effects). Such a contribution is equal to the (capitalized) residual income of the investment. Each period effect is broken down into manager period effect and client period effect. Third, we combine the two perspectives and builds an Attribution Matrix (AM) which contains the attribution values. Each attribution value provides the contribution of a decision made by either decision maker in any period onto the investment's performance in any (same or other) period. In generating the AM we have taken into account the interactions between the manager's decisions, which affect the investment holding period rates, and the client's decisions, which determine the cash injected into or withdrawn from the investment, for each and every period. The procedure may then be summarized in a nuthsell as a three-step procedure: (i) splitting the value added into p decision effects with the clean FCSI, (ii) splitting the value added into n period effects via the residual income, (iii) using (i) and (ii) to calculate the attribution values, build the Attribution Matrix, and calculate the manager period effect and the client period effect. These steps are outlined in Figure 2. Table 21 in Appendix E summarizes all the effects we have measured.

The approach may be employed in other situations where a principal/agent relation arises, such as when a firm's board of directors delegates the investment decisions to a business unit's managers. Future researches may be addressed to generalize the approach and make the attribution analysis in terms of asset classes. This entails splitting up the manager decision effects considering the holdings in the various asset classes.

Investment description:

Cash Flows (determined by Client): F_0, F, with $F = (F_1, F_2, \dots F_{n-1})$

Rates of return (determined by Manager): Benchmark: $i^* = (i_1^*, i_2^*, \dots i_n^*)$,

Realized $i = (i_1, i_2, \dots i_n)$

Function Parameters: benchmark values: $\alpha^0 = (\alpha_1^0, \alpha_2^0, \dots \alpha_p^0) = (i_1^*, i_2^*, \dots i_n^*, 0, 0, \dots 0)$

realized values: $\alpha^1 = (\alpha_1^1, \alpha_2^1, \dots \alpha_p^1) = (i_1, i_2, \dots i_n, F_1, F_2, \dots F_{n-1})$

Terminal value E_n of the investment:

$$E_n = -\sum_{t=0}^{n-1}(1+i_{t+1})(1+i_{t+2})\dots(1+i_n)\cdot F_t \tag{4}$$

(see Section 2 for details)

Value Added VA of the investment:

$$\mathrm{VA} = f(i,F) = \sum_{t=0}^{n-1}\big((1+i_{t+1}^*)(1+i_{t+2}^*)\dots(1+i_n^*) - (1+i_{t+1})(1+i_{t+2})\dots(1+i_n)\big)\cdot F_t \tag{7}$$

(see Section 3 for details)

STEP 1 (use of clean FCSI) – Split the Value Added into p Decision Effects, $\Delta_j^T f, 1 \le j \le p$:

$$\Delta_j^T f = \Delta_j^1 f + \Delta_j^I f \tag{19}$$

where

$$\Delta_j^1 f = f\big(\alpha_j^1, \alpha_{(-j)}^0\big) - f(\alpha^0), \quad \forall j = 1,2,\dots,p. \tag{13}$$

$\big(\alpha_j^1, \alpha_{(-j)}^0\big)$: parameters set at benchmark values $\alpha^0 = (i_1^*, i_2^*, \dots i_n^*, 0,0,\dots 0)$, except $\alpha_j = \alpha_j^1$ (realized value), and

$$\Delta_j^I f = \frac{\Delta_j^\tau f - \Delta_j^1 f}{\sum_{l=1}^p(\Delta_l^\tau f - \Delta_l^1 f)}\cdot\big(\Delta f - \sum_{l=1}^p \Delta_l^1 f\big), \quad \forall j = 1,2,\dots,p \tag{18}$$

with

$$\Delta_j^\tau f = f(\alpha^1) - f\big(\alpha_j^0, \alpha_{(-j)}^1\big), \quad \forall j = 1,2,\dots,p. \tag{16}$$

$\big(\alpha_j^0, \alpha_{(-j)}^1\big)$: parameters set at realized values $\alpha^1 = (i_1, i_2, \dots i_n, F_1, F_2, \dots F_{n-1})$, except $\alpha_j = \alpha_j^0$ (benchmark value)

j	Decision	Decision Effect $\Delta_j^T f$
1	$\boldsymbol{\alpha_1 = i_1}$	$\Delta_1^T f$
2	$\boldsymbol{\alpha_2 = i_2}$	$\Delta_2^T f$
⋮	⋮	⋮
n	$\boldsymbol{\alpha_n = i_n}$	$\Delta_n^T f$
n+1	$\boldsymbol{\alpha_{n+1} = F_1}$	$\Delta_{n+1}^T f$
n+2	$\boldsymbol{\alpha_{n+2} = F_2}$	$\Delta_{n+2}^T f$
⋮	⋮	⋮
p	$\boldsymbol{\alpha_p = F_{n-1}}$	$\Delta_p^T f$
	Total	$\sum_{j=1}^p \Delta_j^T f = \mathbf{VA}$

Manager Effect = $\sum_{j=1}^{n} \Delta_j^T f$

Client Effect = $\sum_{j=n+1}^{p} \Delta_j^T f$

Joint Decision Effect in period $j = \Delta_j^T f + \Delta_{j+n}^T f$, for $j = 1,2,\dots n$ (22)

(see Sections 4.1 and 5 for details)

STEP 2 (use of RI) – Split the Value Added into n Period Effects $\Delta^T f_t, 1 \le t \le n$:

$$\Delta^T f_m = \mathrm{VA}^{(m)} - \mathrm{VA}^{(m-1)} \tag{27}$$

where

$$\mathrm{VA}^{(m)} = \sum_{t=0}^{m-1} F_t\cdot(1+i_{t+1}^*)(1+i_{t+2}^*)\dots(1+i_n^*) + E_m\cdot(1+i_{m+1}^*)(1+i_{m+2}^*)\dots(1+i_n^*) \tag{26}$$

Period	1	2	⋯	n	Total
Period Effect $\Delta^T f_t$	$\Delta^T f_1$	$\Delta^T f_2$	⋯	$\Delta^T f_n$	$\sum_{t=1}^{n}\Delta^T f_t = \mathbf{VA}$

(see Section 4.2 and 6 for details)

STEP 3 (combination of FCSI and RI) – Build the Attribution Matrix:

Sub-step 3.1 - Split the Value Added in $p \times n$ Attribution Values with the clean FCSI, $\Delta_j^T f_m$:

$$\Delta_j^T f_m = \Delta_j^T \mathrm{VA}^{(m)} - \Delta_j^T \mathrm{VA}^{(m-1)} \tag{30}$$

where $\Delta_j^T \mathrm{VA}^{(m)}$ can be calculated using the formulas (13), (16), (18) and (19) in the previous step 1) applied to $\mathrm{VA}^{(m)}$.

Decision \ Period	1	2	⋯	n	Decision Effect $\Delta_j^T f$
$\boldsymbol{\alpha_1 = i_1}$	$\Delta_1^T f_1$	$\Delta_1^T f_2$	⋯	$\Delta_1^T f_n$	$\Delta_1^T f = \sum_{t=1}^n \Delta_1^T f_t$
$\boldsymbol{\alpha_2 = i_2}$	$\Delta_2^T f_1$	$\Delta_2^T f_2$	⋯	$\Delta_2^T f_n$	$\Delta_2^T f = \sum_{t=1}^n \Delta_2^T f_t$
⋮	⋮	⋮		⋮	⋮
$\boldsymbol{\alpha_n = i_n}$	$\Delta_n^T f_1$	$\Delta_n^T f_2$	⋯	$\Delta_n^T f_n$	$\Delta_n^T f = \sum_{t=1}^n \Delta_n^T f_t$
$\boldsymbol{\alpha_{n+1} = F_1}$	$\Delta_{n+1}^T f_1$	$\Delta_{n+1}^T f_2$	⋯	$\Delta_{n+1}^T f_n$	$\Delta_{n+1}^T f = \sum_{t=1}^n \Delta_{n+1}^T f_t$
$\boldsymbol{\alpha_{n+2} = F_2}$	$\Delta_{n+2}^T f_1$	$\Delta_{n+2}^T f_2$	⋯	$\Delta_{n+2}^T f_n$	$\Delta_{n+2}^T f = \sum_{t=1}^n \Delta_{n+2}^T f_t$
⋮	⋮	⋮		⋮	⋮
$\boldsymbol{\alpha_p = F_{n-1}}$	$\Delta_p^T f_1$	$\Delta_p^T f_2$	⋯	$\Delta_p^T f_n$	$\Delta_p^T f = \sum_{t=1}^n \Delta_p^T f_t$
Period Effect $\Delta^T f_t$	$\Delta^T f_1 = \sum_{j=1}^p \Delta_j^T f_1$	$\Delta^T f_2 = \sum_{j=1}^p \Delta_j^T f_2$	⋯	$\Delta^T f_n = \sum_{j=1}^p \Delta_j^T f_n$	Total: $\sum_{j=1}^p \Delta_j^T f = \sum_{t=1}^n \Delta^T f_t = \mathbf{VA}$

Sub-step 3.2 - Split the periods effects $\Delta^T f_m$ in manager period effects and client period effects:

$$\text{Manager period effect} = \sum_{j=1}^n \Delta_j^T f_m$$

$$\text{Client period effect} = \sum_{j=n+1}^{2n-1} \Delta_j^T f_m$$

such that

$$\overbrace{\Delta^T f_m}^{\text{period effect}} = \overbrace{\sum_{j=1}^n \Delta_j^T f_m}^{\text{manager period effect}} + \overbrace{\sum_{j=n+1}^{2n-1} \Delta_j^T f_m}^{\text{client period effect}} \tag{37}$$

where $\Delta_j^T f_m$ can be calculated using formula (30).

(see Section 7 for details)

Figure 2: Attribution Matrix in a nutshell: FCSI and RI combined in a three-step procedure.

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Appendix A: Proof of eq. (31)

e prove that

$$\Delta_j^T f = \sum_{m=1}^{n} \Delta_j^T f_m.$$

It is sufficient to note that $\Delta_j^T \mathrm{VA}^{(0)} = 0$ and $\Delta_j^T \mathrm{VA}^{(n)} = \Delta_j^T f$; therefore,

$$\begin{aligned}
\Delta_j^T f &= \Delta_j^T \mathrm{VA}^{(n)} - \Delta_j^T \mathrm{VA}^{(0)} \\
&= \Delta_j^T \mathrm{VA}^{(1)} - \Delta_j^T \mathrm{VA}^{(0)} \\
&+ \Delta_j^T \mathrm{VA}^{(2)} - \Delta_j^T \mathrm{VA}^{(1)} \\
&+ \Delta_j^T \mathrm{VA}^{(3)} - \Delta_j^T \mathrm{VA}^{(2)} \\
&\quad \vdots \\
&+ \Delta_j^T \mathrm{VA}^{(n-1)} - \Delta_j^T \mathrm{VA}^{(n-2)} \\
&+ \Delta_j^T \mathrm{VA}^{(n)} - \Delta_j^T \mathrm{VA}^{(n-1)} \\
&= [\text{by (30)}] \\
&= \Delta_j^T f_1 + \Delta_j^T f_2 + \ldots + \Delta_j^T f_n \\
&= \sum_{m=1}^{n} \Delta_j^T f_m.
\end{aligned} \tag{38}$$

Appendix B: Proof of eq. (32)

We prove that

$$\sum_{j=1}^{p} \Delta_j^T f_t = \Delta^T f_t$$

We just remind that, for every project truncated at t, the sum of its clean total FCSIs ($\Delta_j^T \mathrm{VA}^{(t)}$) amounts to the value added of the truncated project, $\mathrm{VA}^{(t)}$ (see eq. (20)). Hence,

$$\begin{aligned}
\Delta^T f_t &= \mathrm{VA}^{(t)} - \mathrm{VA}^{(t-1)} \\
&= (\Delta_1^T \mathrm{VA}^{(t)} + \Delta_2^T \mathrm{VA}^{(t)} + \ldots + \Delta_p^T \mathrm{VA}^{(t)}) - (\Delta_1^T \mathrm{VA}^{(t-1)} + \Delta_2^T \mathrm{VA}^{(t-1)} + \ldots + \Delta_p^T \mathrm{VA}^{(t-1)}) \\
&= (\Delta_1^T \mathrm{VA}^{(t)} - \Delta_1^T \mathrm{VA}^{(t-1)}) + (\Delta_2^T \mathrm{VA}^{(t)} - \Delta_2^T \mathrm{VA}^{(t-1)}) + \ldots + (\Delta_p^T \mathrm{VA}^{(t)} - \Delta_p^T \mathrm{VA}^{(t-1)}) \\
&= \sum_{j=1}^{p} (\Delta_j^T \mathrm{VA}^{(t)} - \Delta_j^T \mathrm{VA}^{(t-1)}) \\
&= \sum_{j=1}^{p} \Delta_j^T f_t.
\end{aligned}$$

Appendix C: Proof of Remark 2

Part I. We show that, if $i_k = i_k^*$, then $\Delta_k^T f_1 = \Delta_k^T f_2 = \ldots = \Delta_k^T f_n = 0$.

Since $i_k = i_k^*$, the following equalities hold:

$$\alpha_k^1 = \alpha_k^0, \quad (\alpha_k^1, \alpha_{(-k)}^0) = \alpha^0, \quad (\alpha_k^0, \alpha_{(-k)}^1) = \alpha^1.$$

Therefore, from (13), the individual effect of $\alpha_k = i_k$ on the value added of the truncated investment at each time t is null: $\Delta_k^1 \mathrm{VA}^{(t)} = 0$ for every $t \in \{1, 2, \ldots, n\}$ and, from (16), also the total effect of $\alpha_k = i_k$ on each truncated investment is zero as well: $\Delta_k^{\mathcal{T}} \mathrm{VA}^{(t)} = 0$ for every t. Since $\Delta_k^{\mathcal{I}} \mathrm{VA}^{(t)} = \Delta_k^{\mathcal{T}} \mathrm{VA}^{(t)} - \Delta_k^1 \mathrm{VA}^{(t)}$, then the interaction effect of $\alpha_k = i_k$ is null, i.e. $\Delta_k^{\mathcal{I}} \mathrm{VA}^{(t)} = 0$ for all t. Consequently, remembering (17), even the Clean Interaction effect $\Delta_k^I \mathrm{VA}^{(t)}$ is zero for each truncation date t. Hence, from (19), the Clean total effect of $\alpha_k = i_k$, is $\Delta_k^T \mathrm{VA}^{(t)} = 0, \forall t$. Finally, by (30), the attribution value of parameter α_k in each period t is null: $\Delta_k^T f_t = 0$ for every $t \in \{1, 2, \ldots, n\}$. (QED)

Part II: We show that, if $i_k = i_k^*$, then $\Delta_1^T f_k = \Delta_2^T f_k = \ldots = \Delta_p^T f_k = 0$.

Since $i_k = i_k^*$ and owing to (26), the values added of the truncated investments at $t = k - 1$ and $t = k$ are equal:

$$\mathrm{VA}^{(k)}(\alpha) = \mathrm{VA}^{(k-1)}(\alpha) \tag{39}$$

for every input vector α such that $i_k = i_k^*$.

From (12) and (39), the change in value added from the passive investment α^0 to the active investment α^1 is the same in case of truncation at $t = k - 1$ or at $t = k$, that is, $\Delta \mathrm{VA}^{(k)} = \mathrm{VA}^{(k)}(\alpha^1) - \mathrm{VA}^{(k)}(\alpha^0) = \mathrm{VA}^{(k-1)}(\alpha^1) - \mathrm{VA}^{(k-1)}(\alpha^0) = \Delta \mathrm{VA}^{(k-1)}$.

Furthermore, via (13) and (39), the individual effect of each factor α_j is equal on $\mathrm{VA}^{(k)}(\alpha)$ and on $\mathrm{VA}^{(k-1)}(\alpha)$, that is,

$$\Delta_j^1 \mathrm{VA}^{(k)} = \mathrm{VA}^{(k)}(\alpha_j^1, \alpha_{(-j)}^0) - \mathrm{VA}^{(k)}(\alpha^0) = \mathrm{VA}^{(k-1)}(\alpha_j^1, \alpha_{(-j)}^0) - \mathrm{VA}^{(k-1)}(\alpha^0) = \Delta_j^1 \mathrm{VA}^{(k-1)} \quad \forall j \in \{1, 2, \ldots, p\}.$$

Analogously, each interaction among s-tuples (with $s \geq 2$) is the same from the truncated investments at k or $k - 1$, that is, $\Delta_{j_1, j_2, \ldots, j_s} \mathrm{VA}^{(k)} = \Delta_{j_1, j_2, \ldots, j_s} \mathrm{VA}^{(k-1)}$ and, consequently, the interaction effect of each parameter is equal for the two functions, that is, $\Delta_j^{\mathcal{I}} \mathrm{VA}^{(k)} = \Delta_j^{\mathcal{I}} \mathrm{VA}^{(k-1)}$ for every $j = 1, 2, \ldots, p$. Hence, via (17), the Clean interaction effects coincide: $\Delta_j^I \mathrm{VA}^{(k)} = \Delta_j^I \mathrm{VA}^{(k-1)}$ for every j. Applying (19), the values added of the truncated investments at $t = k-1$ and $t = k$ share the same Clean Total FCSIs: $\Delta_j^T \mathrm{VA}^{(k)} = \Delta_j^T \mathrm{VA}^{(k-1)}$ for every $j = 1, 2, \ldots, p$. Therefore, from (30), the attribution value of each parameter α_j in period k is zero: $\Delta_j^T f_k = \Delta_j^T \mathrm{VA}^{(k)} - \Delta_j^T \mathrm{VA}^{(k-1)} = 0 \; \forall j \in \{1, 2, \ldots, p\}$. (QED)

Appendix D: "Anima Italia A" Fund

The worked example in Section 8 describes an investment management agreement with positive value added, resulting from a positive manager effect which is higher, in absolute value, than the negative client effect. In this section, we propose an empirical illustration with real financial

data, which shows the opposite case of a fund that destroys value due to a negative manager effect, only partially compensated by a positive client effect.

Anima SGR SPA (a branch of ANIMA Holding, a public company listed on the Milan Stock Exchange) is a large independent asset management group in Italy, with total assets under management of ca. 194 billion euros and more than one million customers (source: https://www.animasgr.it/). We analyze one of the investment funds managed by Anima SGR, "Anima Italia A" (hereafter, Anima Italia) in the period 2013-2020 ($t = 0$ denotes year 2013). The fund manager is Anima SGR and we consider the totality of investors of Anima Italia as the client. The model inputs, retrieved from Anima SGR's Annual Reports, are collected in Table 17 (all amounts in this example are in euros).[7] The fund manager overperformed the benchmark in the first, fifth and sixth years (i.e., $i_t > i_t^*$ for $t = 1, 5, 6$), underperforming the benchmark in the remaining periods. Interestingly, the overperformance in year 6 signifies a negative realized return with lower absolute value than the benchmark (-8.63% compared to -12.60%). The highest realized return was in the seventh year, still not achieving the benchmark level ($i_7 = 28.09\% < i_7^* = 30.00\%$).

Table 17: Anima Italia: Inputs

α	$\alpha^0 = (i^*, 0)$	$\alpha^1 = (i, F)$
$\alpha_1 = x_1$	22.50%	23.64%
$\alpha_2 = x_2$	2.20%	1.88%
$\alpha_3 = x_3$	19.30%	17.64%
$\alpha_4 = x_4$	−3.60%	−8.65%
$\alpha_5 = x_5$	17.50%	18.27%
$\alpha_6 = x_6$	−12.60%	−8.63%
$\alpha_7 = x_7$	30.00%	28.09%
$\alpha_8 = x_8$	−4.40%	−9.50%
$\alpha_9 = y_1$	0.00	10,421,031
$\alpha_{10} = y_2$	0.00	−15,462,422
$\alpha_{11} = y_3$	0.00	−13,828,125
$\alpha_{12} = y_4$	0.00	27,658,409
$\alpha_{13} = y_5$	0.00	27,725,289
$\alpha_{14} = y_6$	0.00	22,489,240
$\alpha_{15} = y_7$	0.00	15,096,561

The initial client's contribution is about 135,714,487; the client decided to withdraw cash from the fund on a yearly basis ($F_t > 0$) in all periods except the second and the third year, where the injections ($F_t < 0$) were respectively equal to 15.5 and 13.8 millions of euros. The highest distributions were decided in the fourth and fifth years, around 27.7 millions in both periods. Table 18 compares the active investment in Anima Italia Fund with the corresponding passive investment. The terminal cash flow distributed in $t = 8$, computed via eq. (10), is about 148.7 millions for the active investment and 249.4 millions for the passive investment. The resulting value added of the active investment, computed via eq. (11), is negative, equal to

[7] The benchmark, declared by Anima SGR, is made of 95% MSCI Italy All Cap (Net Total Return - in euros) and 5% ICE BofA Euro Treasury Bill (Gross Total Return - in euros).

-16.9 millions), signifying that the investment in Anima Italia destroyed value for the client.

Table 18: Anima Italia: Passive vs. active investment

	Passive Investment $(i^*, 0)$					**Active Investment** (i, F)					
t	Beginning value $(t-1)$	Rate of return	Ending value (t)	Cash flow	Beginning value (t)	Beginning value $(t-1)$	Rate of return	Ending value (t)	Cash flow	Beginning value (t)	t
0				−135,714,487	135,714,487				−135,714,487	135,714,487	0
1	135,714,487	22.50%	166,250,247	0.00	166,250,247	135,714,487	23.64%	167,790,973	10,421,031	157,369,942	1
2	166,250,247	2.20%	169,907,752	0.00	169,907,752	157,369,942	1.88%	160,325,930	−15,462,422	175,788,352	2
3	169,907,752	19.30%	202,699,948	0.00	202,699,948	175,788,352	17.64%	206,790,938	−13,828,125	220,619,063	3
4	202,699,948	−3.60%	195,402,750	0.00	195,402,750	220,619,063	−8.65%	201,543,175	27,658,409	173,884,766	4
5	195,402,750	17.50%	229,598,231	0.00	229,598,231	173,884,766	18.27%	205,657,521	27,725,289	177,932,232	5
6	229,598,231	−12.60%	200,668,854	0.00	200,668,854	177,932,232	−8.63%	162,577,241	22,489,240	140,088,001	6
7	200,668,854	30.00%	260,869,510	0.00	260,869,510	140,088,001	28.09%	179,434,015	15,096,561	164,337,454	7
8	260,869,510	−4.40%	249,391,252	249,391,252		164,337,454	−9.50%	148,724,750	148,724,750		8
$f(x, y)$				0					−16,945,558		$f(x, y)$

The model presented in this paper allows attributing the overall value destruction of Anima Italia to the managers' investment decisions and the client's cash-flow decisions. The managers decision effects and the client decision effects, computed via Total Order FCSI approach (step 1), are displayed in Table 19. The most infuential decision effect is to be attributed to the manager's investment choices in year 4; in that year, the return rate was equal to -8.65%, five points lower than the benchmark rate equal to -3.60%, and the manager destroyed value by -13.5 millions, accounting for about 80% of the overall (negative) value added. The second most important decision effect is attributable to the manager's investment choices in the last year, $t = 8$, explaining a value loss of -10.9 millions (64% of the VA), due to the significant underperformance of the realized rate, $i_8 = -9.50\%$, as compared to the benchmark, $i_8^* = -4.40\%$. These two effects were partially compensated by the third most influential decision effect, that is, the manager's investment decisions in year 6, which created value by 9.8 millions, since $i_6 = -8.63\% > -12.60\% = i_6^*$. The least influential choices on VA were the client's distribution decisions in years 4, 5 and 7, accounting for less than 400 thousand euros (less than 2.5% of VA). It is worth noting that the manager's decisions were much more impactful than the client's decisions in each year with difference of one order of magnitude except for year 2, where the contributed cash flow $F_2 = -15.5$ millions accounts more than the low difference between the realized rate, $i_2 = 1.88\%$, and the benchmark, $i_2 = 2.20\%$.

Table 19: Anima Italia: Manager decision effects and client decision effects

Effect	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$	$t = 7$	$t = 8$	Sum
Manager decision effect	2,215,465	−734,844	−3,414,505	−13,486,293	1,534,401	9,811,548	−3,068,976	−10,925,951	−18,069,155
Client decision effect	647,776	−908,802	−574,615	315,688	362,598	908,836	372,115	0	1,123,597
Joint decision effect	2,863,240	−1,643,646	−3,989,119	−13,170,605	1,897,000	10,720,384	−2,696,861	−10,925,951	−16,945,558

The joint decision effects of manager and client, as defined in eq. (22) and reported in Table 19, are highest in year 4, 6, and 8: In $t = 4$, the joint effect is significantly negative because the client's positive decisions had little influence compared to value destruction by the manager; in $t = 6$, both the manager and the client positively contributed to value creation, and the value destruction in $t = 8$ is completely attributable to the manager (the last cash flow equals the fund's ending value).

The overall effects of the manager and the client are identified in the last column of Table 19, highlighted in gray: the manager effect is -18.1 millions whereas the client effect is equal to 1.2 millions, meaning that the manager's investment decisions were responsible for a high value destruction, slightly compensated by the value-creating decisions on contribution/distribution by the client.

As for the period effects, obtained via the Residual Income approach (step 2), most of the overall value was destroyed in year 4 (where $i_4 = -8.65\% < i_4^* = -3.60\%$), for a total of -14.2 millions (see Figure 3). The second most impactful period was year 6, which added, overall, a value of 8.8 millions (thwarted by a value loss in year 8 of same magnitude). The lowest impact on VA is attributable to year 2, accountable for a loss of only 743 thousand euros.

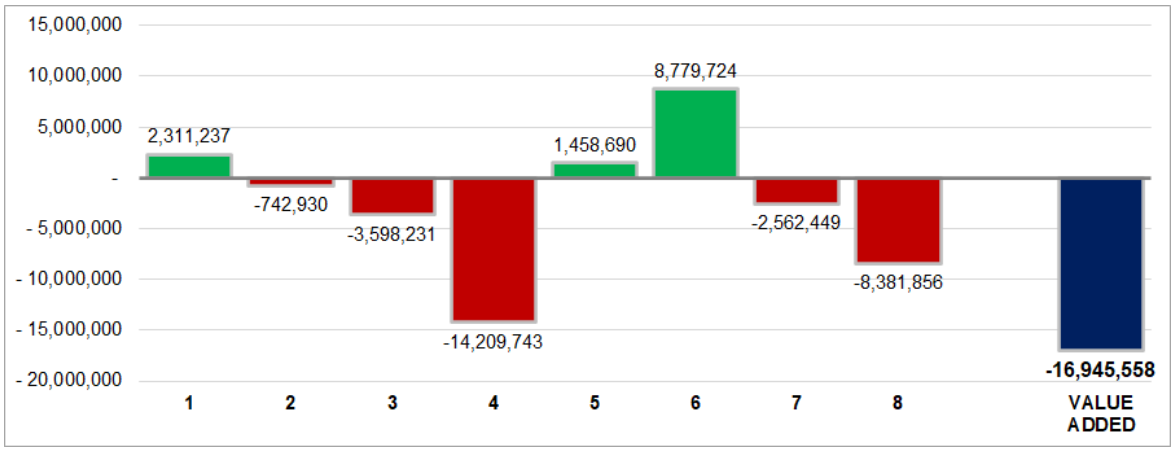

Figure 3: Anima Italia: Decomposition of value added into period effects. A green bar denotes a positive effect, a red bar denotes a negative effect. The blue bar denotes the value added, that is, the algebraic sum of the effects.

Finally, the Attribution Matrix of Anima Italia combines the two decomposition dimensions and reports the attribution values (step 3), which measure the value added in one year by the decisions made by the manager or the client in one (same or different) year (see Table 20). For example, the value added in year 6 by the decisions made by both the manager and the client in year 3 is, respectively, $\Delta_3^T f_6 = -77,447$ and $\Delta_{11}^T f_6 = 368,188$, suggesting that the manager destroyed value, whereas the client created value. The highest attribution values (in absolute value) are the cells on the main diagonal of the manager's submatrix, meaning that the manager's investment decisions greatly impacted on the same period. The lowest attribution value is $\Delta_2^T f_5 = -2,416$ euros associated with the almost negligible influence of the investment decisions made in year 2 onto the performance of year 5.

The sum of the elements of a row of the Attribution Matrix is the manager/client decision effect, whereas the sum of the elements of a column is the period effect. The highest manager decision effect is reported in year 4, equal to -13.5 million euros; its decomposition is contained in the fourth row of the Attribution Matrix. The highest client decision effect is observed in year 6 (equal to 908 thousand euros); the 14th row of the Attribution Matrix breaks it down to the influences on years 7 and 8, equal to 205 and 703 thousand euros, respectively. Finally, the highest period effect is in $t = 4$, whose decomposition is presented in the fourth column of

the Attribution Matrix; the most impacting decisions on $t = 4$ are the manager's investment decision of the same period with attribution value equal to -13.6 million and the contribution decision by the client of the second year, with attribution value equal to -587 thousand euros.

Figure 4 summarizes the various effects.

Table 20: Anima Italia: Attribution Matrix

$\alpha = (x, y)$	$\Delta_j^T f_1$	$\Delta_j^T f_2$	$\Delta_j^T f_3$	$\Delta_j^T f_4$	$\Delta_j^T f_5$	$\Delta_j^T f_6$	$\Delta_j^T f_7$	$\Delta_j^T f_8$	$\Delta_j^T f$
$\alpha_1 = x_1$	2,311,237	–3,637	–15,992	–59,652	7,305	49,303	–16,803	–56,296	2,215,465
$\alpha_2 = x_2$	0	–763,892	5,083	19,337	–2,416	–16,037	5,556	17,525	–734,844
$\alpha_3 = x_3$	0	0	–3,537,822	94,009	–11,292	–77,447	25,978	92,068	–3,414,505
$\alpha_4 = x_4$	0	0	0	–13,633,504	–44,869	–317,692	103,266	406,505	–13,486,293
$\alpha_5 = x_5$	0	0	0	0	1,549,090	33,480	–11,600	–36,569	1,534,401
$\alpha_6 = x_6$	0	0	0	0	0	10,065,394	–73,355	–180,491	9,811,548
$\alpha_7 = x_7$	0	0	0	0	0	0	–3,117,442	48,466	–3,068,976
$\alpha_8 = x_8$	0	0	0	0	0	0	0	–10,925,951	–10,925,951
$\alpha_9 = y_1$	0	24,598	108,163	403,472	–49,408	–333,473	113,650	380,773	647,776
$\alpha_{10} = y_2$	0	0	–157,663	–587,438	71,849	485,423	–165,272	–555,701	–908,802
$\alpha_{11} = y_3$	0	0	0	–445,967	54,256	368,188	–124,809	–426,282	–574,615
$\alpha_{12} = y_4$	0	0	0	0	–115,825	–799,288	266,494	964,308	315,688
$\alpha_{13} = y_5$	0	0	0	0	0	–678,128	226,550	814,176	362,598
$\alpha_{14} = y_6$	0	0	0	0	0	0	205,338	703,498	908,836
$\alpha_{15} = y_7$	0	0	0	0	0	0	0	372,115	372,115
$\Delta^T f_t$	2,311,237	–742,930	–3,598,231	–14,209,743	1,458,690	8,779,724	–2,562,449	–8,381,856	–16,945,558

EFFECT		WORKED EXAMPLE	ANIMA ITALIA	FORMULA	EQUATION
Manager decision effect	$\Delta_1^T f$	1.272	2,215,465	$\Delta_j^T f = \Delta_j^1 f + \Delta_j^I f$ for $j = 1, \dots, n$	eq. (19)
	$\Delta_2^T f$	1.074	-734,844		
	$\Delta_3^T f$	-1.215	-3,414,505		
	$\Delta_4^T f$	-1.905	-13,486,293		
	$\Delta_5^T f$	1.859	1,534,401		
	$\Delta_6^T f$	1.088	9,811,548		
	$\Delta_7^T f$	2.296	-3,068,976		
	$\Delta_8^T f$	-0.648	-10,925,951		
Client decision effect	$\Delta_9^T f$	-0.567	647,776	$\Delta_j^T f = \Delta_j^1 f + \Delta_j^I f$ for $j = n+1, \dots, p$	
	$\Delta_{10}^T f$	0.244	-908,802		
	$\Delta_{11}^T f$	-0.710	-574,615		
	$\Delta_{12}^T f$	-0.277	315,688		
	$\Delta_{13}^T f$	0.488	362,598		
	$\Delta_{14}^T f$	-0.634	908,836		
	$\Delta_{15}^T f$	0.101	372,115		
Joint decision effect	*period 1*	0.705	2,863,240	$\Delta_j^T f + \Delta_{n+j}^T f$ for $j = 1, \dots n$	eq. (21)
	period 2	1.319	-1,643,646		
	period 3	-1.925	-3,989,119		
	period 4	-2.183	-13,170,605		
	period 5	2.347	1,897,000		
	period 6	0.454	10,720,384		
	period 7	2.397	-2,696,861		
	period 8	-0.648	-10,925,951		
Manager effect		3.821	-18,069,155	$\sum_{j=1}^{n} \Delta_j^T f$	eq. (23)
Client effect		-1.355	1,123,597	$\sum_{j=n+1}^{p} \Delta_j^T f$	eq. (23)
Manager period effect	*period 1*	1.253	2,311,237	$\sum_{j=1}^{n} \Delta_j^T f_m$ $m = 1, \dots, n$	eq. (35)
	period 2	1.072	-767,528		
	period 3	-1.209	-3,548,730		
	period 4	-1.886	-13,579,810		
	period 5	1.838	1,497,819		
	period 6	1.086	9,737,001		
	period 7	2.340	-3,084,400		
	period 8	-0.673	-10,634,743		
Client period effect	*period 1*	0.000	0	$\sum_{j=n+1}^{p} \Delta_j^T f_m$ $m = 1, \dots, n$	eq. (35)
	period 2	-0.181	24,598		
	period 3	0.066	-49,500		
	period 4	0.570	-629,933		
	period 5	-0.701	-39,128		
	period 6	-0.190	-957,278		
	period 7	-1.518	521,951		
	period 8	0.599	2,252,887		
Period effect	$\Delta^T f_1$	1.253	2,311,237	$\Delta^T f_m = f^{(m)} - f^{(m-1)}$	eq. (26)
	$\Delta^T f_2$	0.891	-742,930		
	$\Delta^T f_3$	-1.143	-3,598,231		
	$\Delta^T f_4$	-1.316	-14,209,743		
	$\Delta^T f_5$	1.137	1,458,690	$\Delta^T f_m = \sum_{j=1}^{p} \Delta_j^T f_m$ $m = 1, \dots, n$	eq. (32)
	$\Delta^T f_6$	0.895	8,779,724		
	$\Delta^T f_7$	0.822	-2,562,449		
	$\Delta^T f_8$	-0.074	18,381,856		
Value Added		2.466	-16,945,558	$\sum_{j=1}^{p} \Delta_j^T f = \sum_{t=1}^{n} \Delta^T f_t$	eq. (27)

Figure 4: Summary of effects: Worked example and Anima Italia Fund

Appendix E: Summary, symbols, and abbreviations

Table 21: Summary of effects (general)

Effect	Meaning	Computation	Formula	Equation
Manager decision effect	Effect of the manager decisions made in period j on VA	Clean FCSI of α_j on VA, with $j = 1, \ldots, n$	$\Delta_j^T f = \Delta_j^1 f + \Delta_j^I f$, for $j = 1, \ldots, n$	(19)
Client decision effect	Effect of the client decisions made at time j on VA	Clean FCSI of α_j on VA, with $j = n+1, \ldots, 2n-1$	$\Delta_j^T f = \Delta_j^1 f + \Delta_j^I f$, for $j = n+1, \ldots, 2n-1$	(19)
Joint decision effect	Joint effect of the (manager and client) decisions made in period j on VA	Sum of manager decision effect and client decision effect	$\Delta_j^T f + \Delta_{n+j}^T f$, for $j = 1, \ldots, n$	(22)
Manager effect	Effect of all the manager decisions made in the interval $[0, n]$ on VA	Sum of all the manager decision effects	$\sum_{j=1}^{n} \Delta_j^T f$	(24)
Client effect	Effect of all the client decisions made in the interval $[0, n]$ on VA	Sum of all the client decision effects	$\sum_{j=n+1}^{2n-1} \Delta_j^T f$	(24)
Manager period effect	Effect of all the manager decisions made in the interval $[0, n]$ on the value created in period t	Sum of all the attribution values attributable to the manager in period t	$\sum_{j=1}^{n} \Delta_j^T f_m$, for $m = 1, \ldots, n$	(37)
Client period effect	Effect of all the client decisions made in the interval $[0, n]$ on the value created in period t	Sum of all the attribution values attributable to the client in period t	$\sum_{j=n+1}^{2n-1} \Delta_j^T f_m$, for $m = 1, \ldots, n$	(37)
Period effect	Effect of all the (manager and client) decisions made in the interval $[0, n]$ on the value created in period t	Sum of manager period effect and client period effect	$\Delta^T f_m = \mathrm{VA}^{(m)} - \mathrm{VA}^{(m-1)}$ $\Delta^T f_m = \sum_{j=1}^{2n-1} \Delta_j^T f_m$ for $m = 1, \ldots, n$	(27) (32)

Symbols and abbreviations

Symbol	Description
Section 2	
NPV	Net Present Value
n	Liquidation time
VA	Value Added of the financial investment
F_t	Cash flows into and out of the fund
E_t	End-of-period portfolio value at time t
B_t	Beginning-of-period portfolio value at time t
i_t	Rate of return of period t, i.e. the interval $[t-1, t]$
$F = (F_1, F_2, \ldots, F_{n-1})$	Vector collecting the intermediate cash flows
$i = (i_1, i_2, \ldots, i_n)$	Vector of the fund's holding period rates
$i^* = (i_1^*, i_2^*, \ldots, i_n^*)$	Vector of benchmark's holding period rates
Section 3	
$x = (x_1, x_2, \ldots, x_n)$	Vector of single-period rates
$y = (y_1, y_2, \ldots, y_{n-1})$	Vector of interim contributions and distributions
(x^0, y^0)	$(i^*, 0)$
(x^1, y^1)	(i, F)
$f(x, y)$	Value added of a generic investment
$f(i, F)$	Value added of the active investment
$f(i^*, 0)$	Value added of the passive (benchmark) investment
Section 4	
FCSI	Finite Change Sensitivity Index
p	Number of inputs (for an investment, $p = 2n - 1$)
$\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_p)$	Vector of inputs
$\alpha^0 = (\alpha_1^0, \alpha_2^0, \ldots, \alpha_p^0)$	Base value of inputs
$\alpha^1 = (\alpha_1^1, \alpha_2^1, \ldots, \alpha_p^1)$	Realized value of inputs
$f(\alpha^1) - f(\alpha^0)$	Output change when inputs change from α^0 to α^1
$(\alpha_j^1, \alpha_{(-j)}^0)$	Input vector with all the inputs set at their base value α^0, except α_j which is given the realized value α_j^1
$(\alpha_j^0, \alpha_{(-j)}^1)$	Input vector with all the inputs set at their realized value α^1, except α_j which is given the realized value α_j^0
$(\alpha_j^1, \alpha_k^1, \alpha_{(-j,k)}^0)$	Input vector where α_j and α_k are set to the realized values, while the remaining $p-2$ parameters are set at their base value
$\Delta_j^1 f$	First order FCSI of parameter α_j
$\Phi_j^1 f$	Normalized First order FCSI of parameter α_j
$\Delta_{j,k} f$	Interaction between α_j and α_k
$(\alpha_j^1, \alpha_k^1, \alpha_h^1, \alpha_{(-j,k,h)}^0)$	Input vector with α_j, α_k and α_h are set to the realized values, while the remaining $p-3$ parameters are set at their base value
$\Delta_{j,k,h} f$	Interaction between α_j, α_k and α_h
$\Delta_j^{\mathcal{T}} f$	(Borgonovo's) total order FCSI of parameter α_j
$\Delta_j^{\mathcal{I}} f$	(Borgonovo's) interaction effect of parameter α_j
$\Phi_j^{\mathcal{T}} f$	(Borgonovo's) normalized total order FCSI of parameter α_j
$\Delta_j^T f$	Clean total order FCSI of parameter α_j
$\Delta_j^I f$	Clean interaction effect of parameter α_j
$\Phi_j^T f$	Normalized Clean total order FCSI of parameter α_j
RI_m	Residual Income for period m

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Section 5	
$\alpha = (x, y)$	
$= (x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_{n-1})$	Vector of inputs
$\alpha^0 = (i^*, 0)$	
$= (i_1^*, i_2^*, \ldots, i_n^*, 0, 0, \ldots, 0)$	Passive investment policy in the benchmark with zero interim cash flows
$\alpha^1 = (i, F)$	
$= (i_1, i_2, \ldots, i_n, F_1, F_2, \ldots, F_{n-1})$	Active investment policy in the fund with nonzero interim cash flows
$\Delta_j^T f$	Decision effect (i.e. contribution of paramater α_j to VA)
$\Phi_j^T f = \Delta_j^T f/\mathrm{VA}$	Normalized decision effect
Section 6	
$F^{(m)} = (F_0, F_1, F_2, \ldots, F_{m-1}, E_m, 0, 0, \ldots, 0)$	Cash-flow stream of the investment truncated at time m
$\Delta F^{(m)} = F^{(m)} - F^{(m-1)}$	Incremental cash-flow stream if investment is continued from $m-1$ to m
$\mathrm{NPV}^{(m)}$	NPV of the truncated project at time m
$\mathrm{VA}^{(m)}$	Value added (at time $t = n$) of the project truncated at time m
$\Delta^T f_m = \mathrm{VA}^{(m)} - \mathrm{VA}^{(m-1)}$	Period effect (i.e. contribution of period m to VA)
$\Phi^T f_m = \Delta^T f_m/\mathrm{VA}$	Normalized period effect
Section 7	
AM	Attribution Matrix
$\Delta_j^T \mathrm{VA}^{(m)}$	Total effect of α_j on $\mathrm{VA}^{(m)}$ (clean FCSI of $\mathrm{VA}^{(m)}$)
$\Delta_j^T f_m = \Delta_j^T \mathrm{VA}^{(m)} - \Delta_j^T \mathrm{VA}^{(m-1)}$	Attribution value: Part of VA which is generated in period m, i.e. in the interval $[m-1, m]$, by parameter α_j
$\Phi_j^T f_m = \Delta_j^T f_m/\mathrm{VA}$	Normalized attribution value

The Split-Screen approach for project appraisal (Part II: Spreadsheet modelling)

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Work in progress

Ready to be submitted to *The Engineering Economist*

The *split-screen* approach for project appraisal (Part II: Spreadsheet modelling)

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ABSTRACT. This paper employs the newly conceived accounting-and-finance engineering system described in Magni (2020) and Magni (2022, The split-screen approach for project appraisal (Part I: The theory)), addressed to the analysis of capital asset investments. In this second part, we show how to implement this theoretical framework onto a spreadsheet software. We guide the analyst step by step, cell by cell, to the creation of the split-screen matrices describing the project film. Because the AFES is based on two arithmetic relations (law of motion and law of conservation), we can use a minimal approach to modeling, with a frugal use of the most common spreadsheet functions (essentially INDEX() and MATCH()) and no use of the traditional financial functions, yet fulfilling the requisite of clearness, transparency, consistency, and easiness-to-use. Starting from the informal description of the project, we build the model by breaking it down to 7 modules. The spreadsheet model is available online at http://morespace.unimore.it/carloalbertomagni/spreadsheet-modeling/.

INTRODUCTION

This paper is the second, applicative part of a work addressed to illustrate the accounting-and-finance engineering system (AFES) for project appraisal, introduced in Magni (2022). The first part of this work (Magni 2022, henceforth denoted as Part I) describes the theory underlying the AFES for project appraisal.[1] In this second part, we provide a complete translation of the theory into spreadsheet modeling for practical applications. The AFES is based on the *split-screen approach*, which integrates the static and the dynamic dimension of a project overturning the traditional models from three main points of view:

- the creation of pro forma financial statements derived from the model's assumptions is replaced by the so-called split-screen Matrix and split-screen film (displayed in different formats)

[1] We assume the reader is already familiar with Part I or with Magni (2020).

- the new notion of cash flow from (or to) liquid assets is employed (in addition to the classical operating cash flow, cash flow to debt, and cash flow to equity)
- the role of the payout and financing decisions is made explicit
- the project's economic profitability, usually based on cash flows and measured by net present value and internal rate of return, is replaced by a set of new, reciprocally consistent measures, which provide deeper information about the project's economic profitability
- the model is designed so as to act as a diagnostic tool for spotting internal inconsistencies.

These features make the AFES radically different from the approaches to project appraisal currently available in applied corporate finance or engineering economics. For this reason, the new approach requires that the association among accounting and financial magnitudes are remodeled; in other words, the AFES asks for a different-from-usual spreadsheet modeling. This paper aims at filling this gap and offers a detailed step-by-step guide to the AFES construction in spreadsheets, aiming at helping practitioners to employ it in real-life applications. Using a mockup project and following Part I's concepts, notions, formalizations, and algebraic relationships, we build the whole spreadsheet model for the project, starting from the verbal description of it and explaining it in full, cell by cell.[2]

Several organizations propose guidelines in financial modeling with spreadsheets (especially with Microsoft Excel. One of the most popular guidelines is the *FAST Standard* (FAST Standard Organization 2019), a set of rules on the building spreadsheet-based financial models, developed by a non-profit organization. Similarly, two guidelines are well known and freely available which proposed, in the previous decade, some principles and requisites: *Smart Financial Modeling* (Corality financial group, 2015) and *SSRB Best Practice Spreadsheet Modeling Standards* (Spreadsheet Standards Review Board 2016, no longer actively maintained), superseded by Modano Best Practice Modeling Guidelines (*SSRB standards* are no longer actively maintained). The three standards contain some similar principles and rules, which we have implemented in the methodology used in our workbook (described in this paper); in particular, we use separate worksheets for Inputs, Calculations and Outputs, avoid circular references and use concise and simple formula. The result is a spreadsheet model that fulfils the requisite for clear and concise communication and transparency. Indeed, using the rigorous-yet-simple structure of the AFES and implementing the worksheet according to the best practices, we establish a "spreadsheet engineering" discipline that, put it in Thorne's (2009) words, mitigates the problem of spreadsheet errors. Several criteria for spreadsheet accuracy are also applied (some suggestions are, for example, in Powell, Baker, and Lawson 2008) and, more specifically, the AFES articulation is based on the law of motion and law of conservation, which enables the analyst to spot modeling errors (e.g., if numbers in rows and columns do not match according to the law of motion and law of conservation, there is some inconsistencies in the calculation). This zeroes out possible internal inconsistencies.

Traditional financial modeling is based on a vast number of functions (see Avon 2021, Ch. 11 for a list of the most used ones. See also Benninga 2014). Owing to the AFES design, we propose a more frugal and user-friendly model, based as it is on very few spreadsheet functions, which are most common tools for modelers. Specifically, we use only four functions: the numeric function SUM(…), the reference functions INDEX(…) and MATCH(…), and the logical function IF(…); these functions are available in most spreadsheets (with small syntactic differences). We will also use the text string operator &[3] (and, last but not least, the analyst won't need any of the usual financial functions).

We accompany this paper with its associated Microsoft Excel file, named "CAD.xlsx", which is available at the following link: http://morespace.unimore.it/carloalbertomagni/spreadsheet-

[2] Some graphs are included in the spreadsheet file as well, conveying information in a visual form.
[3] In principle, the & operator is not even necessary if one uses homogeneous labels (see section 4).

modeling/). The spreadsheet file, according to best practices described in handbooks (e.g., Swan 2016) and in standard guidelines, has a structure based on separated sheets for inputs assumptions (inputs), calculations (working sheets), and presentation of the results (outputs).

The analyst must be equipped with a spreadsheet software such as Microsoft Excel, Apple Numbers, Google Sheets, LibreOffice, OpenOffice Calc, or equivalents (tools that any financial analyst knows and daily employs in its work).

To present the model, we assume the role of a financial analyst who oversees the economic analysis of a given capital asset investment. We follow the entire process of the analyst, starting from the collection of the pieces of information from the various experts (CFO, CEO, CTO, etc.).

The analyst makes explicit all the assumptions obtained from interviews to the firm's experts (CEO, CFO, CTO, marketing experts, etc.). For illustrative purposes, we model the following project, which has been inspired by an example presented in Magni (2020, Example 4.1):

CAD Inc. project

CAD Inc. faces the opportunity of undertaking a five-year project consisting in the production of a manufactured good. The initial investment in fixed assets is equal to $20,000, to be fully paid in year 0. The investment in fixed assets is depreciated with straight-line depreciation.
The sales projection for the first period (year 1) is 6,000 units of product and the sales growth is equal to 10% yearly. The sale price of each unit of the product is $10 for each year.
The unit manufacturing costs incurred are:
- Material used for production: $2.5,
- Direct and indirect labor: $4.

The production department requires that materials inventory at the end of each period should be 25% of the consumption estimated for the next period.
Selling, General & Administrative Expenses (SGA) are composed of:
- purchases of non-manufacturing materials (assumed to be equal to consumptions), estimated at 15% of sales and
- salaries and wages for non-manufacturing employees, estimated at $6,000 each year.

Finished goods inventory will be zero (i.e., production will equal sales)
The average number of days to fully collect the payment after a sale is 90. All the purchases of materials (both manufacturing and non-manufacturing) will be cashed out in the year after the one in which they are incurred (except for the last period, at the end of which the accounts payable will be $0); salaries and wages will be paid monthly.
The tax rate is τ = 30% and taxes are paid within the year in which they are incurred.
The capital expenditure of $20,000 in $t = 0$ is sustained by:
(i) an equity injection of $6,000,
(ii) a loan of $10,000 at an interest rate of i^d = 2.00%; the loan is reimbursed with 4 principal repayments of equal amount,
(iii) a withdrawal from liquid assets for the residual amount of $4,000.
The interest rate i^l on liquid asset is forecasted to be 3.80%.

CAD Inc. will pay out α = 20% of the net income to its shareholders in each period from 1 to 4.

The required returns on operating assets, on liquid assets and on debt are respectively:
r^o = 15.00% (pre-tax WACC),
r^l = 1.00%,
r^d = 3.00%.

We describe the development of the project model step by step, the creation of the benchmark obtained from the market input data, and the fabrication of the (absolute and relative) measures of economic profitability, which will lead to an accept or reject decision.

The model is divided into 7 modules (see Figure 1 and Table 1):

Module 1: Extracting the data of the project
Module 2: Building the project split-screen strip
Module 3: Building the strips of other matrix representations
Module 4: Building the benchmark split-screen strip
Module 5: Performing the single-period analysis
Module 6: Reshaping the project split-screen strip
(transposed Matrix and financial statements)
Module 7: Assessing value creation and financial efficiency

Starting from the verbal description of the project presented above (see also worksheet "Description"), Module 1 translates it into formal inputs and assumptions (worksheet-"Assumptions") and carries out the preliminary calculations (worksheet "PreCalc") that are necessary for Module 2 (worksheet "SplitScreenStrip"). The latter shows how to build the full-scale project film (split-screen strip). It first carves out the accounting magnitudes for period 0. The formulas for filling the cells in period 0 are in turn divided into (i) cells whose values are provided by the preliminary calculations, (ii) cells whose values are calculated via the law of motion, (iii) cells whose values are calculated with suitable accounting formulas, and (iv) cells whose values are calculated with the law of conservation. Cells of periods 1 through n are obtained by simply dragging right the fill handle. Module 3 starts from the full-scale strip and, in a top-down approach, builds other lower-scale Matrix representations which show different pieces of information and different classification of costs and cash flows, up to the germ strip, which condenses the capital invested in the project, C_t, the project's return, I_t, and the project's cash flow, F_t. Module 4 creates the benchmark split-screen strip (worksheet "SplitScreenStrip"), which is necessary for valuation and decision-making. In Module 5, we build a menu of split-screen Matrices for every period (worksheet "SinglePeriod") which favors period analysis. Module 6 shows how to pass from the split-screen film to the project transposed Matrix and benchmark transposed Matrix (worksheet "TransMatrix"). In the same module, we show how to pass from the project split-screen film to the standard pro forma financial statements (worksheet "FinancStat"). The object of Module 7 is to measure value creation (worksheet "ValueCreation"); in it, we derive the set of absolute measures (quantifying the economic value created by the project) and relative measures (quantifying the financial efficiency of the project) that are described in Part I. Some conclusions follow, and a legend of symbols and abbreviations used in this paper is located at the end of the paper, for the benefit of the reader.

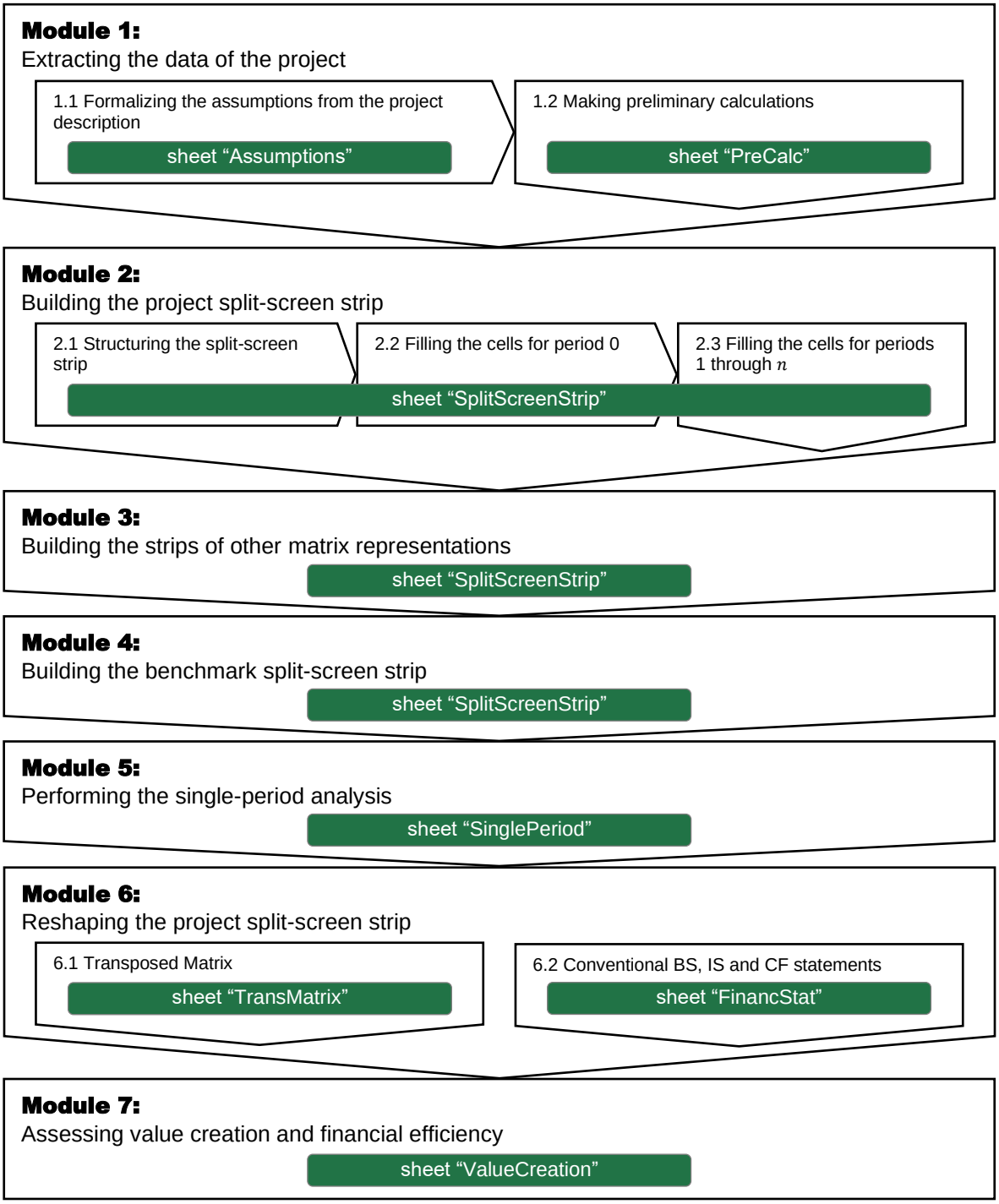


Figure 1. Structure of the paper and of the spreadsheet file.

Table 1. Structure of the paper and of the spreadsheet file.

MODULE	WORKSHEET	SECTION NUMBER	SECTION TITLE
1	Assumptions PrelCalc	1	Extracting the data of the project
2	SplitScreenStrip	2	Building the project split-screen strip
3	SplitScreenStrip	3	Building the strips of other matrix representations
4	SplitScreenStrip	4	Building the benchmark split-screen strip
5	SinglePeriod	5	Performing the single-period analysis
6	TransMatrix FinancStat	6	Reshaping the project split-screen strip
7	Value Creation	7	Assessing value creation and financial efficiency

1 MODULE 1: EXTRACTING THE DATA OF THE PROJECT

1.1 Formalizing the assumptions from the project description

In Module 1, the analyst identifies and formalizes the assumptions and, hence, makes some preliminary calculations, which are necessary for carrying out Module 2. The analyst retrieves all pieces of information regarding the project by the firm's CEO, the CFO, the CTO, and the marketing experts and then need convert them from natural language to figures, in order to use them in a financial model that fully describes the project and the relationships between the economic and financial values. More precisely, the analyst must carve out the figures and the relationships among all the inputs and the resulting accounting and financial magnitudes. As testified by the description above (and explained in detail in Part I), all pieces of information can be classified as capital, income, and cash flow. Also, some of the inputs involve estimation regarding the project (costs, prices, quantities sold, etc.) or the market (required returns on assets, on debt, etc.), some other involve embedded decisions (e.g., the payout policy). The objective of the modeler is to derive, for each magnitude, the capital component, the income, component, and the cash-flow component and, hence, build the pro forma financial statements (balance sheet, BS, income statement, IS, cash-flow statement, CFS) for each period. In other words, the modeler must derive all the formal links among the variables at play.

As a first step of Module 1, the analyst extracts the input variables of the model and the corresponding assumptions from the description conveyed in the natural language. We classify them in project inputs and market inputs; the project inputs are in turn divided into estimation variables and decision (payout) variable.

Project inputs

Estimation variables

Assumption 01	a five-year project
Assumption 02	investment in fixed assets is equal to $20,000, to be fully paid in year 0
Assumption 03	the investment in fixed assets is depreciated with straight-line depreciation
Assumption 04	the sales projection for the first period (year 1) is 6,000 units of product
Assumption 05	the sales growth is equal to 10% yearly
Assumption 06	the sale price of each unit of the product is $10 for each year
Assumption 07	the unit manufacturing costs incurred are: material used for production: $2.5
Assumption 08	the unit manufacturing costs incurred are: direct and indirect labor: $4
Assumption 09	the production department requires that materials inventory at the end of each period should be 25% of the consumption estimated for the next period
Assumption 10	purchases of non-manufacturing materials (assumed to be equal to consumptions), estimated at 15% of sales
Assumption 11	salaries and wages for non-manufacturing employees, estimated at $6,000 each year
Assumption 12	the average number of days to fully collect the payment after a sale is 90
Assumption 13	all the purchases of materials (both manufacturing and non-manufacturing) will be cashed out in the year after the one in which they are incurred (except for the last period, at the end of which the accounts payable are $0)
Assumption 14	salaries and wages will be paid monthly
Assumption 15	the tax rate is τ = 30%
Assumption 16	taxes are paid within the year in which they are incurred
Assumption 17	the capital expenditure in $t = 0$ is sustained by an equity injection of $6,000
Assumption 21	the interest rate i^l on liquid asset is forecasted to be 3.80%

Decision variables

Assumption 18	the capital expenditure in $t = 0$ is sustained by a loan of $10,000
Assumption 19	at an interest rate of i^d = 2.00%
Assumption 20	the loan is reimbursed with 4 principal repayments of equal amount
Assumption 22	CAD Inc. will pay out α=20% of the net income to its shareholders in each period from 1 to 4 and in the last period it will distribute the total amount of the equity

Market inputs

Assumption 23	r^o = 15.00% (pre-tax WACC)
Assumption 24	r^l = 1.00%
Assumption 25	r^d = 3.00%

The description of the project in natural language provides the information needed for populating the split-screen Matrix.

Spreadsheet Modeling: Formalizing assumptions

In a spreadsheet program, the analyst creates a workbook and, in it, adds a first sheet called "Assumptions", where he collects the assumptions.
The analyst can fill the cells of the assumptions-table with the figures extracted from the assumptions. The cells with yellow background are the ones that contain the inputs of the model. Each input value is contained only in one cell, in this sheet: every time it is used in the calculations, the formulas are

linked to this cell. So, the analyst can simulate different scenarios changing only the contents of the sheet “Assumptions”.

	A	B	C	D	E	F	G	H	I
2	assumption						**INPUTS**	**assumptions**	**UoM**
5	01					n	Project length	5	y
6	02				**Fixed Assets**	NFA_0	Investment	20,000	$
7	03						Depreciation	*straight-line*	
8	04				**Sales**	S_0	Sales projection of the first period	6,000	u
9	05						Growth rate (%/year)	10%	% / y
10	06						Unit Price	10	$ / u
11	07				**Manufacturing Costs (COGS)**	COP^m	Unit Cost of Purchases	2.5	$ / u
12	08					LC^m	Unit Cost of Labor	4.0	$ / u
13	09						Raw Materials Inventory (% of next-period consumption)	25%	%
14	10				**NonManufacturing Costs (SGA)**		Cost of purchases (SGA) (% of Sales)	15%	%
15	11					LC^{nm}	Labor costs (SGA)	6,000	$ / y
16	12				**Operating cycle**		DSO (Days Sales Outstanding)	90	dd
17	13						DPO (Days Payable Outstanding)	365	dd
18	14				**Pay period**		Salaries and Wages will be paid	*monthly*	
19	15				**Taxes**	τ	Tax Rate	30%	%
20	16						Taxes are paid	*yearly*	
21	17				**Equity financing**	$C^e{}_0$	Capital Payment (period 0)	6,000	$
22	18				**Debt financing**	$C^d{}_0$	Loan amount	10,000	$
23	19					i^d	Loan Interest Rate	2.00%	%
24	20						Loan reimbursement periods	4	y
25	21				**Liquid Assets**	i^l	Interest rate on liquid assets	3.80%	%
26	22				**Payout policy**	α	Payout ratio (% Net Income)	20%	%
27	23				**Market data**	r^o	pre-tax WACC	15.00%	%
28	24					r^l	required return on liquid assets	1.00%	%
29	25					r^d	required return on debt	3.00%	%
30									

Figure 2. Inputs and assumptions (sheet “Assumptions”)

1.2 Making preliminary calculations

The objective of the analyst is to feed the Full-scale Matrix (Part I, eq. (33)) and build the whole strip of split-screen Matrices (see Part I eq. (66)). However, to fill in the cells of each Matrix, preliminary calculations are needed, which make explicit, period by period, the figures already implicit in the assumptions.

$\mathbf{C}_t$	$\mathbf{C}_{t-1}$	$\mathbf{I}_t$	$-\mathbf{F}_t$
$+\mathrm{AR}_t$	$+\mathrm{AR}_{t-1}$	$+\mathrm{S}_t$	$-F_t^{ar}$
$+\mathrm{Inv}_t$	$+\mathrm{Inv}_{t-1}$	$+\Delta\mathrm{Inv}_t$	-0
$-\mathrm{AP}_t^m$	$-\mathrm{AP}_{t-1}^m$	$-\mathrm{COP}_t^m$	$+F_t^{ap,m}$
$-\mathrm{AP}_t^{nm}$	$-\mathrm{AP}_{t-1}^{nm}$	$-\mathrm{COP}_t^{nm}$	$+F_t^{ap,nm}$
$-\mathrm{SWP}_t^m$	$-\mathrm{SWP}_{t-1}^m$	$-\mathrm{LC}_t^m$	$+F_t^{swp,m}$
$-\mathrm{SWP}_t^{nm}$	$-\mathrm{SWP}_{t-1}^{nm}$	$-\mathrm{LC}_t^{nm}$	$+F_t^{swp,nm}$
$+\mathrm{NFA}_t$	$+\mathrm{NFA}_{t-1}$	$-\mathrm{Dep}_t$	$-F_t^{nfa}$
$-\mathrm{TP}_t$	$-\mathrm{TP}_{t-1}$	$-\mathrm{T}_t$	$+F_t^{tp}$
$+C_t^l$	$+C_{t-1}^l$	$+I_t^l$	$-F_t^l$
$+C_t^d$	$+C_{t-1}^d$	$+I_t^d$	$-F_t^d$
$+C_t^e$	$+C_{t-1}^e$	$+I_t^e$	$-F_t^e$

Figure 3. Full-scale Matrix (compact form. See also Part I, eq. (33))

More precisely, we highlight in yellow the cells in the Full-scale Matrix whose values may be obtained from the assumptions via some simple calculations (see Figure 3). We describe these calculations in this section. The remaining (not highlighted) cells contain amounts that will be calculated in Module 2, as we will see, by making use of the law of motion or the law of conservation or specific accounting formulas.

Fixed Assets

Assumption 02 regarding the NFA provides the following figures:

Period	**0**	**1**	**2**	**3**	**4**	**5**
$-F_t^{nfa}$	20,000	0	0	0	0	0

Assumption 03 enables calculating the (straight-line) depreciation:

$$\mathrm{Dep}_t = \frac{\mathrm{NFA}_0}{n} = \frac{20{,}000}{5} = 4{,}000$$

Period	**0**	**1**	**2**	**3**	**4**	**5**
$-\mathrm{Dep}_t$	0	–4,000	–4,000	–4,000	–4,000	–4,000

Sales

Assumptions 04, 05, and 06 enable the analyst to calculate the sales for each period:

$$\text{Units sold in } t \;=\; (1.1) \cdot \text{Units sold in } t-1$$
$$\mathrm{S}_t = \text{Units sold in } t \cdot \text{unit sale price}$$

Units (produced and) sold in the first year = 6,000
Annual sales growth = 10%
Unit sale price = $10

Period	**0**	**1**	**2**	**3**	**4**	**5**
Units sold in t	0	6,000	6,600	7,260	7,986	8,785
$+\mathrm{S}_t$	0	60,000	66,000	72,600	79,860	87,850

Manufacturing Costs

Assumption 07 enables calculating the material (direct material and overhead) used for production:

$$\text{Material used in } t = \text{units sold in } t \cdot \text{unit cost}$$

Unit cost = $2.5

Period	**0**	**1**	**2**	**3**	**4**	**5**
– Material used in *t*	0	–15,000	–16,500	–18,150	–19,965	–21,963

Assumption 09 enables calculating the raw materials inventory:

$$\mathrm{Inv}_t = (\text{Material used in } t+1) \cdot 25\%$$

Period	**0**	**1**	**2**	**3**	**4**	**5**
$+\mathrm{Inv}_t$	3,750	4,125	4,538	4,991	5,491	0

The Cost of Purchases for the manufacturing activities (COP^m) is the sum of the Material used for the production and the change in the raw materials inventory:[4]

[4] The COGS are computed as $\mathrm{COGS}_t = \mathrm{COP}_t^m + \mathrm{LC}_t^m - \Delta\mathrm{Inv}_t$. However, the relation between material used and manufacturing COP is as follows: Material used in $t = \mathrm{COP}_t^m - \Delta\mathrm{Inv}_t$. Material used in t is an estimated figure, based on the projection of sales (and, therefore, of units produced): Material used in t = units sold in t · unit cost. Hence, COP_t^m = units sold in t · unit cost $+\Delta\mathrm{Inv}_t$ and we can also write

$$\mathrm{COGS}_t = \text{units sold in } t \;\cdot\; \text{unit cost} + \mathrm{LC}_t^m.$$

The latter equality may be used for checking consistency of COGS calculation in the model.

$$\mathrm{COP}_t^m = \text{Material used in } t + \Delta\mathrm{Inv}_t$$

Period	0	1	2	3	4	5
$-\mathrm{COP}_t^m$	–3,750	–15,375	–16,913	–18,604	–20,464	–16,472

Assumption 08 enables calculating the labor costs for manufacturing activities (LC^m):

$$\mathrm{LC}^m = \text{units sold} \cdot \text{unit labor cost}$$

Unit labor cost = $4

Period	0	1	2	3	4	5
$-\mathrm{LC}_t^m$	0	–24,000	–26,400	–29,040	–31,944	–35,140

NonManufacturing Costs (SGA)

Assumption 10 enables calculating the cost of purchases for the nonmanufacturing activities.

$$\mathrm{COP}_t^{nm} = S_t \cdot 15\%$$

Period	0	1	2	3	4	5
$-\mathrm{COP}_t^{nm}$	0	–9,000	–9,900	–10,890	–11,979	–13,178

Assumption 11 enables calculating the cost of labor for the nonmanufacturing activities:

Period	0	1	2	3	4	5
$-\mathrm{LC}_t^{nm}$	0	–6,000	–6,000	–6,000	–6,000	–6,000

Operating cycle

The next set of information extracted from the interviews to the various firm's experts, provides the information about the Operating Cycle of the project. Assumption 12 enables computing the Account Receivables:

$$\mathrm{AR}_t = \mathrm{S}_t \cdot \frac{90}{365}$$

Period	0	1	2	3	4	5
$+S_t$	0	60,000	66,000	72,600	79,860	87,850
$+\mathrm{AR}_t$	0	14,795	16,274	17,901	19,692	0

Assumption 13 enables calculating the Account Payables:

$$\mathrm{AP}_t^m = \mathrm{COP}_t^m \quad \text{and} \quad \mathrm{AP}_t^{nm} = \mathrm{COP}_t^{nm}$$

Period	0	1	2	3	4	5
COP_t^m	–3,750	–15,375	–16,913	–18,604	–20,464	–16,472
AP_t^m	–3,750	–15,375	–16,913	–18,604	–20,464	0

Period	0	1	2	3	4	5
$-\mathrm{COP}_t^{nm}$	0	–9,000	–9,900	–10,890	–11,979	–13,178
AP_t^{nm}	0	–9,000	–9,900	–10,890	–11,979	0

Payroll period

Assumption 14 enables calculating the payments to employees:

$$\mathrm{LC}_t^m = F_t^{swp,m} \quad \text{and} \quad \mathrm{LC}_t^{nm} = F_t^{swp,nm}$$

Period	0	1	2	3	4	5
LC_t^m	0	–24,000	–26,400	–29,040	–31,944	–35,140
$+F_t^{swp,m}$	0	24,000	26,400	29,040	31,944	35,140

Period	0	1	2	3	4	5
LC_t^{nm}	0	–6,000	–6,000	–6,000	–6,000	–6,000
$+F_t^{swp,nm}$	0	6,000	6,000	6,000	6,000	6,000

Taxes

Assumption 16 enables calculating the Taxes Payables, which will be 0 in each period:

$$\mathrm{T}_t = F_t^{tp} \Longrightarrow \mathrm{TP}_t = 0$$

Period	0	1	2	3	4	5
$-\mathrm{TP}_t$	0	0	0	0	0	0

Debt financing

Assumptions 18 enables calculating the debt's principal repayment and the principal outstanding:

$$\text{principal repayment} = C_{t-1}^d - C_t^d = \frac{C_0^d}{4} = \frac{10{,}000}{4} = 2{,}500$$

whence

$$C_t^d = C_{t-1}^d - 2{,}500$$

Period	0	1	2	3	4	5
$+C_t^d$	10,000	7,500	5,000	2,500	0	0

Spreadsheet modelling: Preliminary Calculations

The analyst adds, in his workbook, a new sheet, named "PreCalc", where he calculates the figures obtained from the assumptions.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
2											0	1	2	3	4	5
4	assumption			C	I	F	PRELIMINARY CALCULATIONS	const	UoM		2022	2023	2024	2025	2026	2027
5																
18							Fixed Assets									
19																
20	02						Investment	20,000	$							
21							Investment period flag		flag		1	-	-	-	-	-
22						$-F^{ab}$	– Asset disposal (net of capital expenditures)		$		20,000	-	-	-	-	-
23																
24	02						Investment	20,000	$							
25	01						Project length n	5	Y							
26							Period depreciation	4,000	$							
27																
28	03						Depreciation	straight-line								
29							Period depreciation	4,000	$							
30							Project period flag		flag		-	1	1	1	1	1
31					− Dep		– Depreciation		$		-	- 4,000	- 4,000	- 4,000	- 4,000	- 4,000
32																
33																
34							Sales									
35																
36	04						Sales projection of the first period	6,000	u							
37	05						sales growth (%/year)	10%	% / year							
38							Project period flag		flag		-	1	1	1	1	1
39							**Sales (units of product)**		**u**		**-**	**6,000**	**6,600**	**7,260**	**7,986**	**8,785**
40																
41							Sales (units of product)		u		-	6,000	6,600	7,260	7,986	8,785
42	06						Unit Price	10	$ / u							
43					+ S		+ Sales		$		-	60,000	66,000	72,600	79,860	87,850
44																
45																
46							Manufacturing Costs (COGS)									
47																
48							Sales (units of product)		u		-	6,000	6,600	7,260	7,986	8,785
49	07						Unit Cost of Purchases	2.5	$ / u							
50							– Consumption of Raw Materials		$		-	- 15,000	- 16,500	- 18,150	- 19,965	- 21,963
51																
52							– Consumption of Raw Materials		$		-	- 15,000	- 16,500	- 18,150	- 19,965	- 21,963
53	09						Raw Materials Inventory (% of next-period consumptio	25%	%							
54				+ Inv			+ Inventory		$		3,750	4,125	4,538	4,991	5,491	-
55																
56							+ Inventory		$		3,750	4,125	4,538	4,991	5,491	-
57							– Δ inventory		$		- 3,750	- 375	- 413	- 454	- 499	5,491
58																

Figure 4. Examples of preliminary calculations (sheet "PreCalc")

Each column represents one period, and the rows contain the values taken on by the various variables. Columns A:G contain descriptions related to the variables; column H contains the constant values (i.e. the values that are independent of the period); column I contains the unit of measurement of the

variables; column K and the n columns on the right (L, M, N…) contain the amounts of periods 0 through n (last period of the project).
Each row has a label in column G: this label contains the description of the variable. Columns D, E, and F report the variables that are expected to be recorded (with proper sign) in the split-screen Matrix, classified, respectively, as:

capital (AR, Inv, $-\mathrm{AP}^{m}$, $-\mathrm{AP}^{nm}$, $-\mathrm{TP}$, C^{d}),
income (S, $-\mathrm{COP}^{m}$, $-\mathrm{COP}^{nm}$, $-\mathrm{LC}^{m}$, $-\mathrm{LC}^{nm}$, $-\mathrm{Dep}$),
cash flow ($F^{swp,m}$, $F^{swp,nm}$, $-F^{nfa}$).

The amounts in the rows with a label in columns D, E, or F, are the ones that will be linked in the split-screen Matrix in the next steps.
We use different font colors for the cells that are linked to other cells:

- blue for the rows that are linked to rows in the previous sheet "Assumptions"
- grey for the rows that are linked to other rows in the same sheet
- red for the cells that will be used for building the split-screen Matrix and will be linked to cells in sheet "SplitScreenStrip".

2 MODULE 2: BUILDING THE PROJECT SPLIT-SCREEN STRIP

2.1 Structuring the split-screen strip

As seen in Part I, four main areas are involved in a project: operating assets, non-operating (i.e., liquid) asset, debt capital, and equity capital. The law of motion holds for each of the four classes and may be represented with the four-area Matrix for each one of the n periods, giving rise to a split-screen film or split-screen strip.

$$
\begin{array}{c}
t=1 \\
\left[\begin{array}{ccc|c}
C_0^o & +I_1^o & -F_1^o & C_1^o \\
C_0^l & +I_1^l & -F_1^l & C_1^l \\
\hline
C_0^d & +I_1^d & -F_1^d & C_1^d \\
C_0^e & +I_1^e & -F_1^e & C_1^e
\end{array}\right]
\end{array}
\quad
\begin{array}{c}
t=2 \\
\left[\begin{array}{ccc|c}
C_1^o & +I_2^o & -F_2^o & C_2^o \\
C_1^l & +I_2^l & -F_2^l & C_2^l \\
\hline
C_1^d & +I_2^d & -F_2^d & C_2^d \\
C_1^e & +I_2^e & -F_2^e & C_2^e
\end{array}\right]
\end{array}
\quad \ldots \quad
\begin{array}{c}
t=n \\
\left[\begin{array}{ccc|c}
C_{n-1}^o & +I_n^o & -F_n^o & C_n^o \\
C_{n-1}^l & +I_n^l & -F_n^l & C_n^l \\
\hline
C_{n-1}^d & +I_n^d & -F_n^d & C_n^d \\
C_{n-1}^e & +I_n^e & -F_n^e & C_n^e
\end{array}\right]
\end{array}
$$

Figure 5. The split-screen film or strip (discrete format) (see Part I, eq. (64))

In Figure 5, the project film is displayed in a discrete format (sequence of matrices). Merging the last and first column of any two consecutive matrices, the project film may be reframed in a continuous format (Figure 6):[5]

[5] The vertical bar in Figure 6. The split-screen film or strip (continuous format) (see Part I, eq. (65)) is now in Indian Red and its meaning changes as follows: for each row, the sum of the three elements preceding the bar ($C_{t-1} + I_t - F_t$) is equal to the element following the bar (C_t).

$$
\begin{array}{|ccc|ccc|ccc|c|}
\hline
C_0^o & +I_1^o & -F_1^o & C_1^o & +I_2^o & -F_2^o & C_2^o & +I_3^o & -F_3^o & C_3^o \\
C_0^l & +I_1^l & -F_1^l & C_1^l & +I_2^l & -F_2^l & C_2^l & +I_3^l & -F_3^l & C_3^l \\
\hline
C_0^d & +I_1^d & -F_1^d & C_1^d & +I_2^d & -F_2^d & C_2^d & +I_3^d & -F_3^d & C_3^d \\
C_0^e & +I_1^e & -F_1^e & C_1^e & +I_2^e & -F_2^e & C_2^e & +I_3^e & -F_3^e & C_3^e \\
\hline
\end{array}
\cdots
\begin{array}{|cc|c|}
\hline
+I_n^o & -F_n^o & 0 \\
+I_n^l & -F_n^l & 0 \\
\hline
+I_n^d & -F_n^d & 0 \\
+I_n^e & -F_n^e & 0 \\
\hline
\end{array}
$$

Figure 6. The split-screen film or strip (continuous format) (see Part I, eq. (65))

This compact representation of the film of split-screen Matrices in a continuous format is particularly convenient for spreadsheet modeling. The assumptions and the preliminary calculations enable to build the project film in full scale, that is, the strip of the n full-scale split-screen Matrices (Figure 7).

$$
\begin{array}{|ccc|ccc|c}
\hline
+\mathrm{AR}_0 & +\mathrm{S}_1 & -F_1^{ar} & +\mathrm{AR}_1 & +\mathrm{S}_2 & -F_2^{ar} & +\mathrm{AR}_2 \\
+\mathrm{Inv}_0 & +\Delta\mathrm{Inv}_1 & +0 & +\mathrm{Inv}_1 & +\Delta\mathrm{Inv}_2 & +0 & +\mathrm{Inv}_2 \\
-\mathrm{AP}_0^m & -\mathrm{COP}_1^m & +F_1^{ap,m} & +\mathrm{AP}_1^m & -\mathrm{COP}_2^m & +F_2^{ap,m} & +\mathrm{AP}_2^m \\
-\mathrm{AP}_0^{nm} & -\mathrm{COP}_1^{nm} & +F_1^{ap,nm} & -\mathrm{AP}_1^{nm} & -\mathrm{COP}_2^{nm} & +F_2^{ap,nm} & -\mathrm{AP}_2^{nm} \\
-\mathrm{SWP}_0^m & -\mathrm{LC}_1^m & +F_1^{swp,m} & -\mathrm{SWP}_1^m & -\mathrm{LC}_2^m & +F_2^{swp,m} & -\mathrm{SWP}_2^m \\
-\mathrm{SWP}_0^{nm} & -\mathrm{LC}_1^{nm} & +F_1^{swp,nm} & -\mathrm{SWP}_1^{nm} & -\mathrm{LC}_2^{nm} & +F_2^{swp,nm} & -\mathrm{SWP}_2^{nm} \\
+\mathrm{NFA}_0 & -\mathrm{Dep}_1 & -F_1^{nfa} & +\mathrm{NFA}_1 & -\mathrm{Dep}_2 & -F_2^{nfa} & +\mathrm{NFA}_2 \\
-\mathrm{TP}_0 & -\mathrm{T}_1 & +F_1^{tp} & -\mathrm{TP}_1 & -\mathrm{T}_2 & +F_2^{tp} & -\mathrm{TP}_2 \\
+C_0^l & +I_1^l & -F_1^l & +C_1^l & +I_2^l & -F_2^l & +C_2^l \\
\hline
+C_0^d & +I_1^d & -F_1^d & C_1^d & +I_2^d & -F_2^d & +C_2^d \\
+C_0^e & +I_1^e & -F_1^e & C_1^e & +I_2^e & -F_2^e & +C_2^e \\
\hline
\end{array}
\cdots
\begin{array}{|cc|c}
\hline
+\mathrm{S}_n & -F_n^{ar} & 0 \\
+\Delta\mathrm{Inv}_n & +0 & 0 \\
-\mathrm{COP}_n^m & +F_n^{ap,m} & 0 \\
-\mathrm{COP}_n^{nm} & +F_n^{ap,nm} & 0 \\
-\mathrm{LC}_n^m & +F_n^{swp,m} & 0 \\
-\mathrm{LC}_n^{nm} & +F_n^{swp,nm} & 0 \\
-\mathrm{Dep}_n & -F_n^{nfa} & 0 \\
-\mathrm{T}_n & +F_n^{tp} & 0 \\
+I_n^l & -F_n^l & 0 \\
\hline
+I_n^d & -F_n^d & 0 \\
+I_n^e & -F_n^e & 0 \\
\hline
\end{array}
$$

Figure 7. The full-scale strip (see Part I, eq. (66))

This structure fully describes the project from $t = 0$ to $t = n$. The description of the project may require the analyst to add values for $t = -1$. In this case, the strip starts with the capital component in $t = -1$ (See Figure 8. See also Part I, eq. (67)).

Spreadsheet Modeling: Structure of the split-screen strip

The analyst adds, in his workbook, a new sheet, named “SplitScreenStrip”, where he lays out a range of cells with the structure of the previous table.

8						C	I	F	C	I	F	C	I	F	C	I	F	C	I	F	C	I	F	C
9						-1	0	0	0	1	1	1	2	2	2	3	3	3	4	4	4	5	5	5
12		C	I	–F	C_{-1}	$+I_0$	$-F_0$	C_0	$+I_1$	$-F_1$	C_1	$+I_2$	$-F_2$	C_2	$+I_3$	$-F_3$	C_3	$+I_4$	$-F_4$	C_4	$+I_5$	$-F_5$	C_5
13		+ AR	+ S	$-F^{ar}$	0																			
14		+ Inv	+ ΔInv	− 0	0																			
15		$-AP^{m}$	$-COP^{m}$	$+F^{ap,m}$	0																			
16		$-AP^{nm}$	$-COP^{nm}$	$+F^{ap,nm}$	0																			
17		$-SWP^{m}$	$-LC^{m}$	$+F^{swp,m}$	0																			
18		$-SWP^{nm}$	$-LC^{nm}$	$+F^{swp,nm}$	0																			
19		+ NFA	− Dep	$-F^{nfa}$	0																			
20		− TP	− T	$+F^{tp}$	0																			
21		$+C^{l}$	$+I^{l}$	$-F^{l}$	0																			
22		$+C^{d}$	$+I^{d}$	$-F^{d}$	0																			
23		$+C^{e}$	$+I^{e}$	$-F^{e}$	0																			

Figure 8. Structure of the full-scale strip in the spreadsheet (sheet "SplitScreenStrip")

The table has a first group of columns representing a generic full-scale split-screen Matrix: column D, E and F report, respectively, the capital amounts, the incomes, and the cash flows.[6] The following columns (from column G on) will contain the figures of the project, starting from period $t = -1$, before the beginning of the project. The total number of columns containing numerical values is $(3 \cdot (n + 1) + 1)$. Each value is 0 in column G since the project has not started yet ($C_{-1} = 0$).

Row 8 describes the kind of column (C, I or F) and row 9 contains the periods. Row 12 contains the labels of the columns, with specification of the category to which they belong (C, I or F) and of the period $(-1,0, \dots, n)$. Each cell is identified by the labels. For example, cell K13 will contain the sales (S) of period 1; cell M13 will contain the Accounts Receivable (AR) of period 1 (see Figure 9).

8						C	I	F	C	I	F	C	I	F	C	I	F	C	I	F	C	I	F	C
9						-1	0	0	0	1	1	1	2	2	2	3	3	3	4	4	4	5	5	5
12		C	I	–F	C_{-1}	$+I_0$	$-F_0$	C_0	$+I_1$	$-F_1$	C_1	$+I_2$	$-F_2$	C_2	$+I_3$	$-F_3$	C_3	$+I_4$	$-F_4$	C_4	$+I_5$	$-F_5$	C_5
13		+ AR	+ S	$-F^{ar}$	0																			
14		+ Inv	+ ΔInv	− 0	0																			
15		$-AP^{m}$	$-COP^{m}$	$+F^{ap,m}$	0																			
16		$-AP^{nm}$	$-COP^{nm}$	$+F^{ap,nm}$	0																			
17		$-SWP^{m}$	$-LC^{m}$	$+F^{swp,m}$	0																			
18		$-SWP^{nm}$	$-LC^{nm}$	$+F^{swp,nm}$	0																			
19		+ NFA	− Dep	$-F^{nfa}$	0																			
20		− TP	− T	$+F^{tp}$	0																			
21		$+C^{l}$	$+I^{l}$	$-F^{l}$	0																			
22		$+C^{d}$	$+I^{d}$	$-F^{d}$	0																			
23		$+C^{e}$	$+I^{e}$	$-F^{e}$	0																			

Figure 9. Cell M13 will contain the Accounts Receivable at t=1 (sheet "SplitScreenStrip")

To build the whole split-screen film, one must follow two steps: in the first step, the analyst fills in the cells of array H13:J23, which contain the values of all the accounting and financial magnitudes that refer to period 0. These cells may be divided into four classes:

- Cells whose values are provided in "PreCalc"
- Cells whose values are calculated with the law of motion
- Cells whose values are calculated with suitable formulas
- Cells whose values are calculated with the law of conservation

[6] We note, again, that column D describes a statement of capital amounts (equality of investments and financings), column E describes a statement of incomes (equality of income from investments and income to capital provider), column F describes a statement of cash flows (equality of cash flow from investments and cash flow distributed to capital providers).

We describe the calculations for all these cells in sections 2.2.

In the second step, the analyst fills the whole strip by copying the range H13:J23, which refer to the remaining periods (periods 1 to n). This is done in few seconds by copying and pasting it in the adjacent ranges, or by dragging the fill handle (we show the procedure in section 2.3).

2.2 Filling the cells for period 0

2.2.1 Cells whose values are provided in "PreCalc"

Preliminary calculations feed into the split-screen strip. To fill the cells of array H13:J23, the analyst should use nested functions INDEX and MATCH.[7] This function is necessary because it makes the formulas parameterized and dependent on the cell where they are (using the references to the row and column labels): it will enable the analyst to drag and fill the whole strip, thereby completing the project film in few seconds (see section 2.3).
In the spreadsheet-modeling box below, we first provide the code for the general formula and then show how to use it for each one of the three economic elements: capital, income, and cash flow.
(For those who are not familiar with spreadsheet formulas, we suggest reading the online support pages of the software employed.) There are 15 cells to be filled (highlighted with yellow background in Figure 3): 6 of them are capital amounts, 6 are incomes, and 3 are cash flows (see Table 2 and Figure 12).

Table 2. Cells whose values are provided in "PreCalc"

Cell	Symbol	Magnitude	Type
H13	$+\mathrm{S}_0$	Sales	Income
J13	$+\mathrm{AR}_0$	Accounts Receivable	Capital
J14	$+\mathrm{Inv}_0$	Inventory	Capital
H15	$-\mathrm{COP}_0^m$	Cost of Purchases (manufacturing)	Income
J15	$-\mathrm{AP}_0^m$	Accounts Payable (manufacturing)	Capital
H16	$-\mathrm{COP}_0^{nm}$	Cost of Purchases (nonmanufacturing)	Income
J16	$-\mathrm{AP}_0^{mn}$	Accounts Payable (nonmanufacturing)	Capital
H17	$-\mathrm{LC}_0^m$	Labor costs (manufacturing)	Income
I17	$+F_0^{swp,m}$	Payments to employees (manufacturing)	Cash flow
H18	$-\mathrm{LC}_0^{nm}$	Labor costs (nonmanufacturing)	Income
I18	$+F_0^{swp,nm}$	Payments to employees (nonmanufacturing)	Cash flow
H19	$-\mathrm{Dep}_0$	Depreciation	Income
I19	$-F_0^{nfa}$	Asset disposals (net of capital expenditures)	Cash flow
J20	$-\mathrm{TP}_0$	Taxes Payable	Capital
J22	$+C_0^d$	Debt	Capital

[7] Those unfamiliar with these functions may turn to Avon (2021. Ch. 11) or to any guide to spreadsheets.

Spreadsheet Modeling: Cells whose values are provided in "PreCalc"

The analyst must pick the figures from the sheet "PreCalc" and insert them into the split-screen strip. For this operation, the analyst must use the nested formulas INDEX-MATCH.

The INDEX function

$$\text{INDEX}(\text{array}, \text{row\_num}, \text{column\_num})$$

returns the value in the cell of the array at the intersection of row_num and column_num.
The MATCH function

$$\text{MATCH}(\text{lookup\_value}, \text{lookup\_array}, \text{match\_type})$$

returns the position of the matched value within lookup_array, finding the first value that is exactly equal to lookup_value, if the optional argument is 0.

Hence, the combined formula is

$$= \text{INDEX}(\ \text{PreCalc!}\ \$\text{K}{:}\ \$\text{P}, \text{MATCH}(\text{lookup\_value}, \text{lookup\_array}, 0\ ), \text{column\_num})$$

returns a value within the range PreCalc!K:P, that is, the groups of columns which contain the preliminary calculated amounts in sheet "PreCalc". The row of this value must be the one where the label in columns D, E or F matches with the corresponding label in the split-screen strip. The column must be the $(t+1)$-th column, where t is the period (first column for the period $t = 0$).
The formulas are similar for three types of components: capital, income, and cash flow. Following are some examples of spreadsheets codes.

For an income component: lookup_value is the label in column SplitScreenStrip!E and lookup_array is the column PreCalc!E. For example, the code for cell H13 is

$$\text{H13} = \text{INDEX}(\ \underbrace{\text{PreCalc!}\ \$\text{K}{:}\ \$\text{P}}_{\substack{\text{lookup}_{\text{value}}\text{:}\\ \text{columns group of the}\\ \text{amounts calculated in}\\ \text{worksheet "PreCalc"}}}\ , \underbrace{\text{MATCH}(\ \$\text{E13}, \text{PreCalc!}\ \$\text{E}{:}\ \$\text{E}, 0\ )}_{\substack{\text{the result of this formula is the number}\\ \text{of the row in the worksheet "PreCalc" with}\\ \text{the same label of cell E13}}}\ , \underbrace{\text{H}\$9 + 1}_{\substack{\text{the } (t+1)\text{−th column,}\\ \text{where } t \text{ is the period}\\ \text{(first column for period 0)}\\ \text{in the columns group of the}\\ \text{worksheet "PreCalc"}}}\ )$$

(see Figure 10 and Figure 11).

For a cash-flow component: lookup_value is the label in column SplitScreenStrip!F, and lookup_array is the column PreCalc!F. For example, the code for cell I17 is

$$\text{I17} = \text{INDEX}(\ \text{PreCalc!}\ \$\text{K}{:}\ \$\text{P}, \text{MATCH}(\ \$\text{F17}, \text{PreCalc!}\ \$\text{F}{:}\ \$\text{F}, 0\ ), \text{I}\$9 + 1\ )$$

For a capital component: lookup_value is the label in column SplitScreenStrip!D, and lookup_array is the column PreCalc!D. For example, the code for cell J15 is

$$\text{J15} = \text{INDEX}(\ \text{PreCalc!}\ \$\text{K}{:}\ \$\text{P}, \text{MATCH}(\ \$\text{D15}, \text{PreCalc!}\ \$\text{D}{:}\ \$\text{D}, 0\ ), \text{J}\$9 + 1\ )$$

(see also Figure 12).

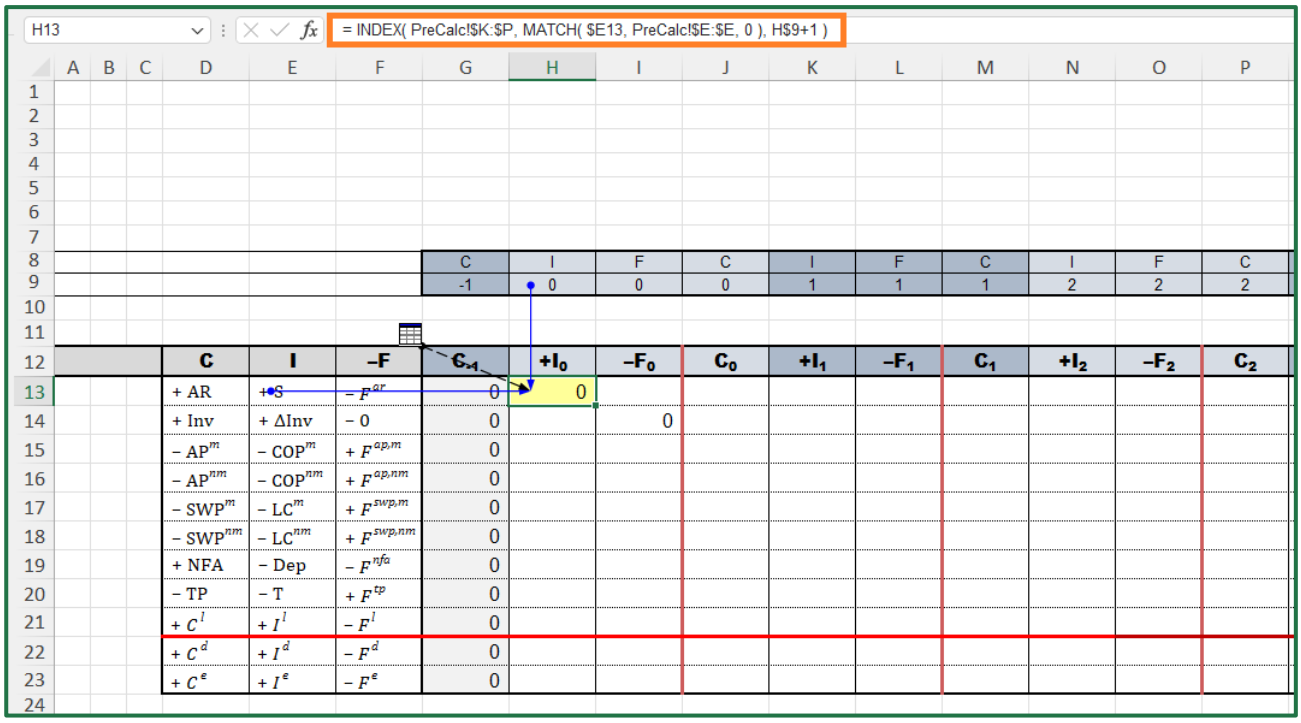


Figure 10. Cell H13 contains the formula for calculating the income component of AR (i.e., sales). Cell I14 is zero, by definition (sheet "SplitScreenStrip")

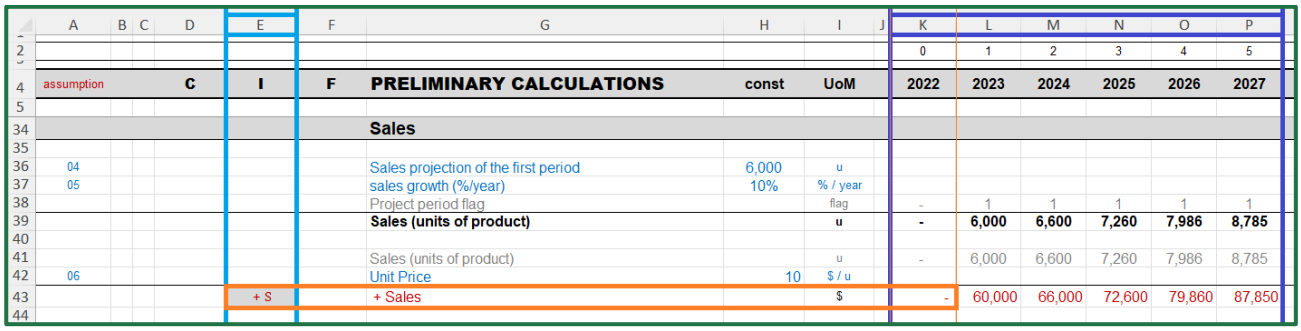


Figure 11. Cell K43 contains the value that has the same label of SplitScreenStrip!E13 and that is in the (H$9+1)-th column (sheet "PreCalc")

Every cell in Table 2 should be filled using the formula INDEX-MATCH. Figure 12 shows the output of this step.

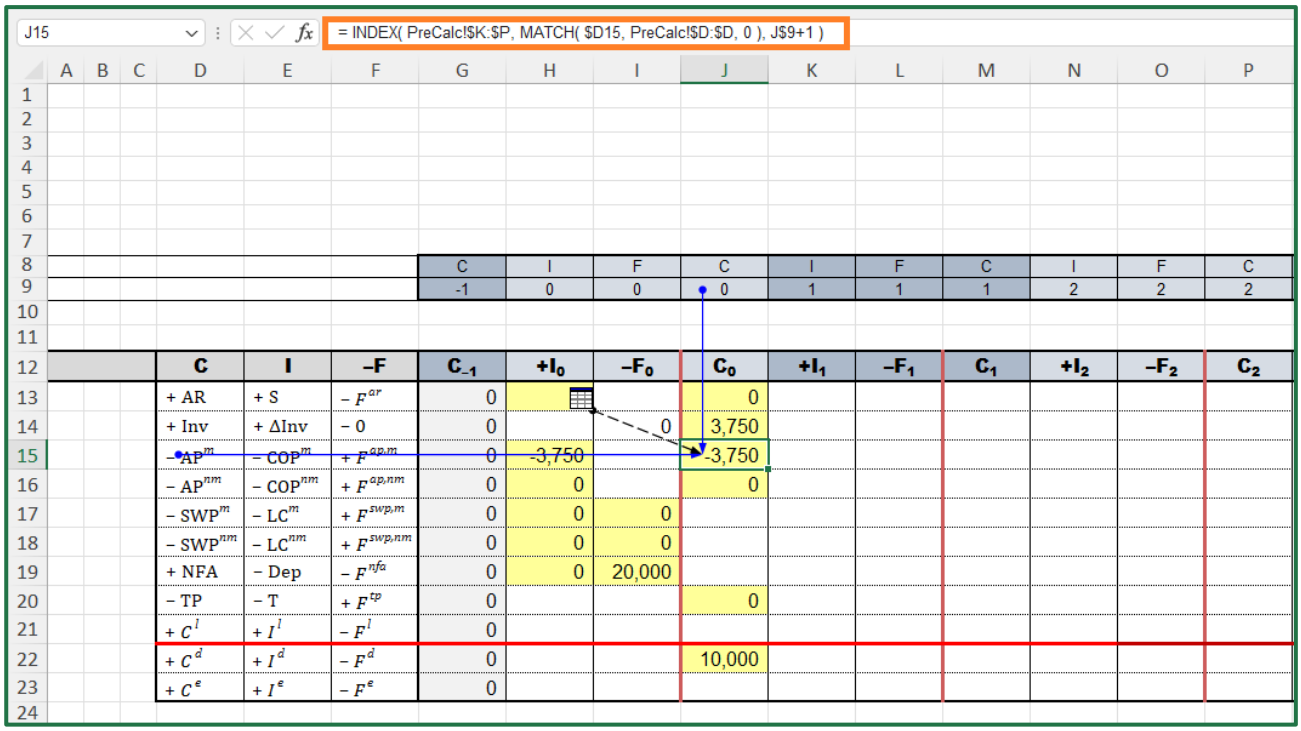


Figure 12. The 15 cells whose values are provided in "PreCalc" are in yellow. Cell J15 calculates the accounts payable for manufacturing activities at $t = 0$ (sheet "SplitScreenStrip")

2.2.2 Cells whose values are calculated with the law of motion

The second class of cells in array H13:J23 is the class of those cells which must be filled with the law of motion. To fill in these values, the analyst should use the law of motion for t=0 (see Part I, eq. (2)). Specifically, to find a capital amount, the law of motion is expressed as

$$C_0 = C_{-1} + I_0 - F_0.$$

To find an income amount, the law of motion becomes

$$I_0 = C_0 - C_{-1} + F_0.$$

To find a cash-flow component, the law of motion becomes

$$F_0 = C_0 - C_{-1} + I_0.$$

Applying these laws of motion, the cells of this class are calculated, with their own sign (see Table 3):

Table 3. Cells whose values are calculated with the law of motion

Cell	Symbol	Magnitude	Law of motion	Type
H14	$+\Delta\mathrm{Inv}_0$	Income from inventory	$+\Delta\mathrm{Inv}_0 = \mathrm{Inv}_0 - \mathrm{Inv}_{-1} + (-0)$	Income
I13	$-F_0^{ar}$	– Cash receipts from customers	$-F_0^{ar} = \mathrm{AR}_0 - \mathrm{AR}_{-1} - \mathrm{S}_0$	Cash flow
I15	$+F_0^{ap,m}$	Payments to suppliers (manufacturing)	$+F_0^{ap,m} = -\mathrm{AP}_0^m + \mathrm{AP}_{-1}^m + \mathrm{COP}_t^m$	Cash flow
I16	$+F_0^{ap,nm}$	Payments to suppliers (manufacturing)	$+F_0^{ap,nm} = -\mathrm{AP}_0^{nm} + \mathrm{AP}_{-1}^{nm} + \mathrm{COP}_0^{nm}$	Cash flow
I20	$+F_0^{tp}$	Tax payments	$+F_0^{tp} = -\mathrm{TP}_0 + \mathrm{TP}_{-1} + \mathrm{T}_0$	Cash flow
I22	$-F_0^d$	– Cash flow to debt	$-F_0^d = C_0^d - C_{-1}^d - I_0^d$	Cash flow
J17	$-\mathrm{SWP}_0^m$	– Salaries & Wages Payable (manufacturing)	$-\mathrm{SWP}_0^m = -\mathrm{SWP}_{-1}^m - \mathrm{LC}_0^m + F_0^{swp,m}$	Capital
J18	$-\mathrm{SWP}_0^{nm}$	– Salaries & Wages Payable (nonmanufacturing)	$-\mathrm{SWP}_0^{nm} = -\mathrm{SWP}_{-1}^{nm} - \mathrm{LC}_0^{nm} + F_0^{swp,nm}$	Capital
J19	$+\mathrm{NFA}_0$	Net Fixed Asset	$+\mathrm{NFA}_0 = \mathrm{NFA}_{-1} - \mathrm{Dep}_0 + (-F_0^{nfa})$	Capital
J21	$+C_t^l$	Liquid assets	$+C_t^l = C_{t-1}^l + I_t^l - F_t^l$	Capital
J23	$+C_t^e$	Equity	$+C_t^e = C_{t-1}^e + I_t^e - F_t^e$	Capital

Note that the application of the law of motion for J21 supplies a value equal to zero, because I_0^l and F_0^l (respectively, cell H21 and I21) have not been calculated yet. However, the values of J21 will be automatically updated as soon as H21 and I21 will be calculated in the next steps. The same holds for J23, and I20 and I22, cells that temporarily can exhibit an incorrect value, because one of the cells linked in the formula is not yet calculated. The output of this step is displayed in Figure 13 (the cells of the class are highlighted in green).

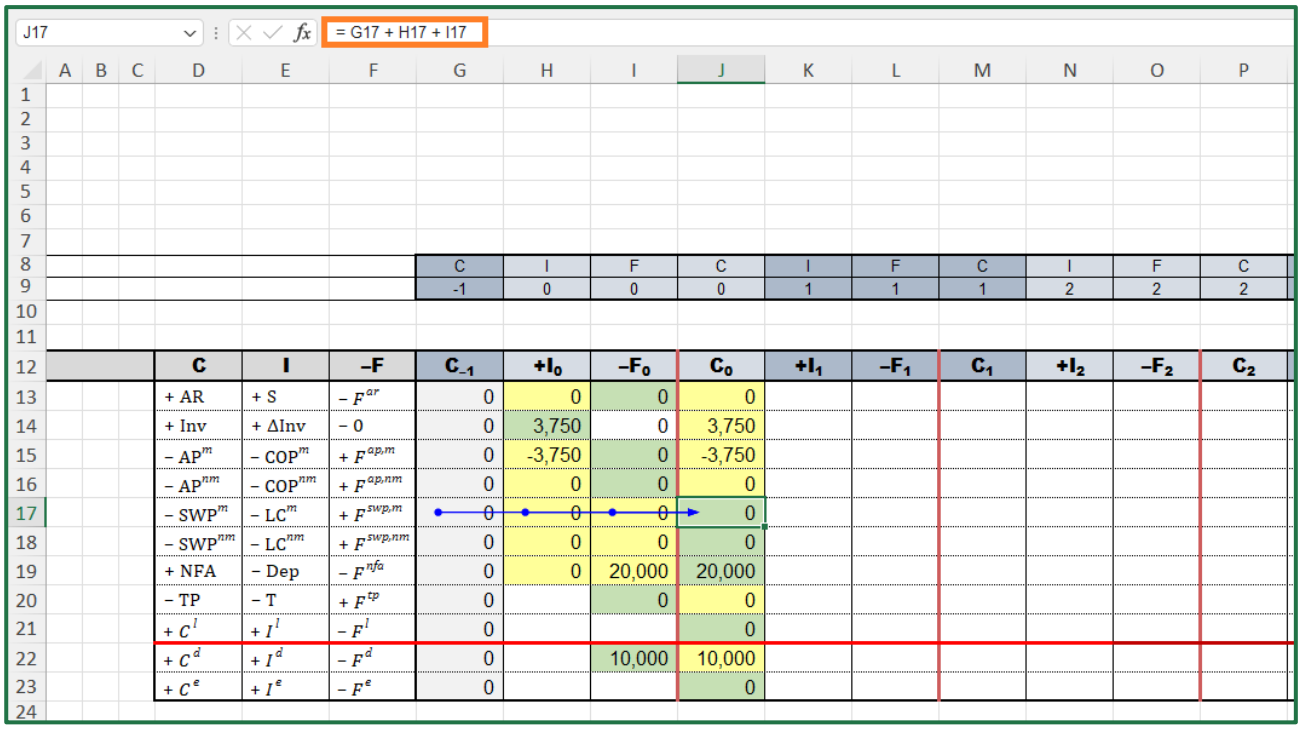
Spreadsheet Modeling: Cells whose values are calculated with the law of motion

Example of capital component

The (changed-in-sign) salaries and wages payable at t=0 in cell J17 is obtained as follows:

$$\underbrace{\mathrm{J17}}_{-\mathrm{SWP}_0^m} = \underbrace{\mathrm{G17}}_{-\mathrm{SWP}_{-1}^m} + \underbrace{\mathrm{H17}}_{-\mathrm{LC}_0^m} + \underbrace{\mathrm{I17}}_{+F_0^{swp,m}}$$

(see Figure 13).



J17 = G17 + H17 + I17

	C	I	–F	C_{-1}	$+I_0$	$-F_0$	C_0	$+I_1$	$-F_1$	C_1	$+I_2$	$-F_2$	C_2
				C	I	F	C	I	F	C	I	F	C
				-1	0	0	0	1	1	1	2	2	2
+ AR	+ S	$-F^{ar}$		0	0	0	0						
+ Inv	+ ΔInv	− 0		0	3,750	0	3,750						
$-AP^{m}$	$-COP^{m}$	$+F^{ap,m}$		0	-3,750	0	-3,750						
$-AP^{nm}$	$-COP^{nm}$	$+F^{ap,nm}$		0	0	0	0						
$-SWP^{m}$	$-LC^{m}$	$+F^{swp,m}$		0	0	0	0						
$-SWP^{nm}$	$-LC^{nm}$	$+F^{swp,nm}$		0	0	0	0						
+ NFA	− Dep	$-F^{nfa}$		0	0	20,000	20,000						
− TP	− T	$+F^{tp}$		0		0	0						
$+C^{l}$	$+I^{l}$	$-F^{l}$		0			0						
$+C^{d}$	$+I^{d}$	$-F^{d}$		0		10,000	10,000						
$+C^{e}$	$+I^{e}$	$-F^{e}$		0			0						

Figure 13. The cells whose values are calculated with the law of motion are in green. Cell J17 contains the law of motion for SWP_0^m (sheet "SplitScreenStrip")

The analyst can copy cell J17 and paste it in cells J18, J19, J21, and J23 (the equations share the same structure).

Example of income component

The income from inventory in cell H14 is just equal to ΔInv_0 (because the cash-flow component of inventory is zero):

$$\underbrace{H14}_{+\Delta Inv_0} = \underbrace{J14}_{+Inv_0} - \underbrace{G14}_{+Inv_{-1}} + \underbrace{I14}_{-F_0^{inv}}$$

with $F_0^{inv} = 0$ (see Figure 14).

H14 = J14 - G14 + I14

	C	I	–F	C_{-1}	$+I_0$	$-F_0$	C_0	$+I_1$	$-F_1$	C_1	$+I_2$	$-F_2$	C_2
	C	I	F	C	I	F	C	I	F	C	I	F	C
	-1	0	0	0	1	1	1	2	2	2			
+ AR	+ S	$-F^{ar}$	0	0	0	0							
+ Inv	+ ΔInv	− 0	0	0	3,750	0	3,750						
− AP^{m}	− COP^{m}	$+F^{ap,m}$	0	-3,750	0	-3,750							
− AP^{nm}	− COP^{nm}	$+F^{ap,nm}$	0	0	0	0							
− SWP^{m}	− LC^{m}	$+F^{swp,m}$	0	0	0	0							
− SWP^{nm}	− LC^{nm}	$+F^{swp,nm}$	0	0	0	0							
+ NFA	− Dep	$-F^{nfa}$	0	0	20,000	20,000							
− TP	− T	$+F^{tp}$	0		0	0							
$+C^{l}$	$+I^{l}$	$-F^{l}$	0			0							
$+C^{d}$	$+I^{d}$	$-F^{d}$	0		10,000	10,000							
$+C^{e}$	$+I^{e}$	$-F^{e}$	0			0							

Figure 14. Cell H14 contains the law of motion for ΔInv_0 (sheet "SplitScreenStrip")

Example of cash-flow component

The (changed-in-sign) cash receipts from customers $-F_0^{ar}$ in cell I13 are obtained as follows:

$$\underbrace{\text{I13}}_{-F_0^{ar}} = \underbrace{\text{J13}}_{+\text{AR}_0} - \underbrace{\text{G13}}_{+\text{AR}_{-1}} - \underbrace{\text{H13}}_{+\text{S}}$$

(see Figure 15).

I13 = J13 - G13 - H13

	C	I	–F	C_{-1}	$+I_0$	$-F_0$	C_0	$+I_1$	$-F_1$	C_1	$+I_2$	$-F_2$	C_2
	C	I	F	C	I	F	C	I	F	C	I	F	C
	-1	0	0	0	1	1	1	2	2	2			
+ AR	+ S	$-F^{ar}$	0	0	0	0							
+ Inv	+ ΔInv	− 0	0	3,750	0	3,750							
− AP^{m}	− COP^{m}	$+F^{ap,m}$	0	-3,750	0	-3,750							
− AP^{nm}	− COP^{nm}	$+F^{ap,nm}$	0	0	0	0							
− SWP^{m}	− LC^{m}	$+F^{swp,m}$	0	0	0	0							
− SWP^{nm}	− LC^{nm}	$+F^{swp,nm}$	0	0	0	0							
+ NFA	− Dep	$-F^{nfa}$	0	0	20,000	20,000							
− TP	− T	$+F^{tp}$	0		0	0							
$+C^{l}$	$+I^{l}$	$-F^{l}$	0			0							
$+C^{d}$	$+I^{d}$	$-F^{d}$	0		10,000	10,000							
$+C^{e}$	$+I^{e}$	$-F^{e}$	0			0							

Figure 15. Cell I13 contains the law of motion for the cash receipts from customers (sheet "SplitScreenStrip")

The analyst can copy cell I13 and paste it in cells I15, I16, I20, and I22 (the equations share the same structure).

2.2.3 Cells whose values are calculated with suitable formulas

The third class of cells consists of those cells which can be calculated via suitable, specific formulas. These are the taxes, the interest income, the interest expenses, and the CFE (see Table 4 and description below).

Table 4. Cells whose values are calculated via suitable formulas

Cell	Symbol	Magnitude	Formula	Type
H20	T_t	Taxes	$\mathrm{T}_t = \mathrm{EBT}_t \cdot \tau$	Income
H21	I_t^l	Interest income	$I_t^l = C_{t-1}^l \cdot i^l$	Income
H22	I_t^d	Interest expenses	$I_t^d = C_{t-1}^d \cdot i^d$	Income
I23	F_t^e	Cash flow to equity	depends on the period	Cash flow

TAXES

The amount of Taxes (T_t) is not known, but the assumptions provide the necessary information fort calculating it. The tax amount is computed applying the tax rate to earnings before taxes (EBT):

$$\mathrm{T}_t = \mathrm{EBT}_t \cdot \tau$$

where τ is the tax rate, equal to 30% (by Assumption 15) and EBT is equal to $\mathrm{EBT}_t = \mathrm{EBIT}_t + I_t^l - I_t^d$, so that

$$\mathrm{T}_t = \left(\mathrm{EBIT}_t + I_t^l - I_t^d\right) \cdot \tau$$

where EBIT_t is obtained as $\mathrm{EBIT}_t = \mathrm{S}_t - \mathrm{COGS}_t - \mathrm{SGA}_t - \mathrm{Dep}_t$ (Part I, eq. (47)). Hence,

$$\mathrm{T}_t = (\mathrm{S}_t - (\overbrace{\mathrm{COP}_t^m + \mathrm{LC}_t^m - \Delta\mathrm{Inv}_t}^{\mathrm{COGS}_t}) - (\overbrace{\mathrm{COP}_t^{nm} + \mathrm{LC}_t^{nm}}^{\mathrm{SGA}_t}) - \mathrm{Dep}_t + I_t^l - I_t^d) \cdot \tau$$

(see also Part I, footnote 7). All the values in this formula have already been computed or will be computed in the next steps (i.e., I_t^l and I_t^d).

Spreadsheet Modeling: Taxes

The tax rate τ is recorded in "Assumption" sheet: the analyst can show it in a cell above the Matrix in the "SplitScreenStrip" sheet (e.g., in cell D3) and then use it in the formulas (see Figure 16).
Cell D3 is linked to the cell that contains the value of τ with the formula

$$= \mathrm{Assumptions!\,\$F\$19}.$$

The (changed-in-sign) taxes, $-\mathrm{T}_t$, are calculated in cell H20:

$$\underbrace{\mathrm{H20}}_{-\mathrm{T}_0} = (-1) * \mathrm{SUM}(\underbrace{\mathrm{H13{:}H19}}_{\substack{+\mathrm{S}_0+\Delta\mathrm{Inv}_0 \\ -\mathrm{COP}_0^m-\mathrm{COP}_0^{nm} \\ -\mathrm{LC}_0^m-\mathrm{LC}_0^{nm} \\ -\mathrm{Dep}_0}}, \underbrace{\mathrm{H21}}_{+I_0^l}, \underbrace{-\mathrm{H22}}_{-(+I_0^d)}) * \underbrace{\mathrm{\$D\$3}}_{\tau}.$$

While the equation is correct, the value obtained in H20 is not yet the correct one, because cells H21 ($+I_0^l$) and H22 ($-I_0^d$) have not yet been occupied by formulas; however, they will be filled with the

amounts of interest income and expenses in the next step, and the value taken on by cell H20 will be automatically updated (see Figure 16).

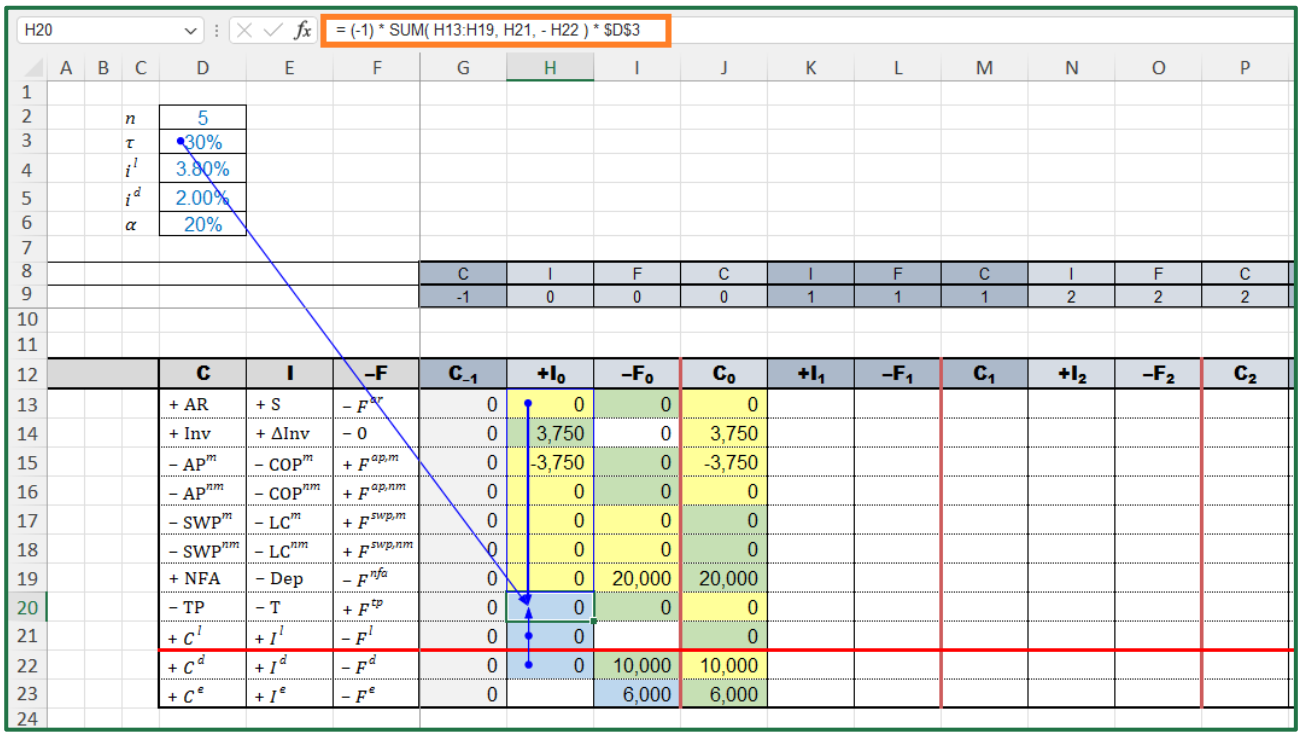


Figure 16. The cells whose values are calculated with suitable formulas are shown in blue. Cell H20 contains the specific formula for calculating the taxes (sheet "SplitScreenStrip")

INTEREST INCOME

The amounts of interest expenses (I_t^l) are not known, but the assumptions provide sufficient information for calculating it by applying the interest rate to the beginning-of-period balance of liquid assets:

$$I_t^l = C_{t-1}^l \cdot i^l$$

where i^l is the interest rate on liquid assets, equal to 3.80% (by assumption 21).

Spreadsheet Modeling: Interest income

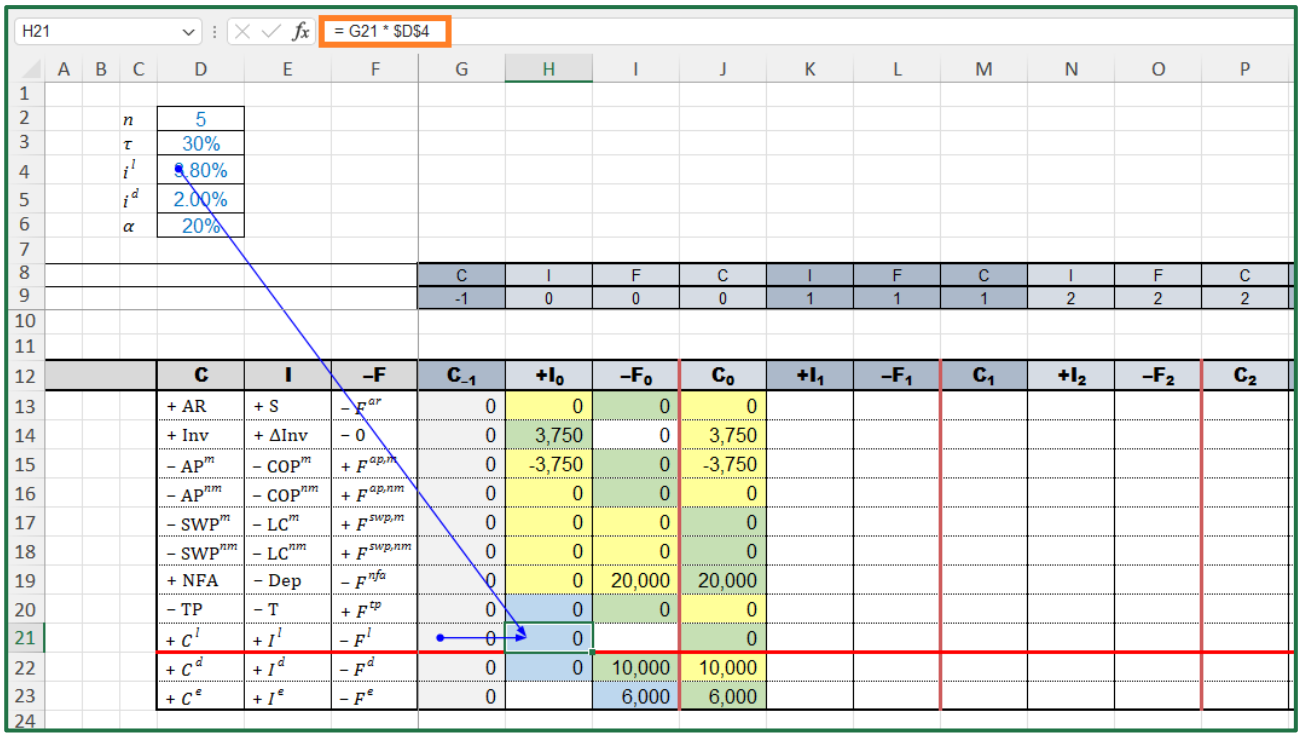
The interest rate i^l is recorded in the sheet "Assumption": the analyst can show it in a cell above the Matrix in the "SplitScreenStrip" sheet (e.g., in cell D4) and then use it in the formulas (see Figure 15). Cell D4 is linked to the cell that contains the value of interest rate i^l with the formula

$$\text{D4} = \text{Assumptions!}\,\$\text{F}\$25.$$

The interest income I_t^l is calculated in cell H22:

$$\underbrace{\text{H21}}_{+I_1^l} = \underbrace{\text{G21}}_{+C_0^l} * \underbrace{\$\text{D}\$4}_{i^l}$$

(see Figure 17).



H21 = G21 * $D$4

n	5	
τ	30%	
i^l	4.80%	
i^d	2.00%	
α	20%	

				C	I	F	C	I	F	C	I	F	C
				-1	0	0	0	1	1	1	2	2	2
C	I	–F	C_{-1}	$+I_0$	$-F_0$	C_0	$+I_1$	$-F_1$	C_1	$+I_2$	$-F_2$	C_2
+ AR	+ S	$-F^{ar}$	0	0	0	0						
+ Inv	+ ΔInv	− 0	0	3,750	0	3,750						
$-AP^m$	$-COP^m$	$+F^{ap,m}$	0	-3,750	0	-3,750						
$-AP^{nm}$	$-COP^{nm}$	$+F^{ap,nm}$	0	0	0	0						
$-SWP^m$	$-LC^m$	$+F^{swp,m}$	0	0	0	0						
$-SWP^{nm}$	$-LC^{nm}$	$+F^{swp,nm}$	0	0	0	0						
+ NFA	− Dep	$-F^{nfa}$	0	0	20,000	20,000						
− TP	− T	$+F^{tp}$	0	0	0	0						
$+C^l$	$+I^l$	$-F^l$	0	0		0						
$+C^d$	$+I^d$	$-F^d$	0	0	10,000	10,000						
$+C^e$	$+I^e$	$-F^e$	0		6,000	6,000						

Figure 17. Cell H21 contains the specific formula for calculating the interest income (sheet "SplitScreenStrip")

INTEREST EXPENSES

The amounts of interest expenses (I_t^d) are not known, but the assumptions provide sufficient information for calculating it by applying the interest rate to the beginning-of-period principal outstanding:

$$I_t^d = C_{t-1}^d \cdot i^d$$

where i^d is the interest rate of the loan, equal to 2% (by assumption 19).

Spreadsheet Modeling: Interest expenses

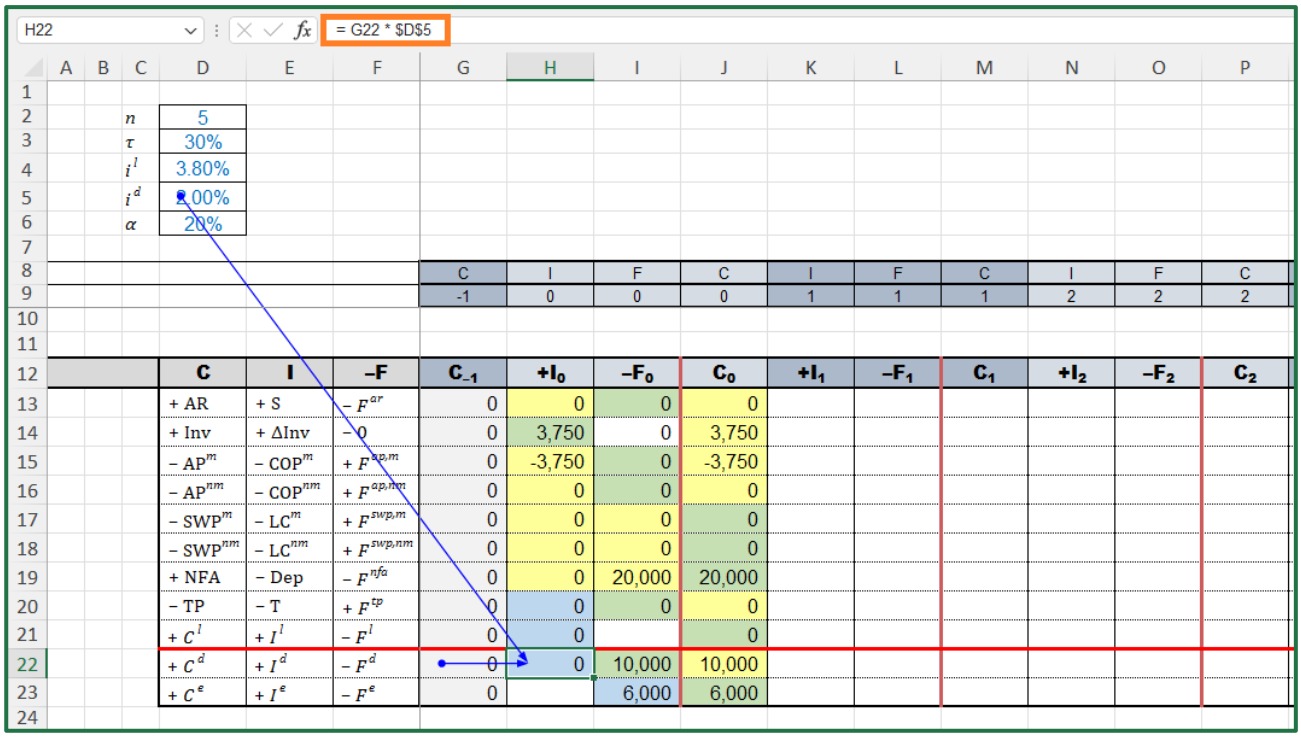
The interest rate i^d is recorded in the sheet "Assumption": the analyst can show it in a cell above the Matrix in the "SplitScreenStrip" sheet (e.g., in cell D5) and then use it in the formulas (see Figure 18). Cell D5 is linked to the cell that contains the value of interest rate i^d with the formula

$$\text{D5} = \text{Assumptions!}\,\$\text{F}\$23.$$

The interest expenses I_t^d is calculated in cell H22:

$$\underbrace{\text{H22}}_{+I_1^d} = \underbrace{\text{G22}}_{+C_0^d} * \underbrace{\$\text{D}\$5}_{i^d}$$

(see Figure 18).



H22 = G22 * $D$5

n	5	
τ	30%	
i^l	3.80%	
i^d	8.00%	
α	20%	

				C	I	F	C	I	F	C	I	F	C
				-1	0	0	0	1	1	1	2	2	2
C	I	–F	C_{-1}	$+I_0$	$-F_0$	C_0	$+I_1$	$-F_1$	C_1	$+I_2$	$-F_2$	C_2
+ AR	+ S	$-F^{ar}$	0	0	0	0						
+ Inv	+ ΔInv	− 0	0	3,750	0	3,750						
$-AP^{m}$	$-COP^{m}$	$+F^{ap,m}$	0	-3,750	0	-3,750						
$-AP^{nm}$	$-COP^{nm}$	$+F^{ap,nm}$	0	0	0	0						
$-SWP^{m}$	$-LC^{m}$	$+F^{swp,m}$	0	0	0	0						
$-SWP^{nm}$	$-LC^{nm}$	$+F^{swp,nm}$	0	0	0	0						
+ NFA	− Dep	$-F^{nfa}$	0	0	20,000	20,000						
− TP	− T	$+F^{tp}$	0	0	0	0						
$+C^{l}$	$+I^{l}$	$-F^{l}$	0	0		0						
$+C^{d}$	$+I^{d}$	$-F^{d}$	0	0	10,000	10,000						
$+C^{e}$	$+I^{e}$	$-F^{e}$	0		6,000	6,000						

Figure 18. Cell H22 contains the interest expenses (sheet "SplitScreenStrip")

CASH FLOW TO EQUITY

For simplifying the spreadsheet calculation for the CFE (F_t^e), it is convenient to break down the CFE into three categories (equity financing, interim payout, and liquidation dividend) and, hence, build the stream of cash flows for each such category. Equity financing is equal to \$6,000 for $t = 0$ (i.e., the cash flow initially contributed by shareholders is $F_0^e = -6{,}000$) and equal to 0 for $t > 0$ (the firm's shareholders will not be required to make any other contribution) (assumption 17). The interim payout distributions are equal to α =20% of the project's net income for $1 < t < 5$ (assumption 22). The liquidation dividend, F_5^e, may be obtained as the sum of the equity at time $t = 4$ and the net income at time $t = 5$ (because $C_5^e = 0$ and, by the law of motion, $C_5^e = C_4^e + I_5^e - F_5^e$) (see Table 5. Breakdown of CFE).

Remark. We remind that the interim payout is a decision variable which affects the project's value and, as such, the project's economic profitability. Such a variable is often neglected in traditional financial modelling, where the project's value is calculated (not by discounting the actual project CFEs but) by discounting the project FCFEs. However, this boils down to assuming that the payout ratio is 100% and that such a ratio is applied to FCFE. In real life, the payout ratio may be different (even 0%) and may be applied, more commonly, to the net income or other basis (e.g., the minimum between net income and FCFE. See Part I, section 9).

Table 5. Breakdown of CFE (payout ratio = 20% of net income)

	TIME	0	1	2	3	4	5
CASH FLOW TO EQUITY (F_t^e)	Shareholders' investment	$-6{,}000$	0	0	0	0	0
	Interim payout (20% of net income)	0	$0.2 \cdot I_1^e$	$0.2 \cdot I_2^e$	$0.2 \cdot I_3^e$	$0.2 \cdot I_4^e$	0
	Liquidation dividend	0	0	0	0	0	$C_4^e + I_5^e$

Spreadsheet Modeling: Cash Flow to Equity

The analyst must fill cell I23 with a single formula that should consider that, depending on time t, either of the three categories of CFE applies. The project length and the pay-out ratio are recorded in the sheet "Assumption": the analyst can show it in a cell above the Matrix in the "SplitScreenStrip" sheet (e.g., cell D2 and cell D6) and then use it in the formulas (see Figure 19). Cells D2 and D6 are linked to the cells that contain the values of project length and the pay-out ratio with the formulas = Assumptions! $F$5 and = Assumptions! $F$26 respectively.

The spreadsheet code for I23 is:

$$\overbrace{\text{I23}}^{-F_0^e} = \text{IF}(\overbrace{\text{I9}}^{\text{period}} = 0, \underbrace{\overbrace{\text{Assumptions!\$H\$21}}^{\text{equity financing}}}_{\text{case } t=0}, \underbrace{\text{IF}(\overbrace{\text{I9}}^{\text{period}} < \overbrace{\$\text{D}\$2}^{n}, \underbrace{\overbrace{-\text{H23} * \$\text{D}\$6}^{\text{interim payout}}}_{\text{case } 0<t<n}, \underbrace{\overbrace{-(\text{G23} + \text{H23})}^{\text{liquidation dividend}}}_{\text{case } t=n})}_{\text{case } t\neq 0})$$

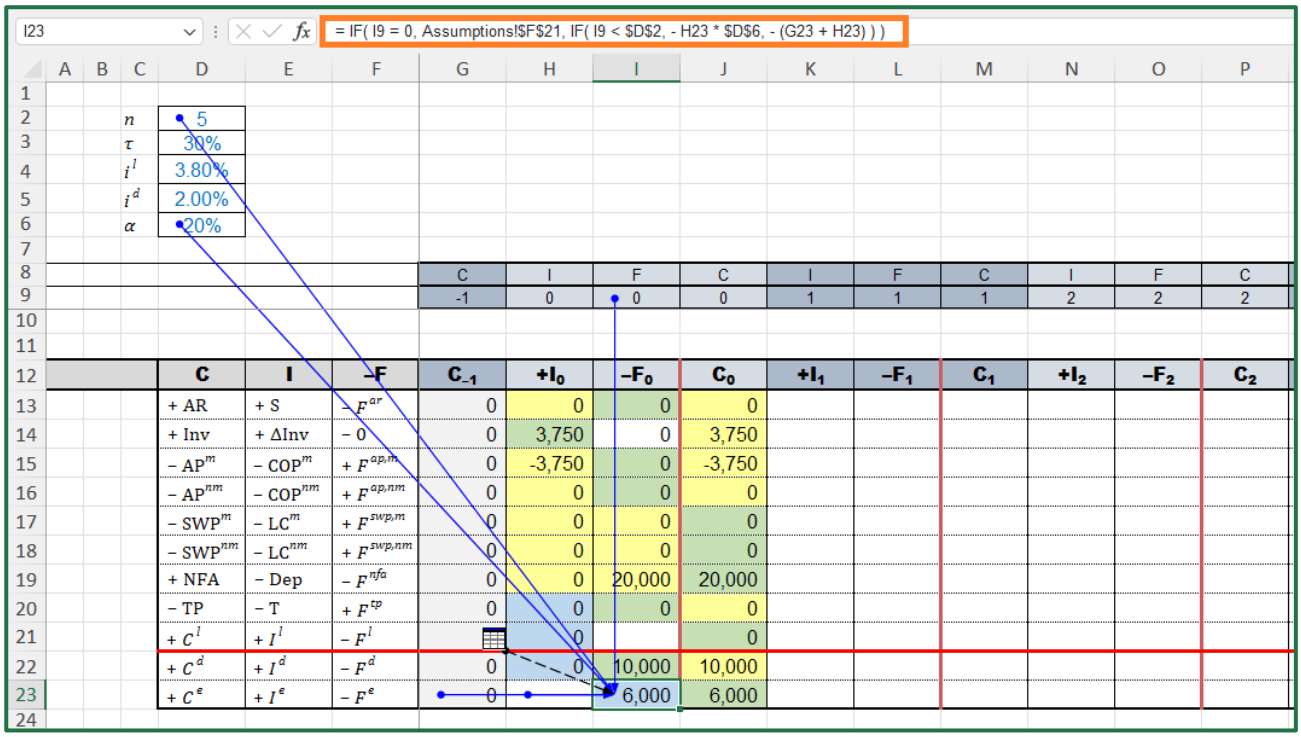


Figure 19. Cell I23 reports the Cash flow to equity (sheet "SplitScreenStrip")

The payout is calculated as a function of the net income. However, as shown in Part I, other schemes are possible. For example, the payout may be a fraction of the FCFE or a fraction of the minimum between net income and FCFE (see section 9 and, in particular, eq. (60)). In the Appendix of this paper, we show how to cope with these alternative schemes.

2.2.4 Cells whose values are calculated with the law of conservation

The fourth class of cells in array H13:J23 consists of those cells which can be calculated by means of the law of conservation. These are the net income and the cash flow from non-operating (liquid) assets (see Table 6).

Table 6. Cells whose values are calculated with the law of conservation

Cell	Symbol	Magnitude	Law of conservation)	Type
H20	I_t^e	Net Income	$I_t^e = I_t^o + I_t^l - I_t^d$	Income
H21	$-F_t^l$	−Cash flow from liquid assets	$-F_t^l = F_t^o - F_t^d - F_t^e$	Cash flow

NET INCOME

Owing to the law of conservation, $I_t^o + I_t^l = I_t^d + I_t^e$ (see Part I, eq. (10)) or, breaking down the after-tax operating profit,

$$\overbrace{\mathrm{S}_t + \Delta\mathrm{Inv}_t - \mathrm{COP}_t^m - \mathrm{COP}_t^{nm} - \mathrm{LC}_t^m - \mathrm{LC}_t^{nm} - \mathrm{Dep}_t - \mathrm{T}_t}^{I_t^o} + I_t^l = +I_t^d + I_t^e$$

(see also Part I, eq. (15)), whence

$$I_t^e = +\mathrm{S}_t + \Delta\mathrm{Inv}_t - \mathrm{COP}_t^m - \mathrm{COP}_t^{nm} - \mathrm{LC}_t^m - \mathrm{LC}_t^{nm} - \mathrm{Dep}_t - \mathrm{T}_t + I_t^l - I_t^d$$

Spreadsheet Modeling: Net Income

Net Income I_t^e is calculated in cell H23:

$$\underbrace{\mathrm{H23}}_{I_0^e} = \mathrm{SUM}\Big(\underbrace{\mathrm{H13{:}H21}}_{\substack{+\mathrm{S}_0+\Delta\mathrm{Inv}_0 \\ -\mathrm{COP}_0^m-\mathrm{COP}_0^{nm} \\ -\mathrm{LC}_0^m-\mathrm{LC}_0^{nm} \\ -\mathrm{Dep}_0-T_0+I_0^l}} , -\underbrace{\mathrm{H22}}_{I_0^d} \Big)$$

(see Figure 20).

H23 = SUM(H13:H21, - H22)

		n	5												
		τ	30%												
		i^l	3.80%												
		i^d	2.00%												
		α	20%												
						C	I	F	C	I	F	C	I	F	C
						-1	0	0	0	1	1	1	2	2	2
		C	I	–F	C_{-1}	$+I_0$	$-F_0$	C_0	$+I_1$	$-F_1$	C_1	$+I_2$	$-F_2$	C_2
		+ AR	+ S	$-F^{ar}$	0	0	0	0						
		+ Inv	+ ΔInv	− 0	0	3,750	0	3,750						
		$-AP^m$	$-COP^m$	$+F^{ap,m}$	0	-3,750	0	-3,750						
		$-AP^{nm}$	$-COP^{nm}$	$+F^{ap,nm}$	0	0	0	0						
		$-SWP^m$	$-LC^m$	$+F^{swp,m}$	0	0	0	0						
		$-SWP^{nm}$	$-LC^{nm}$	$+F^{swp,nm}$	0	0	0	0						
		+ NFA	− Dep	$-F^{nfa}$	0	0	20,000	20,000						
		− TP	− T	$+F^{tp}$	0	0	0	0						
		$+C^l$	$+I^l$	$-F^l$	0	0	-4,000	-4,000						
		$+C^d$	$+I^d$	$-F^d$	0	0	10,000	10,000						
		$+C^e$	$+I^e$	$-F^e$	0	0	6,000	6,000						

Figure 20. The cells whose values are calculated with the law of conservation are shown in light pink. Cell H23 reports the net income (sheet "SplitScreenStrip")

CASH FLOW FROM LIQUID ASSETS

Owing to the law of conservation (see Part I, eq. (11)), $F_t^o + F_t^l = F_t^d + F_t^e$ or, breaking down the operating cash flow,

$$\overbrace{-F_t^{ar} - 0 + F_t^{ap,m} + F_t^{ap,nm} + F_t^{swp,m} + F_t^{swp,nm} - F_t^{nfa} + F_t^{tp}}^{-F_t^o} - F_t^l = -F_t^d - F_t^e$$

(see also Part I, eq. (16)), whence

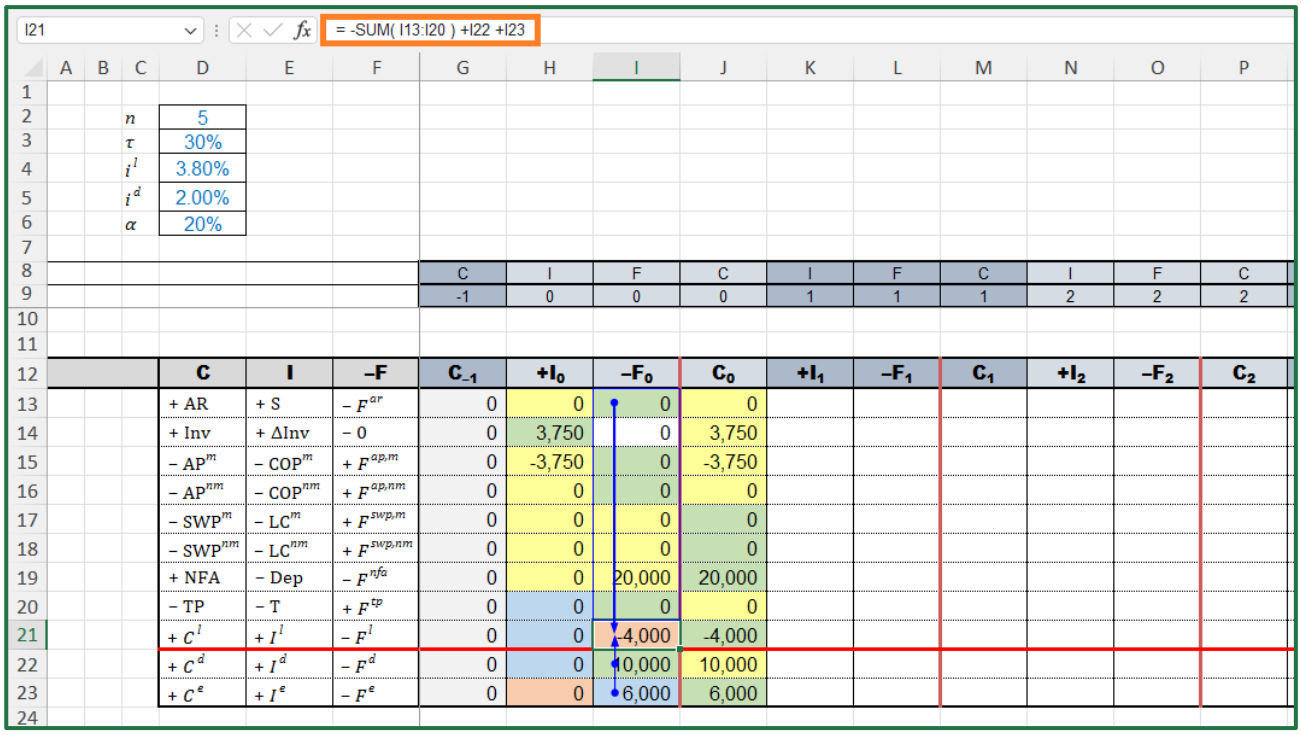
$$-F_t^l = F_t^{ar} - F_t^{ap,m} - F_t^{ap,nm} - F_t^{swp,m} - F_t^{swp,nm} + F_t^{nfa} - F_t^{tp} - F_t^d - F_t^e.$$

Spreadsheet Modeling: Cash flow from liquid assets

Here is an example of how to calculate the cash flow from/to liquid assets F_t^l in cell I21:

$$\underbrace{\text{I21}}_{-F_0^l} = -\,\text{SUM}(\underbrace{\text{I13:I20}}_{\substack{-F_0^{ar}+F_0^{ap,m} \\ +F_0^{ap,nm}+F_0^{swp,m} \\ +F_0^{swp,nm}-F_0^{nfa} \\ +F_0^{tp}}})+\underbrace{\text{I22}}_{-F_0^d}+\underbrace{\text{I23}}_{-F_0^e}$$

(see Figure 21).



I21 = -SUM(I13:I20) +I22 +I23

n	5	
τ	30%	
i^l	3.80%	
i^d	2.00%	
α	20%	

					C	I	F	C	I	F	C	I	F	C
					-1	0	0	0	1	1	1	2	2	2
	C	I	–F	C_{-1}	$+I_0$	$-F_0$	C_0	$+I_1$	$-F_1$	C_1	$+I_2$	$-F_2$	C_2
	+ AR	+ S	$-F^{ar}$	0	0	0	0						
	+ Inv	+ ΔInv	− 0	0	3,750	0	3,750						
	$-AP^{m}$	$-COP^{m}$	$+F^{ap,m}$	0	-3,750	0	-3,750						
	$-AP^{nm}$	$-COP^{nm}$	$+F^{ap,nm}$	0	0	0	0						
	$-SWP^{m}$	$-LC^{m}$	$+F^{swp,m}$	0	0	0	0						
	$-SWP^{nm}$	$-LC^{nm}$	$+F^{swp,nm}$	0	0	0	0						
	+ NFA	− Dep	$-F^{nfa}$	0	0	20,000	20,000						
	− TP	− T	$+F^{tp}$	0	0	0	0						
	$+C^{l}$	$+I^{l}$	$-F^{l}$	0	0	-4,000	-4,000						
	$+C^{d}$	$+I^{d}$	$-F^{d}$	0	0	10,000	10,000						
	$+C^{e}$	$+I^{e}$	$-F^{e}$	0	0	6,000	6,000						

Figure 21. Cell I21 contains the cash flow from/to liquid assets (sheet "SplitScreenStrip")

2.3 Filling the cells for periods 1 through n

The law of motion and the law of conservation hold for all $t = 0,1, \dots, n$: the structure of all the split-screen Matrices is the same. Hence, once the cells for period 0 have been filled, it suffices to extend the same formulas for periods 1 through n to complete the project split-screen film.

Spreadsheet Modeling: From the matrix to the strip

The analyst can copy the range H13:J23 and paste it in the ranges K13:M23, N13:P23, etc. Alternatively, one can copy the formulas into adjacent cells by using the fill handle (the procedure is explained in the support web page of the software employed by the analyst).
This technique works properly because the formulas don't contain embedded data or constants (the formulas must contain the links to the cells in the "Assumptions" or in "PreCalc" sheet, and they must be parameterized using the labels of row and of column).

1. Select the range of cells H13:J23
2. Hover your cursor over the lower-right corner so that it turns into a plus sign (+) (the "fill handle")
3. Drag the fill handle right to cover all the cells up to column Y
4. Let it go and the formulas get automatically filled in the other columns of the split-screen strip

(see Figure 22).

n	5
τ	30%
i^l	3.80%
i^d	2.00%
α	20%

				C	I	F	C	I	F	C	I	F	C	
				-1	0	0	0	1	1	1	2	2	2	
	C	I	–F	C_{-1}	$+I_0$	$-F_0$	C_0	$+I_1$	$-F_1$	C_1	$+I_2$	$-F_2$	C_2	
	+ AR	+ S	$-F^{ar}$	0	0	0	0							
	+ Inv	+ ΔInv	− 0	0	3,750	0	3,750							
	− AP^{m}	− COP^{m}	$+F^{ap,m}$	0	-3,750	0	-3,750							
	− AP^{nm}	− COP^{nm}	$+F^{ap,nm}$	0	0	0	0							
	− SWP^{m}	− LC^{m}	$+F^{swp,m}$	0	0	0	0							
	− SWP^{nm}	− LC^{nm}	$+F^{swp,nm}$	0	0	0	0							
	+ NFA	− Dep	$-F^{nfa}$	0	0	20,000	20,000							
	− TP	− T	$+F^{tp}$	0	0	0	0							
	$+C^{i}$	$+I^{i}$	$-F^{i}$	0	0	-4,000	-4,000							
	$+C^{d}$	$+I^{d}$	$-F^{d}$	0	0	10,000	10,000							
	$+C^{e}$	$+I^{e}$	$-F^{e}$	0	0	6,000	6,000							

Figure 22. Drag the fill handle to fill the strip (sheet “SplitScreenStrip”)

The result of this step is the whole strip of the project, populated with the correct figures (Figure 23).

n	5
τ	30%
i^l	3.80%
i^d	2.00%
α	20%

				C	I	F	C	I	F	C	I	F	C	I	F	C	I	F	C	I	F	C	I	F	C
				-1	0	0	0	1	1	1	2	2	2	3	3	3	4	4	4	5	5	5
	C	I	–F	C_{-1}	$+I_0$	$-F_0$	C_0	$+I_1$	$-F_1$	C_1	$+I_2$	$-F_2$	C_2	$+I_3$	$-F_3$	C_3	$+I_4$	$-F_4$	C_4	$+I_5$	$-F_5$	C_5
	+ AR	+ S	$-F^{ar}$	0	0	0	0	60,000	-45,205	14,795	66,000	-64,521	16,274	72,600	-70,973	17,901	79,860	-78,070	19,692	87,850	-107,542	0
	+ Inv	+ ΔInv	− 0	0	3,750	0	3,750	375	0	4,125	413	0	4,538	454	0	4,991	499	0	5,491	-5,491	0	0
	− AP^{m}	− COP^{m}	$+F^{ap,m}$	0	-3,750	0	-3,750	-15,375	3,750	-15,375	-16,913	15,375	-16,913	-18,604	16,913	-18,604	-20,464	18,604	-20,464	-16,472	36,936	0
	− AP^{nm}	− COP^{nm}	$+F^{ap,nm}$	0	0	0	0	-9,000	0	-9,000	-9,900	9,000	-9,900	-10,890	9,900	-10,890	-11,979	10,890	-11,979	-13,178	25,157	0
	− SWP^{m}	− LC^{m}	$+F^{swp,m}$	0	0	0	0	-24,000	24,000	0	-26,400	26,400	0	-29,040	29,040	0	-31,944	31,944	0	-35,140	35,140	0
	− SWP^{nm}	− LC^{nm}	$+F^{swp,nm}$	0	0	0	0	-6,000	6,000	0	-6,000	6,000	0	-6,000	6,000	0	-6,000	6,000	0	-6,000	6,000	0
	+ NFA	− Dep	$-F^{nfa}$	0	0	20,000	20,000	-4,000	0	16,000	-4,000	0	12,000	-4,000	0	8,000	-4,000	0	4,000	-4,000	0	0
	− TP	− T	$+F^{tp}$	0	0	0	0	-494	494	0	-959	959	0	-1,414	1,414	0	-1,919	1,919	0	-2,478	2,478	0
	$+C^{i}$	$+I^{i}$	$-F^{i}$	0	0	-4,000	-4,000	-152	8,030	3,878	147	3,689	7,714	293	4,446	12,454	473	5,268	18,195	691	-18,887	0
	$+C^{d}$	$+I^{d}$	$-F^{d}$	0	0	10,000	10,000	200	-2,700	7,500	150	-2,650	5,000	100	-2,600	2,500	50	-2,550	0	0	0	0
	$+C^{e}$	$+I^{e}$	$-F^{e}$	0	0	6,000	6,000	1,154	-231	6,923	2,238	-448	8,713	3,299	-660	11,353	4,477	-895	14,934	5,783	-20,717	0

Figure 23. Full-scale project strip with all the figures (sheet “SplitScreenStrip”)

3 MODULE 3: BUILDING THE STRIPS OF OTHER MATRIX REPRESENTATIONS

Once constructed the full-scale strip, more compact representations of the strip can be drawn from it to act as summarized statements highlighting specific accounting and/or financial magnitudes that may be of interest to the analyst. Among the many framings that the split-screen Matrix (and Strip) can take, we show how to draw the Matrix with costs classified by nature, the Matrix with costs

classified by function, the Matrix showing the net operating liabilities, and the Matrix highlighting the net operating working capital (see also Part I, sections 6-8).

3.1 Expanded Matrices

If the analyst wants to build an expanded Matrix and the related strip, it suffices to consolidate the relevant items of the full-scale Matrix and related strip. Alternatively, the analyst may start from some other (higher-scale) Matrix representation to get the same result.

The expanded Matrix by nature

Suppose the analyst wants to build the expanded Matrix by nature and the associated strip (Part I, eq. (27)). To do so, it suffices to consolidate, respectively, the rows pertaining to the firm's suppliers and the items pertaining with the firm's employees.

Full-scale Matrix

$\mathbf{C}_t$	$\mathbf{C}_{t-1}$	$+\mathbf{I}_t$	$-\mathbf{F}_t$
$+\mathrm{AR}_t$	$+\mathrm{AR}_{t-1}$	$+\mathrm{S}_t$	$-F_t^{ar}$
$+\mathrm{Inv}_t$	$+\mathrm{Inv}_{t-1}$	$+\Delta\mathrm{Inv}_t$	-0
$-\mathrm{AP}_t^m$	$-\mathrm{AP}_{t-1}^m$	$-\mathrm{COP}_t^m$	$+F_t^{ap,m}$
$-\mathrm{AP}_t^{nm}$	$-\mathrm{AP}_{t-1}^{nm}$	$-\mathrm{COP}_t^{nm}$	$+F_t^{ap,nm}$
$-\mathrm{SWP}_t^m$	$-\mathrm{SWP}_{t-1}^m$	$-\mathrm{LC}_t^m$	$+F_t^{swp,m}$
$-\mathrm{SWP}_t^{nm}$	$-\mathrm{SWP}_{t-1}^{nm}$	$-\mathrm{LC}_t^{nm}$	$+F_t^{swp,nm}$
$+\mathrm{NFA}_t$	$+\mathrm{NFA}_{t-1}$	$-\mathrm{Dep}_t$	$-F_t^{nfa}$
$-\mathrm{TP}_t$	$-\mathrm{TP}_{t-1}$	$-\mathrm{T}_t$	$+F_t^{tp}$
$+C_t^l$	$+C_{t-1}^l$	$+I_t^l$	$-F_t^l$
$+C_t^d$	$+C_{t-1}^d$	$+I_t^d$	$-F_t^d$
$+C_t^e$	$+C_{t-1}^e$	$+I_t^e$	$-F_t^e$

$\Rightarrow$

Expanded Matrix (by nature)

$\mathbf{C}_t$	$\mathbf{C}_{t-1}$	$+\mathbf{I}_t$	$-\mathbf{F}_t$
$+\mathrm{AR}_t$	$+\mathrm{AR}_{t-1}$	$+\mathrm{S}_t$	$-F_t^{ar}$
$+\mathrm{Inv}_t$	$+\mathrm{Inv}_{t-1}$	$+\Delta\mathrm{Inv}_t$	-0
$-\mathrm{AP}_t$	$-\mathrm{AP}_{t-1}$	$-\mathrm{COP}_t$	$+F_t^{ap}$
$-\mathrm{SWP}_t$	$-\mathrm{SWP}_{t-1}$	$-\mathrm{LC}_t$	$+F_t^{swp}$
$+\mathrm{NFA}_t$	$+\mathrm{NFA}_{t-1}$	$-\mathrm{Dep}_t$	$-F_t^{nfa}$
$-\mathrm{TP}_t$	$-\mathrm{TP}_{t-1}$	$-\mathrm{T}_t$	$+F_t^{tp}$
$+C_t^l$	$+C_{t-1}^l$	$+I_t^l$	$-F_t^l$
$+C_t^d$	$+C_{t-1}^d$	$+I_t^d$	$-F_t^d$
$+C_t^e$	$+C_{t-1}^e$	$+I_t^e$	$-F_t^e$

Figure 24. From full-scale Matrix to expanded Matrix by nature

Spreadsheet Modeling: From full-scale Matrix to expanded Matrix (by nature)

The analyst lays out a range of cells in the sheet "SplitScreenStrip" (rows 25:35) with the structure of the full-scale Matrix (rows 11:23), but with fewer rows. More precisely, on one hand, the analyst merges the rows pertaining to the suppliers (rows 15:16) into one row (row 29); on the other hand, the analyst merges the rows pertaining to the employees (rows 17:18) into one row (row 30) by means of a simple sum of the relevant items.

Operationally, one first fills column G (cells G27:G35). To do so, one consolidates the manufacturing and nonmanufacturing AP:

$$\underbrace{\mathrm{G29}}_{-\mathrm{AP}_{-1}} = \underbrace{\mathrm{G15}}_{-\mathrm{AP}_{-1}^m} + \underbrace{\mathrm{G16}}_{-\mathrm{AP}_{-1}^{nm}},$$

and the manufacturing and nonmanufacturing SWP:

$$\underbrace{\mathrm{G30}}_{-\mathrm{SWP}_{-1}} = \underbrace{\mathrm{G17}}_{-\mathrm{SWP}_{-1}^m} + \underbrace{\mathrm{G18}}_{-\mathrm{SWP}_{-1}^{nm}}.$$

The remaining cells of column G remain unvaried, so one just need to add the proper link. For example, as regards the AR,

$$\underbrace{\text{G27}}_{\text{AR}_{-1}} = \underbrace{\text{G13}}_{\text{AR}_{-1}}.$$

Once column G has been filled, the analyst needs only copy and paste (or drag the fill handle) to fill the cells of all periods (see Figure 25).

n	5
τ	30%
i^l	3.80%
i^d	2.00%
α	20%

				G	H	I	J	K	L	M	N	O	P
				C	I	F	C	I	F	C	I	F	C
				-1	0	0	0	1	1	1	2	2	2
Project: Full-scale Matrix													
C	**I**	**−F**		**C_{-1}**	**$+I_0$**	**$-F_0$**	**C_0**	**$+I_1$**	**$-F_1$**	**C_1**	**$+I_2$**	**$-F_2$**	**C_2**
+ AR	+ S	$-F^{ar}$		0	0	0	0	60,000	-45,205	14,795	66,000	-64,521	16,274
+ Inv	+ ΔInv	− 0		0	3,750	0	3,750	375	0	4,125	413	0	4,538
$-AP^{m}$	$-COP^{m}$	$+F^{ap,m}$		0	-3,750	0	-3,750	-15,375	3,750	-15,375	-16,913	15,375	-16,913
$-AP^{nm}$	$-COP^{nm}$	$+F^{ap,nm}$		0	0	0	0	-9,000	0	-9,000	-9,900	9,000	-9,900
$-SWP^{m}$	$-LC^{m}$	$+F^{swp,m}$		0	0	0	0	-24,000	24,000	0	-26,400	26,400	0
$-SWP^{nm}$	$-LC^{nm}$	$+F^{swp,nm}$		0	0	0	0	-6,000	6,000	0	-6,000	6,000	0
+ NFA	− Dep	$-F^{nfa}$		0	0	20,000	20,000	-4,000	0	16,000	-4,000	0	12,000
− TP	− T	$+F^{tp}$		0	0	0	0	-494	494	0	-959	959	0
$+C^{l}$	$+I^{l}$	$-F^{l}$		0	0	-4,000	-4,000	-152	8,030	3,878	147	3,689	7,714
$+C^{d}$	$+I^{d}$	$-F^{d}$		0	0	10,000	10,000	200	-2,700	7,500	150	-2,650	5,000
$+C^{e}$	$+I^{e}$	$-F^{e}$		0	0	6,000	6,000	1,154	-231	6,923	2,238	-448	8,713
Expanded Matrix (by nature)													
C	**I**	**−F**		**C_{-1}**	**$+I_0$**	**$-F_0$**	**C_0**	**$+I_1$**	**$-F_1$**	**C_1**	**$+I_2$**	**$-F_2$**	**C_2**
+ AR	+ S	$-F^{ar}$		0	0	0	0	60,000	-45,205	14,795	66,000	-64,521	16,274
+ Inv	+ ΔInv	− 0		0	3,750	0	3,750	375	0	4,125	413	0	4,538
− AP	− COP	$+F^{ap}$		0	-3,750	0	-3,750	-24,375	3,750	-24,375	-26,813	24,375	-26,813
− SWP	− LC	$+F^{swp}$		0	0	0	0	-30,000	30,000	0	-32,400	32,400	0
+ NFA	− Dep	$-F^{nfa}$		0	0	20,000	20,000	-4,000	0	16,000	-4,000	0	12,000
− TP	− T	$+F^{tp}$		0	0	0	0	-494	494	0	-959	959	0
$+C^{l}$	$+I^{l}$	$-F^{l}$		0	0	-4,000	-4,000	-152	8,030	3,878	147	3,689	7,714
$+C^{d}$	$+I^{d}$	$-F^{d}$		0	0	10,000	10,000	200	-2,700	7,500	150	-2,650	5,000
$+C^{e}$	$+I^{e}$	$-F^{e}$		0	0	6,000	6,000	1,154	-231	6,923	2,238	-448	8,713

Figure 25. The strip of expanded Matrices (by nature) (sheet "SplitScreenStrip")

The expanded Matrix by function

Suppose the analyst wants to build the expanded Matrix and the associated strip by function (Part I, eq. (36) and Figure 3 therein). To do so, it suffices to consolidate the items row pertaining to the firm's suppliers and the items pertaining with the firm's employees.

Full-scale Matrix

$\mathbf{C_t}$	$\mathbf{C_{t-1}}$	$\mathbf{+I_t}$	$\mathbf{-F_t}$
$+\mathrm{AR}_t$	$+\mathrm{AR}_{t-1}$	$+\mathrm{S}_t$	$-F_t^{ar}$
$+\mathrm{Inv}_t$	$+\mathrm{Inv}_{t-1}$	$+\Delta\mathrm{Inv}_t$	-0
$-\mathrm{AP}_t^m$	$-\mathrm{AP}_{t-1}^m$	$-\mathrm{COP}_t^m$	$+F_t^{ap,m}$
$-\mathrm{AP}_t^{nm}$	$-\mathrm{AP}_{t-1}^{nm}$	$-\mathrm{COP}_t^{nm}$	$+F_t^{ap,nm}$
$-\mathrm{SWP}_t^m$	$-\mathrm{SWP}_{t-1}^m$	$-\mathrm{LC}_t^m$	$+F_t^{swp,m}$
$-\mathrm{SWP}_t^{nm}$	$-\mathrm{SWP}_{t-1}^{nm}$	$-\mathrm{LC}_t^{nm}$	$+F_t^{swp,nm}$
$+\mathrm{NFA}_t$	$+\mathrm{NFA}_{t-1}$	$-\mathrm{Dep}_t$	$-F_t^{nfa}$
$-\mathrm{TP}_t$	$-\mathrm{TP}_{t-1}$	$-\mathrm{T}_t$	$+F_t^{tp}$
$+C_t^l$	$+C_{t-1}^l$	$+I_t^l$	$-F_t^l$
$+C_t^d$	$+C_{t-1}^d$	$+I_t^d$	$-F_t^d$
$+C_t^e$	$+C_{t-1}^e$	$+I_t^e$	$-F_t^e$

$\Rightarrow$

Expanded Matrix (by function)

$\mathbf{C_t}$	$\mathbf{C_{t-1}}$	$\mathbf{+I_t}$	$\mathbf{-F_t}$
$+\mathrm{AR}_t$	$+\mathrm{AR}_{t-1}$	$+\mathrm{S}_t$	$-F_t^{ar}$
$-\mathrm{NOL}_t^m$	$-\mathrm{NOL}_{t-1}^m$	$-\mathrm{COGS}_t$	$+F_t^{cogs}$
$-\mathrm{NOL}_t^{nm}$	$-\mathrm{NOL}_{t-1}^{nm}$	$-\mathrm{SGA}_t$	$+F_t^{swp}$
$+\mathrm{NFA}_t$	$+\mathrm{NFA}_{t-1}$	$-\mathrm{Dep}_t$	$-F_t^{nfa}$
$-\mathrm{TP}_t$	$-\mathrm{TP}_{t-1}$	$-\mathrm{T}_t$	$+F_t^{tp}$
$+C_t^l$	$+C_{t-1}^l$	$+I_t^l$	$-F_t^l$
$+C_t^d$	$+C_{t-1}^d$	$+I_t^d$	$-F_t^d$
$+C_t^e$	$+C_{t-1}^e$	$+I_t^e$	$-F_t^e$

Figure 26. From full-scale Matrix to expanded Matrix by function

Spreadsheet Modeling: From full-scale Matrix to expanded Matrix (by function)

The analyst prepares a range of cells in the sheet "SplitScreenStrip" (rows 37:46) with the structure of the full-scale Matrix (rows 11:23), but with fewer rows. More precisely, on one hand, the analyst merges the inventory and the manufacturing costs (rows 14, 15 and 17) into one row (row 40); on the other hand, the analyst merges the nonmanufacturing costs (rows 16 and 18) into one row (row 41). Consolidation is obtained by means of a simple sum of the relevant items. Operationally, one first fills column G (cells G39:G46). To do so, one consolidates the manufacturing book values,

$$\underbrace{\mathrm{G40}}_{-\mathrm{NOL}_{-1}^m} = \underbrace{\mathrm{G14}}_{\mathrm{Inv}_{-1}} + \underbrace{\mathrm{G15}}_{-\mathrm{AP}_{-1}^m} + \underbrace{\mathrm{G17}}_{-\mathrm{SWP}_{-1}^m}$$

and the nonmanufacturing book values,

$$\underbrace{\mathrm{G41}}_{-\mathrm{NOL}_{-1}^{nm}} = \underbrace{\mathrm{G16}}_{-\mathrm{AP}_{-1}^{nm}} + \underbrace{\mathrm{G18}}_{-\mathrm{SWP}_{-1}^{nm}}$$

The remaining cells of column G remain unvaried, so one just need to add the proper link. For example, as regards the AR,

$$\underbrace{\mathrm{G39}}_{\mathrm{AR}_{-1}} = \underbrace{\mathrm{G13}}_{\mathrm{AR}_{-1}}$$

Once columns G is filled, the analyst needs only copy and paste (or drag the fill handle) to fill the cells of all periods (see Figure 1Figure 27).

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1																
2			n	5												
3			τ	30%												
4			i^l	3.80%												
5			i^d	2.00%												
6			α	20%												
7																
8							C	I	F	C	I	F	C	I	F	C
9							-1	0	0	0	1	1	1	2	2	2
10																
11	**Project: Full-scale Matrix**															
12				**C**	**I**	**−F**	**C_{-1}**	**$+I_0$**	**$-F_0$**	**C_0**	**$+I_1$**	**$-F_1$**	**C_1**	**$+I_2$**	**$-F_2$**	**C_2**
13				+ AR	+ S	$-F^{ar}$	0	0	0	0	60,000	-45,205	14,795	66,000	-64,521	16,274
14				+ Inv	+ ΔInv	− 0	0	3,750	0	3,750	375	0	4,125	413	0	4,538
15				$-AP^m$	$-COP^m$	$+F^{ap,m}$	0	-3,750	0	-3,750	-15,375	3,750	-15,375	-16,913	15,375	-16,913
16				$-AP^{nm}$	$-COP^{nm}$	$+F^{ap,nm}$	0	0	0	0	-9,000	0	-9,000	-9,900	9,000	-9,900
17				$-SWP^m$	$-LC^m$	$+F^{swp,m}$	0	0	0	0	-24,000	24,000	0	-26,400	26,400	0
18				$-SWP^{nm}$	$-LC^{nm}$	$+F^{swp,nm}$	0	0	0	0	-6,000	6,000	0	-6,000	6,000	0
19				+ NFA	− Dep	$-F^{nfa}$	0	0	20,000	20,000	-4,000	0	16,000	-4,000	0	12,000
20				− TP	− T	$+F^{tp}$	0	0	0	0	-494	494	0	-959	959	0
21				$+C^l$	$+I^l$	$-F^l$	0	0	-4,000	-4,000	-152	8,030	3,878	147	3,689	7,714
22				$+C^d$	$+I^d$	$-F^d$	0	0	10,000	10,000	200	-2,700	7,500	150	-2,650	5,000
23				$+C^e$	$+I^e$	$-F^e$	0	0	6,000	6,000	1,154	-231	6,923	2,238	-448	8,713
24																
37	**Expanded Matrix (by function)**															
38				**C**	**I**	**−F**	**C_{-1}**	**$+I_0$**	**$-F_0$**	**C_0**	**$+I_1$**	**$-F_1$**	**C_1**	**$+I_2$**	**$-F_2$**	**C_2**
39				+ AR	+ S	$-F^{ar}$	0	0	0	0	60,000	-45,205	14,795	66,000	-64,521	16,274
40				$-NOL^m$	− COGS	$+F^{cogs}$	0	0	0	0	-39,000	27,750	-11,250	-42,900	41,775	-12,375
41				$-NOL^{nm}$	− SGA	$+F^{sga}$	0	0	0	0	-15,000	6,000	-9,000	-15,900	15,000	-9,900
42				+ NFA	− Dep	$-F^{nfa}$	0	0	20,000	20,000	-4,000	0	16,000	-4,000	0	12,000
43				− TP	− T	$+F^{tp}$	0	0	0	0	-494	494	0	-959	959	0
44				$+C^l$	$+I^l$	$-F^l$	0	0	-4,000	-4,000	-152	8,030	3,878	147	3,689	7,714
45				$+C^d$	$+I^d$	$-F^d$	0	0	10,000	10,000	200	-2,700	7,500	150	-2,650	5,000
46				$+C^e$	$+I^e$	$-F^e$	0	0	6,000	6,000	1,154	-231	6,923	2,238	-448	8,713
47																

Figure 27. The strip of expanded Matrices (by function) (sheet "SplitScreenStrip")

3.2 NOL-framed and NOWC-framed Matrices

The NOL-framed Matrix

If one wants to build a NOL-framed Matrix and associated strip, one may simply aggregate the items of the manufacturing and nonmanufacturing NOL obtained in the expanded Matrix (Figure 27, rows 40 and 41. See also Part I, eq. (39)) Alternatively, starting from the full-scale Matrix, one consolidates the rows pertaining to the net operating liabilities.

Spreadsheet Modeling: From Expanded Matrix (by function) to NOL-framed Matrix

The analyst lays out a range of cells in the sheet "SplitScreenStrip" (rows 48:56) with the structure of the Expanded Matrix (by function) (rows 37:46), but with fewer rows. Specifically, the analyst merges the rows pertaining to the net operating liabilities (rows 40:41) into one single row (row 51). Consolidation is obtained by means of a simple sum of the relevant items. Operationally, one proceeds as made for the previous matrix representations by first filling column G (cells G50:G56). To do so, one consolidates the rows of the Expanded Matrix (by function) which form the group of net operating liabilities:

$$\underbrace{\text{G51}}_{-\text{NOL}_{-1}} = \text{SUM}\left(\underbrace{\text{G41:G42}}_{-\text{NOL}_{-1}^{m}-\text{NOL}_{-1}^{nm}}\right)$$

The remaining cells of column G remain unvaried, so one just needs to add the proper links. For example, as regards the AR,

$$\underbrace{\text{G50}}_{\text{AR}_{-1}} = \underbrace{\text{G39}}_{\text{AR}_{-1}}$$

Once column G has been filled, the analyst need only copy and paste (or drag the fill handle) to fill the cells of all periods (see Figure 28).

n	5	
τ	30%	
i^l	3.80%	
i^d	2.00%	
α	20%	

				F	C	I	F	C	I	F	C	
				0	0	1	1	1	2	2	2	
Expanded Matrix (by function)												
C	**I**	**–F**	**$-F_0$**	**C_0**	**$+I_1$**	**$-F_1$**	**C_1**	**$+I_2$**	**$-F_2$**	**C_2**		
+ AR	+ S	$-F^{ar}$	0	0	60,000	-45,205	14,795	66,000	-64,521	16,274		
$-\text{NOL}^m$	− COGS	$+F^{cogs}$	0	0	-39,000	27,750	-11,250	-42,900	41,775	-12,375		
$-\text{NOL}^{nm}$	− SGA	$+F^{sga}$	0	0	-15,000	6,000	-9,000	-15,900	15,000	-9,900		
+ NFA	− Dep	$-F^{nfa}$	20,000	20,000	-4,000	0	16,000	-4,000	0	12,000		
− TP	− T	$+F^{tp}$	0	0	-494	494	0	-959	959	0		
$+C^l$	$+I^l$	$-F^l$	-4,000	-4,000	-152	8,030	3,878	147	3,689	7,714		
$+C^d$	$+I^d$	$-F^d$	10,000	10,000	200	-2,700	7,500	150	-2,650	5,000		
$+C^e$	$+I^e$	$-F^e$	6,000	6,000	1,154	-231	6,923	2,238	-448	8,713		
NOL-framed Matrix												
C	**I**	**–F**	**$-F_0$**	**C_0**	**$+I_1$**	**$-F_1$**	**C_1**	**$+I_2$**	**$-F_2$**	**C_2**		
+ AR	+ S	$-F^{ar}$	0	0	60,000	-45,205	14,795	66,000	-64,521	16,274		
− NOL	+ OpC	$+F^{nol}$	0	0	-54,000	33,750	-20,250	-58,800	56,775	-22,275		
+ NFA	− Dep	$-F^{nfa}$	20,000	20,000	-4,000	0	16,000	-4,000	0	12,000		
− TP	− T	$+F^{tp}$	0	0	-494	494	0	-959	959	0		
$+C^l$	$+I^l$	$-F^l$	-4,000	-4,000	-152	8,030	3,878	147	3,689	7,714		
$+C^d$	$+I^d$	$-F^d$	10,000	10,000	200	-2,700	7,500	150	-2,650	5,000		
$+C^e$	$+I^e$	$-F^e$	6,000	6,000	1,154	-231	6,923	2,238	-448	8,713		

Figure 28. The strip of NOL-framed Matrices (sheet "SplitScreenStrip")

The NOWC-framed Matrix

If one wants to build a NOWC-framed Matrix and corresponding strip, one may simply aggregate the relevant rows starting from the NOL-framed Matrix. This result in a 5-area Matrix (see Part I, eq. (50)).

Spreadsheet Modeling: From NOL-framed Matrix to NOWC-framed Matrix

The analyst lays out a range of cells (rows 58:64) in the sheet "SplitScreenStrip" with the structure of the NOL-framed Matrix (rows 48:56), but with fewer rows. Specifically, the analyst merges the rows pertaining to net operating working capital (rows 50:51 and 53) into one single row (row 60). Consolidation is obtained by means of a simple sum of the relevant items. Operationally, one proceeds as made for the previous matrix representations by first filling column G (cells G60:G64). To do so, one consolidates the rows in the full-scale Matrix which form the group net operating working capital:

$$\underbrace{\text{G60}}_{+\text{NOWC}_{-1}} = \text{SUM}(\ \underbrace{\text{G50:G51, G53}}_{\text{AR}_{-1}-\text{NOL}_{-1}-\text{TP}_{-1}}\ )$$

The remaining cells of column G remain unvaried, so one just need to add the proper links. For example, as regards NFA,

$$\underbrace{\text{G61}}_{\text{NFA}_{-1}} = \underbrace{\text{G52}}_{\text{NFA}_{-1}}$$

Once column G is filled, the analyst need only copy and paste (or drag the fill handle) to fill the cells of all periods (see Figure 29).

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1																
2			n	5												
3			τ	30%												
4			i^l	3.80%												
5			i^d	2.00%												
6			α	20%												
7																
8							C	I	F	C	I	F	C	I	F	C
9							-1	0	0	0	1	1	1	2	2	2
47																
48	**NOL-framed Matrix**															
49				**C**	**I**	**–F**	**C_{-1}**	**$+I_0$**	**$-F_0$**	**C_0**	**$+I_1$**	**$-F_1$**	**C_1**	**$+I_2$**	**$-F_2$**	**C_2**
50				+ AR	+ S	$-F^{ar}$	0	0	0	0	60,000	-45,205	14,795	66,000	-64,521	16,274
51				– NOL	+ OpC	$+F^{nol}$	0	0	0	0	-54,000	33,750	-20,250	-58,800	56,775	-22,275
52				+ NFA	– Dep	$-F^{nfa}$	0	0	20,000	20,000	-4,000	0	16,000	-4,000	0	12,000
53				– TP	– T	$+F^{tp}$	0	0	0	0	-494	494	0	-959	959	0
54				$+C^l$	$+I^l$	$-F^l$	0	0	-4,000	-4,000	-152	8,030	3,878	147	3,689	7,714
55				$+C^d$	$+I^d$	$-F^d$	0	0	10,000	10,000	200	-2,700	7,500	150	-2,650	5,000
56				$+C^e$	$+I^e$	$-F^e$	0	0	6,000	6,000	1,154	-231	6,923	2,238	-448	8,713
57																
58	**NOWC-framed Matrix**															
59				**C**	**I**	**–F**	**C_{-1}**	**$+I_0$**	**$-F_0$**	**C_0**	**$+I_1$**	**$-F_1$**	**C_1**	**$+I_2$**	**$-F_2$**	**C_2**
60				+ NOWC	+ EBITDA	$-F^{nowc}$	0	0	0	0	5,506	-10,961	-5,455	6,241	-6,786	-6,001
61				+ NFA	– Dep	$-F^{nfa}$	0	0	20,000	20,000	-4,000	0	16,000	-4,000	0	12,000
62				$+C^l$	$+I^l$	$-F^l$	0	0	-4,000	-4,000	-152	8,030	3,878	147	3,689	7,714
63				$+C^d$	$+I^d$	$-F^d$	0	0	10,000	10,000	200	-2,700	7,500	150	-2,650	5,000
64				$+C^e$	$+I^e$	$-F^e$	0	0	6,000	6,000	1,154	-231	6,923	2,238	-448	8,713
65																

Figure 29. The strip of NOWC-framed Matrices (sheet "SplitScreenStrip")

3.3 Four-area and germ Matrices

It is possible to further shrink the strip and have more compact forms. For example, to favor a synthetic analysis of the project, the analyst may be willing to build a four-area matrix and corresponding strip, where one only the elements related to operating assets, non-operating assets, debt, and equity are reported (Part I, right matrix in eq. (12)). Also, merging the two classes of assets

and the two classes of financings, one may even build the germ strip, where all the accounting and financial items of the project are aggregated (Part I, left matrix in eq. (12)).

The four-area Matrix

To build the four-area Matrix and corresponding strip, it suffices to sum the relevant items from some higher-scale Matrix (NOWC-framed strip, NOL-framed matrix, expanded Matrix, full-scale Matrix).

Spreadsheet Modeling: From NOWC-framed Matrix to four-area Matrix

The analyst lays out in the sheet "SplitScreenStrip" a range of cells (rows 66:71) with the structure of the NOWC-framed Matrix (rows 58:64), but with only four rows, one for each area: operations, liquidity, debt, equity. The analyst proceeds as in the previous Matrices by filling the cells of column G and then copy and paste for the whole strip. The operating capital is obtained as

$$\underbrace{\text{G68}}_{+C^{o}_{-1}} = \text{SUM}(\ \underbrace{\text{G60:G61}}_{\text{NOWC}_{-1}+\text{NFA}_{-1}}\ )$$

For the three remaining cells of column G, it suffices to provide the proper links (see Figure 29).

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1																
2			n	5												
3			τ	30%												
4			i^l	3.80%												
5			i^d	2.00%												
6			α	20%												
7																
8							C	I	F	C	I	F	C	I	F	C
9							-1	0	0	0	1	1	1	2	2	2
57																
58	**NOWC-framed Matrix**															
59				**C**	**I**	**–F**	**C_{-1}**	**$+I_0$**	**$-F_0$**	**C_0**	**$+I_1$**	**$-F_1$**	**C_1**	**$+I_2$**	**$-F_2$**	**C_2**
60				+ NOWC	+ EBITDA	$-F^{nowc}$	0	0	0	0	5,506	-10,961	-5,455	6,241	-6,786	-6,001
61				+ NFA	− Dep	$-F^{nfa}$	0	0	20,000	20,000	-4,000	0	16,000	-4,000	0	12,000
62				$+C^l$	$+I^l$	$-F^l$	0	0	-4,000	-4,000	-152	8,030	3,878	147	3,689	7,714
63				$+C^d$	$+I^d$	$-F^d$	0	0	10,000	10,000	200	-2,700	7,500	150	-2,650	5,000
64				$+C^e$	$+I^e$	$-F^e$	0	0	6,000	6,000	1,154	-231	6,923	2,238	-448	8,713
65																
66	**Four-area Matrix**															
67				**C**	**I**	**–F**	**C_{-1}**	**$+I_0$**	**$-F_0$**	**C_0**	**$+I_1$**	**$-F_1$**	**C_1**	**$+I_2$**	**$-F_2$**	**C_2**
68				$+C^o$	$+I^o$	$-F^o$	0	0	20,000	20,000	1,506	-10,961	10,545	2,241	-6,786	5,999
69				$+C^l$	$+I^l$	$-F^l$	0	0	-4,000	-4,000	-152	8,030	3,878	147	3,689	7,714
70				$+C^d$	$+I^d$	$-F^d$	0	0	10,000	10,000	200	-2,700	7,500	150	-2,650	5,000
71				$+C^e$	$+I^e$	$-F^e$	0	0	6,000	6,000	1,154	-231	6,923	2,238	-448	8,713
72																

Figure 30. Four-area Matrix (sheet "SplitScreenStrip")

The germ Matrix

The germ Matrix is easily built, as usual, by consolidating the rows from some higher-scale Matrix.

Spreadsheet Modeling: From four-area Matrix to germ Matrix

The analyst sets up a range of cells (rows 73:76) in the sheet "SplitScreenStrip" with the same structure of the four-area Matrix (rows 66:71), but with only two rows, one for each area: investment and financing. As in the previous representations, the analyst may start from the full-scale Matrix and

fill column G by aggregation. Specifically, the two cells of column G of the germ Matrix are calculated as follows:

$$\underbrace{\text{G75}}_{c_{-1}^{\text{inv}}} = \text{SUM}(\underbrace{\text{G68:G69}}_{c_{-1}^{o}+c_{-1}^{l}}) \qquad \underbrace{\text{G76}}_{c_{-1}^{\text{fin}}} = \text{SUM}(\underbrace{\text{G70:G71}}_{c_{-1}^{d}+c_{-1}^{e}})$$

Then, one copies and paste to fill the whole strip.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1																
2			n	5												
3			τ	30%												
4			i^l	3.80%												
5			i^d	2.00%												
6			α	20%												
7																
8							C	I	F	C	I	F	C	I	F	C
9							-1	0	0	0	1	1	1	2	2	2
65																
66	**Four-area Matrix**															
67				**C**	**I**	**–F**	**C_{-1}**	**$+I_0$**	**$-F_0$**	**C_0**	**$+I_1$**	**$-F_1$**	**C_1**	**$+I_2$**	**$-F_2$**	**C_2**
68				$+c^o$	$+I^o$	$-F^o$	0	0	20,000	20,000	1,506	-10,961	10,545	2,241	-6,786	5,999
69				$+c^l$	$+I^l$	$-F^l$	0	0	-4,000	-4,000	-152	8,030	3,878	147	3,689	7,714
70				$+c^d$	$+I^d$	$-F^d$	0	0	10,000	10,000	200	-2,700	7,500	150	-2,650	5,000
71				$+c^e$	$+I^e$	$-F^e$	0	0	6,000	6,000	1,154	-231	6,923	2,238	-448	8,713
72																
73	**Germ Matrix**															
74				**C**	**I**	**–F**	**C_{-1}**	**$+I_0$**	**$-F_0$**	**C_0**	**$+I_1$**	**$-F_1$**	**C_1**	**$+I_2$**	**$-F_2$**	**C_2**
75				$+c^{\text{inv}}$	$+I^{\text{inv}}$	$-F^{\text{inv}}$	0	0	16,000	16,000	1,354	-2,931	14,423	2,388	-3,098	13,713
76				$+c^{\text{fin}}$	$+I^{\text{fin}}$	$-F^{\text{fin}}$	0	0	16,000	16,000	1,354	-2,931	14,423	2,388	-3,098	13,713
77																

Figure 31. Germ Matrix (sheet "SplitScreenStrip")

4 MODULE 4: BUILDING THE BENCHMARK SPLIT-SCREEN STRIP

Project appraisal requires the comparison of the project with a benchmark which collects the values, the profits, and the cash flows of the benchmark portfolio. This module is precisely devoted to building the benchmark film (or strip) from the market input data. The mathematical relations are equivalent to the ones used for the project four-area Matrix and strip, but, in this case, we need to flesh out the benchmark values from the prospective cash flows (which are just equal to the project's cash flows), so we use backward induction.

4.1 Benchmark four-area Matrix

To design the benchmark Matrix, the analyst requires the MARRs as well as the cash flows. Specifically, the economic value of each area may be calculated with the backward formula

$$V_{t-1}^{j} = \frac{V_t^j + F_t^{Vj}}{1 + r_t^j}, \qquad j = o, l, d, \qquad t = 1, \dots n$$

with $V_n^o = V_n^l = V_n^d = V_n^e = 0$ and where $F_0^{Vj} = -V_0^j$ and $F_0^{Vj} = F_t^j$ for $t \geq 1$ (see Part I, section 11). The economic value of the equity can be calculated via the law of conservation: $V_t^e = V_t^o + V_t^l - V_t^d$. The cash flows in the benchmark Matrix are the same as in the full-scale Matrix of the project, except for period 0. The column of benchmark profits is calculated as

$$I_t^{Vj} = r^j \cdot V_{t-1}^j, \quad j = o, l, d$$

for the of operating assets, the non-operating assets, and the debt. The benchmark profit for the equity can be calculated with the law of conservation as a residual amount: $I_t^{Ve} = I_t^{Vo} + I_t^{Vl} - I_t^{Vd}$. The operating cash flows can be obtained as sum of the cash-flow components of the operating area in some higher-scale Matrix and the other cash flows are merely linked from some other higher-scale Matrix.

Spreadsheet Modeling: Benchmark Matrix scheme

The analyst prepares a table in a range of cells (rows 81:86) in the sheet "SplitScreenStrip" having the structure of the project four-area Matrix (rows 66:71). The rates r^o, r^l and r^d are recorded in the sheet "Assumption". The analyst can position them in a space above the full-scale Matrix (cells F3:F5) in the sheet "SplitScreenStrip" and use them in the relevant formulas. Cells F3:F5 are linked to the cells that contain the value of r^o, r^l and r^d with the formulas $=$ Assumptions! \$F\$27, $=$ Assumptions! \$F\$28, and $=$ Assumptions! \$F\$29.

Spreadsheet Modeling: Economic values in the benchmark Matrix

For the operating class, the analyst starts by inserting the value 0 for V_5^o, V_5^l, V_5^d, and V_5^e and then calculates the economic value of AR as of period $t = n - 1 = 4$ with the backward formula:

$$\underbrace{\text{V83}}_{V_4^o} = (\underbrace{\text{Y83}}_{V_5^o} - \underbrace{\text{X83}}_{-F_5^{Vo}(=-F_5^o)})/(1 + \underbrace{\$\text{F}\$3}_{r^o})$$

(see Figure 32). For the liquid assets and the debt, it suffices to use:

$$\underbrace{\text{V84}}_{V_4^l} = (\underbrace{\text{Y84}}_{V_5^l} - \underbrace{\text{X84}}_{-F_5^{Vl}(=-F_5^l)})/(1 + \underbrace{\$\text{F}\$4}_{r^l})$$

$$\underbrace{\text{V85}}_{V_4^d} = (\underbrace{\text{Y85}}_{V_5^d} - \underbrace{\text{X85}}_{-F_5^{Vd}(=-F_5^d)})/(1 + \underbrace{\$\text{F}\$5}_{r^d})$$

The equity value is calculated with the law of conservation:

$$\underbrace{\text{V86}}_{V_4^e} = \underbrace{\text{V83}}_{V_4^o} + \underbrace{\text{V84}}_{V_4^l} - \underbrace{\text{V85}}_{V_4^d}$$

While the equations are correct, the value obtained in the cell range V83:V86 are not yet the correct ones, because cells X83:X86 have not yet been occupied by formulas; however, they will be filled with the amounts of cash flows in the next steps, and the value taken on by cells V83:V86 will be automatically updated.

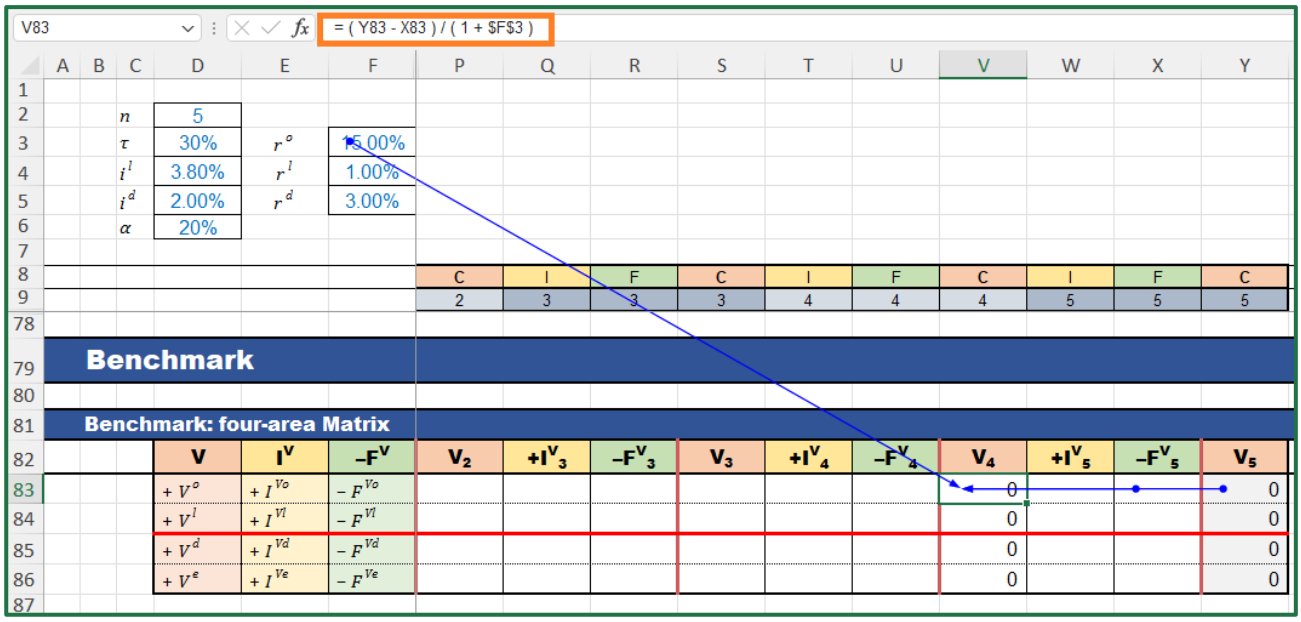


Figure 32. Cell V83 contains the formula for calculating the economic value V_4^o as a function of next-period value V_5^o and cash flow F_5^o (sheet "SplitScreenStrip")

Spreadsheet Modeling: Incomes in the benchmark Matrix

The analyst may start from the operating area and, in particular, from cell W83:

$$\underbrace{\text{W83}}_{I_4^{Vo}} = \underbrace{\text{V83}}_{V_4^o} * \underbrace{\$\text{F}\$3}_{r^o}$$

(see Figure 33).
For the non-operating area and the debt area, respectively, one uses

$$\underbrace{\text{W84}}_{I_4^{Vl}} = \underbrace{\text{V84}}_{V_4^l} * \underbrace{\$\text{F}\$4}_{r^l} \quad \text{and} \quad \underbrace{\text{W85}}_{I_4^{Vd}} = \underbrace{\text{V85}}_{V_4^d} * \underbrace{\$\text{F}\$5}_{r^d}$$

The cash flow of the equity area can be calculated via the law of conservation:

$$\underbrace{\text{W86}}_{I_4^{Ve}} = \underbrace{\text{W83}}_{I_4^{Vo}} + \underbrace{\text{W84}}_{I_4^{Vl}} - \underbrace{\text{W85}}_{I_4^{Vd}}$$

While the equations are correct, the value obtained in the cell range W84:W86 are not yet the correct ones, because cells X83:X86 have not yet been occupied by formulas; however, they will be filled with the amounts of cash flows in the next steps, and the value taken on by cells W84:W86 will be automatically updated.

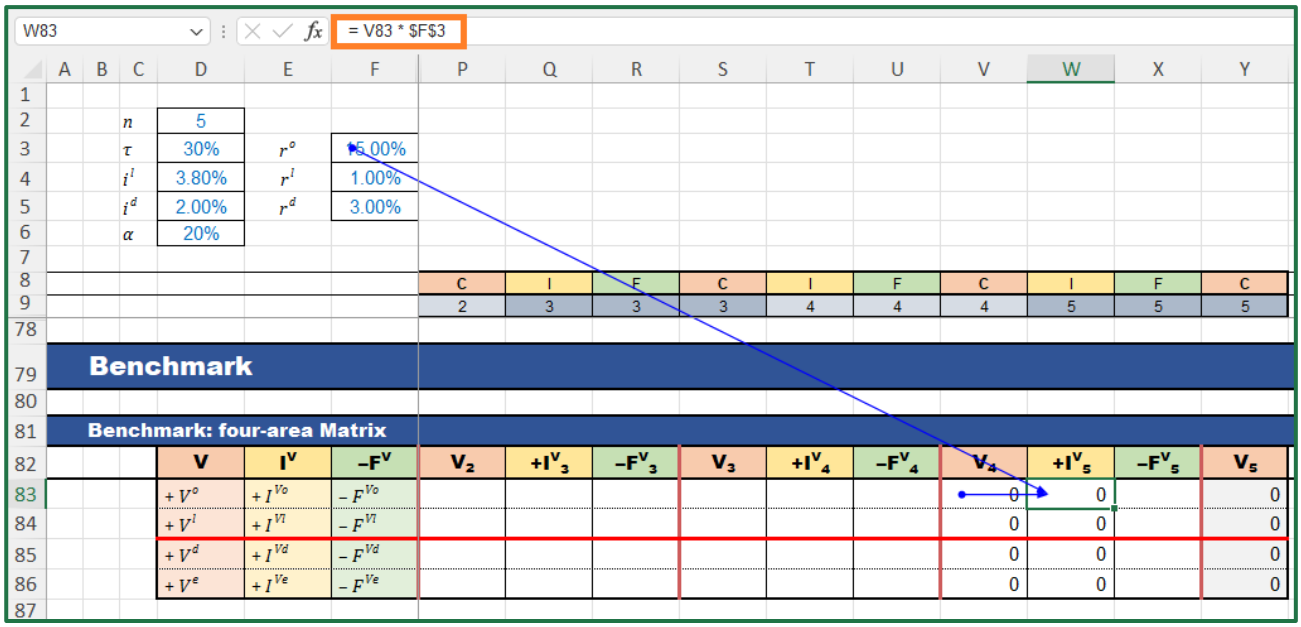


Figure 33. Cell W83 contains the formula for calculating the benchmark profit (sheet "SplitScreenStrip")

Spreadsheet Modeling: Cash flows in the benchmark Matrix

The (changed-in-sign) cash flow may be grabbed by the split-screen strip for $t > 0$, whereas it is equal to the economic value for $t = 0$. So, the analyst may use the following formula for cell X83:

$$\underbrace{\text{X83}}_{-F_4^{Vo}} = \text{IF}(\underbrace{\text{X\$9}}_{\text{period}} = 0, \quad \underbrace{\text{Y83}}_{V_5^o}, \quad \underbrace{\text{X68}}_{-F_4^o})$$

(In such a way, the analyst may later copy and paste the formula to complete the whole strip.)

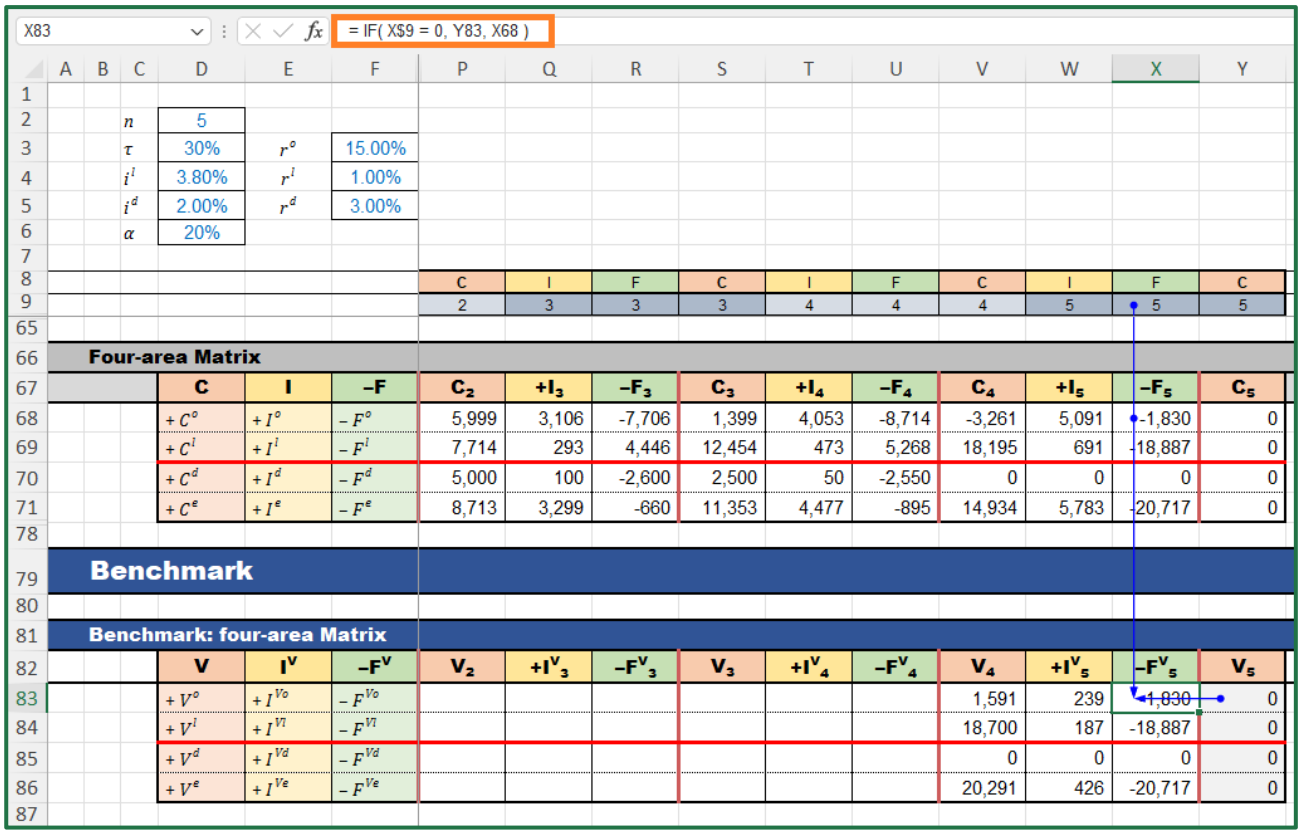
The analyst then copies the formula in X83 and paste it in cell X84 (the non-operating area) and in cell X85 (the debt area).

$$\underbrace{\text{X84}}_{-F_4^{Vl}} = \text{IF}(\underbrace{\text{X\$9}}_{\text{period}} = 0, \underbrace{\text{Y84}}_{V_5^l}, \underbrace{\text{X69}}_{-F_4^l}) \quad \text{and} \quad \underbrace{\text{X85}}_{-F_4^{Vd}} = \text{IF}(\underbrace{\text{X\$9}}_{\text{period}} = 0, \underbrace{\text{Y85}}_{V_5^d}, \underbrace{\text{X70}}_{-F_4^d})$$

The (changed-in-sign) CFE can be calculated with the same formula or via the law of conservation:

$$\underbrace{\text{X86}}_{-F_4^{Ve}} = \underbrace{\text{X83}}_{-F_4^{Vo}} + \underbrace{\text{X84}}_{-F_4^{Vl}} - \underbrace{\text{X85}}_{-F_4^{Vd}}$$

(see Figure 34).



X83 = IF(X$9 = 0, Y83, X68)

		n	5												
		τ	30%	r^o	15.00%										
		i^l	3.80%	r^l	1.00%										
		i^d	2.00%	r^d	3.00%										
		α	20%												
						C	I	F	C	I	F	C	I	F	C
						2	3	3	3	4	4	4	5	5	5
Four-area Matrix															
		C	**I**	**–F**		**C_2**	**$+I_3$**	**$-F_3$**	**C_3**	**$+I_4$**	**$-F_4$**	**C_4**	**$+I_5$**	**$-F_5$**	**C_5**
		$+C^o$	$+I^o$	$-F^o$		5,999	3,106	-7,706	1,399	4,053	-8,714	-3,261	5,091	-1,830	0
		$+C^l$	$+I^l$	$-F^l$		7,714	293	4,446	12,454	473	5,268	18,195	691	-18,887	0
		$+C^d$	$+I^d$	$-F^d$		5,000	100	-2,600	2,500	50	-2,550	0	0	0	0
		$+C^e$	$+I^e$	$-F^e$		8,713	3,299	-660	11,353	4,477	-895	14,934	5,783	-20,717	0
Benchmark															
Benchmark: four-area Matrix															
		V	**I^V**	**$-F^V$**		**V_2**	**$+I^V_3$**	**$-F^V_3$**	**V_3**	**$+I^V_4$**	**$-F^V_4$**	**V_4**	**$+I^V_5$**	**$-F^V_5$**	**V_5**
		$+V^o$	$+I^{Vo}$	$-F^{Vo}$								1,591	239	-1,830	0
		$+V^l$	$+I^{Vl}$	$-F^{Vl}$								18,700	187	-18,887	0
		$+V^d$	$+I^{Vd}$	$-F^{Vd}$								0	0	0	0
		$+V^e$	$+I^{Ve}$	$-F^{Ve}$								20,291	426	-20,717	0

Figure 34. Cell X83 contains the formula for calculating the OCF (sheet "SplitScreenStrip")

The analyst can now copy the cell range V83: X86 and paste it in the cell range G83: 86 (or, more simply, use backwards the fill handle).

4.2 Benchmark: germ Matrix

Just as with the project Matrix, it is possible to construct the benchmark germ Matrix.

Spreadsheet Modeling: From benchmark four-area Matrix to benchmark germ Matrix

The analyst lays out a range of cells (rows 88:91) in the sheet "SplitScreenStrip" with the structure of the project germ Matrix (rows 73:76). The formulas for producing the benchmark germ Matrix are trivial:

$$\underbrace{\mathrm{G90}}_{V_{-1}^{\mathrm{inv}}} = \mathrm{SUM}(\underbrace{\mathrm{G83:G84}}_{V_{-1}^{o}+V_{-1}^{l}}) \qquad \underbrace{\mathrm{G91}}_{V_{-1}^{\mathrm{fin}}} = \mathrm{SUM}(\underbrace{\mathrm{G85:G86}}_{V_{-1}^{d}+V_{-1}^{e}})$$

whence the whole strip is obtained by means of copying and pasting.

4.3 Market Value Added Matrix

The analyst can build the Market Value Added (MVA) Matrix and the related strip by computing the differences between the profits of the projects and the benchmark profits (see Part I, eq. (84)).

Spreadsheet Modeling: MVA four-area Matrix

The analyst prepares a cell range (rows 96:101) in the "SplitScreenStrip" sheet with the structure of the four-area Matrices and fills each cell of it with the differences between the corresponding cells of the benchmark four-area Matrix (rows 83:86) and the project four-area Matrix (rows 68:71). For example, starting from the first cell of the table,

$$\text{G98} = \underbrace{\text{G83}}_{V^o_{-1}} - \underbrace{\text{G68}}_{C^o_{-1}}, \quad \text{G99} = \underbrace{\text{G84}}_{V^l_{-1}} - \underbrace{\text{G69}}_{C^l_{-1}}, \quad \text{G100} = \underbrace{\text{G85}}_{V^d_{-1}} - \underbrace{\text{G70}}_{C^d_{-1}}, \quad \text{G101} = \underbrace{\text{G86}}_{V^e_{-1}} - \underbrace{\text{G71}}_{C^e_{-1}}$$

Then, it suffices to copy G98:G101 and paste for the whole strip H98:Y101 (or, more simply, use backwards the fill handle).

As the reader can see, the range I98:I101 shows the NPV for the various areas: $\text{NPV}^o = 5{,}622$ $\text{NPV}^l = 1{,}025$, $\text{NPV}^d = -236$, and $\text{NPV}^e = 6{,}882$ (see also section 7.2).
The cells reporting the difference $I_t^{Vj} - I_t^j$ (H98:H101, K98:K101, N98:N101, Q98:Q101, T98:T101, W98:W101) give voice to the economic residual income (changed in sign. See next section for details).

Spreadsheet Modeling: MVA germ Matrix

The analyst prepares a range of cells (rows 103:106) in the sheet "SplitScreenStrip" with the structure of the germ Matrices. To get the two values one merely turns to the MVA four-area Matrix and sums:

$$\underbrace{\text{G105}}_{\text{MVA}^{\text{inv}}_{-1}} = \text{SUM}(\underbrace{\text{G98:G99}}_{\text{MVA}^o_{-1}+\text{MVA}^l_{-1}}) \qquad \underbrace{\text{G106}}_{\text{MVA}^{\text{fin}}_{-1}} = \text{SUM}(\underbrace{\text{G100:G101}}_{\text{MVA}^d_{-1}+\text{MVA}^e_{-1}})$$

Then, it suffices to copy G105:G106 and paste for the whole strip H105:Y106 (or, more simply, use the fill handle).

n	5			
τ	30%		r^o	15.00%
i^l	3.80%		r^l	1.00%
i^d	2.00%		r^d	3.00%
α	20%			

	G	H	I	J	K	L	M	N	O
8	C	I	F	C	I	F	C	I	F
9	-1	0	0	0	1	1	1	2	2

	G	H	I	J	K	L	M	N	O	P
8	C	I	F	C	I	F	C	I	F	C

	P	Q	R	S	T	U	V	W	X	Y
8	C	I	F	C	I	F	C	I	F	C
9	2	3	3	3	4	4	4	5	5	5

Market Value Added

Market Value Added: Four-area Matrix

			$V_{-1}-C_{-1}$	$+I^V_0-I_0$	$-(F^V_0-F_0)$	V_0-C_0	$+I^V_1-I_1$	$-(F^V_1-F_1)$	V_1-C_1	$+I^V_2-I_2$	$-(F^V_2-F_2)$
MVA^o	ERI^o	$-(F^{Vo}-F^o)$	0	0	5,622	5,622	2,338	0	7,959	535	0
MVA^l	ERI^l	$-(F^{Vl}-F^l)$	0	0	1,025	1,025	122	0	1,147	-97	0
MVA^d	ERI^d	$-(F^{Vd}-F^d)$	0	0	-236	-236	93	0	-143	71	0
MVA^e	ERI^e	$-(F^{Ve}-F^e)$	0	0	6,882	6,882	2,367	-0	9,249	367	0

	V_2-C_2	$+I^V_3-I_3$	$-(F^V_3-F_3)$	V_3-C_3	$+I^V_4-I_4$	$-(F^V_4-F_4)$	V_4-C_4	$+I^V_5-I_5$	$-(F^V_5-F_5)$	V_5-C_5
MVA^o	8,494	-932	0	7,562	-2,709	0	4,853	-4,853	0	0
MVA^l	1,050	-206	0	845	-340	0	504	-504	0	0
MVA^d	-72	48	0	-24	24	0	0	0	0	0
MVA^e	9,616	-1,185	0	8,431	-3,074	0	5,357	-5,357	0	0

Market Value Added: Germ Matrix

			$V_{-1}-C_{-1}$	$+I^V_0-I_0$	$-(F^V_0-F_0)$	V_0-C_0	$+I^V_1-I_1$	$-(F^V_1-F_1)$	V_1-C_1	$+I^V_2-I_2$	$-(F^V_2-F_2)$
MVA^{inv}	ERI^{fin}	$-(F^{V\text{fin}}-F^{\text{fin}})$	0	0	6,647	6,647	2,460	0	9,107	438	0
MVA^{inv}	ERI^{inv}	$-(F^{V\text{inv}}-F^{\text{inv}})$	0	0	6,647	6,647	2,460	-0	9,107	438	0

	V_2-C_2	$+I^V_3-I_3$	$-(F^V_3-F_3)$	V_3-C_3	$+I^V_4-I_4$	$-(F^V_4-F_4)$	V_4-C_4	$+I^V_5-I_5$	$-(F^V_5-F_5)$	V_5-C_5
MVA^{inv} (105)	9,544	-1,138	0	8,407	-3,050	0	5,357	-5,357	0	0
MVA^{inv} (106)	9,544	-1,138	0	8,407	-3,050	0	5,357	-5,357	0	0

Figure 35. Market Value Added: four-area Matrix and germ Matrix (sheet "SplitScreenStrip")

5 MODULE 5: PERFORMING THE SINGLE-PERIOD ANALYSIS

The whole project is described in detail in the split-screen film. However, a convenient representation of it is possible which favors the detailed analysis of a single period. One creates a worksheet named

“SinglePeriod” where the Matrix will be located. In it, one can use the preferred framing of the split-screen Matrix. For example, we propose a framing where the BS at a given date is illustrated and one where the CFS is illustrated. The former facilitates the comprehension of how the project’s book value is apportioned across the various areas (customers, suppliers, employees, etc.) and how it is affected diachronically by the income components and the cash-flow components; the latter facilitates the comprehension of how the project’s cash flow is apportioned across the various areas and how it is diachronically determined by income and change of book values. These configurations turn the project’s split-screen strip into a sequence of snapshots describing the accounting and financial magnitudes and their relationships for one period.

Spreadsheet Modeling: Building the set of Matrices for single-period analysis

The analyst builds the structure of two split-screen Matrices, in the configuration of full-scale Matrix and in the form of BS and CFS (see Figure 36). Cell E2 indicates the period. The analyst can apply the Data Validation to restrict the values to the periods of the project, allowing only the values that are in the list “0, 1, 2, 3, 4, 5” or creating a drop-down list with these values.8 Some spreadsheets software allow to use a range as source list, even if it is in another worksheet: in this case, it is possible to use the range “=PreCalc!$K$2:$P$2”. Three columns on the left (columns D, E and F, same as in the sheets “PreCalc” and “SplitScreenStrip”) contain the labels of the rows (“+AR”, “+Inv”, etc.). Below the structure of the matrix, a row (row 4) contains the descriptor of the column content. This content is C for Capital, I for Income, and F for Cash Flow. It is variable and depends on the value of cell E2: “C-1”, “+I0”, “–F0”, and “C0” if number 0 is selected for cell E2; “C0”, “+I1”, “–F1”, and “C1” if number 1 is selected for cell E2, and so on.

$$\mathrm{I4} = \text{"C"} \,\&\, (\$\mathrm{E}\$2 - 1)$$

$$\mathrm{J4} = \text{" + I"} \,\&\, \$\mathrm{E}\$2$$

$$\mathrm{K4} = \text{"– F"} \,\&\, \$\mathrm{E}\$2$$

$$\mathrm{L4} = \text{"C"} \,\&\, \$\mathrm{E}\$2$$

(see Figure 36).

To fill the cells of the Matrix, we again employ the INDEX and MATCH combination. The technique is like the one described in section 2.2, used in the split-screen strip for picking the amounts from the worksheet “PreCalc”.

[8] The methods of applying data validation are different in the various spreadsheets’ software; the instruction is generally available in the on-line help for the users.

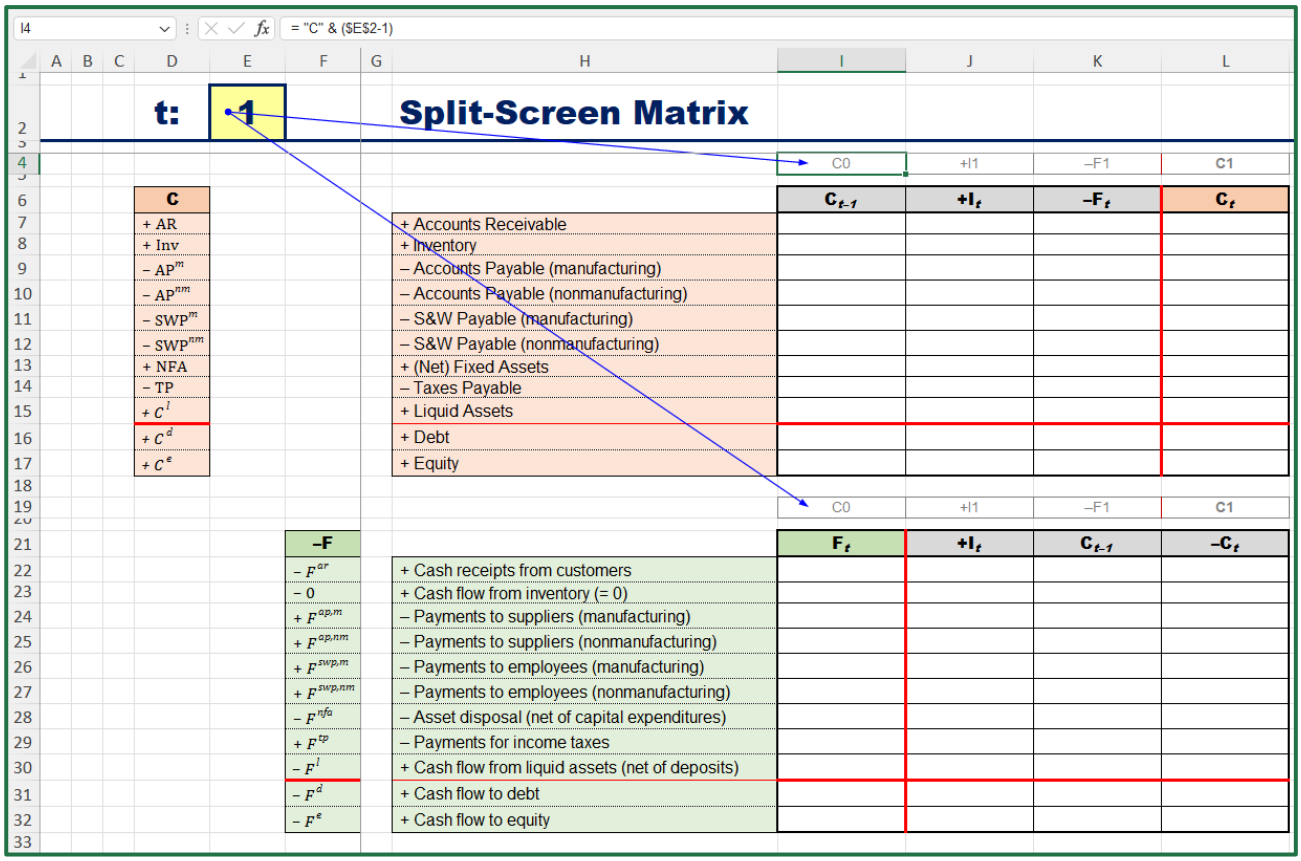


Figure 36. Cell I4 depends on cell E2: if E2 value is t, then I4 value is Ct-1 (e.g., if E2=1, then I4 is "C0"). Analogous code for row 19 (sheet "SinglePeriod")

The more external formula is:

$$\text{INDEX}(\text{FullScaleStrip}, \text{row\_num}, \text{column\_num})$$

where

"FullScaleStrip" is the range of cells where the full-scale strip is located (i.e. the range SplitScreenStrip!$13:$23);

"row_num" is the row-number of the range "FullScaleStrip" where the item that contains the sought value is located: it can be selected by matching the label of the row in the Matrix and the label in the split-screen strip, using the function MATCH;

"column_num" is the column-number of the range "FullScaleStrip" where the category (capital, income, or cash flow) and the period that contains the sought value are indicated: it can be selected by matching the label of the column in the Matrix and the labels in the split-screen strip, using the function MATCH.

One starts from cell I7:

$$\text{I7} = \text{INDEX}(\text{SplitScreenStrip!}\$13{:}\$23, \underbrace{\text{MATCH}(\$\text{D7}, \text{SplitScreenStrip!}\$\text{D}\$13{:}\$\text{D}\$23, 0)}_{\text{the result of this formula is the number of the row in the four-area strip that has the same label as cell \$D7 ("+AR")}}, \underbrace{\text{MATCH}(\text{I}\$4, \text{SplitScreenStrip!}\$12{:}\$12, 0)}_{\text{the result of this formula is the number of the column in the four-area strip that has the same label as cell I\$4 ("C0")}})$$

(see Figure 37 and Figure 38).

I7 = INDEX(SplitScreenStrip!$13:$23, MATCH($D7, SplitScreenStrip!$D$13:$D$23, 0), MATCH(I$4, SplitScreenStrip!$12:$12, 0))

t: 1

Split-Screen Matrix

C		C0	+I1	–F1	C1
		C_{t-1}	$+I_t$	$-F_t$	C_t
+AR	+ Accounts Receivable	0	60,000	-45,205	**14,795**
+ Inv	+ Inventory	3,750	375	0	**4,125**
– AP^m	– Accounts Payable (manufacturing)	-3,750	-15,375	3,750	**-15,375**
– AP^{nm}	– Accounts Payable (nonmanufacturing)	0	-9,000	0	**-9,000**
– SWP^m	– S&W Payable (manufacturing)	0	-24,000	24,000	**0**
– SWP^{nm}	– S&W Payable (nonmanufacturing)	0	-6,000	6,000	**0**
+ NFA	+ (Net) Fixed Assets	20,000	-4,000	0	**16,000**
– TP	– Taxes Payable	0	-494	494	**0**
$+ C^l$	+ Liquid Assets	-4,000	-152	8,030	**3,878**
$+ C^d$	+ Debt	10,000	200	-2,700	**7,500**
$+ C^e$	+ Equity	6,000	1,154	-231	**6,923**

Figure 37. Cell I7 contains the formula for picking up the Account Receivable AR as of period t=0 (sheet "SinglePeriod")

n	5			
τ	30%	r^o	15.00%	
i^l	3.80%	r^l	1.00%	
i^d	2.00%	r^d	3.00%	
α	20%			

Project: Full-scale Matrix

	C	I	–F	C_{-1}	$+I_0$	$-F_0$	C_0	$+I_1$	$-F_1$	C_1	$+I_2$	$-F_2$	C_2	$+I_3$	$-F_3$	C_3	$+I_4$	$-F_4$	C_4	$+I_5$	$-F_5$	C_5
	C	I	F	C	I	F	C	I	F	C	I	F	C	I	F	C	I	F	C	I	F	C
	-1	0	0	0	1	1	1	2	2	2	3	3	3	4	4	4	5	5	5			
13	+ AR	+ S	$-F^{ar}$	0	0	0	0	60,000	-45,205	14,795	66,000	-64,521	16,274	72,600	-70,973	17,901	79,860	-78,070	19,692	87,850	-107,542	0
14	+ Inv	+ ΔInv	– 0	0	3,750	0	3,750	375	0	4,125	413	0	4,538	454	0	4,991	499	0	5,491	-5,491	0	0
15	– AP^m	– COP^m	$+F^{sp,m}$	0	-3,750	0	-3,750	-15,375	3,750	-15,375	-16,913	15,375	-16,913	-18,604	16,913	-18,604	-20,464	18,604	-20,464	-16,472	36,936	0
16	– AP^{nm}	– COP^{nm}	$+F^{sp,nm}$	0	0	0	0	-9,000	0	-9,000	-9,900	9,000	-9,900	-10,890	9,900	-10,890	-11,979	10,890	-11,979	-13,178	25,157	0
17	– SWP^m	– LC^m	$+F^{inv,p,m}$	0	0	0	0	-24,000	24,000	0	-26,400	26,400	0	-29,040	29,040	0	-31,944	31,944	0	-35,140	35,140	0
18	– SWP^{nm}	– LC^{nm}	$+F^{inv,p,nm}$	0	0	0	0	-6,000	6,000	0	-6,000	6,000	0	-6,000	6,000	0	-6,000	6,000	0	-6,000	6,000	0
19	+ NFA	– Dep	$-F^{ng}$	0	0	20,000	20,000	-4,000	0	16,000	-4,000	0	12,000	-4,000	0	8,000	-4,000	0	4,000	-4,000	0	0
20	– TP	– T	$+F^{tp}$	0	0	0	0	-494	494	0	-959	959	0	-1,414	1,414	0	-1,919	1,919	0	-2,478	2,478	0
21	$+ C^l$	$+ I^l$	$- F^l$	0	0	-4,000	-4,000	-152	8,030	3,878	147	3,689	7,714	293	4,446	12,454	473	5,268	18,195	691	-18,887	0
22	$+ C^d$	$+ I^d$	$- F^d$	0	0	10,000	10,000	200	-2,700	7,500	150	-2,650	5,000	100	-2,600	2,500	50	-2,550	0	0	0	0
23	$+ C^e$	$+ I^e$	$- F^e$	0	0	6,000	6,000	1,154	-231	6,923	2,238	-448	8,713	3,299	-660	11,353	4,477	-895	14,934	5,783	-20,717	0

Figure 38. Cell J13 contains the value that has the same row-label as SiglePeriod!D7 (+AR) and the same column-label as SiglePeriod!I4 (C_0) (sheet "SplitScreenStrip")

The formula written for cell I7 can then be used for cells J7 and K7 as well:

$$
\begin{aligned}
\text{J7} \;=\; \text{INDEX(}\; & \text{SplitScreenStrip!}\,\$13{:}\,\$23, \\
& \text{MATCH(}\; \$\text{D7}, \text{SplitScreenStrip!}\,\$\text{D}\$13{:}\,\$\text{D}\$23, 0\;\text{),} \\
& \text{MATCH(}\; \text{J}\$4, \text{SplitScreenStrip!}\,\$12{:}\,\$12, 0\;\text{)} \\
& \text{)}
\end{aligned}
$$

The analyst can then copy cell I7 and paste it in cells J7 and K7. Cell L7 contains the formula that applies the law of motion:

$$C_t = C_{t-1} + I_t - F_t$$

$$\underbrace{\text{L7}}_{\text{AR}_t} = \underbrace{\text{I7}}_{\text{AR}_{t-1}} + \underbrace{\text{J7}}_{\text{S}_t} + \underbrace{\text{K7}}_{-F_t^{ar}}$$

The analyst can copy range I7:L7 and paste it in range I8:L17 (or drag the fill handle).

The realization of the Matrix where CFS statement is illustrated is analogous to the one described above. The labels of the rows are in column F (column of cash-flow labels) and the labels of the columns are in row 19. The figures of the Matrix are grabbed from the full-scale strip with the formulas:

```
J22 = INDEX( SplitScreenStrip! $13: $23,
             MATCH( $F22, SplitScreenStrip! $F$13: $F$23, 0 ),
             MATCH( J$19, SplitScreenStrip! $12: $12, 0 )
             )
K22 = INDEX( SplitScreenStrip! $13: $23,
             MATCH( $F22, SplitScreenStrip! $F$13: $F$23, 0 ),
             MATCH( K$19, SplitScreenStrip! $12: $12, 0 )
             )
L22 = (−1) ∗ INDEX( SplitScreenStrip! $13: $23,
                    MATCH( $F22, SplitScreenStrip! $F$13: $F$23, 0 ),
                    MATCH( K$19, SplitScreenStrip! $12: $12, 0 )
                    )
```

Cell I22 contains the formula that applies the law of motion:

$$F_t = I_t + C_{t-1} - C_t$$

$$\underbrace{\mathrm{L22}}_{F_t^{ar}} = \underbrace{\mathrm{J22}}_{\mathrm{S}_t} + \underbrace{\mathrm{K22}}_{\mathrm{AR}_{t-1}} + \underbrace{\mathrm{L22}}_{-\mathrm{AR}_t}$$

The analyst can copy range I22:L22 and paste it in range I23:L32 (or drag the fill handle).

The analyst can build in the same way any preferred framing of the split-screen Matrix, with higher or lower scale, depending on the objective of the analysis.

6 MODULE 6: RESHAPING THE PROJECT SPLIT-SCREEN STRIP

6.1 Reshaping the strip as a transposed Matrix

The model's main purpose is to communicate the result of the numeric analysis: the recipients of the reports can include readers that are not familiar with the compact structure of the split-screen strip. The transposed Matrix and a set of standard financial statements may be easily extracted from the split-screen strip. The transposed Matrix, presented in Part I, section 10.2, is a compact form of the project strip which provides an intuitive picture of the whole project in terms of three submatrices, respectively reporting the book values (C_t), the incomes (I_t), and the cash flows (F_t), each one divided into the main four areas (operating, non-operating, debt, and equity) (see also Figure 39):

Symbol	Description
$+C^o$	+ Operating Assets
$+C^l$	+ Liquid Assets
$\boldsymbol{C}^{\text{inv}}$	**Investments**
$+C^d$	+ Debt
$+C^e$	+ Equity
$\boldsymbol{C}^{\text{fin}}$	**Financings**

$+I^o$	+ Operating income
$+I^l$	+ Interest income
$+\boldsymbol{I}^{\text{inv}}$	**Income from assets**
$+I^d$	+ Interest expenses
$+I^e$	+ Net Income
$\boldsymbol{I}^{\text{fin}}$	**Income to capital providers**

$+F^o$	+ OCF (cash flow from operations)
$+F^l$	+ CFL (cash flow from/to non-operating assets)
$\boldsymbol{F}^{\text{inv}}$	**Cash flow from assets**
$+F^d$	+ CFD (cash flow to debtholders)
$+F^e$	+ CFE (cash flow to equity)
$\boldsymbol{F}^{\text{fin}}$	**Cash flow to capital providers**

Figure 39. Structure of a column of the transposed Matrix: capital amounts, incomes, and cash flows are in vertical format (symbol on the left, description on the right; see the whole transposed Matrix in Part I, eq. (69))

Spreadsheet Modeling: Structure of transposed Matrix

The analyst adds, in his workbook, a new sheet "TransMatrix" and creates three ranges of cells representing the three submatrices presented in Part I, eq. (69) (see also Figure 40 below).

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1														
2				C	I	F			2022	2023	2024	2025	2026	2027
3														
4									C_0	C_1	C_2	C_3	C_4	C_5
6				$+C^o$				$+C^o$						
7				$+C^l$				$+C^l$						
8								$+C^{inv}$						
10				$+C^d$				$+C^d$						
11				$+C^e$				$+C^e$						
12								$+C^{fin}$						
13														
14									I_0	I_1	I_2	I_3	I_4	I_5
16					$+I^o$			$+I^o$						
17					$+I^l$			$+I^l$						
18								$+I^{inv}$						
20					$+I^d$			$+I^d$						
21					$+I^e$			$+I^{fin}$						
22								$+I^{fin}$						
23														
24									F_0	F_1	F_2	F_3	F_4	F_5
26						$-F^o$		$+F^o$						
27						$-F^l$		$+F^l$						
28								$+F^{inv}$						
30						$-F^d$		$+F^d$						
31						$-F^e$		$+F^e$						
32								$+F^{fin}$						
33														

Figure 40. Structure of the transposed Matrix (sheet "TransMatrix")

Each row of the transposed Matrix contains the label of the corresponding row in the four-area Matrix (SplitScreenStrip!D68:F71): columns D, E, and F report the labels that pertains, respectively, capital, income and cash flow.

Spreadsheet Modeling: Filling the capital submatrix of transposed Matrix

We again employ the INDEX and MATCH combination. The technique is like the one described in section 2.2, used in the split-screen strip for picking the amounts from the worksheet "PreCalc".
The more external formula is:

INDEX(FourAreaStrip, row_num, column_num)

where

"FourAreaStrip" is the range of cells where the four-area Matrix is located (i.e. the range SplitScreenStrip!$G$68:$Y$71);

"row_num" is the row-number of the range "FourAreaStrip" where the item that contains the sought value is located: it can be selected by matching the label of the row in the transposed Matrix and the label in the split-screen strip, using the function MATCH;

"column_num" is the column-number of the range "FourAreaStrip" where the category (capital, income, or cash flow) and the period that contains the sought value are indicated: it can be selected by matching the label of the column in the transposed Matrix and the labels in the split-screen strip, using the function MATCH. One starts from cell I6:

$$
\begin{aligned}
\text{I6} \;=\; \text{INDEX(}\; & \text{SplitScreenStrip!}\,\$G\$68{:}\,\$Y\$71, \\
& \underbrace{\text{MATCH(}\;\$D6, \text{SplitScreenStrip!}\,\$D\$68{:}\,\$D\$71, 0\;\text{)}}_{\substack{\text{the result of this formula is the number of the row in the} \\ \text{four-area strip that has the same label as cell \$D6 (“}+C^{o}\text{”)}}}, \\
& \underbrace{\text{MATCH(}\;\text{I\$4}, \text{SplitScreenStrip!}\,\$G\$67{:}\,\$Y\$67, 0\;\text{)}}_{\substack{\text{the result of this formula is the number of the column} \\ \text{in the four-area strip that has the same label as cell I\$4 (“}C_0\text{”)}}} \\
& \text{)}
\end{aligned}
$$

The formula written for cell I6 can then be used for cells I7, I10, and I11 as well.
Rows 8 and 12 contain the sums of the previous rows:

$$
\underbrace{\text{I8}}_{C_0^{\text{inv}}} = \text{SUM(}\underbrace{\text{I6:I7}}_{C_0^{o}+C_0^{l}}\text{)} \qquad\qquad \underbrace{\text{I12}}_{C_0^{\text{fin}}} = \text{SUM(}\underbrace{\text{I10:I11}}_{C_0^{d}+C_0^{e}}\text{)}
$$

Hence, the analyst can copy the range of cells I6:I12 and paste it in the range J6:N12 (or, more simply, use the fill handle). See Figure 41 and Figure 42.

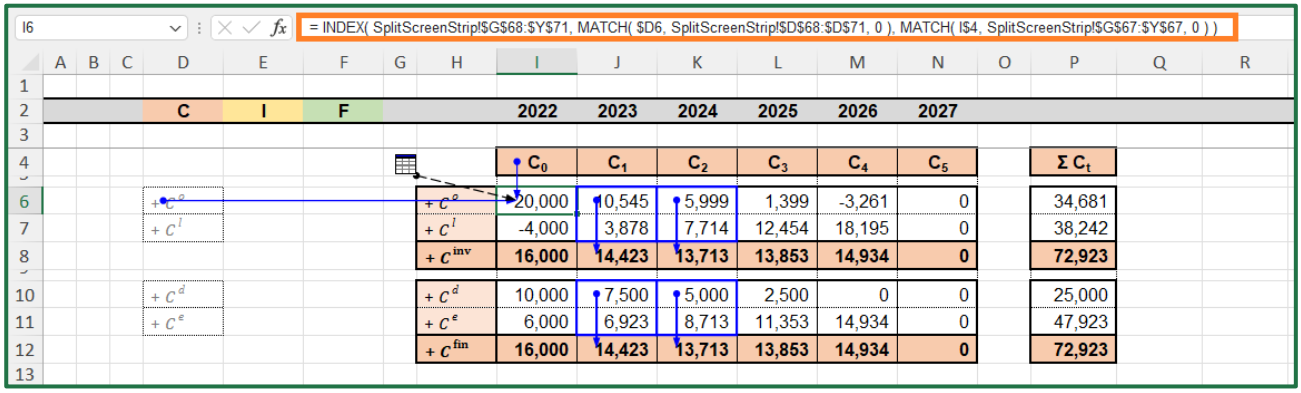


Figure 41. Cell I6 contains the formula for picking up the operating capital C^o as of period t=0; cells I8:N8 and I12:N12 contain the sums of the two rows above (sheet “TransMatrix”)

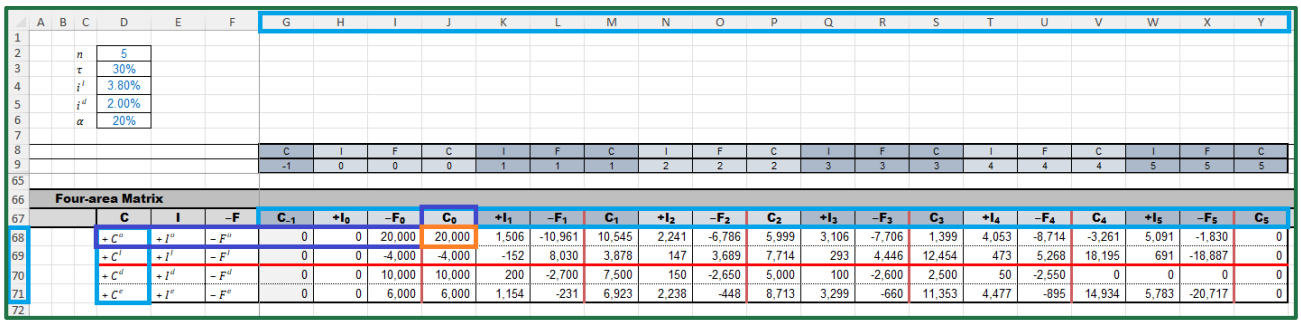


Figure 42. Cell J68 contains the value that has the same row-label as TransMatrix!D6 ($+C_0$) and the same column-label as TransMatrix!I4 (C_0) (sheet “SplitScreenStrip”)

Spreadsheet Modeling: Filling the income submatrix of transposed Matrix

The technique useful for filling the cells of the income submatrix is the same employed for the capital submatrix. The only difference in the formula is in the column where the row-labels are reported: the column of incomes is E. Cell I16 is calculated as follows:

$$
\begin{aligned}
\text{I16} \;=\; \text{INDEX(}\; & \text{SplitScreenStrip!}\,\$G\$68{:}\,\$Y\$71, \\
& \underbrace{\text{MATCH(}\;\$E16, \text{SplitScreenStrip!}\,\$E\$68{:}\,\$E\$71, 0\;\text{)}}_{\substack{\text{the result of this formula is the number of the row in the} \\ \text{four-area strip that has the same label as cell \$E11 (“}+I^{o}\text{”)}}}, \\
& \underbrace{\text{MATCH(}\;\text{" + "\&I\$14}, \text{SplitScreenStrip!}\,\$G\$67{:}\,\$Y\$67, 0\;\text{)}}_{\substack{\text{the result of this formula is the number of the column} \\ \text{in the four-area strip that has the same label} \\ \text{(preceded by "+") as cell I\$14 (“}+I_0\text{”)}}} \\
& \text{)}
\end{aligned}
$$

(see Figure 43).

The operator “&” joins the text strings, so the formula " + "&I$16 returns the symbol “+” alongside the content of cell I16, that is, it returns the value $+I_0$. This operation is necessary, because the labels in the sheet “TransMatrix” are “I_0”, “I_1”, “I_2”, “I_3”, “I_4”, and “I_5”, but the labels in the sheet “SplitScreenStrip” are “$+I_0$”, “$+I_1$”, “$+I_2$”, “$+I_3$”, “$+I_4$”, and “$+I_5$”.[9]

The formula written for cell I16 can be used for I17, I20, and I21 as well. Rows 18 and 22 contain the sums of the previous rows:

$$
\underbrace{\text{I18}}_{I_0^{\text{inv}}} = \text{SUM}\left(\underbrace{\text{I16:I17}}_{I_0^{o}+I_0^{l}}\right) \qquad\qquad \underbrace{\text{I22}}_{I_0^{\text{fin}}} = \text{SUM}\left(\underbrace{\text{I20:I21}}_{I_0^{d}+I_0^{e}}\right)
$$

The analyst can then copy the cell range of cells I16: I22 and paste it in the range J16: N22 (or, more simply, use the fill handle. See Figure 43).

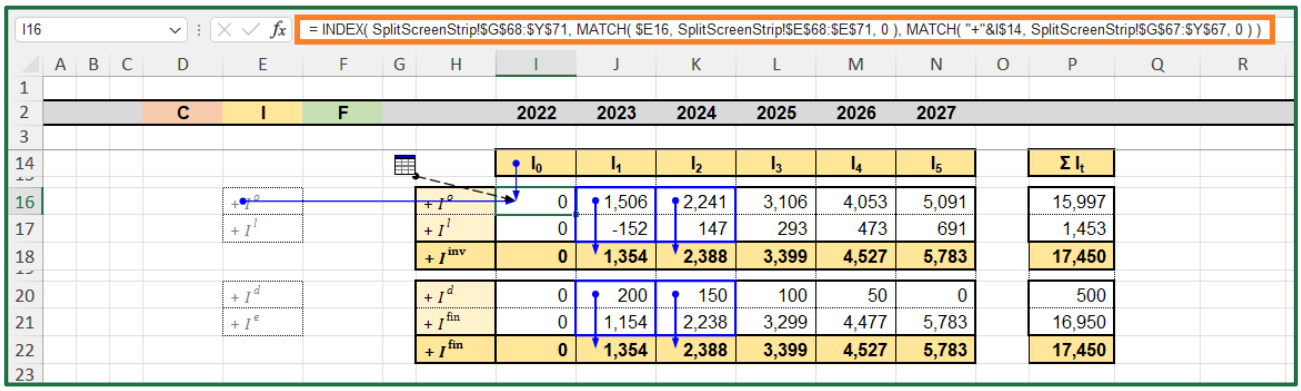


Figure 43. Cell I16 contains the formula for picking the operating income I^o as of period t=0; cells I18:N18 and I22:N22 contain the sums of the two rows above (sheet “TransMatrix”)

Spreadsheet Modeling: Filling the cash-flow submatrix of transposed Matrix

The technique useful for filling the cells of the cash-flow submatrix is the same employed for the capital submatrix and the income submatrix. The column where the row-labels are is column F. In the sheet “TransMatrix”, the rows of cash flows have positive sign, whereas in the four-area Matrix in the sheet “SplitScreenStrip” they have negative sign. For this reason, when we pick the value from the four-area Matrix, we must change it in sign (i.e., one multiplies by -1). One starts from the OCF, in cell I26:

[9] In principle, labels in the “TransMatrix” sheet may be adjusted with the addition of “+” so that the “&” operator is not necessary.

$$\mathrm{I26} = (-1) * \mathrm{INDEX}(\ \mathrm{SplitScreenStrip!\,\$G\$68{:}\,\$Y\$71},\ \underbrace{\mathrm{MATCH}(\ \mathrm{\$F26, SplitScreenStrip!\,\$F\$68{:}\,\$F\$71, 0}\ )}_{\substack{\text{the result of this formula is the number of the row in the}\\ \text{four-area strip that has the same label as cell \$F26 (“}-F^o\text{”)}}},\ \underbrace{\mathrm{MATCH}(\ \text{"}-\text{"\&I\$24, SplitScreenStrip!\,\$G\$67{:}\,\$Y\$67, 0}\ )}_{\substack{\text{the result of this formula is the number of the column}\\ \text{in the four-area strip that has the same label}\\ \text{(preceded by "}-\text{") as cell I\$24 (“}-F_0\text{”)}}}\ )$$

(see Figure 44).
The operator “&” joins the text strings, so the formula " − "&I$24 returns the symbol “–” alongside the content of the cell I24, i.e. it returns the value $-F_0$: this operation is necessary, because the labels in the sheet “TransMatrix” are “F_0”, “F_1”, “F_2”, “F_3”, “F_4”, “F_5”, but in the sheet “SplitScreenStrip” the labels are “$-F_0$”, “$-F_1$”, “$-F_2$”, “$-F_3$”, “$-F_4$”, “$-F_5$”.
The formula written for the cell I26 can be used for I27, I30, and I31 as well.
Rows 28 and 32 contain the sums of the previous rows:

$$\underbrace{\mathrm{I28}}_{F_0^{\mathrm{inv}}} = \mathrm{SUM}(\ \underbrace{\mathrm{I26{:}\,I27}}_{F_0^o+F_0^l}\ ) \qquad \underbrace{\mathrm{I32}}_{F_0^{\mathrm{fin}}} = \mathrm{SUM}(\ \underbrace{\mathrm{I30{:}\,I31}}_{F_0^d+F_0^e}\ )$$

The analyst can then copy the cell range I26: I32 and paste it in the range J26: N32 (or, more simply, use the fill handle).

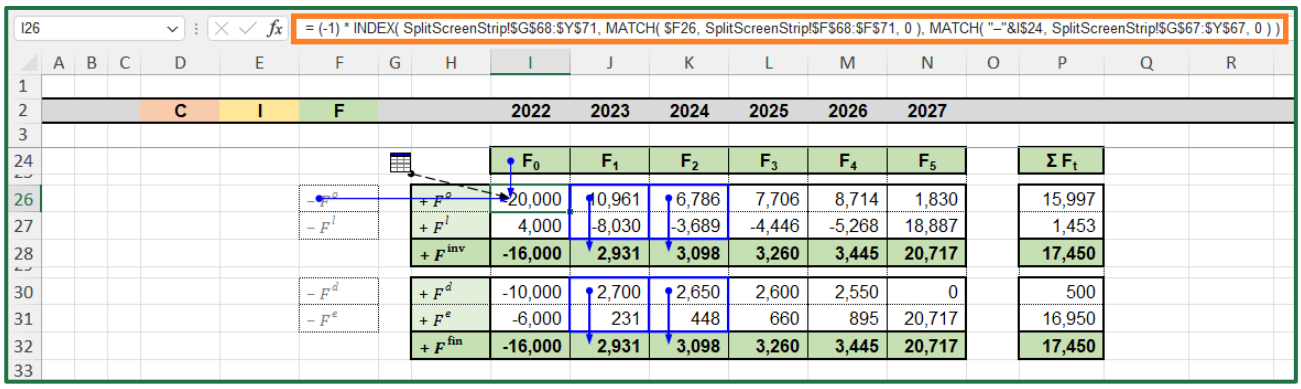


Figure 44. Cell I26 contains the formula for picking the operating cash flow F^o as of the period t=0; cells I28:N28 and I32:N32 contain the sums of the two rows above (sheet “TransMatrix”)

FCFE and CFL

The CFS in the transposed Matrix enables the analyst to disclose an important piece of information: the Free Cash Flow from (or to) Equity (FCFE). It is obtained by consolidating the OCF and CFD row (i.e., $\mathrm{FCFE}_t = F_t^o - F_t^d$). A positive FCFE indicates that the operations sustain the debt and leave some extra cash that may be distributed to shareholders or reinvested in liquid assets; a negative FCFE indicates that the operations cannot service the debt, so a financial deficit exists which must be covered either by equity or by liquid assets or by new borrowing (see Figure 45 below. See also Part I, section 9). In both cases, it is necessary to make a decision on the CFL; in particular, if FCFE is positive, it is necessary to make a payout decision (how much will be retained in the firm and how much will be distributed to equityholders?); if FCFE is negative, it is necessary to make a financing decisions (how much will be financed by cash withdrawals, by issuance of new shares, or by new borrowing?). In other words, CFE is a decision variable from $t = 0$ to $t = n - 1$ (at $t = n$, the project ends and the CFE is equal to the sum of the last FCFE and the end-of-period balance of liquid assets).

Spreadsheet Modeling: Filling the CFL strip

The FCFE Matrix is simply a different representation of the data in the cash-flow submatrix of transposed Matrix (see Part I, right matrix in eq. (58)). It can be placed in the "TransMatrix" sheet, rows 37:40, below the transposed Matrix:

$$\underbrace{\text{I38}}_{\text{FCFE}_0} = \underbrace{\text{I26}}_{F_0^o} - \underbrace{\text{I30}}_{F_0^d}$$

$$\underbrace{\text{I39}}_{-F_0^e} = (-1) * \underbrace{\text{I31}}_{F_0^e}$$

$$\underbrace{\text{I40}}_{-F_0^l} = \underbrace{\text{I38}}_{\text{FCFE}_0} + \underbrace{\text{I39}}_{-F_0^e}$$

(see Figure 45).

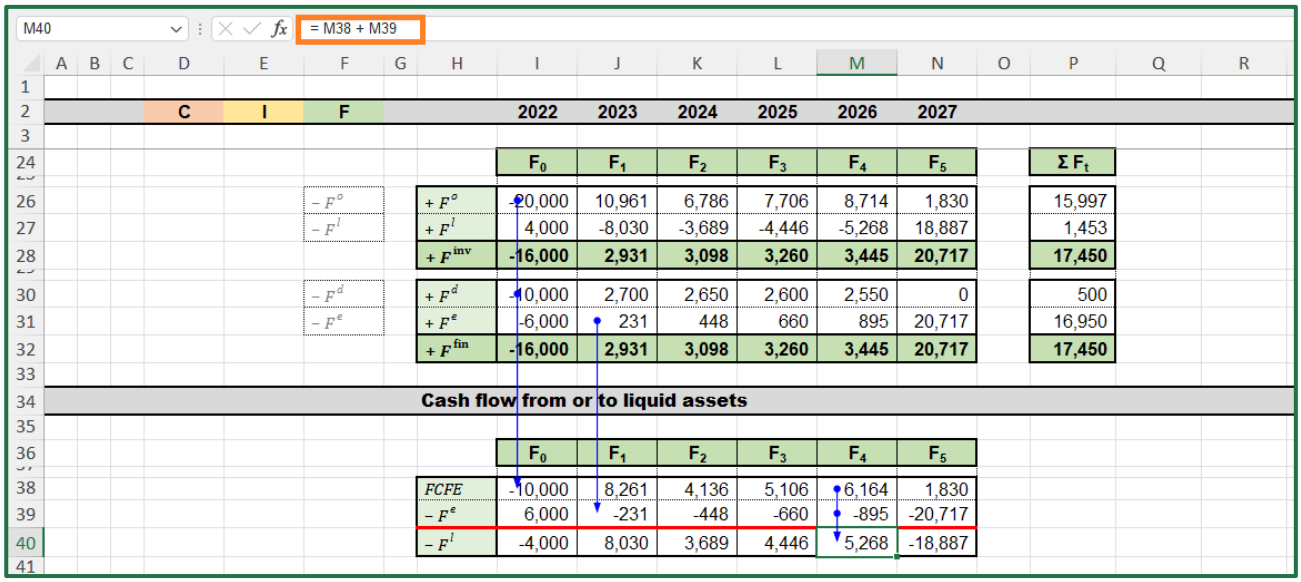


Figure 45. The FCFE at time $t = 4$ (cell M38) is positive and equal to \$6,164. Part of it is distributed to the fimr's shareholders (F_4^e =\$895), the remaining part is retained by the firm and reinvested in liquid assets ($-F_4^l$ = \$5,268). This figure is necessary for completing the BS as of period 4. The CFE at time $t = 5$, when the project ends, is F_5^e =\$20,717, equal to the FCFE (\$1,830) plus the end-of-period balance of liquid assets (\$18,887).

The benchmark transposed Matrix

It is convenient to build the transposed Matrix for the benchmark as well, which will turn out to be computationally helpful when appraising the project.

Spreadsheet Modeling: Building the benchmark transposed Matrix

The analyst creates, in rows 45:73 of sheet "TransMatrix", a structure like the one created for the transposed Matrix in rows 4:32. Each cell of this new structure contains the same formula as the corresponding cell of transposed Matrix, but different in the strip where the amounts are picked up: the source of data of project transposed Matrix is the project strip (rows 68:71 of sheet

"SplitScreenStrip") whereas the source of data of benchmark transposed Matrix is the benchmark strip (rows 83:86 of sheet "SplitScreenStrip"). For example, the value V_0^o in cell I47:

$$\begin{aligned}\mathrm{I47} = \mathrm{INDEX}(\ & \mathrm{SplitScreenStrip!\,\$G\$83{:}\,\$Y\$86},\\ & \underbrace{\mathrm{MATCH}(\ \$\mathrm{D47}, \mathrm{SplitScreenStrip!\,\$D\$83{:}\,\$D\$86}, 0\ )}_{\substack{\text{the result of this formula is the number of the row}\\ \text{in the benchmark four−area strip that has}\\ \text{the same label as cell \$D47 (“}+V^o\text{”)}}},\\ & \underbrace{\mathrm{MATCH}(\ \mathrm{I\$45}, \mathrm{SplitScreenStrip!\,\$G\$82{:}\,\$Y\$82}, 0\ )}_{\substack{\text{the result of this formula is the number of the column}\\ \text{in the benchmark four−area strip that has}\\ \text{the same label as cell I\$45 (“}V_0\text{”)}}}\\ &)\end{aligned}$$

(see Figure 46).

The formula written for the cell I47 can be used for I48, I51, and I52 as well. Rows 49 and 53 contain the sums of the previous rows:

$$\underbrace{\mathrm{I49}}_{V_0^{\mathrm{inv}}} = \mathrm{SUM}(\ \underbrace{\mathrm{I47{:}\,I48}}_{V_0^o+V_0^l}\ ) \qquad \underbrace{\mathrm{I53}}_{V_0^{\mathrm{fin}}} = \mathrm{SUM}(\ \underbrace{\mathrm{I51{:}\,I52}}_{V_0^d+V_0^e}\ )$$

The value I_0^{Vo} in cell I57 is obtained as

$$\begin{aligned}\mathrm{I57} = \mathrm{INDEX}(\ & \mathrm{SplitScreenStrip!\,\$G\$83{:}\,\$Y\$86},\\ & \mathrm{MATCH}(\ \$\mathrm{E57}, \mathrm{SplitScreenStrip!\,\$D\$83{:}\,\$D\$86}, 0\ ),\\ & \mathrm{MATCH}(\ "+"\&\mathrm{I\$55}, \mathrm{SplitScreenStrip!\,\$G\$82{:}\,\$Y\$82}, 0\ )\\ &)\end{aligned}$$

The formula written for cell I57 can be used for I58, I61, and I62 as well. Rows 59 and 63 contain the sums of the previous rows:

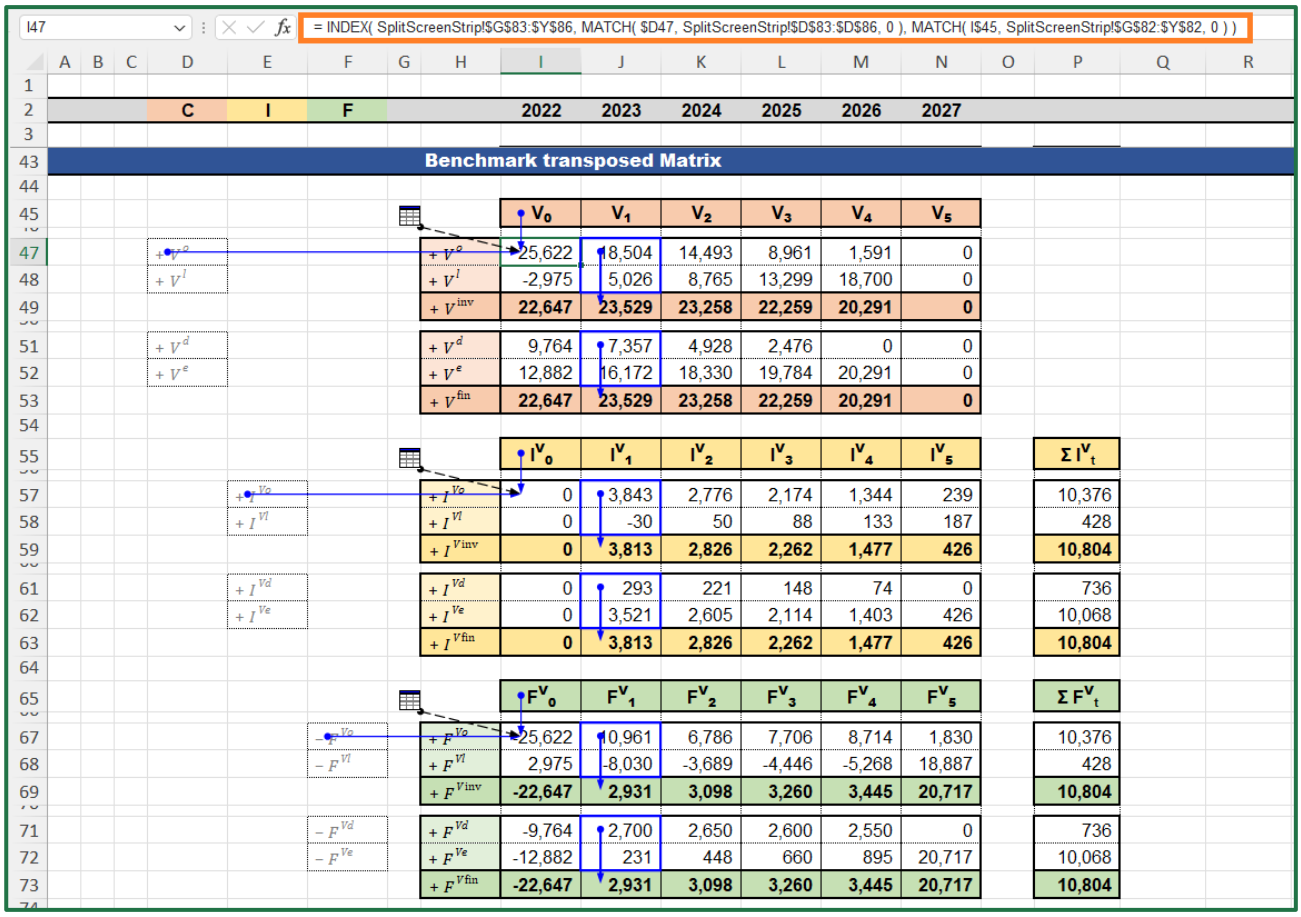
$$\underbrace{\mathrm{I59}}_{I_0^{V\mathrm{inv}}} = \mathrm{SUM}(\ \underbrace{\mathrm{I57{:}\,I58}}_{I_0^{Vo}+I_0^{Vl}}\ ) \qquad \underbrace{\mathrm{I63}}_{I_0^{V\mathrm{fin}}} = \mathrm{SUM}(\ \underbrace{\mathrm{I61{:}\,I62}}_{I_0^{Vd}+I_0^{Ve}}\ )$$

The value F_0^{Vo} in the cell I67 is obtained as

$$\begin{aligned}\mathrm{I67} = (-1) * \mathrm{INDEX}(\ & \mathrm{SplitScreenStrip!\,\$G\$83{:}\,\$Y\$86},\\ & \mathrm{MATCH}(\ \$\mathrm{F67}, \mathrm{SplitScreenStrip!\,\$D\$83{:}\,\$D\$86}, 0\ ),\\ & \mathrm{MATCH}(\ "-"\&\mathrm{I\$65}, \mathrm{SplitScreenStrip!\,\$G\$82{:}\,\$Y\$82}, 0\ )\\ &)\end{aligned}$$

The formula written for cell I67 can be used for I68, I71, and I72 as well. Rows 69 and 73 contain the sums of the previous rows:

$$\underbrace{\mathrm{I69}}_{F_0^{V\mathrm{inv}}} = \mathrm{SUM}(\ \underbrace{\mathrm{I67{:}\,I68}}_{F_0^{Vo}+F_0^{Vl}}\ ) \qquad \underbrace{\mathrm{I73}}_{F_0^{V\mathrm{fin}}} = \mathrm{SUM}(\ \underbrace{\mathrm{I71{:}\,I72}}_{F_0^{Vd}+F_0^{Ve}}\ )$$



I47 = INDEX(SplitScreenStrip!$G$83:$Y$86, MATCH($D47, SplitScreenStrip!$D$83:$D$86, 0), MATCH(I$45, SplitScreenStrip!$G$82:$Y$82, 0))

	C	I	F	2022	2023	2024	2025	2026	2027

Benchmark transposed Matrix

				V_0	V_1	V_2	V_3	V_4	V_5
47	$+V^o$			$+V^o$	25,622	18,504	14,493	8,961	1,591	0
48	$+V^I$			$+V^I$	-2,975	5,026	8,765	13,299	18,700	0
49				$+V^{inv}$	**22,647**	**23,529**	**23,258**	**22,259**	**20,291**	**0**
51	$+V^d$			$+V^d$	9,764	7,357	4,928	2,476	0	0
52	$+V^e$			$+V^e$	12,882	16,172	18,330	19,784	20,291	0
53				$+V^{fin}$	**22,647**	**23,529**	**23,258**	**22,259**	**20,291**	**0**

				I^V_0	I^V_1	I^V_2	I^V_3	I^V_4	I^V_5	ΣI^V_t
57		$+I^{Vo}$		$+I^{Vo}$	0	3,843	2,776	2,174	1,344	239	10,376
58		$+I^{VI}$		$+I^{VI}$	0	-30	50	88	133	187	428
59				$+I^{Vinv}$	**0**	**3,813**	**2,826**	**2,262**	**1,477**	**426**	**10,804**
61		$+I^{Vd}$		$+I^{Vd}$	0	293	221	148	74	0	736
62		$+I^{Ve}$		$+I^{Ve}$	0	3,521	2,605	2,114	1,403	426	10,068
63				$+I^{Vfin}$	**0**	**3,813**	**2,826**	**2,262**	**1,477**	**426**	**10,804**

				F^V_0	F^V_1	F^V_2	F^V_3	F^V_4	F^V_5	ΣF^V_t
67			$+F^{Vo}$	$+F^{Vo}$	25,622	10,961	6,786	7,706	8,714	1,830	10,376
68		$-F^{VI}$		$+F^{VI}$	2,975	-8,030	-3,689	-4,446	-5,268	18,887	428
69				$+F^{Vinv}$	**-22,647**	**2,931**	**3,098**	**3,260**	**3,445**	**20,717**	**10,804**
71		$-F^{Vd}$		$+F^{Vd}$	-9,764	2,700	2,650	2,600	2,550	0	736
72		$-F^{Ve}$		$+F^{Ve}$	-12,882	231	448	660	895	20,717	10,068
73				$+F^{Vfin}$	**-22,647**	**2,931**	**3,098**	**3,260**	**3,445**	**20,717**	**10,804**

Figure 46. Benchmark transposed Matrix. Cell I47 contains the formula for picking up the economic value of operating assets V^o as of period t=0 (sheet "TransMatrix")

The transposed Matrix makes it easy to calculate the sums of all the rows, which will be employed for computing the measures of worth in Module 7. The analyst uses column P in sheet "TransMatrix" for calculating the sums of the rows. For example, one can calculate the sum of book values, C $(= \sum_{t=0}^{n} C_t)$ in cell P6:

$$\text{P6} = \underbrace{\text{SUM}(\underbrace{\text{I6:N6}}_{C_0^o, C_1^o, \ldots C_n^o})}_{\sum_{t=0}^{n} C_t^o}$$

(see Figure 47).

P26 = SUM(I26:N26)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1																		
2				C	I	F			2022	2023	2024	2025	2026	2027				
3																		
4									C_0	C_1	C_2	C_3	C_4	C_5		ΣC_t		
6				$+C^o$				$+C^o$	20,000	10,545	5,999	1,399	-3,261	0		34,681		
7				$+C^l$				$+C^l$	-4,000	3,878	7,714	12,454	18,195	0		38,242		
8								$+C^{inv}$	**16,000**	**14,423**	**13,713**	**13,853**	**14,934**	**0**		**72,923**		
10				$+C^d$				$+C^d$	10,000	7,500	5,000	2,500	0	0		25,000		
11				$+C^e$				$+C^e$	6,000	6,923	8,713	11,353	14,934	0		47,923		
12								$+C^{fin}$	**16,000**	**14,423**	**13,713**	**13,853**	**14,934**	**0**		**72,923**		
13																		
14									I_0	I_1	I_2	I_3	I_4	I_5		ΣI_t		
16					$+I^o$			$+I^o$	0	1,506	2,241	3,106	4,053	5,091		15,997		
17					$+I^l$			$+I^l$	0	-152	147	293	473	691		1,453		
18								$+I^{inv}$	**0**	**1,354**	**2,388**	**3,399**	**4,527**	**5,783**		**17,450**		
20					$+I^d$			$+I^d$	0	200	150	100	50	0		500		
21					$+I^e$			$+I^{fin}$	0	1,154	2,238	3,299	4,477	5,783		16,950		
22								$+I^{fin}$	**0**	**1,354**	**2,388**	**3,399**	**4,527**	**5,783**		**17,450**		
23																		
24									F_0	F_1	F_2	F_3	F_4	F_5		ΣF_t		
26						$-F^o$		$+F^o$	-20,000	10,961	6,786	7,706	8,714	1,830		15,997		
27						$-F^l$		$+F^l$	4,000	-8,030	-3,689	-4,446	-5,268	18,887		1,453		
28								$+F^{inv}$	**-16,000**	**2,931**	**3,098**	**3,260**	**3,445**	**20,717**		**17,450**		
30						$-F^d$		$+F^d$	-10,000	2,700	2,650	2,600	2,550	0		500		
31						$-F^e$		$+F^e$	-6,000	231	448	660	895	20,717		16,950		
32								$+F^{fin}$	**-16,000**	**2,931**	**3,098**	**3,260**	**3,445**	**20,717**		**17,450**		
33																		

Figure 47. Column P contains the sums of the rows (sheet "TransMatrix")

The formula written for cell P6 can be used for P7, P8, P10, P11, and P12 as well. In the same way, the calculation of the sum of incomes and cash flows is possible in the other submatrices of the project (cells P16:P18, P20:P22, P26:P28, and P30:P32) and of the benchmark (cells P57:P59, P61:P63, P67:P69, and P71:P73).

6.2 Reshaping the strip as a set of conventional financial statements

A different representation of the output is in the form of standard financial statements. Using report forms, the BS, the IS, and the CFS are laid out as in Figure 48 (see also Part I, section 5).

	Balance Sheet
$+C^o$	**+ Operating Assets**
+NOWC	+ Net Operating WC
+AR	+ Accounts Receivable
+Inv	+ Inventory
−AP	– Accounts Payable
−SWP	– S&W Payable
−TP	– Taxes Payable
+NFA	+ (Net) Fixed Assets
$+C^l$	**+ Liquid Assets**
$+C^{inv}$	**Investments**
$+C^d$	**+ Debt**
$+C^e$	**+ Equity**
$+C^{fin}$	**Financings**

	Income Statement
+S	+ Sales
(−) COGS	(–) Cost of Goods Sold
+ΔInv	+ Income from inventory (Δ Inventory)
$-\mathrm{COP}^m$	– Cost of Purchases (manufacturing)
$-\mathrm{LC}^m$	– Labor costs (manufacturing)
	Gross Profit
(−) SGA	(–) Selling, General and Administrative Expenses
$-\mathrm{COP}^{nm}$	– Cost of Purchases (nonmanufacturing)
$-\mathrm{LC}^{nm}$	– Labor costs (nonmanufacturing)
EBITDA	**Earnings Before Interest, Taxes, Depreciation and Amortization**
−Dep	– Depreciation
EBIT	**Earnings Before Interest and Taxes**
$+I^l$	+ Interest income
$-I^d$	– Interest expenses
EBT	**Earnings Before Taxes**
−T	– Taxes
$+I^e$	**Net Income**

	Statement of cash flows (Direct method)	
$+F^{ar}$	+ Cash receipts from customers	
$+I^l$	+ Interest income	
$-F^{ap}$	– Payment to suppliers	
$-F^{swp}$	– Payments to employees	
$-I^d$	– Interest expenses	
$-F^{tp}$	– Payments for income taxes	
	Cash flow from operating activities	**(a)**
$+F^{nfa}$	+ Asset disposals – Capital expenditures	
	Cash flow from investing activities	**(b)**
$+\Delta C^d$	+ New borrowing – Principal repayments	
$-F^e$	+ Issuance of new stock – Distributions to equityholders	
	Cash flow from financing activities	**(c)**
	Change in cash and cash equivalents	**(a) + (b) + (c)**

Figure 48. Conventional financial statements: Balance Sheet, Income Statement and Statement of cash flows (direct method)

Spreadsheet Modeling: Structure of Financial Statements

The analyst creates, in his workbook, a new sheet "FinancStat", and sets up some ranges of cells to host the pro forma BSs, ISs, and CFs (Figure 49-Figure 51).

2											0	1	2	3	4	5
4			C	I	F			Financial Statements	UoM	2022	2023	2024	2025	2026	2027
7								**Balance Sheet**		C_0	C_1	C_2	C_3	C_4	C_5
9							$+C^o$	**+ Operating Assets**	$						
10							+ NOWC	+ Net Operating WC	$						
11			+ AR				+ AR	+ Accounts Receivable	$						
12			+ Inv				+ Inv	+ Inventory	$						
13			$-AP^m$				$-AP^m$	– Accounts Payable (manufacturing)	$						
14			$-AP^{nm}$				$-AP^{nm}$	– Accounts Payable (nonmanufacturing)	$						
15			$-SWP^m$				$-SWP^m$	– S&W Payable (manufacturing)	$						
16			$-SWP^{nm}$				$-SWP^{nm}$	– S&W Payable (nonmanufacturing)	$						
17			– TP				– TP	– Taxes Payable	$						
18			+ NFA				+ NFA	+ (Net) Fixed Assets	$						
19			$+C^l$				$+C^l$	**+ Liquid Assets**	$						
20							$+C^{inv}$	**Investments**	$						
22			$+C^d$				$+C^d$	**+ Debt**	$						
23			$+C^e$				$+C^e$	**+ Equity**	$						
24							$+C^{fin}$	**Financings**	$						

Figure 49. Structure of the Balance Sheets in the spreadsheet (sheet "FinancStat")

2											0	1	2	3	4	5
4			C	I	F			Financial Statements	UoM	2022	2023	2024	2025	2026	2027
26								**Income Statement**		I_0	I_1	I_2	I_3	I_4	I_5
28				+ S			+ S	**+ Sales**	$						
29							(–) COGS	– Cost Of Goods Sold	$						
30				+ ΔInv			+ ΔInv	+ Income from inventory (Δ Inventory)	$						
31				$-COP^m$			$-COP^m$	– Cost of Purchases (manufacturing)	$						
32				$-LC^m$			$-LC^m$	– Labor costs (manufacturing)	$						
33								**Gross Profit**	$						
34							(–) SGA	– Selling General and Administrative Expenses	$						
35				$-COP^{nm}$			$-COP^{nm}$	– Cost of Purchases (nonmanufacturing)	$						
36				$-LC^{nm}$			$-LC^{nm}$	– Labor costs (nonmanufacturing)	$						
37							EBITDA	**Earnings Before Interest, Taxes, Depreciation and Amortization**	$						
38				– Dep			– Dep	– Depreciation	$						
39							EBIT	**Earnings Before Interest and Taxes**	$						
40				$+I^i$			$+I^i$	+ Interest income	$						
41				$+I^d$			$-I^d$	+ Interest expenses	$						
42							EBT	**Earnings BeforeTaxes**	$						
43				– T			– T	– Taxes	$						
44				$+I^e$			$+I^e$	**Net Income**	$						

Figure 50. Structure of the Income Statements in the spreadsheet (sheet "FinancStat")

2											0	1	2	3	4	5
4			C	I	F			Financial Statements	UoM	2022	2023	2024	2025	2026	2027
46								**Statement of cash flows (Direct method)**		F_0	F_1	F_2	F_3	F_4	F_5
48					$-F^{ar}$		$+F^{ar}$	+ Cash receipts from customers	$						
49				$+I^i$			$+I^i$	+ Interest income	$						
50					$+F^{ap,m}$		$-F^{ap,m}$	– Payments to suppliers (manufacturing)	$						
51					$+F^{ap,nm}$		$-F^{ap,nm}$	– Payments to suppliers (nonmanufacturing)	$						
52					$+F^{swp,m}$		$-F^{swp,m}$	– Payments to employees (manufacturing)	$						
53					$+F^{swp,nm}$		$-F^{swp,nm}$	– Payments to employees (nonmanufacturing)	$						
54				$+I^d$			$-I^d$	– Interest expenses	$						
55					$+F^{tp}$		$-F^{tp}$	– Payments for income taxes	$						
56							(a)	**Cash flow from operating activities**	$						
58					$-F^{nfa}$		$+F^{nfa}$	+ Asset disposal – Capital expenditures	$						
59							(b)	**Cash flow from investing activities**	$						
61			$+C^d$				$+C^d$	+ Debt	$						
62							$+\Delta C^d$	+ New borrowing – Principal repayments	$						
63					$-F^e$		$-F^e$	+ Issuance of New stock – Distributions to equityholders	$						
64							(c)	**Cash flow from financing activities**	$						
66							(a)+(b)+(c)	**Change in cash and cash equivalents (ΔC^l)**	$						

Figure 51. Structure of the Cash Flow Statements in the spreadsheet (sheet "FinancStat")

Each row contains a label in column D, E, or F: they are references of the values in the split-screen strip and report the capital amounts, the incomes, and the cash flows.

The standard pro forma financial statements are just a different way to show the results of the project. The analyst picks up the values for period 0 from the "SplitScreenStrip" sheet and feed them into the

conventional BS, IS and CFS. Then, it suffices to copy and paste (or drag the fill handle) to fill the strip of all the project's BSs.

Spreadsheet Modeling: Filling the Balance Sheet

An effective technique for catching the figures from the split-screen strip is the use of the combinate functions INDEX and MATCH. The technique is the same described in the previous sections. For example, the AR are obtained as follows:

$$\begin{aligned} \text{L11} \;=\; \text{INDEX(}\; & \text{SplitScreenStrip!}\,\$13{:}\,\$23, \\ & \underbrace{\text{MATCH(}\;\$\text{D11, SplitScreenStrip!}\,\$\text{D}\$13{:}\,\$\text{D}\$23, 0\;\text{)}}_{\substack{\text{the result of this formula is the number of the row in the} \\ \text{split-screen strip that has the same label as cell } \$\text{D11 (“+AR”)}}}, \\ & \underbrace{\text{MATCH(}\;\text{L}\$7, \text{SplitScreenStrip!}\,\$12{:}\,\$12, 0\;\text{)}}_{\substack{\text{the result of this formula is the number of the column} \\ \text{in the split-screen strip that has the same label as cell L}\$7\ (\text{“C}_0\text{”})}} \\ & \text{)} \end{aligned}$$

(see Figure 52).

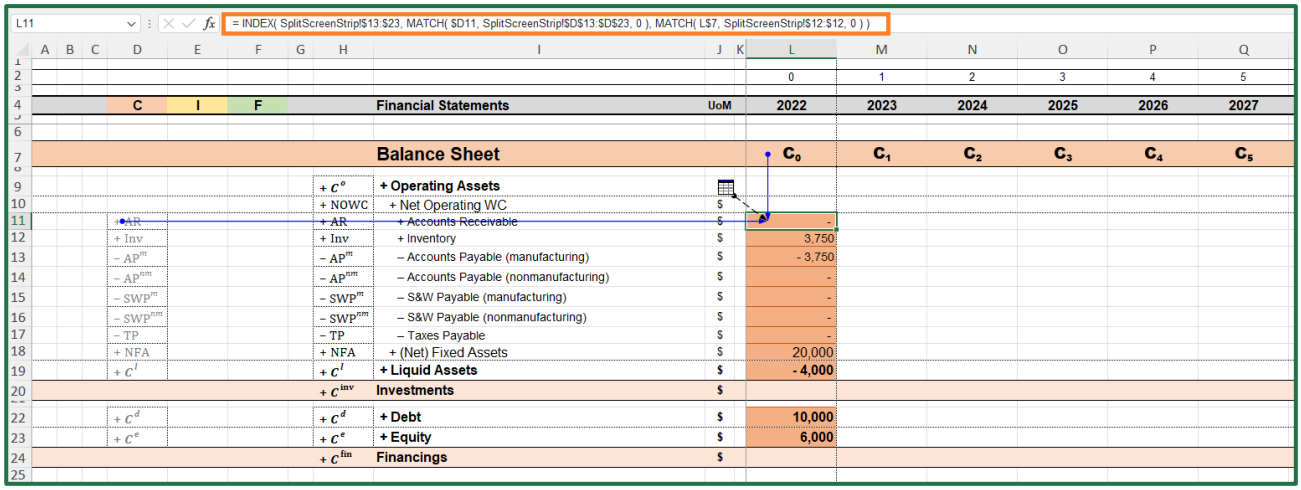


Figure 52. Cell L11 contains the formula for picking the Account Receivables (+AR) of period t=0 from the split-screen strip (sheet "FinancStat")

Every cell of the BS that contains a value calculated in the split-screen strip can be filled using the function INDEX-MATCH: the analyst can copy cell L11 and paste it in cells L12:L19 and L22:L23, as shown in Figure 52.

Other cells are partial sums of the values in the split-screen strip. An example is cell L10, which contains the value of the net operating working capital, NOWC:

$$\underbrace{\text{L10}}_{+\text{NOWC}_0} \;=\; \text{SUM(}\; \underbrace{\text{L11:L17}}_{+\text{AR}_0+\text{Inv}_0-\text{AP}_0^{m}-\text{AP}_0^{nm}-\text{SWP}_0^{m}-\text{SWP}_0^{nm}-\text{TP}_0} \;\text{)}$$

or cells L9, L20, L24:

$$\underbrace{\text{L9}}_{C_0^o} \;=\; \underbrace{\text{L10}}_{+\text{NOWC}_0} \;+\; \underbrace{\text{L18}}_{+\text{NFA}_0}$$

$$\underbrace{\text{L20}}_{C_0^{\text{inv}}} \;=\; \underbrace{\text{L9}}_{C_0^o} + \underbrace{\text{L19}}_{C_0^l}$$

$$\underbrace{\text{L24}}_{C_0^{\text{fin}}} = \underbrace{\text{L22}}_{C_0^{d}} + \underbrace{\text{L23}}_{C_0^{e}}$$

as represented in Figure 53.

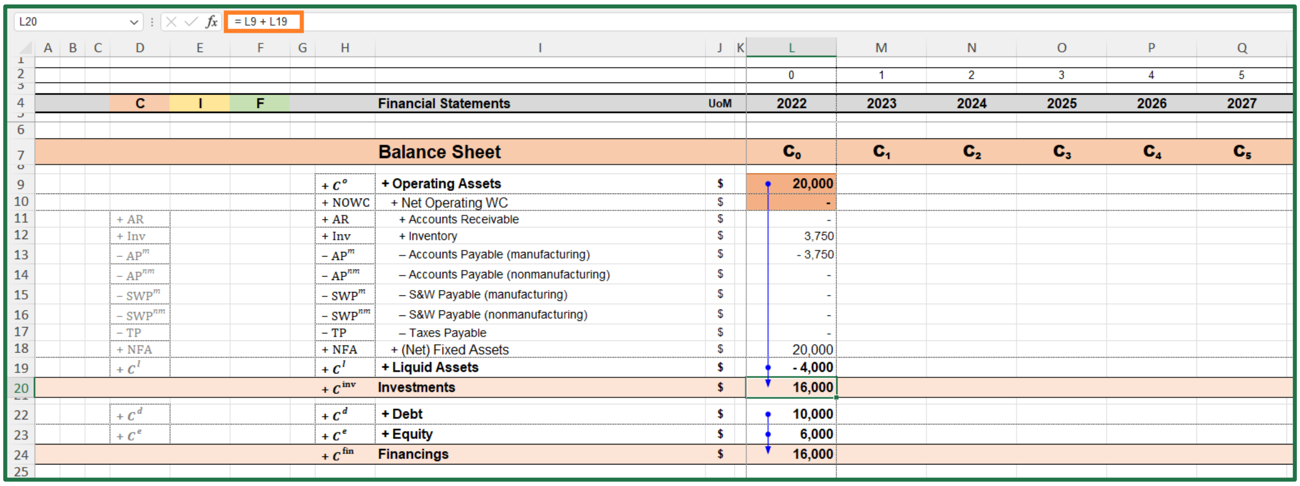


Figure 53. Cell L20 contains the formula fur summarizing C^o and C^l, which determine C^{inv} (sheet "FinancStat")

Once the BS for $t = 0$ has been completed, the analyst can copy the cell range L9: L24 and paste it in the cell range M9: Q24 (or, more simply, use the fill handle).

Spreadsheet Modeling: Filling the Income Statement

The structure of the formula used for filling the BS can be used for filling the IS. An example of the formula is

$$\begin{aligned}\text{L30} = \text{INDEX}(&\ \text{SplitScreenStrip!}\,\$13{:}\,\$23,\\ &\underbrace{\text{MATCH}(\ \$\text{E30}, \text{SplitScreenStrip!}\,\$\text{E}\$13{:}\,\$\text{E}\$23, 0\ )}_{\substack{\text{the result of this formula is the number of the row in the}\\ \text{split-screen strip that has the same label as cell \$E30 ("+}\Delta\text{Inv")}}},\\ &\underbrace{\text{MATCH}(\ "+"\&\text{L}\$26, \text{SplitScreenStrip!}\,\$12{:}\,\$12, 0\ )}_{\substack{\text{the result of this formula is the number of the column in the}\\ \text{split-screen strip that has the same label (preceded by +) as cell L\$26 ("}I_0\text{")}}}\\ &)\end{aligned}$$

(see Figure 54).

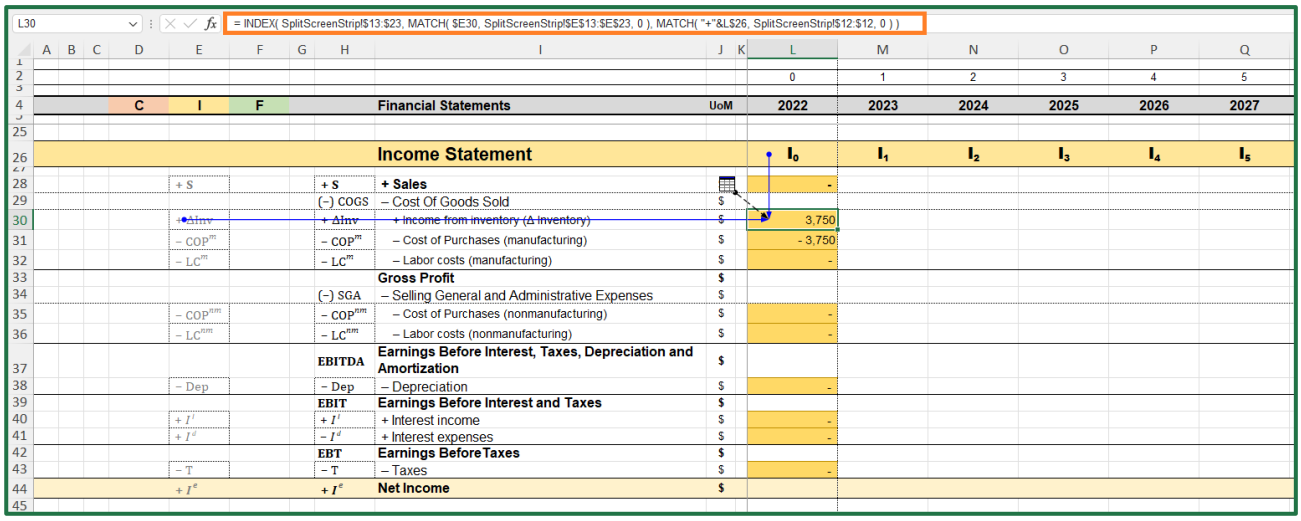


Figure 54. Cell L30 contains the formula for picking the inventory variation, +ΔInv, of period t=0 (sheet "FinancStat")

I^d is a special case, in the sense that, while $+I^d$ is available in the split-screen strip, the value $-I^d = (-1) \cdot I^d$ is actually required in the IS scheme:

$$\begin{aligned} \text{L41} = (-1) * \text{INDEX}(\ & \text{SplitScreenStrip!}\,\$13{:}\,\$23, \\ & \text{MATCH}(\ \$\text{E41}, \text{SplitScreenStrip!}\,\$\text{E}\$13{:}\,\$\text{E}\$23, 0\ ), \\ & \text{MATCH}(\ "+"\&\text{L}\$26, \text{SplitScreenStrip!}\,\$12{:}\,\$12, 0\ ) \\ &) \end{aligned}$$

The intermediate components of the income statements are not in the split-screen strip, so the analyst must calculate them in this layout:

$$\underbrace{\text{L29}}_{-\text{COGS}_0} = \text{SUM}(\ \underbrace{\text{L30:L32}}_{+\Delta\text{Inv}_0 - \text{COP}_0^m - \text{LC}_0^m}\ )$$

$$\underbrace{\text{L33}}_{\text{Gross Profit}_0} = \underbrace{\text{L28}}_{+\text{Sales}_0} + \underbrace{\text{L29}}_{-\text{COGS}_0}$$

$$\underbrace{\text{L34}}_{-\text{SGA}_0} = \text{SUM}(\ \underbrace{\text{L35:L36}}_{-\text{COP}_0^{nm} - \text{LC}_0^{nm}}\ )$$

$$\underbrace{\text{L37}}_{\text{EBITDA}_0} = \underbrace{\text{L33}}_{\text{Gross Profit}_0} + \underbrace{\text{L34}}_{-\text{SGA}_0}$$

$$\underbrace{\text{L39}}_{\text{EBIT}_0} = \text{SUM}(\ \underbrace{\text{L37:L38}}_{+\text{EBITDA}_0 - \text{Dep}_0}\ )$$

$$\underbrace{\text{L42}}_{\text{EBT}_0} = \text{SUM}(\ \underbrace{\text{L39:L41}}_{+\text{EBIT}_0 + I_0^l - I_0^d}\ )$$

The net income, I^e, is available in the split-screen strip as well. Of course, the analyst can pick it up and feed it into the conventional scheme (with the combined function INDEX-MATCH) or calculate it as

$$\underbrace{\text{L44}}_{I_0^e} = \text{SUM}(\ \underbrace{\text{L42:L43}}_{+\text{EBT}_0 - \text{T}_0}\ ).$$

The analyst can now copy the range L28: L44 and paste it in the cell range M28: Q44 (or, more simply, use the fill handle).

Spreadsheet Modeling: Filling the Cash Flow Statement

The structure of the formula used for filling the BS and for the IS can be used for filling the Cash Flow Statement. An example of the formula is

$$\begin{aligned}\text{L48} = (-1) * \text{INDEX(}\ &\text{SplitScreenStrip!}\ \$13{:}\ \$23,\\ &\underbrace{\text{MATCH(}\ \$\text{F48}, \text{SplitScreenStrip!}\ \$\text{F}\$13{:}\ \$\text{F}\$23, 0\ \text{)}}_{\substack{\text{the result of this formula is the number of the row in the}\\ \text{split-screen strip that has the same label as cell \$F48 (“}-F^{ar}\text{”)}}},\\ &\underbrace{\text{MATCH(}\ \text{"– "\&L\$46}, \text{SplitScreenStrip!}\ \$12{:}\ \$12, 0\ \text{)}}_{\substack{\text{the result of this formula is the number of the column in the}\\ \text{split-screen strip that has the same label (preceded by–) as cell L\$46 (“}-F_0\text{”)}}}\\ &\text{)}\end{aligned}$$

(see Figure 55).

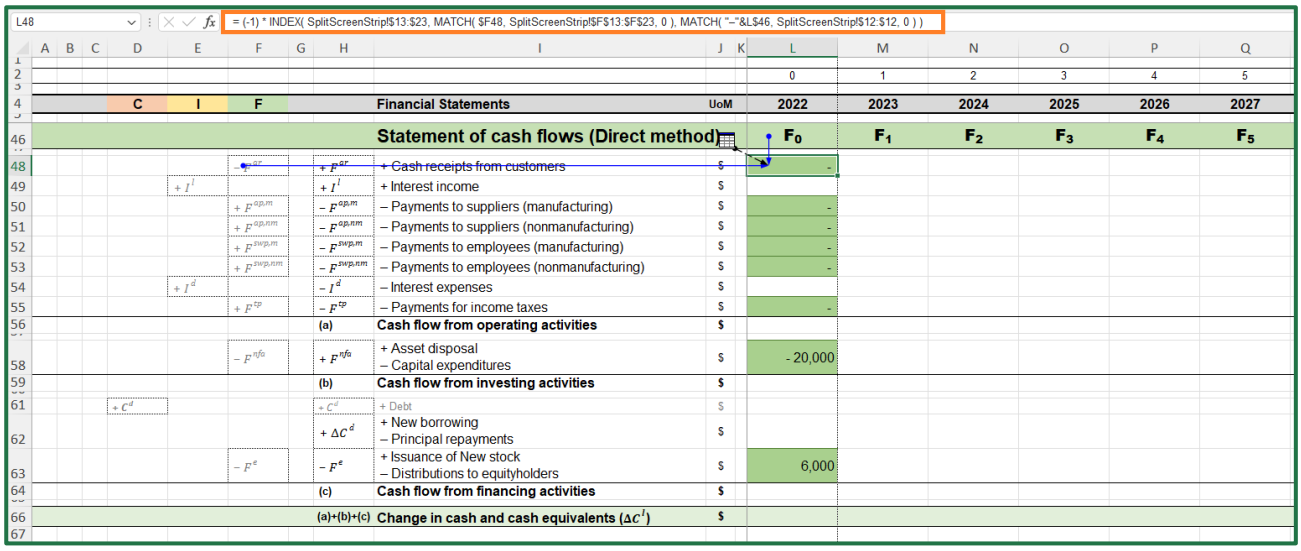


Figure 55. Cell L48 contains the formula for picking up the Cash receipts from customers, $+F^{ar}$, of period t=0 (sheet “FinancStat”)

The statement of cash flows contains values that the analyst can find in the cash-flow column of the split-screen strip (e.g., F^{ar}, $F^{ap,m}$, $F^{ap,nm}$), but it also contains values available in the income columns (I^l, I^d). For these figures, the analyst can use the same formula employed for the IS. An example of the formula is

$$\begin{aligned}\text{L49} = \text{INDEX(}\ &\text{SplitScreenStrip!}\ \$13{:}\ \$23,\\ &\text{MATCH(}\ \$\text{E49}, \text{SplitScreenStrip!}\ \$\text{F}\$13{:}\ \$\text{F}\$23, 0\ \text{)},\\ &\text{MATCH(}\ \text{"– "\&L\$26}, \text{SplitScreenStrip!}\ \$12{:}\ \$12, 0\ \text{)}\\ &\text{)}\end{aligned}$$

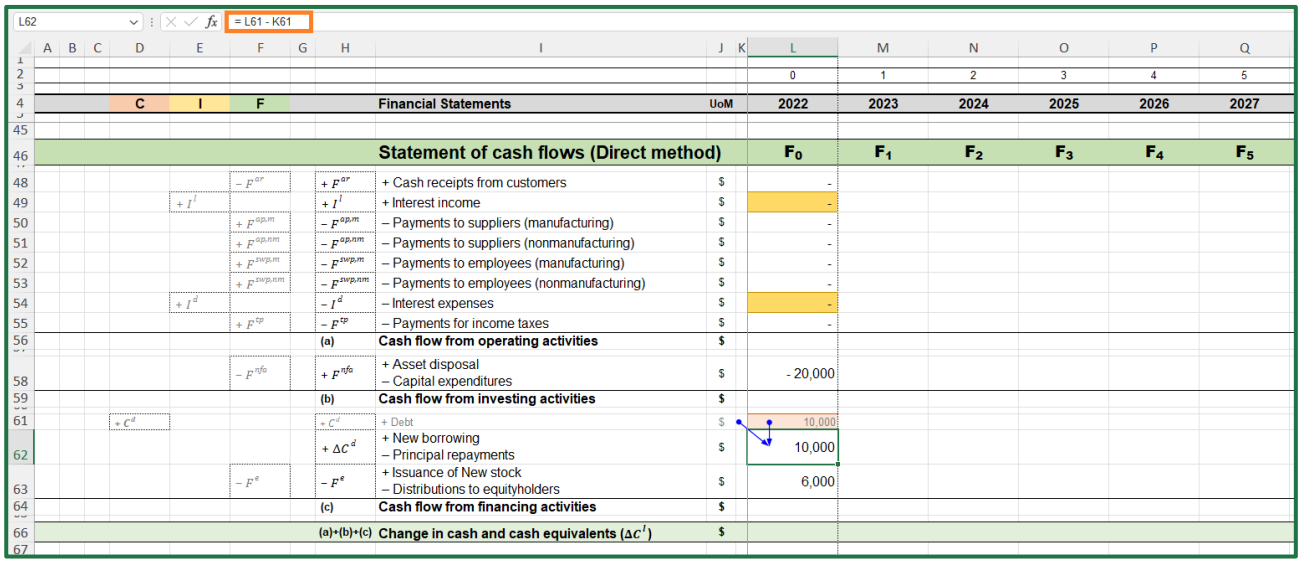
ΔC^d is a special case because it is not available in the split-screen Strip. However, the value C^d does appear in the Strip, so the analyst can pick it up with the formula already used for the BS:

$$\begin{aligned}\text{L61} = \text{INDEX(}\ &\text{SplitScreenStrip!}\ \$13{:}\ \$23,\\ &\text{MATCH(}\ \$\text{D61}, \text{SplitScreenStrip!}\ \$\text{F}\$13{:}\ \$\text{F}\$23, 0\ \text{)},\\ &\text{MATCH(}\ \text{L\$7}, \text{SplitScreenStrip!}\ \$12{:}\ \$12, 0\ \text{)}\\ &\text{)}.\end{aligned}$$

The variation ΔC_t^d is $\Delta C_t^d = C_t^d - C_{t-1}^d$; in spreadsheet formula, this means, for period 0,

$$\underbrace{\text{L62}}_{\Delta C_0^d} = \underbrace{\text{L61}}_{C_0^d} - \underbrace{\text{K61}}_{C_{-1}^d}$$

(see Figure 56).



L62								= L61 - K61															
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
2												0	1	2	3	4	5
4				C	I	F			Financial Statements	UoM		2022	2023	2024	2025	2026	2027
46									Statement of cash flows (Direct method)			F_0	F_1	F_2	F_3	F_4	F_5
48						$-F^{ar}$		$+F^{ar}$	+ Cash receipts from customers	$		-					
49					$+I^{l}$			$+I^{l}$	+ Interest income	$		-					
50						$+F^{ap,m}$		$-F^{ap,m}$	– Payments to suppliers (manufacturing)	$		-					
51						$+F^{ap,nm}$		$-F^{ap,nm}$	– Payments to suppliers (nonmanufacturing)	$		-					
52						$+F^{swp,m}$		$-F^{swp,m}$	– Payments to employees (manufacturing)	$		-					
53						$+F^{swp,nm}$		$-F^{swp,nm}$	– Payments to employees (nonmanufacturing)	$		-					
54					$+I^{d}$			$-I^{d}$	– Interest expenses	$		-					
55						$+F^{tp}$		$-F^{tp}$	– Payments for income taxes	$		-					
56								(a)	Cash flow from operating activities	$							
58						$-F^{nfa}$		$+F^{nfa}$	+ Asset disposal – Capital expenditures	$		- 20,000					
59								(b)	Cash flow from investing activities	$							
61				$+C^{d}$				$+C^{d}$	+ Debt	$		10,000					
62								$+\Delta C^{d}$	+ New borrowing – Principal repayments	$		10,000					
63						$-F^{e}$		$-F^{e}$	+ Issuance of New stock – Distributions to equityholders	$		6,000					
64								(c)	Cash flow from financing activities	$							
66								(a)+(b)+(c)	Change in cash and cash equivalents (ΔC^{l})	$							

Figure 56. Cell L62 contains the formula for calculating the variation ΔC_0^d as the difference between values in row 61 (sheet "FinancStat")

The three classes of cash flows of the standard CFS are obtained by summing the individual items:

$$\underbrace{\text{L56}}_{\text{Cash flow from operating activities}} = \text{SUM}(\underbrace{\text{L48:L55}}_{+F_0^{ar}+I_0^{l}-F_0^{ap,m}-F_0^{ap,nm}-F_0^{swp,m}-F_0^{swp,nm}-I_0^{d}-F_0^{tp}})$$

$$\underbrace{\text{L59}}_{\text{Cash flow from investing activities}} = \underbrace{\text{L58}}_{+F_0^{nfa}}$$

$$\underbrace{\text{L64}}_{\text{Cash flow from financing activities}} = \text{SUM}(\underbrace{\text{L62:L63}}_{+\Delta C_t^d - F_t^e}).$$

The sum of three components above is the change in cash and cash equivalents (ΔC^l):

$$\text{L66} = \text{L56} + \text{L59} + \text{L64}$$

The analyst can copy the range L48: L66 and paste it in range M48: Q66 (or, more simply, use the fill handle).

7 MODULE 7: ASSESSING VALUE CREATION AND FINANCIAL EFFICIENCY

As we have seen, the NPV(s) can be found in the MVA Matrix. This section is addressed to calculating some measures of economic profitability which provide different pieces of information from the NPV but are NPV-consistent and, therefore, reciprocally consistent. They offer an alternative but logically equivalent view on the project's value creation and financial efficiency (see section Part I, section 13). A new sheet for the presentation of the absolute and relative measures of worth must be created, called "ValueCreation". In it, we report the NPV and the economic residual income (ERI), which are absolute measures of worth, and, in addition, the rate of return (i) and the

cash-flow return on capital (CFROC), along with the corresponding benchmark rates of return (respectively, ρ and CFROC^V), which are relative measures of worth.

7.1 Economic Residual Income (ERI)

The ERI informs about how value creation may be apportioned across the various periods, that is, how the project has contributed to increase shareholders' wealth in the various periods. The total of such ERIs provide the overall value created.

Spreadsheet Modeling: ERI table

The analyst creates, in his workbook, a new sheet called "ValueCreation", where he prepares a range of cells which can contain the table of the ERIs (see Figure 57). Columns A and B contain the labels of the area (o, l, d, e, inv, and fin), and a header row (row 6) which contains the numbers of the periods. Each cell of this range contains the ERI of one area, $I_t^i - I_t^{Vi},\ i = o, l, d, e,$ inv, fin. These values are already available (changed in sign) in the MVA Matrix (SplitScreenStrip!98:101), so the analyst can pick them up with the following formulas:

$$\mathrm{C9} = (-1) * \mathrm{INDEX}(\underbrace{\mathrm{SplitScreenStrip!\,\$98{:}\,\$101}}_{\substack{\text{the MVA four-area Matrix}\\ \text{that contains the wanted values}}},$$
$$\underbrace{\mathrm{MATCH}(\ \$\mathrm{B9}, \mathrm{SplitScreenStrip!\,\$B\$98{:}\,\$B\$101}, 0\ )}_{\substack{\text{the result of this formula is the number of the row in}\\ \text{MVA four-area Matrix that has the same label as cell \$B9 ("o")}}},$$
$$\underbrace{\mathrm{MATCH}(\ \mathrm{C\$6}, \mathrm{SplitScreenStrip!\,\$9{:}\,\$9}, 0\ )}_{\substack{\text{the result of this formula is the number of the first column in the}\\ \text{MVA four-area Matrix that has the same label as cell C\$6 ("0")}}}$$
$$)$$

Then, the analyst copies and pastes the same formula for the whole strip C9:H12. The same table can also be framed in terms of investments vs financings. For this purpose, it suffices to use the previous ERI strip. For example, for the ERI generated by the investments is calculated as

$$\underbrace{\mathrm{C14}}_{\mathrm{ERI^{inv}}} = \mathrm{SUM}(\ \underbrace{\mathrm{C9{:}\,C10}}_{\mathrm{ERI}^o + \mathrm{ERI}^l}\ )$$

The ERI informs that the project subtracts value for the firm's shareholders in the first two periods ($-2{,}367$ and -367) but adds value in the remaining three periods (1,185, 3,074, and 5,357). Overall, the Total ERI is positive and equal to 6,882. It is calculated in the last column of the tables (column I) as $\mathrm{ERI} = \sum_{t=0}^{n}(I_t - I_t^V)$. In the spreadsheet,

$$\mathrm{I9} = \mathrm{SUM(C9{:}\,H9)}.$$

As explained in Part I, section 13, the Total ERI is equal to the NPV. Therefore, calculating the ERIs represent a way for decomposing the NPV by periods.

		0	1	2	3	4	5	
		$I_0-I^V_0$	$I_1-I^V_1$	$I_2-I^V_2$	$I_3-I^V_3$	$I_4-I^V_4$	$I_5-I^V_5$	Total ERI
	o	0	-2,338	-535	932	2,709	4,853	**5,622**
	l	0	-122	97	206	340	504	**1,025**
	d	0	-93	-71	-48	-24	0	**-236**
	e	0	-2,367	-367	1,185	3,074	5,357	**6,882**
inv		0	-2,460	-438	1,138	3,050	5,357	**6,647**
fin		0	-2,460	-438	1,138	3,050	5,357	**6,647**

Figure 57. Economic Residual Income (sheet "ValueCreation")

7.2 Rate of return, average ERI, and cash-flow return on capital

Beside (the NPV and the) Total ERI, which are absolute measures of worth, some useful information is provided by the relative measures of worth, which can be extracted with simple formulas (see Part I, section 13). The formulas of some measures employ the sum of project book values and others contain the sum of benchmark amounts. It is convenient to pick up such figures from the "TransMatrix" sheet, where they have already been calculated there.

Spreadsheet Modeling: Sums of rows of the transposed Matrices

The analyst builds a table in sheet "ValueCreation" and reports all the sums calculated in the sheet "TransMatrix" (see Figure 58). For example:

$$\text{E20} = \text{TransMatrix! P6}$$

$$\text{G20} = \text{TransMatrix! P16}$$

$$\text{I20} = \text{TransMatrix! P26}$$

$$\text{K20} = \text{TransMatrix! P57}$$

$$\text{M20} = \text{TransMatrix! P67}$$

These values are used for assessing the project's economic profitability (see box below).

E20 = TransMatrix!P6

	$\sum_{t=0}^{n} C_t$		$\sum_{t=0}^{n} I_t$		$\sum_{t=0}^{n} F_t$		$\sum_{t=0}^{n} I_t^V$		$\sum_{t=0}^{n} F_t^V$	
ΣC^o	34,681	ΣI^o	15,997	ΣF^o	15,997	ΣI^{Vo}	10,376	ΣF^{Vo}	10,376
ΣC^l	38,242	ΣI^l	1,453	ΣF^l	1,453	ΣI^{Vl}	428	ΣF^{Vl}	428
ΣC^{inv}	72,923	ΣI^{inv}	17,450	ΣF^{inv}	17,450	$\Sigma I^{V\,inv}$	10,804	$\Sigma F^{V\,inv}$	10,804
ΣC^d	25,000	ΣI^d	500	ΣF^d	500	ΣI^{Vd}	736	ΣF^{Vd}	736
ΣC^e	47,923	ΣI^e	16,950	ΣF^e	16,950	ΣI^{Ve}	10,068	ΣF^{Ve}	10,068
ΣC^{fin}	72,923	ΣI^{fin}	17,450	ΣF^{fin}	17,450	$\Sigma I^{V\,fin}$	10,804	$\Sigma F^{V\,fin}$	10,804

Figure 58. The table in the rows 18:26 contains the sums of the transposed matrices rows (sheet "ValueCreation")

Spreadsheet Modeling: Absolute and relative measures of economic profitability

The analyst sets up a range of cells that can host the values of the (absolute and relative) measures of worth, and the benchmark rate of return (see Figure 59).

NPV (column D)

As noted in section 4.3, the NPV value is equal to the MVA at $t = 0$, which can be picked up from the MVA Matrix (rows 98:101 of "SplitScreenStrip" sheet). For example, the NPV generated by the operations is obtained as

$$\text{D31} = \text{SplitScreenStrip!J98}$$

(as we already know, this output is also equal to the operating Total ERI in cell I9). The analyst can then copy cell D31 and paste it in cells D32, D35 and D36. The NPV of the investment and financing areas are the sums of the previous rows. For example, the investment NPV is calculated as

$$\underbrace{\text{D33}}_{\text{NPV}^{\text{inv}}} = \text{SUM}(\underbrace{\text{D31:D32}}_{\text{NPV}^{o}+\text{NPV}^{l}})$$

AERI (column F)

The Average Economic Residual Income (AERI) is the ratio of Total ERI and the project's lifespan:

$$\underbrace{\text{F31}}_{\text{AERI}^{o}} = \underbrace{\text{I9}}_{\text{total ERI}^{o}} / \underbrace{\text{Assumptions!\$H\$5}}_{n}$$

The analyst can copy cell F31 and paste it in cells F32, F35 and F36. The AERI of the investment and financing areas are the sums of the previous rows. For example, the investment AERI is

$$\underbrace{\text{F33}}_{\text{AERI}^{\text{inv}}} = \text{SUM}(\underbrace{\text{F31:F32}}_{\text{AERI}^{o}+\text{AERI}^{l}})$$

CFROC (column H)

The Cash-flow return on capital (CFROC) is the ratio of the sum of cash-flows, F $(= \sum_{t=0}^{n} F_t)$, and the sum of book values, C $(= \sum_{t=0}^{n} C_t)$.

$$\text{CFROC} = \frac{\sum_{t=0}^{n} F_t}{\sum_{t=0}^{n} C_t}$$

The analyst can pick up the values of F and C from the table in rows 18:26:

$$\underbrace{\text{H31}}_{\text{CFROC}^{o}} = \underbrace{\text{I20}}_{\sum_{t=0}^{n} F_t^{o}} / \underbrace{\text{E20}}_{\sum_{t=0}^{n} C_t^{o}}$$

The analyst can then copy cell H31 and paste it in ranges H32:H33 and H35:H37.

Rate of return (column J)

The rate of return of the project, i, is the ratio of the sum of project incomes, I $(= \sum_{t=0}^{n} I_t)$ and the sum of the project book values, C:

$$i = \frac{\sum_{t=0}^{n} I_t}{\sum_{t=0}^{n} C_t}.$$

The analyst can grab the values of I and C from the table in rows 18:26:

$$\underbrace{\text{J31}}_{i^o} = \underbrace{\text{G20}}_{\sum_{t=0}^{n} I_t^o} \;/\; \underbrace{\text{E20}}_{\sum_{t=0}^{n} C_t^o}.$$

(As explained in Part I, section 13, i and CFROC must coincide, so J31 = H31, as we expect.) The analyst can then copy cell J31 and paste it in ranges J32:J33 and J35:J37. These rates of return refer to all the areas of the project. They are, respectively: the average ROI, the average ROL, the average ROA, the average ROD, the average ROE (see also Table 5 in Part I).

CFROC^V (column L)

The cash-flow return on capital of the benchmark, CFROC^V, is the cutoff rate associated with CFROC which signal value creation (i.e., project acceptance) or value destruction (i.e., project rejection): it is the ratio of the sum of the cash flows generated by the benchmark ($\sum_{t=0}^{n} F_t^V$) and the sum of the book values, C.

$$\text{CFROC}^V = \frac{\sum_{t=0}^{n} F_t^V}{\sum_{t=0}^{n} C_t}$$

The analyst can grab the values of F^V and C from the table in rows 18:26:

$$\underbrace{\text{L31}}_{\text{CFROC}^{Vo}} = \underbrace{\text{M20}}_{\sum_{t=0}^{n} F_t^{Vo}} \;/\; \underbrace{\text{E20}}_{\sum_{t=0}^{n} C_t^o}$$

The analyst can then copy the cell L31 and paste it in ranges L32:L33 and L35:L37.

Benchmark rate of return (column N)

The benchmark rate of return ρ, associated with i, is the ratio of the sum of benchmark profits $I^V (= \sum_{t=1}^{n} I_t^V)$, and the sum of book values, C:

$$\rho = \frac{\sum_{t=0}^{n} I_t^V}{\sum_{t=0}^{n} C_t}$$

The analyst can grab the values of I^V and C from the table in rows 18:26:

$$\underbrace{\text{N31}}_{\rho^o} = \underbrace{\text{K20}}_{\sum_{t=0}^{n} I_t^{Vo}} \;/\; \underbrace{\text{E20}}_{\sum_{t=0}^{n} C_t^o}$$

The analyst can then copy the cell N31 and paste it in ranges N32:N33 and N35:N37. (As explained in Part I, section 13, CFROC^V must coincide with ρ, so $\text{L31} = \text{N31}$, as we expect.)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1														
2			**Measuring value creation and financial efficiency**											
3														
28			**NPV**		**AERI**		**CFROC**		**project rate of return**		**CFROC^V**		**MARR**	
29			$V_0 - C_0$		$\frac{\text{Total ERI}}{n}$		$\frac{\sum_{t=0}^{n} F_t}{\sum_{t=0}^{n} C_t}$		$\frac{\sum_{t=0}^{n} I_t}{\sum_{t=0}^{n} C_t}$		$\frac{\sum_{t=0}^{n} F_t^V}{\sum_{t=0}^{n} C_t}$		$\frac{\sum_{t=0}^{n} I_t^V}{\sum_{t=0}^{n} V_t}$	
31			**NPV^o**	5,622	**AERI^o**	1,124	**CFROC^o**	46.1%	i^o **average ROI**	46.1%	**CFROC^{Vo}**	29.9%	ρ^o **operating MARR**	29.9%
32			**NPV^l**	1,025	**AERI^l**	205	**CFROC^l**	3.8%	i^l **average ROL**	3.8%	**CFROC^{Vl}**	1.1%	ρ^l **non-operating MARR**	1.1%
33			**NPV^{inv}**	6,647	**AERI^{inv}**	1,329	**CFROC^{inv}**	23.9%	i^{inv} **average ROA**	23.9%	**CFROC$^{V\,inv}$**	14.8%	ρ^{inv} **project MARR**	14.8%
35			**NPV^d**	-236	**AERI^d**	-47	**CFROC^d**	2.0%	i^d **average ROI**	2.0%	**CFROC^{Vd}**	2.9%	ρ^d **debt MARR**	2.9%
36			**NPV^e**	6,882	**AERI^e**	1,376	**CFROC^e**	35.4%	i^e **average ROE**	35.4%	**CFROC^{Ve}**	21.0%	ρ^e **equity MARR**	21.0%
37			**NPV^{fin}**	6,647	**AERI^{fin}**	1,329	**CFROC^{fin}**	23.9%	i^{fin} **average ROA**	23.9%	**CFROC$^{V\,fin}$**	14.8%	ρ^{fin} **project MARR**	14.8%
38														

Figure 59. Measures of economic profitability (sheet "ValueCreation")

As the reader can see, areas D31:D32 and D35:D36 of the "ValueCreation" sheet show the NPV for the various areas: $\text{NPV}^o = 5{,}622$ (value created by the project's operations), $\text{NPV}^l = 1{,}025$ (value created by the reinvestment in liquid assets), $\text{NPV}^d = -236$ (value lost by the debtholders at the expense of the firm's shareholders), and $\text{NPV}^e = 6{,}882$ (value created for the firm's shareholders). Cells D33 and D37 report the project NPV: the value created by the project is $\text{NPV} = 6{,}647$, part of which is generated by the liquid assets (because assumptions 21 and 24 imply that $i^l < r^l$, that is, the interest rate on liquid assets is greater than the required return on them). However, the equity NPV ($6,882) is greater than $6,647 because part of it ($236) is lost by the debtholders. This occurs because assumptions 19 and 25 imply $i^d < r^d$, that is, the interest rate on debt is smaller than the required return on debt (i.e., the maximum attractive financing rate for the firm).

An alternative way of calculating the NPV consists in computing the product of the total invested capital (C) and the financial efficiency of the investment, which is equal to the difference between the rate of return and the benchmark rate of return. This represents an explicit link between the capital, the rate of return and the MARR (see box below).

Spreadsheet Modeling: Alternative way of calculating the NPV

The analyst sets up a range of cells that can host the alternative way of calculating the NPV.

Total capital

The first column contains the total capital of the project ($\sum_{t=0}^{n} C_t$), grabbed from the transposed Matrix:

$$\underbrace{\text{D42}}_{\substack{\text{total investment}\\ \text{in operating assets}}} = \text{TransMatrix! P6}$$

The analyst can then copy the cell D42 and paste it in ranges D43:D44 and D46:D48.

Financial efficiency

The second column contains the financial efficiency of the investment $(i - \rho)$, obtained from the values already calculated in the table above.

$$\underbrace{\text{F42}}_{\substack{\text{financial efficiency}\\ \text{of the operating assets}}} = \underbrace{\text{J31}}_{i^o} - \underbrace{\text{N31}}_{\rho^o}$$

The analyst can then copy the cell F42 and paste it in ranges F43:F44 and F46:F48.

NPV

The last column contains the new calculation of NPV:

$$\underbrace{\text{H42}}_{\text{NPV}^o} = \underbrace{\text{D42}}_{\substack{\text{total investment}\\ \text{in operating assets}}} * \underbrace{\text{F42}}_{\substack{\text{financial efficiency}\\ \text{of the operating assets}}}$$

The analyst can then copy the cell H42 and paste it in the ranges H43:H44 and H46:H48.

	A	B	C	D	E	F	G	H	I
1									
2			**Measuring value creation and financial efficiency**						
3									
40			**total capital**		**financial efficiency**		**NPV**		
42			ΣC^o	34,681	$i^o - \rho^o$	16.2%	NPV^o	5,622	
43			ΣC^l	38,242	$i^l - \rho^l$	2.7%	NPV^l	1,025	
44			ΣC^{inv}	72,923	$i^{inv} - \rho^{inv}$	9.1%	NPV^{inv}	6,647	
46			ΣC^d	25,000	$i^d - \rho^d$	-0.9%	NPV^d	-236	
47			ΣC^e	47,923	$i^e - \rho^e$	14.4%	NPV^e	6,882	
48			ΣC^{fin}	72,923	$i^{fin} - \rho^{fin}$	9.1%	NPV^{fin}	6,647	
49									

Figure 60. Alternative way of calculating the NPV (sheet "ValueCreation")

In particular, the total capital invested in the project is C =\$72,923 at an above-normal rate of return of $i - \rho = 9.1\%$, which makes a \$6,647 gain. The total capital borrowed is C^d =\$25,000 at a rate of return which is lower than the benchmark rate of return by $|i^d - \rho^d|$ =0.9%, so the debt NPV is slightly negative and equal to $\text{NPV}^d = -236$. The total capital invested by shareholders is $C^e =$ \$47,923, invested at an above-normal rate of return of $i^e - \rho^e = 14.4\%$, which makes a \$6,882 increase in wealth.

CONCLUSIONS

This paper provides with practitioners a guide to innovative financial modeling. Specifically, it implements the accounting-and-finance engineering system (AFES) described in Magni (2020, 2022) on spreadsheet software. The AFES is based on the split-screen approach, and we guide the modeler step by step showing how to build a frugal, transparent, and easy-to-use model for performing a detailed financial analysis of a capital asset project. The AFES articulation is such that it acts as a diagnostic tool, spotting modeling errors and internal inconsistencies model. In particular, if numbers in rows and columns do not match, there is some inconsistencies in the calculation. Also, if the absolute measures of worth are not reciprocally consistent, then some problems arise in some parts of the model (either in the evaluation process or, earlier, in processing the accounting and financial magnitudes). We start from the verbal description of the project, then carry out preliminary

calculation and use them to obtain the split-screen strip, whereby the pro forma financial statements are chronologically chained. We divide the model into 7+1 modules with 8 worksheets associated with them and make the file available for downloading at the following address: http://morespace.unimore.it/carloalbertomagni/spreadsheet-modeling/.

SYMBOLS AND ABBREVIATIONS

AERI	average economic residual income
AP_t	accounts payable
AP_t^m	accounts payable (manufacturing)
AP_t^{nm}	accounts payable (nonmanufacturing)
AR_t	accounts receivable
C_t^d	debt capital
C_t^e	equity capital
C_t^{fin}	capital raised from capital providers (claimholders)
C_t^{inv}	capital invested in the project
C_t^l	liquid assets
C_t^o	operating assets
CFD	cash flow to debt
CFE	cash flow to equity
CFL	cash flow from/to liquid assets
CFROC	project's cash-flow return on capital
CFROC^V	benchmark's cash-flow return on capital
COGS_t	cost of goods sold
COP_t	cost of purchases
COP_t^m	cost of purchases (manufacturing)
COP_t^{nm}	cost of purchases (nonmanufacturing)
Dep_t	depreciation
EBIT_t	earnings before interest and taxes
EBITDA_t	earnings before interest, taxes, depreciation, and amortization
EBT_t	earnings before taxes
F_t^{ap}	payments to suppliers
$F_t^{ap,m}$	payments to suppliers (manufacturing)
$F_t^{ap,nm}$	payments to suppliers (nonmanufacturing)
$F_t^{swp,m}$	payments to employees (manufacturing)
$F_t^{swp,nm}$	payments to employees (nonmanufacturing)
F_t^{ar}	cash receipts from customers
F_t^{cogs}	payments for COGS
F_t^d	cash flow to debt
F_t^e	cash flow to equity
F_t^{fin}	cash flow to capital providers (claimholders)
F_t^{inv}	cash flow from project's assets
F_t^l	cash flow from liquid assets (net of deposits)
F_t^{nfa}	asset disposal (net of capital expenditures)
F_t^o	cash flow from operations
F_t^{swp}	payments to employees

F_t^{tp}	payments for income taxes
F_t^V	benchmark's cash flow
FCFE	free cash flow to equity
I_t^d	interest expenses
I_t^e	net income
I_t^{fin}	income from the project's assets
I_t^{inv}	income to capital providers
I_t^l	interest income
I_t^o	operating income
I_t^V	benchmark profit
i^d	return on debt (loan interest rate)
i^l	return on non-operating assets (interest rate on liquid assets)
Inv_t	inventory
ΔInv_t	income from inventory (change in inventory)
LC_t	labor costs
LC_t^m	labor costs (manufacturing)
LC_t^{nm}	labor costs (nonmanufacturing)
MARR	Minimum Attractive Rate of Return
NFA_t	net fixed assets
NOL	net operating liabilities
NOL_t^m	net operating liabilities (manufacturing)
NOL_t^{nm}	net operating liabilities (nonmanufacturing)
NOWC_t	net operating working capital
NPV	net present value
OCF	cash flow from operations
r	project cost of capital (required return on assets)
r^d	cost of debt (required return on debt)
r^l	cost of non-operating assets (required return on non-operating assets)
r^o	cost of operating assets, pre-tax WACC (required return on operating assets)
ρ	MARR for the project
ρ^d	MARR for the debt
ρ^e	MARR for the equity
ρ^l	MARR for liquid assets
ρ^o	MARR for the operating assets
S_t	sales
SGA_t	selling, general, and administrative expenses
SWP_t	salaries and wages payable
SWP_t^m	salaries and wages payable (manufacturing)
SWP_t^{nm}	salaries and wages payable (nonmanufacturing)
T_t	taxes
TP_t	taxes payable
T_t	income taxes
V_t	project's economic value
V_t^d	economic value of debt
V_t^e	economic value of equity
V_t^l	economic value of non-operating assets
V_t^o	economic value of operating assets
WACC	weighted average cost of capital

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APPENDIX: ALTERNATIVE SCHEMES FOR CFE CALCULATION

In this paper, we have described the case where the firm applies a payout ratio to the net income for determining the CFE. In real-life applications, many different schemes may be employed. In this Appendix, we illustrate two alternative schemes, based on FCFE.

Let us assume that the firm determines the CFE by applying the payout ratio (20%) to the FCF, instead of the net income. This implies that Table 5 turns to Table 7.

Table 7. Breakdown of CFE (payout ratio: 20% of FCFE).

	TIME	**0**	**1**	**2**	**3**	**4**	**5**
CASH FLOW TO EQUITY (F_t^e)	Shareholders' investment	$-6{,}000$	0	0	0	0	0
	Interim payout (20% of FCFE)	0	$0.2 \cdot FCFE_1$	$0.2 \cdot FCFE_2$	$0.2 \cdot FCFE_3$	$0.2 \cdot FCFE_4$	0
	Liquidation dividend	0	0	0	0	0	$C_4^e + I_5^e$

The difference between Table 5 and Table 7 lies in the "Interim payout" row.

Spreadsheet Modeling: Cash Flow to Equity calculated as a fraction of FCFE

Section 2.2.3 contains the formula used for filling the cells $-F_t^e$ of the Full-scale Matrix. The spreadsheet code for the cell I23 ($-F_0^e$) is:

$$\overbrace{\text{I23}}^{-F_0^e} = \text{IF}(\overbrace{\text{I9}}^{\text{period}} = 0, \underbrace{\overbrace{\text{Assumptions!\$H\$21}}^{\text{equity financing}}}_{\text{case } t=0}, \underbrace{\text{IF}(\overbrace{\text{I9}}^{\text{period}} < \overbrace{\$\text{D}\$2}^{n}, \underbrace{\overbrace{-\overbrace{\text{H23}}^{I_0^e} * \overbrace{\$\text{D}\$6}^{20\%}}^{\text{interim payout (20\% of net income)}}}_{\text{case } 0<t<n}, \underbrace{\overbrace{-(\text{G23} + \text{H23})}^{\text{liquidation dividend}}}_{\text{case } t=n})}_{\text{case } t\neq 0})$$

Assuming FCFE is the calculation base for the CFE, one only needs change the part of the formula that calculates the interim payout ("$-\text{H23} * \$\text{D}\$6$" in the example above). Specifically, net income ($-\text{H23}$) must be replaced by FCFE. The latter is equal to the difference between OCF and CFD ($\text{FCFE}_t = F_t^o - F_t^d$) and the OCF is equal to

$$F_t^o = F_t^{ar} - F_t^{ap,m} - F_t^{ap,nm} - F_t^{swp,m} - F_t^{swp,nm} + F_t^{nfa} - F_t^{tp}.$$

The new spreadsheet code for I23 is

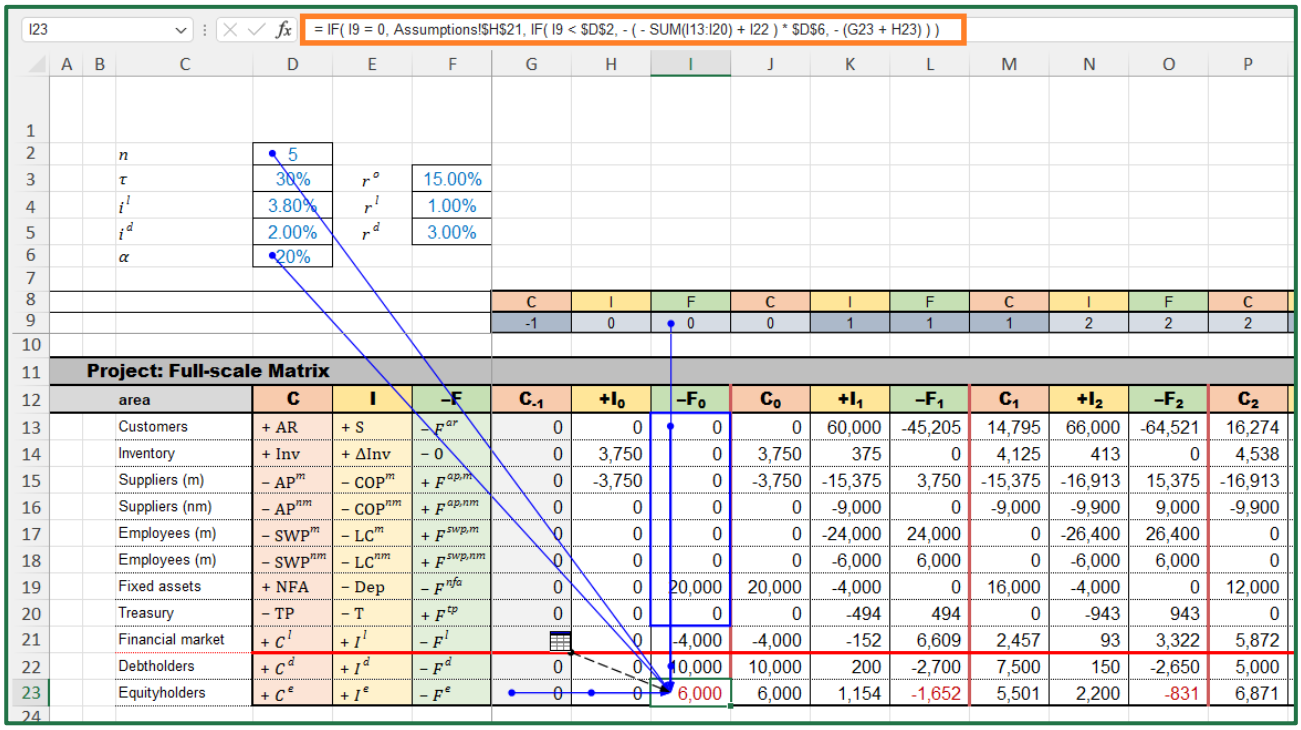
$$\overbrace{\text{I23}}^{-F_0^e} = \text{IF}(\overbrace{\text{I9}}^{\text{period}} = 0, \underbrace{\overbrace{\text{Assumptions!\$H\$21}}^{\text{equity financing}}}_{\text{case } t=0},$$

$$\underbrace{\text{IF}(\overbrace{\text{I9}}^{\text{period}} < \overbrace{\text{\$D\$2}}^{n}, \underbrace{-(-\overbrace{\text{SUM(I13:I20)}}^{\text{OCF}_0} + \overbrace{\text{I22}}^{-F_0^d}) * \overbrace{\text{\$D\$6}}^{20\%}}_{\text{case } 0<t<n}, \underbrace{\overbrace{-(\text{G23} + \text{H23})}^{\text{liquidation dividend}}}_{\text{case } t=n})}_{\text{case } t\neq 0}$$

interim payout (20% of FCFE)

FCFE

$-F_0^{ar} - 0$
$+F_0^{ap,m} + F_0^{ap,nm}$
$+F_0^{swp,m} + F_0^{swp,nm}$
$-F_0^{nfa} + F_0^{tp}$



area	C	I	–F	C_{-1}	$+I_0$	$-F_0$	C_0	$+I_1$	$-F_1$	C_1	$+I_2$	$-F_2$	C_2
Customers	+ AR	+ S	$-F^{ar}$	0	0	0	0	60,000	-45,205	14,795	66,000	-64,521	16,274
Inventory	+ Inv	+ ΔInv	− 0	0	3,750	0	3,750	375	0	4,125	413	0	4,538
Suppliers (m)	$-AP^m$	$-COP^m$	$+F^{ap,m}$	0	-3,750	0	-3,750	-15,375	3,750	-15,375	-16,913	15,375	-16,913
Suppliers (nm)	$-AP^{nm}$	$-COP^{nm}$	$+F^{ap,nm}$	0	0	0	0	-9,000	0	-9,000	-9,900	9,000	-9,900
Employees (m)	$-SWP^m$	$-LC^m$	$+F^{swp,m}$	0	0	0	0	-24,000	24,000	0	-26,400	26,400	0
Employees (m)	$-SWP^{nm}$	$-LC^{nm}$	$+F^{swp,nm}$	0	0	0	0	-6,000	6,000	0	-6,000	6,000	0
Fixed assets	+ NFA	− Dep	$-F^{nfa}$	0	0	20,000	20,000	-4,000	0	16,000	-4,000	0	12,000
Treasury	− TP	− T	$+F^{tp}$	0	0	0	0	-494	494	0	-943	943	0
Financial market	$+C^l$	$+I^l$	$-F^l$		0	-4,000	-4,000	-152	6,609	2,457	93	3,322	5,872
Debtholders	$+C^d$	$+I^d$	$-F^d$	0	0	10,000	10,000	200	-2,700	7,500	150	-2,650	5,000
Equityholders	$+C^e$	$+I^e$	$-F^e$	0	0	6,000	6,000	1,154	-1,652	5,501	2,200	-831	6,871

Figure 61. An alternative way to determine the Cash Flow to Equity in cell I23 (sheet "SplitScreenStrip")

Other ways to determine the distribution policy are possible and one just has to replace the nested formula that calculates the interim payout. For example, the payout to equityholders might be calculated as the minimum value between net income and FCFE:

$$\text{interim payout (period 0)} = \overbrace{-\text{MIN}(\overbrace{\text{H23}}^{I_0^e}, \overbrace{(-\overbrace{\text{SUM(I13:I20)}}^{\text{OCF}_0} + \overbrace{\text{I22}}^{-F_0^d})}^{\text{FCFE}_0})}^{\text{MIN}(I_0^e,\ \text{FCFE}_0)} * \overbrace{\text{\$D\$6}}^{20\%}$$

More generally, a firm may not be willing (or may not be allowed) to distribute cash to its shareholders if either the net income or the FCFE is negative. In this case, the previous formula changes to

$$\text{interim payout (period 0)} = -\,\text{MAX}(\,\overbrace{\text{MIN}(\,\overbrace{\text{H23}}^{I_0^e}\,,\,\overbrace{(-\text{SUM}(\text{I13:I20}) + \text{I22}\,)}^{\text{FCFE}_0}\,)}^{\text{MIN}(\,I_0^e,\ \text{FCFE}_0\,)}\,,0\,)\ *\ \overbrace{\$\text{D}\$6}^{20\%}$$

which implies that the firm will pay the minimum between net income and FCFE only if both are positive (if one of them is zero or negative, then $\text{MIN}(\,\text{H23}, -\text{SUM}(\text{I13:I20}) + \text{I22}), 0) \leq 0$ and $\text{MAX}(\,\leq 0\,, 0\,) = 0$).

Smart-Meter Installation Scheduling in the Context of Water Distribution

Dario Vezzali

Manuel Iori

Carlo Alberto Magni

Andrea Marchioni

Davide Baschieri

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the cleaning operation has to be complete. To tackle the problem, we propose a constructive heuristic followed by several local search procedures that are used one after the other in an iterative way. Extensive tests on real-world instances provided by the company prove that the proposed algorithms can obtain very good-quality solutions within a limited computing time.

3 - Smart-Meter Installation Scheduling in the Context of Water Distribution
Dario Vezzali, Manuel Iori, Carlo Alberto Magni, Andrea Marchioni, Davide Baschieri

In this work, we propose a Mixed Integer Linear Programming (MILP) formulation to model a Smart-Meter Installation Scheduling Problem (SMISP) in the context of water distribution. The model has been used to solve a real case study from a multi-utility company operating in the Italian market. Specifically, in compliance with the European and the Italian regulations on metering, a distribution company is obligated to periodically control meters and substitute them in case they have reached their lifespan. In the examined case study, the multi-utility company has opted for a massive substitution plan in order to install innovative "walk-by smart-meters" in place of traditional mechanical meters. The MILP formulation aims at integrating both the operational and the financial perspective of the SMISP. In particular, the objective function has been carefully defined in order to maximize the Net Present Value (NPV) of the massive substitution plan, including the operational savings produced by using the walk-by smart-meters, the additional incomes originating from the gradual charge of substitution costs on customers' invoices as considered by the Italian Authority, the depreciation of walk-by smart-meters, the investment costs, and the impact of income taxes on the objective function. The final goal of the proposed formulation is to define a scheduling for the massive substitution plan that satisfies a number of operational constraints and produces the maximum NPV.

■ WB-62

Wednesday, 10:30-12:00 - Virtual Room 62

Sırma Zeynep Alparslan- Gök

Stream: Keynotes
Keynote session
Chair: *Gerhard-Wilhelm Weber*

1 - Recent advances in Cooperative games and their potential on Economics and Operations Research situations under uncertainty
Sirma Zeynep Alparslan Gök

Cooperative game theory has been enriched in recent years with several models which provide decision-making support in collaborative situations under uncertainty. Such models are generalizations of the classical model regarding the type of coalition values. Thus, the characteristic functions are not real valued as in classical case; meaning that payoffs to coalitions of players are known with certainty; but they capture the uncertainty on the outcome of cooperation in its different forms such as interval uncertainty, grey uncertainty, fuzzy uncertainty. To incorporate uncertainty in cooperative games is motivated by the real world, where noise in observation and experimental design, incomplete information and further vagueness in preference structures and decision making play an important role. This causes a great mathematical challenge which is approached and well understood in the case of interval uncertainty. This talk surveys and improves recent advances in understanding the mathematical foundations and interdisciplinary implications of cooperative games under uncertainty as well as the connections of Operations Research approach to economics and its related instruments.

Wednesday, 12:30-14:00

■ WC-02

Wednesday, 12:30-14:00 - Bulding A, Room A5

MCDA Methods 2

Stream: Multiple Criteria Decision Analysis
Invited session
Chair: *Sarah Ben Amor*

1 - Consolidation of criteria weights and veto related preference structures in multi-attribute utility models for sorting
Andrej Bregar

Multi-attribute utility models have been recently extended with veto related preference structures, which are adopted from the outranking approach and allow for non-compensation of unsatisfactory preferences. The more selective that the effect of veto is with regard to a criterion, the more this criterion influences the evaluation of alternatives. Veto hence reflects the importance of criteria and exhibits similar preference structures as criteria weights. In this study, the correlation between veto and criteria weights is introduced and methodologically applied to MAUT based sorting models. An algorithm to infer criteria weights from veto related information is proposed, with which selective strengths of veto degrees are calculated to compare the magnitudes of veto, while strengths of veto assignments are used to determine the influence of veto criteria on the deterioration of categories into which alternatives are sorted. Strengths of non-compensatory veto criteria are then projected into weights of compensatory utility criteria. To sort alternatives, hierarchical and linear ascending procedures are applied and assessed. The experimental study reveals the characteristics of indirectly derived criteria weights and the influence of veto. Several quality factors are considered, such as the validity of weights, accuracy of results, richness of information and ability to discriminate conflicting alternatives. Weights are also compared to standard ROC and RS surrogate weights.

2 - A robust multiple-criteria aggregation procedure for mixed evaluations
Sarah Ben Amor, Ahmet Kandakoglu, Anissa Frini

In most multicriteria decision aiding situations, the presence of different types of information imperfections such as uncertainty, imprecision and ambiguity, is inevitable. MCAPmix, a Multiple-criteria aggregation procedure for mixed evaluations, is a ranking procedure that provides a framework to deal with these imperfections on the alternative evaluations using stochastic dominance concepts and distance measures between pairs of alternatives. In this paper, a new multiple-criteria approach addressing the robustness concern is proposed by extending MCAPmix to deal with imperfections on the technical parameters. Different plausible parameters sets are generated to define different versions of the decision problem. The preorders obtained for the different versions are combined into a synthesis preorder. A robustness criterion is then defined based on a distance measure to assess the robustness of the synthesis preorder. In addition, the simple weighted sum to aggregate the local preferences into global ones between the pairs of alternatives, is replaced by an ELECTRE type aggregation in the new procedure. The proposed approach is illustrated through a case study.

3 - Developing an Exact Zoning Optimization Model for Marine Spatial Planning(MSP)
Mohadese Basirati, Patrick Meyer, Romain Billot, Erwan Bocher

Systematic approaches to marine spatial planning (MSP) have recently attracted more attention as an efficient planning tool. MSP is a process gathering multiple users of the ocean with the objective to simplify decisions regarding the sustainable use of marine resources. One of the challenges in MSP is to determine an optimal zone for the activity of

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