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Experiments on shells under base excitation

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Highlights

- The nonlinear dynamics and instability of shells under base excitation are studied experimentally
- The dynamic scenario is analysed for different shell geometries
- Complexity of the dynamics is evidenced: quasi-periodic and chaotic vibrations
- A saturation phenomenon associated with huge lateral vibration levels is observed
- The instability is proven to be quite robust as different parameters are varied

Abstract

The aim of the present paper is a deep experimental investigation of the nonlinear dynamics of circular cylindrical shells. The specific problem regards the response of circular cylindrical shells subjected to base excitation. The shells are mounted on a shaking table that furnishes a vertical vibration parallel to the cylinder axis; a heavy rigid disk is mounted on the top of the shells. The base vibration induces a rigid body motion, which mainly causes huge inertia forces exerted by the top disk to the shell. In-plane stresses due to the aforementioned inertias give rise to impressively large vibration on the shell. An extremely violent dynamic phenomenon suddenly appears as the excitation frequency varies up and down close to the linear resonant frequency of the first axisymmetric mode. The dynamics are deeply investigated by varying excitation level and frequency. Moreover, in order to generalize the investigation, two different geometries are analysed. The paper furnishes a complete dynamic scenario by means of: i) amplitude frequency diagrams, ii) bifurcation diagrams, iii) time histories and spectra, iv) phase portraits and Poincaré maps. It is to be stressed that all the results presented here are experimental.

Introduction

The complex dynamics of shells, under axial base excitation, is the object of the present work, which is part of a long research activity covering theoretical and experimental aspects of the shell behaviour. More specifically, here an intense campaign of experiments is presented in order to reveal the extreme complexity of simple structures such as circular cylindrical shells.

Predicting the mechanical properties of shells, panels and plates is the main concern of structural engineers; shell elements present complicated stability behaviours, rich linear vibration spectra (high modal density), high sensitivity to perturbations, strong interactions with surrounding elements (fluid structure interactions, structure born sound).

Naïve operators are often convinced that any structural problem can be attacked using CAE technologies, for these people only the computational power appears a limitation. Nonetheless, experienced engineers are aware about the limits of traditional computational tools and more generally about limits of virtualization; lab testing appears almost mandatory when one is dealing with shell vibration, this is true *a fortiori* in the case of nonlinear vibration.

Shell structures are likely to vibrate in resonance due to the high modal density, this can induce moderately large amplitude of vibration. Even though the amplitude is moderately high, shells can exhibit surprisingly strong nonlinear phenomena. This is well documented by a large scientific literature, mostly focused on modelling and much less on experimentation.

In Refs. [1]-[8] a comprehensive literature analysis of the western and eastern scientific production can be found, as well as details regarding theories and the most representative experimental findings.

Here a short description of the literature strictly related to the present paper is given. The literature review is subdivided into experimental and theoretical studies.

Experimental studies

In Ref. [9], 1977, one of the first experimental studies regarding shells subjected to base excitation was presented; an extremely violent dynamic instability, occurring when the vibration modes are parametrically excited in principal and secondary instability regions, was described. In Refs. [10],[11] theoretical models were presented in order to give an initial explanation of several experimental results related to axially loaded shells; discrepancies between the theoretical forecasts and the experimental evidence were justified by imperfections (geometrical, material, loading) combined with the presence of conjugate modes that can split due to the perturbations. In Ref. [12] further experiments were presented about shells subjected to base motion (clamped free); both instability regions and nonlinear regimes were investigated; in particular, it was found a hardening behaviour for modes having two circumferential diameters (n=2) and softening type for n>2.

In Ref. [13] a new phenomenon of instability was discovered when a circular cylindrical shell connected with a rigid body on the top is excited by a base (axial) vibration. Again, a violent dynamic response was detected when the excitation frequency approaches the natural frequency of the first axisymmetric mode; out-of-plane (radial) huge vibration levels were measured, up to 2000g (acceleration), with a moderately high base excitation (10g). No subharmonic response was detected and a high frequency energy transfer was observed; conjectures about the possible excitation of high frequency shell-like modes (n>1) were made.

In Refs. [14],[15],[16] an intense theoretical and experimental activity regarding shells connected with rigid bodies under axial excitation was presented; the authors found interesting dynamic phenomena having chaotic character, beating phenomena as well as sensitivity to imperfections. They considered both isotropic and orthotropic models to simulate the experimental results. Moreover, they supposed that some of the complex dynamics found experimentally were due to the interaction between the vibrating structure

and the excitation source; therefore, they developed an interactional theoretical model that considers both the shell and the shaker dynamics; by combining the structural and electro-dynamic equations they furnished an interpretation to their experimental results.
It is to be noted that, at our knowledge, the first publication regarding the modelling of shaker-structure interaction is due to Krasnopol'skaya [17], she was the first scientist who clarified the instability aspects due to such interaction.
In Refs. [18],[19], [20] the problem of shells connected with a top mass was analysed in detail both from a theoretical and experimental point of view. An accurate experimental modal analysis was used for setting up and validate a new semi-analytical modelling technique based on the Sanders-Koiter theory, capable of handling complex boundary conditions and nonlinear modelling; new experimental data were presented and compared with the new models, which allowed to clarify that the instability and the complexity of the response is due to a combination of effects: interaction between shaker and structure; parametric excitation; high modal density and presence of double modes.

Models
Regarding the theoretical and numerical studies, the scientific production is huge; for the sake of brevity and completeness, here a short list of papers strictly related to the present work is commented.
Since the second half of the last century several papers attacked the problem of shells under axial periodic loads. Initially the research was focused on linear modelling, see e.g. Refs. [21]-[25], the analysis of parametric stability was the first step for interpreting and explaining experiments. Such studies were extremely important as a starting point for further more sophisticated models; for example it was clarified that neglecting the Poisson's effect in the axisymmetric bending vibration, i.e. considering only the membrane approach for in-plane stresses, can lead to highly inaccurate results, see also Refs. [26]- [33].
In Refs. [19], [20] a theoretical model, based on the Sanders-Koiter nonlinear theory combined with the electro-dynamic shaker model, clarified that important non-stationary responses can be predicted if: 1) the energy source is not assumed as infinite (exciter modelling); 2) a nonlinear shell model is considered; 3) complex modal interactions are allowed, including conjugate modes activation.
Recent models correlated with the present work were published in the following papers: in Ref. [34] a theoretical approach based on Reissner–Naghdi's shell theory was presented and solved using a mixed series expansion similar to Refs. [18]-[20], this approach considers a modified variational principle, the work was limited to linear vibrations and no experiments were presented; in Ref. [35] a theoretical approach based on Von Kármán non-linear theory and the first-order shear deformation theory was presented, the study was limited to simply supported shells, no experimental results were presented; in Refs. [36] and [37] classical and higher-order shear deformation theories with von Kármán type non-linearities were used for studying sandwich plates in large amplitude regime, Chebyshev polynomials were considered, a full experimental analysis was published.
Other interesting papers related to the present work in terms of modelling and theory are Refs. [38], [39], where the analytical approaches, used in the past for analysing the phenomena reported here, have been applied to different problems, i.e. functionally graded shells and nanotube dynamics.
The goal of the present work is to furnish an extensive series of experimental data referred to the problem of shells under base excitation. The shells are mounted on a shaking table in vertical position, on the top of the shell a rigid disk is mounted. The base of the shell is excited vertically by a shaker (LDS V806, 13kN force) with moderately low amplitude sinewave signal (few grms). Due to the base motion, a rigid body motion is imposed to the shell and

consequently to the top disk, the latter one has a mass considerably larger than the shell, its vertical motion induces membrane stresses on the shell, mainly along the vertical axis. On the other hand the top disk motion itself is influenced by the modal response of the shell. Eventually, the system made of shell and top disk has a non-trivial interaction with the shaker, even though the shaker moving mass is largely bigger than the whole system mass (shell and top disk). The combination of top disk, shell and shaker dynamics gives rise to extremely violent and complex dynamic responses: energy transfer from low to high frequency, energy transfer from axisymmetric to asymmetric modes, huge lateral vibration levels, non-stationary and chaotic responses.

The dynamic scenario is deeply analysed by varying the frequency and amplitude of excitation; amplitude-frequency and bifurcation diagrams are presented as well as time histories, spectra and Poincaré maps. Two shells having different length are analysed, in order to clarify the effect of the geometry.

It is to point out that this work enlarges and completes the experimental analysis presented in Refs. [18]-[20]; in particular the main differences and novelties with respect to the past publications are: i) it is proven that the complex dynamics appear for different geometries and maintain their general character; ii) a full experimental analysis is carried out by varying the excitation levels; iii) more detailed experimental bifurcation diagrams are presented; iv) Poincaré maps and phase portraits are shown. Therefore, the general achievement of this work is the presentation of the complete experimental dynamic scenario of the systems under investigation.

Experimental setup

The experimental setup is represented in Figure 1, a circular cylindrical shell is mounted vertically on a high power shaking table, shortly called here "shaker" (LDS V806, 13000N peak force, 100g maximum acceleration, 300kg payload, 1–3000Hz band frequency); on the top of the shell a thick aluminium disk is mounted.

The shaker provides a base excitation, which induces inertia forces on the shell and on the top disk, here the disk mass is greatly prevalent with respect to the shell.

A signal generator ONO SOKKI CF-5220, Figure 2, is used for controlling the shaker by means of an open loop scheme; it is to be pointed out that it was impossible to use closed loop control strategies when strongly nonlinear phenomena take place, these phenomena appear suddenly and produce a complex response with broad-band spectrum, in this case the shaker control algorithms present in the commercial controllers are not able to control the base vibration (shaker) as they are designed for regular types of vibration.

The base vibration is measured by a Wilcoxon Research S 100 C accelerometer located on the shell fixture, the disk vibration is measured through a PCB M352C65 micro accelerometer, see Figure 3. A Micro Epsilon optoNCDT 2200 Laser displacement sensor (1μm resolution, sensitivity 2mm/V) is used for measuring the shell radial vibration, this kind of sensors is considered for two reasons: i) it does not perturb the shell response, ii) as the shell vibration can exceed thousand g of acceleration, the use of accelerometers is inadvisable; the counterpart is that the Laser measurement is polluted by the vertical motion of the surface measured (along the shell axis).

The acquisition and control chain of Figure 1 consists of: i) a drive signal generated by the signal generator ONO-SOKKY, which is the input of the power amplifier of the shaker; ii) an acquisition card (NI 4472) that acquires signals of sensors and the drive. The drive signal is acquired in order to take trace of the actual drive synchronized with the system response.

The drive signal is sinusoidal, the amplitude and frequency are controlled manually during the experiments following a sine-step procedure.

The shell is clamped at the base to the fixture (Figure 4), which is a thick aluminium disk bolted to the shaker; the clamping is obtained by bonding the lateral surface of the shell with the corresponding cylindrical protuberance of the fixture. A similar fixing is carried out on the top of the shell, which is connected to a free rigid disk. Therefore, the boundary conditions are: i) bottom, clamping with imposed vertical motion; ii) top, free rigid body motion.

Two specimens having different shell lengths are considered in the present work (see Figure 3 and Figure 4).

The shell material is PET (polymer) with: mass density ρ=1366kg/m^3; Poisson's ratio ν=0.4; Young's modulus E=46×10^{8}N/m^2.

The dimensions of the shells are: mean radius R=43.9×10^{-3}m; effective length $\tilde{L}$=97×10^{-3}m or $\tilde{L}$=57×10^{-3}m (long or short shell); thickness h=0.25×10^{-3}m. The top disk mass is M=0.82kg.

The shaker structure is represented in Figure 5: this is an electro-mechanical machine, its main body is suspended on the ground by means of gas suspensions. The main body (about 10^{3}kg mass) contains a field coil: when the current provided by the amplifier passes on the coil, then a constant magnetic flux is generated in the air gap (stator flux). The shaker vibration is generated by a time variant electric current in the armature coil, such a current is proportional to the drive voltage provided to the shaker amplifier, but it is also influenced by the armature motion, i.e. the actual shaker vibration. The moving element is connected with springs to the stator (main body), therefore it is a mechanical oscillator. The analysis of the shaker structure clarifies that the actual shaker vibration is correlated to the drive excitation by means of a complex equation and it is influenced by the force exerted by the specimen; when the specimen vibration is non-linear, there is not linear proportionality between drive and shaker vibration, with possibility of spectral distortion.

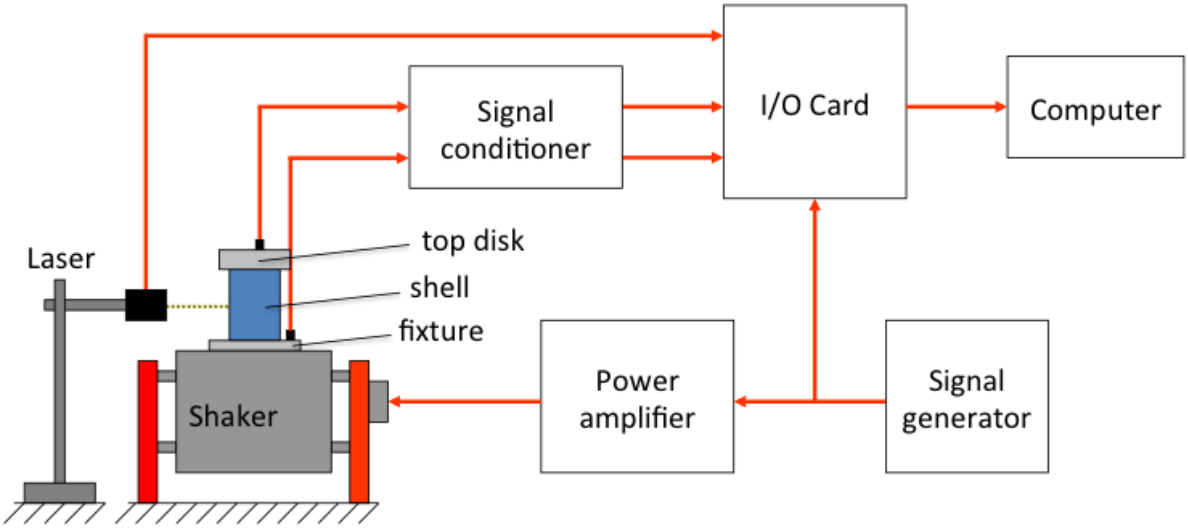

Figure 1 Experimental setup: acquisition and control chain

Figure 2 Signal generator ONO SOKKI CF-5220

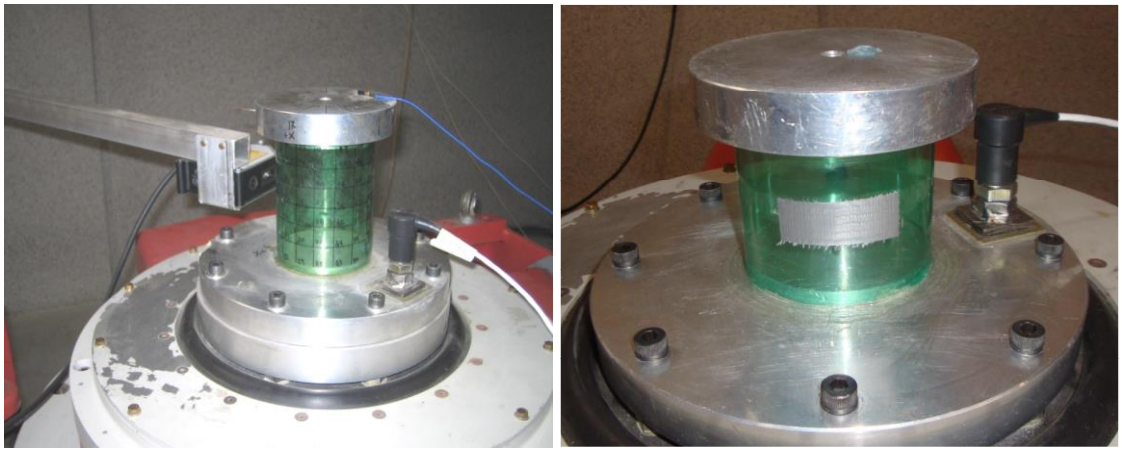

Figure 3 Specimens and sensors

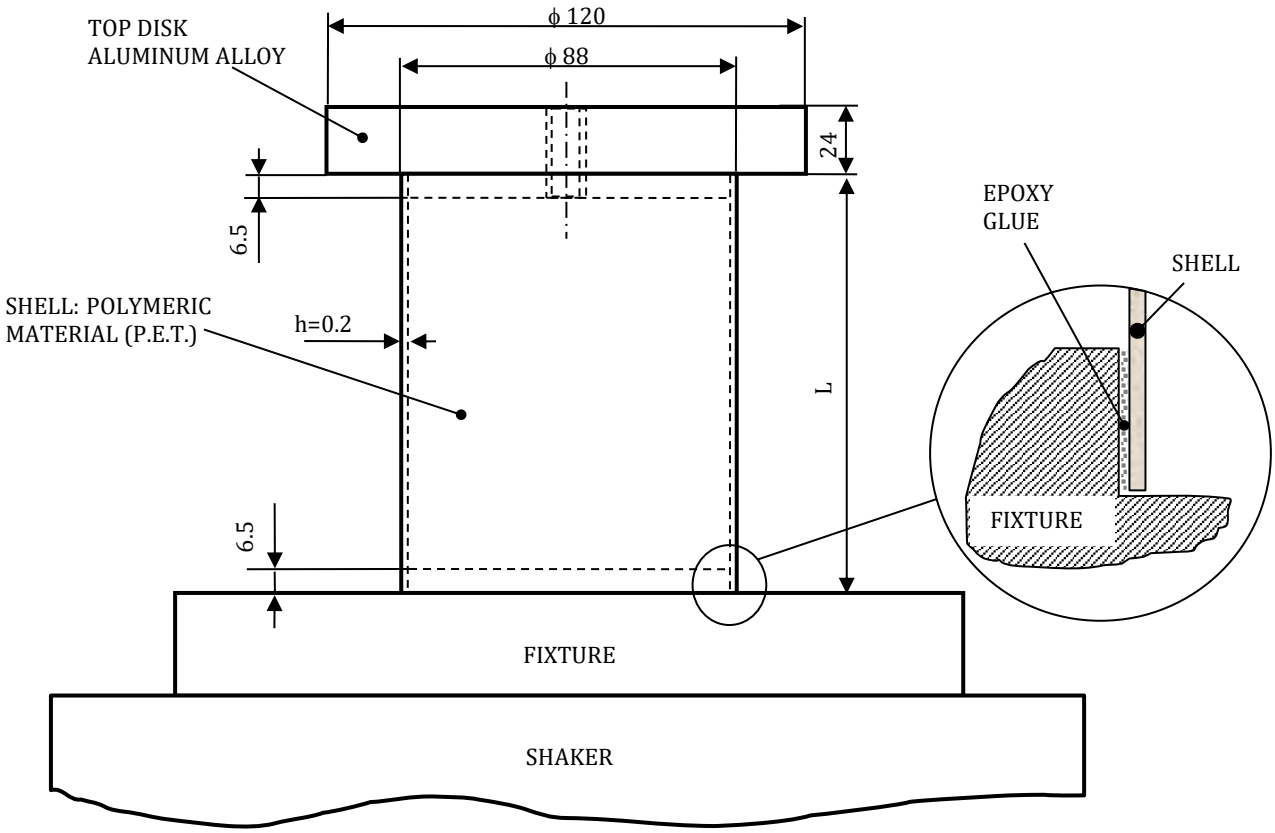

Figure 4 Scheme of the shell-top disk system (dimensions mm): Case long shell L=110mm, Case short shell L=70mm

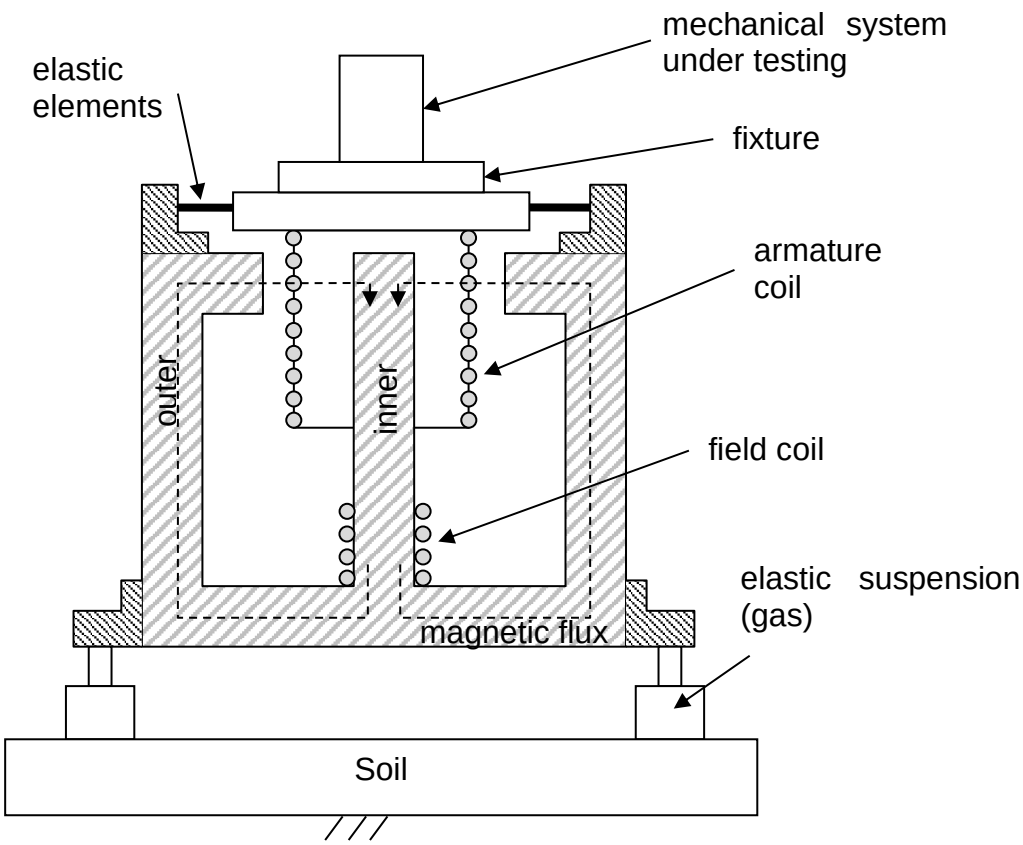

Figure 5 Shaker structure

Details of the testing procedure

The goal of the present experimental campaign is a deeper investigation of the complex dynamic scenario observed in the past [18], where initial tests were limited to a specified excitation level (drive voltage) and a specific shell; the main findings of Ref. [18] are discussed in the introduction, so they are not repeated here. However, it is worthwhile to stress that the violent dynamic phenomena induced by a strong energy transfer from the top mass resonant vibration to the high frequency shell-like modes, having large amplitudes, broad band and non-stationarity characteristics, do not allow to use a closed loop control of the shaker, even though this technology is available in the lab and continually used for standard testing on the shaking table.

As the intent is the analysis of the system subjected to a harmonic-like base excitation and the investigation of complex dynamics, a sine step procedure is considered (sine excitation with the frequency varied stepwise). In this work the step sine procedure is carried out fully manually: the output of the signal generator is gradually varied in order to pass from one frequency to the next, then few seconds of delay are considered for eliminating the transient, eventually the acquisition is carried out.

Measurements relate to lateral shell displacement, top disk acceleration, base acceleration and drive signal. The first two quantities are the most important ones: the top disk vibration is magnified by the first axisymmetric mode resonance; the shell vibration should be negligible when compared with the top disk, as it is due to the Poisson's effect, but it can become huge when a nonlinear energy transfer takes place. In addition to the main physical quantities, also the base acceleration is measured; indeed, the interaction between the shaker and the structure under testing and the use of an open loop control induce a variation of the base amplitude; moreover, the nonlinear phenomena in the shell vibration cause a spectrum distortion on the base vibration, which should be ideally sinusoidal. Eventually, also the drive signal is acquired; this acquisition could appear useless, but its uselessness is only apparent for the following reasons: i) reacquiring the generated signal allows to check the quality of the signal and in particular the noise level; ii) the actual drive frequency can be estimated with great accuracy; iii) the drive signal allows to synchronize the other signals and build up Poincaré maps and their bifurcation diagrams.

For all the tests, a 10,000Hz sampling rate is considered with 1s of acquisition time, i.e. 10,000 samples are recorded for each acquisition. The NI4472 acquisition card is provided with anti-aliasing filters, so the available bandwidth is less than 5,000Hz.

The excitation frequency is varied in the neighbourhood of the resonance of the first axisymmetric mode (approximately 320Hz for the long shell and 370Hz for the short shell) using about 20-30 steps.

The environmental temperature was about 25°C during experiments, which were carried out without temperature control; therefore, a temperature variation of the specimen at the beginning of the test was unavoidable: in order to overcome this problem, the tests were carried out one hour after switching on the shaker.

Overview on the system spectrum

The frequency spectrum of the shells under investigation displays a non-standard behaviour due to the presence of the heavy rigid body on the top having a mass (0.82kg) much bigger than that of the shells (8.94×10^{-3}kg for the long and 4.96×10^{-3}kg for the short shell). Even though this topic was deeply analysed in Ref. [18] using analytical, numerical and experimental tools, here some comments are needed; in fact, the linear character of the system is extremely useful for understanding the complicated nonlinear phenomena observed in this experimental research.

In Figure 6 and Figure 7 the first five mode shapes are presented for long and short shells, respectively; these modes are obtained by the theoretical model of Ref. [18] fully validated experimentally. The first three modes are strongly influenced by the presence of the top disk; the first mode (95.8Hz for the long shell and 151.5Hz for the short shell) is quite similar to the one of a cantilever beam, it is extremely influenced by the lateral displacement of the disk and its rotation about a horizontal axis; the second mode (322.0Hz for the long shell and 433.6Hz for the short shell) is axisymmetric, it is virtually the first longitudinal bar-like mode, the disk motion is a pure translation along the shell axis; the third mode (431.8Hz for the long shell and 517Hz for the short shell) is the last mode involving the disk motion (except torsional modes which are out of interest here) that mainly rotates and marginally moves horizontally and vertically, so it appears as a combination of the first two modes, this is quite interesting as it can justify modal interactions in nonlinear field; the fourth mode (802.2Hz for the long shell and 1601.4Hz for the short shell, with six nodal diameters and one longitudinal wave) and the fifth mode (926.9Hz for the long shell and 1894.8Hz for the short shell, with five nodal diameters and one longitudinal wave) are the usual mode shapes of clamped-clamped shells; indeed, for such modes the rigid body displacement or rotation play no role, so the free top disk acts as a pure clamping .

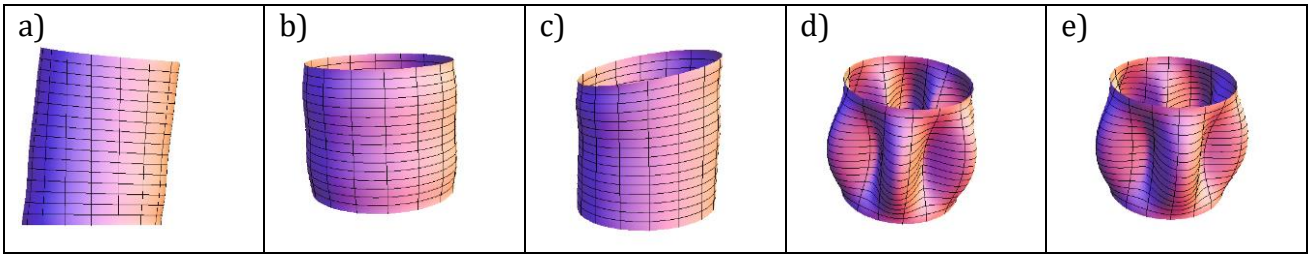

Figure 6 Mode shapes: long shell: a) 95.8Hz beam-like, b) 322.0Hz axisymmetric, c) 431.8Hz combined beam-axisymmetric, d) 802.2Hz shell-like (1,6), e) 926.9Hz shell-like (1,5).

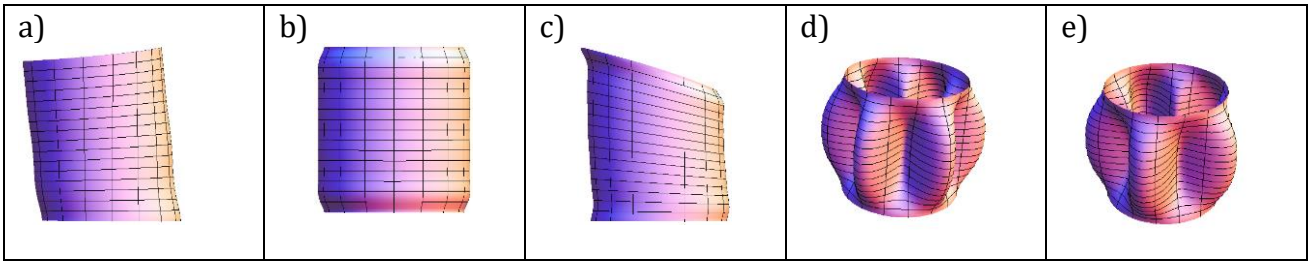

Figure 7 Mode shapes: short shell: a) 151.5Hz beam-like, b) 433.6Hz axisymmetric, c) 517Hz combined beam-axisymmetric, d) 1601.4Hz shell-like (1,6), e) 1894.8Hz shell-like (1,5).

Dynamic scenario: effect of the excitation level and geometry

In order to have a comprehensive overview on the system response and its dependence on excitation and physical parameters, in this section the amplitude-frequency response is analysed in detail.

Long shell

The levels of excitation are set taking advantage from previous studies ([13], [18], [30], [31]) where it was evidenced that the long shell experiences strong vibration levels when a base vibration excites the first axisymmetric mode, with an amplitude of the order of 10g (about 0.1V drive). Here the range of excitation is explored more accurately and the analysis is repeated for the short shell, in order to check if complex behaviours are independent from the geometry.

In Figure 8 the base motion is analysed, the ranges of interest are 295-345Hz and 0.03-0.1V. The base amplitude of vibration is not constant due to the interaction between the shaker and the shell-disk system. In this range, the resonance of the first axisymmetric mode is met. It is to be noted that the amplitude drops down below 320Hz at low voltage, such a valley in frequency changes location (moves toward lower frequency) as the voltage increases; this is due to nonlinear phenomena, which can be evidenced by analysing the top disk and the shell vibration.

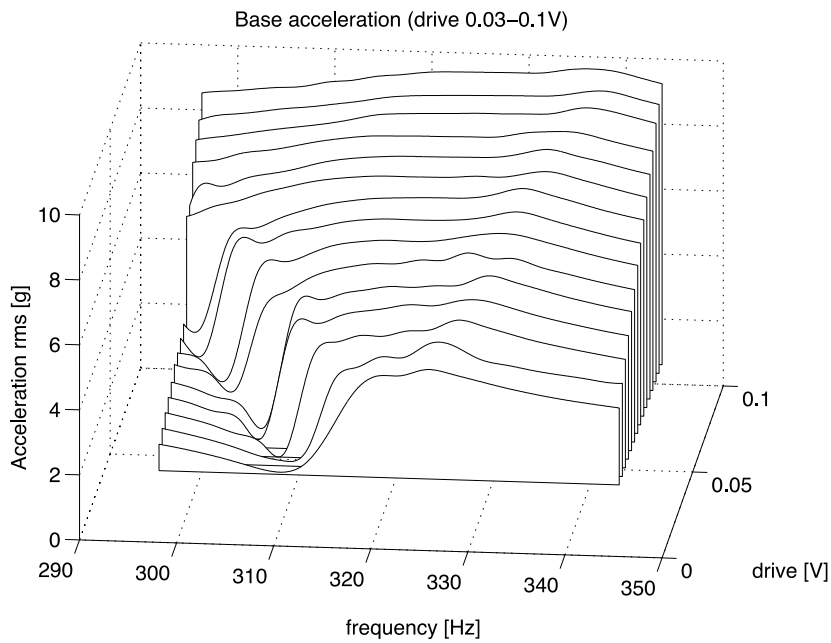

Figure 8 Base acceleration vs. excitation frequency and amplitude of drive voltage. Long shell

The top disk vibration is shown in Figure 9, at low voltage (0.03V) a symmetric curve is visible, showing an almost linear classical resonant behaviour; however, a flat region is present in the central part. Increasing the voltage, this kind of behaviour is systematically repeated, but the width of the flat region increases with the voltage level; moreover, at about 0.06V a jump is evident. Such kind of behaviour is due to an energy transfer from the axisymmetric to the shell-like modes, which are excited by an auto-parametric resonance. An interpretative model was presented in Ref. [18], where also some experiments referred to a single excitation voltage were discussed and compared with the model. The results presented here are referred to the same geometry of Ref. [18] and similar conditions; the experimental results are similar both qualitatively and quantitatively, with differences in terms of top disk vibration, probably due to the accelerometer location. Here the full scenario is presented by varying both frequency and excitation amplitude, it confirms the saturation phenomenon

analysed in Ref. [18], and proves that the same kind of behaviour takes place at different excitation levels.

When the instability takes place, a big amplitude of vibration is induced on the shell, Figure 10, the amplitude reaches almost half millimetre RMS (more than one millimetre MAX); this is the amplitude of a broadband spectrum signal, having energy up to 1500Hz, its second derivative gives an acceleration of the order of thousands g, as confirmed by the early experiments of Ref. [13]. The combined analysis of base, top mass and shell vibrations shows that the instability takes place at a certain level of the top mass vibration, i.e., the axial load of the shell; such axial load induces an auto-parametric resonance of high frequency shell-like modes, which is the reason of the energy transfer from the base and top vibration to the shell.

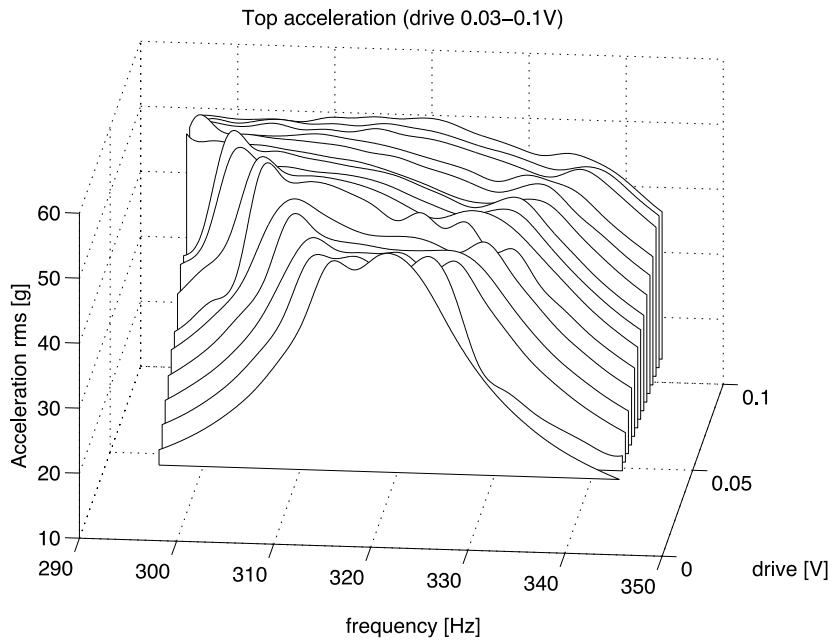

Figure 9 Top acceleration vs. excitation frequency and amplitude of drive voltage. Long shell

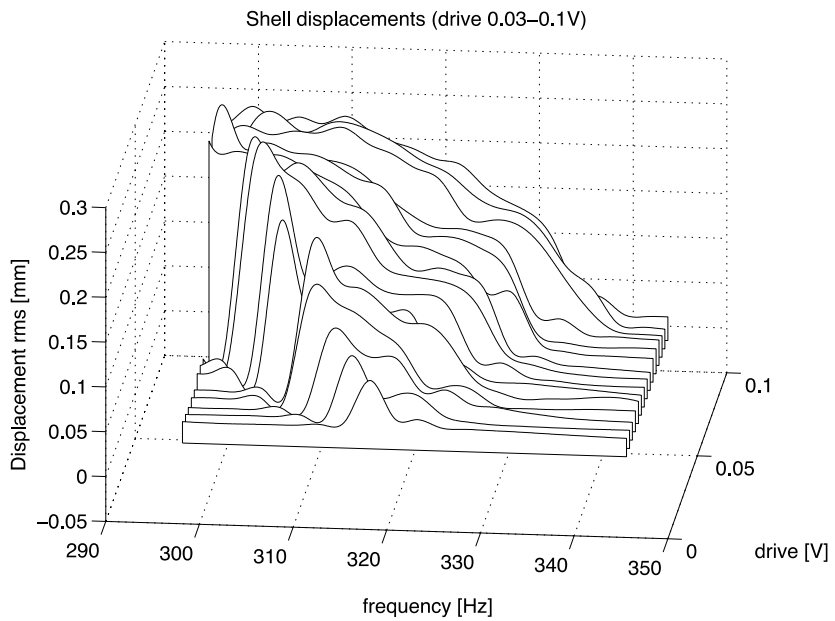

Figure 10 Lateral shell displacement vs. excitation frequency and amplitude of drive voltage. Long shell

From Figure 11 to Figure 13 the dynamic scenario is analysed by comparing amplitude-frequency diagrams in terms of RMS and maxima. The base acceleration shows a strong dependence from the frequency due to the resonance of the axisymmetric mode, the effect of the drive level is a regular increment of the amplitude. The analysis of the top disk vibration is much more interesting; it appears that there are two saturation regions (flat), partially dependent on the frequency and partially on the drive level. From 0.03 to 0.045V only one flat region is present, it is centred at about 320Hz and the corresponding levels of acceleration are 45grms and 65g(max); when the drive level is increased, a second saturation level appears at 50grms and 80g(max). It is to be stressed that the maximum of the top disk vibration is not influenced by the drive amplitude.
The lateral shell vibration shows a huge amplitude of vibration in the instability region, up to 0.25mm(rms) and 2mm(max), it must be compared with the few μm of amplitude outside the region; the big difference between maximum of the signal and its RMS, in the case of the shell lateral vibration, is due to the fact that the time history is quite irregular. It is to be noted that a 2mm amplitude of vibration at 300Hz means 724g acceleration for a pure sinusoidal signal and higher acceleration levels if the signal contains super-harmonics (or generally energy on the higher part of the spectrum); such estimation is of the same order of magnitude measured during the early experiments of Ref. [13]. Moreover, the lateral vibration of Figure 13 clearly shows a softening type response, typical of thin shell structures vibrating at large amplitude (see [7]).

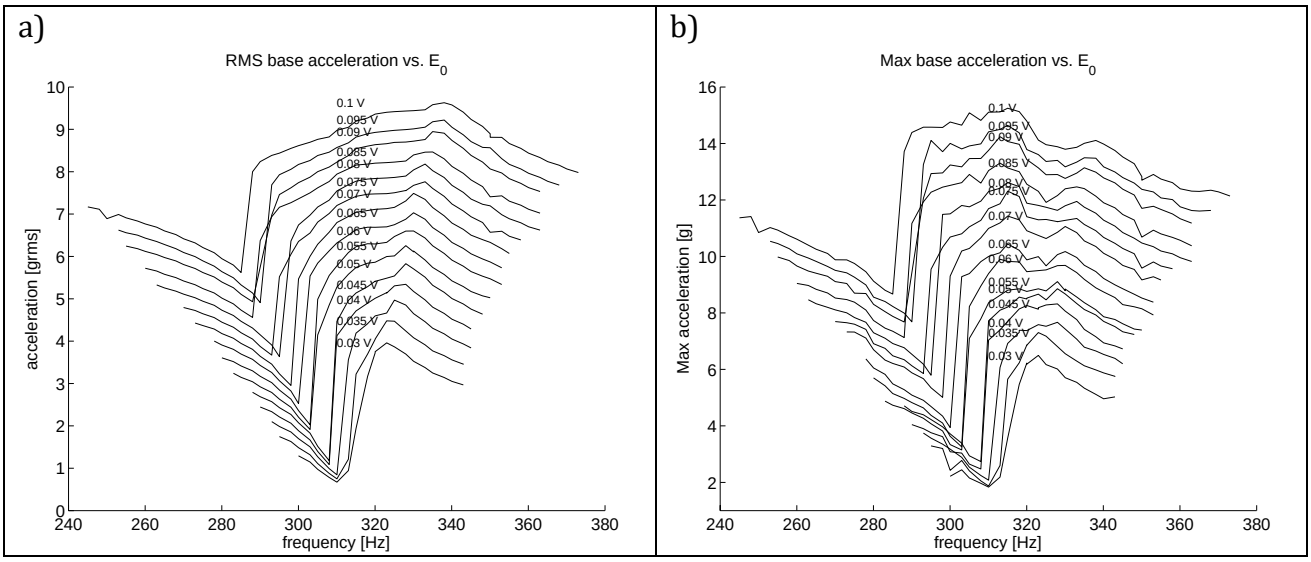

Figure 11 Base acceleration vs. excitation frequency and amplitude of drive voltage. Long shell. a) RMS, b) MAX

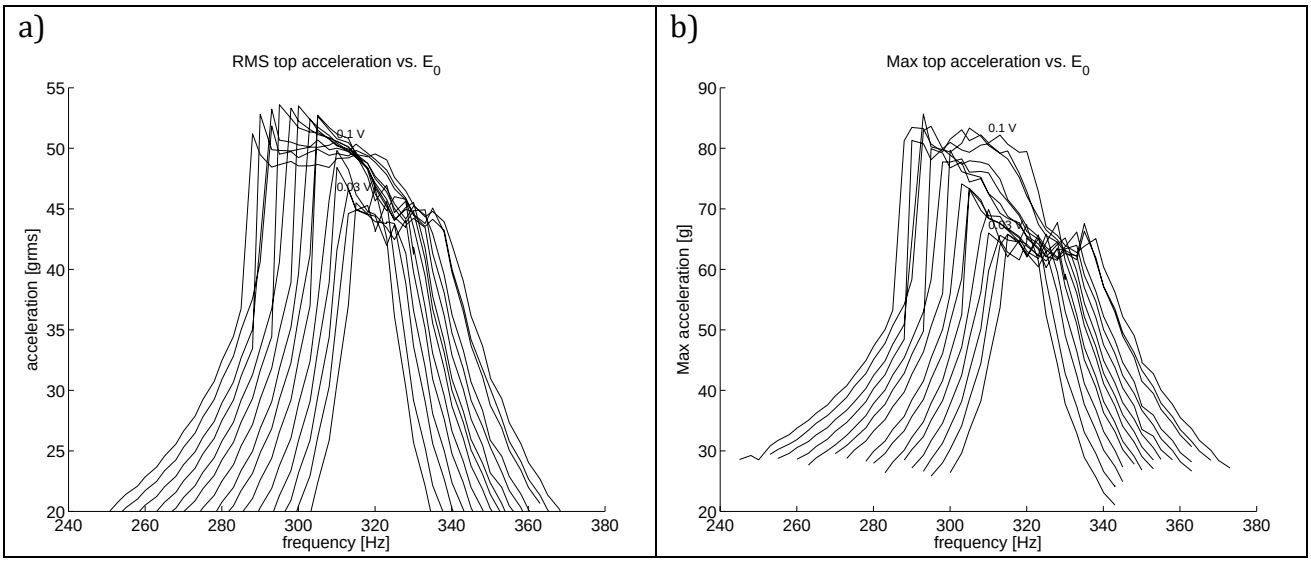

Figure 12 Top acceleration vs. excitation frequency and amplitude of drive voltage. Long shell. a) RMS, b) MAX

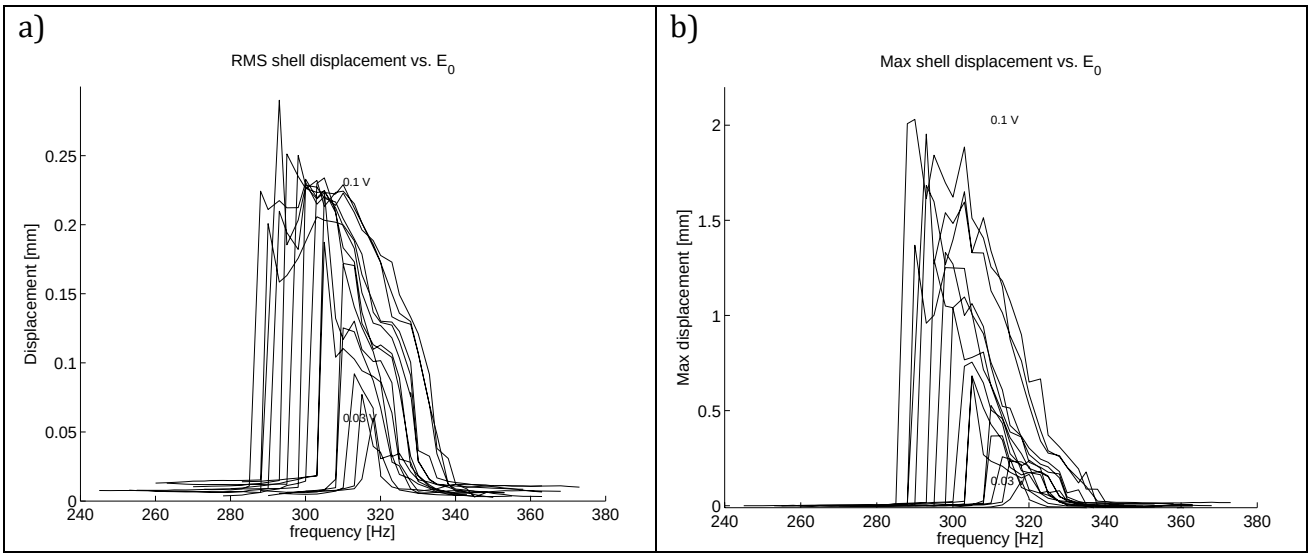

Figure 13 Lateral shell displacement vs. excitation frequency and amplitude of drive voltage. Long shell. a) RMS, b) MAX

This analysis extends and completes the experimental campaign presented in Ref. [18], where only a specific level of excitation was analysed; the use of different excitation levels clarifies that there is a threshold level of the top disk vibration which gives rise to instability.
On the other hand, one could object that the phenomenon observed depends on the specific shell under investigation, which perhaps presents special values of the fundamental ratios. Therefore, a further analysis is carried out on a shorter shell, having about one half the length of the previous one.

Short shell

As pointed out in the section devoted to the system frequency spectrum, the short shell presents a generalized increment of the natural frequencies, without mode switching, i.e., the sequence of modes does not change, even though the increment is not uniform for each mode; this could clearly change the overall scenario.

For the short shell the resonance of the first axisymmetric mode is about 436Hz; also in this case the resonance strongly influences the base excitation, i.e., the shaker performance, as shown in Figure 14, where the base RMS acceleration vs. excitation frequency is represented for different levels of drive amplitude.
The behaviour of the top disk RMS acceleration vs. the excitation frequency, for different levels of the drive amplitude, is represented in Figure 15; the resonance of the first axisymmetric mode is evident in particular for low levels of excitation. Moreover, a saturation phenomenon, quite similar to that of the long shell, is clearly evidenced in the top vibration. For this short shell it was possible to increase the excitation up to 0.13V without damaging the shell. In this case of short shell only one saturation level is found.
When the instability region is met, the shell vibration switches suddenly to huge amplitude: up to 0.5mm(RMS) and 1mm(MAX) for a drive of 0.1-0.13V, see Figure 16.
The overall behaviour of the short shell is qualitatively very similar to that of the long one, proving that the same kind of instability persists after a big variation of the shell geometry.

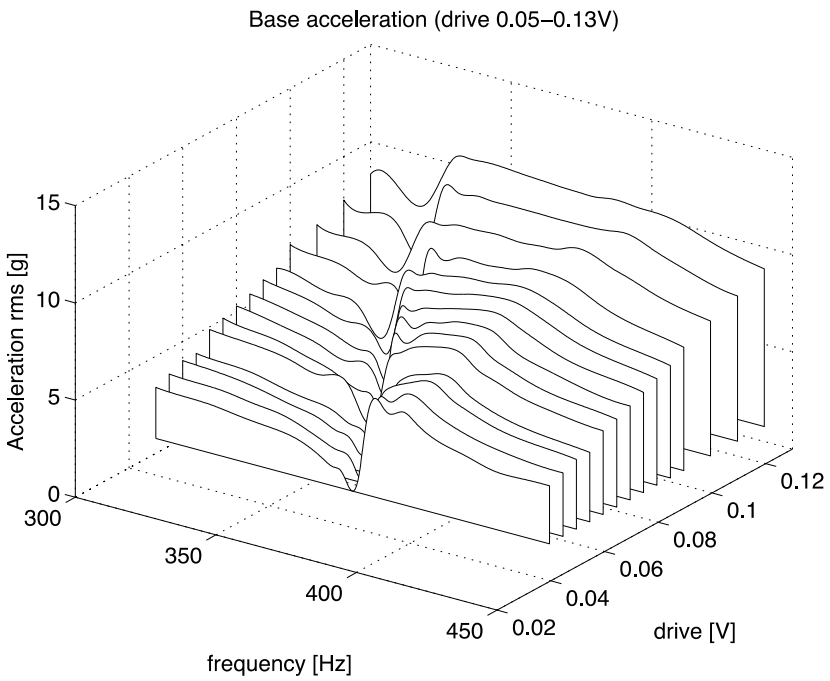

Figure 14 Base acceleration vs. excitation frequency and amplitude of drive voltage. Short shell

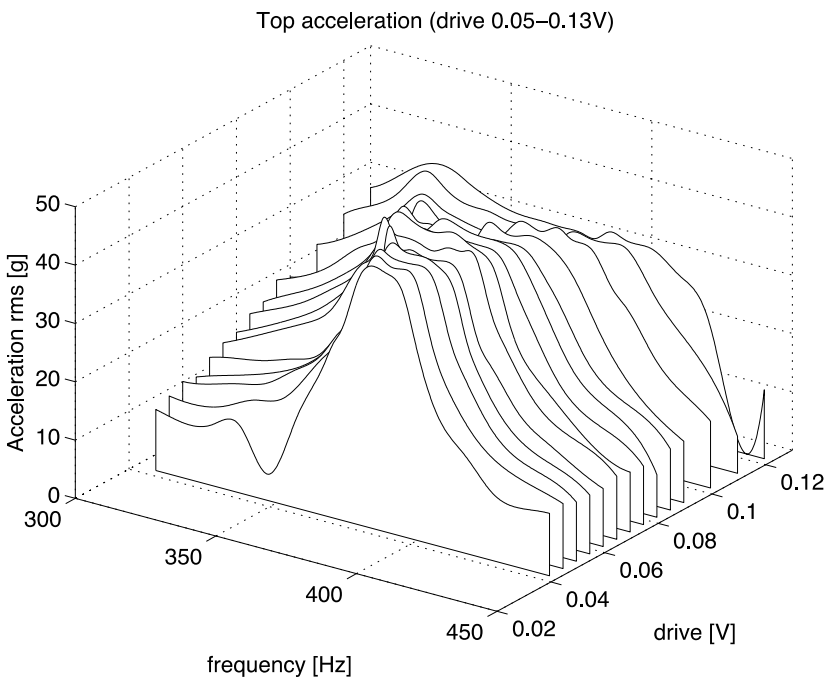

Figure 15 Top acceleration vs. excitation frequency and amplitude of drive voltage. Short shell

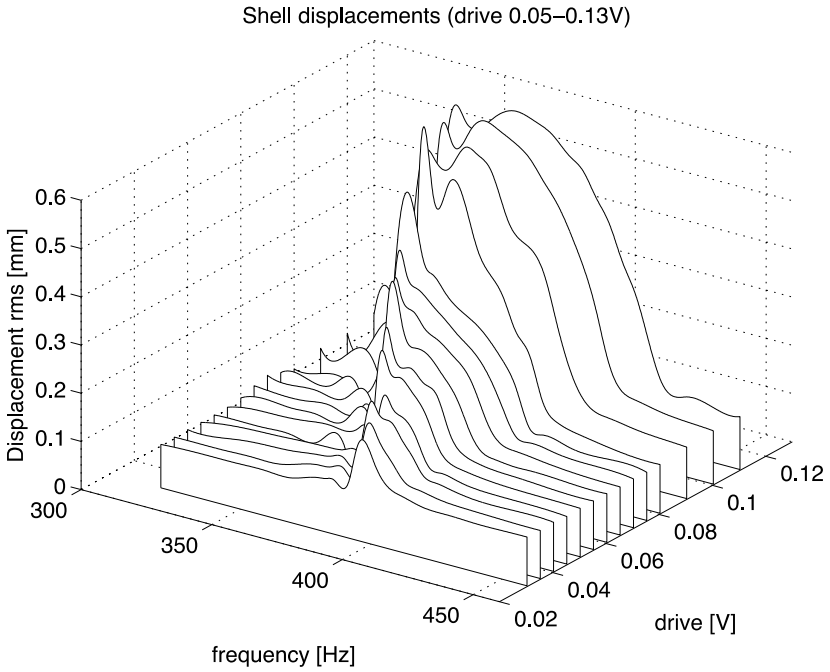

Figure 16 Lateral shell displacement vs. excitation frequency and amplitude of drive voltage. Short shell

Bifurcation analysis

In order to obtain a deeper knowledge of the system response, the experimental data achieved are now processed by means of different tools. This is necessary because the mere presentation of the scenario in terms of RMS or maximum of amplitude is not exhaustive and it does not allow a sufficient understanding of the dynamic scenario.

Here, each time history used in the previous section for the amplitude-frequency diagrams is manipulated for building up Poincaré maps and their bifurcation diagrams.

The Poincaré map of a harmonically forced system is generally obtained by sampling the time history at the frequency of excitation; in this way, for periodic responses with the same period of excitation, the sampling returns always the same value, i.e., one point on the Poincaré map; when the response is one half sub-harmonic, it returns two points and so on.

The procedure used in the present work for extracting Poincaré maps from experimental time histories is the following.

- An additional channel is recorded, see Figure 1, i.e., the drive signal; this is an electric signal having very low pollution and distortion.
- The drive signal is numerically derived with respect to the time by using standard formulae of finite differentiation.
- The crossings of zero of the derived signal are detected as well as their slope, in order to find the maxima of the original signal (positive slope).
- All the physical signals are under-sampled by recording data in correspondence of the maxima of the drive signal (also the drive signal is under-sampled for checking purposes).

The previous procedure is repeated for all the measurements carried out with different amplitude and frequency of the drive signal.

For the long shell, the results are reported in Figure 17, where, for the sake of brevity, only two drive levels are shown (0.1V, 0.065V), even though experimental data are available from 0.05V up to 0.1V, with step of 0.005V.

The shell response appears periodic outside the instability region, while inside the instability region two different dynamic behaviours can be observed: at 0.1V from 288Hz to 323Hz the response is strongly non stationary, then it becomes more regular 323-340Hz (total width 288-340Hz), over 340Hz the instability disappears; at 0.065V the nonlinear behaviour is similar, but the instability region is smaller (300-330 Hz) and divided into two parts with a different degree of non-stationarity.

The top disk vibration displays a similar scenario with an evidence of non-steady response and two distinct frequency intervals. It is to be noted that the nonlinear character is not so evident as for the shell, probably due to the inertial filtering properties of the big top mass.

The short shell is analysed for the same voltage amplitudes, Figure 18, i.e., 0.1V and 0.065V; the bifurcation diagrams for the shell response and top disk vibration show that also for the short shell the instability region is characterized by a strongly unsteady response. For the short shell, the bifurcation diagrams show different character of the response as the excitation frequency varies, it is to be noted that this change of nonlinear character was not evident from the previous amplitude-frequency diagrams.

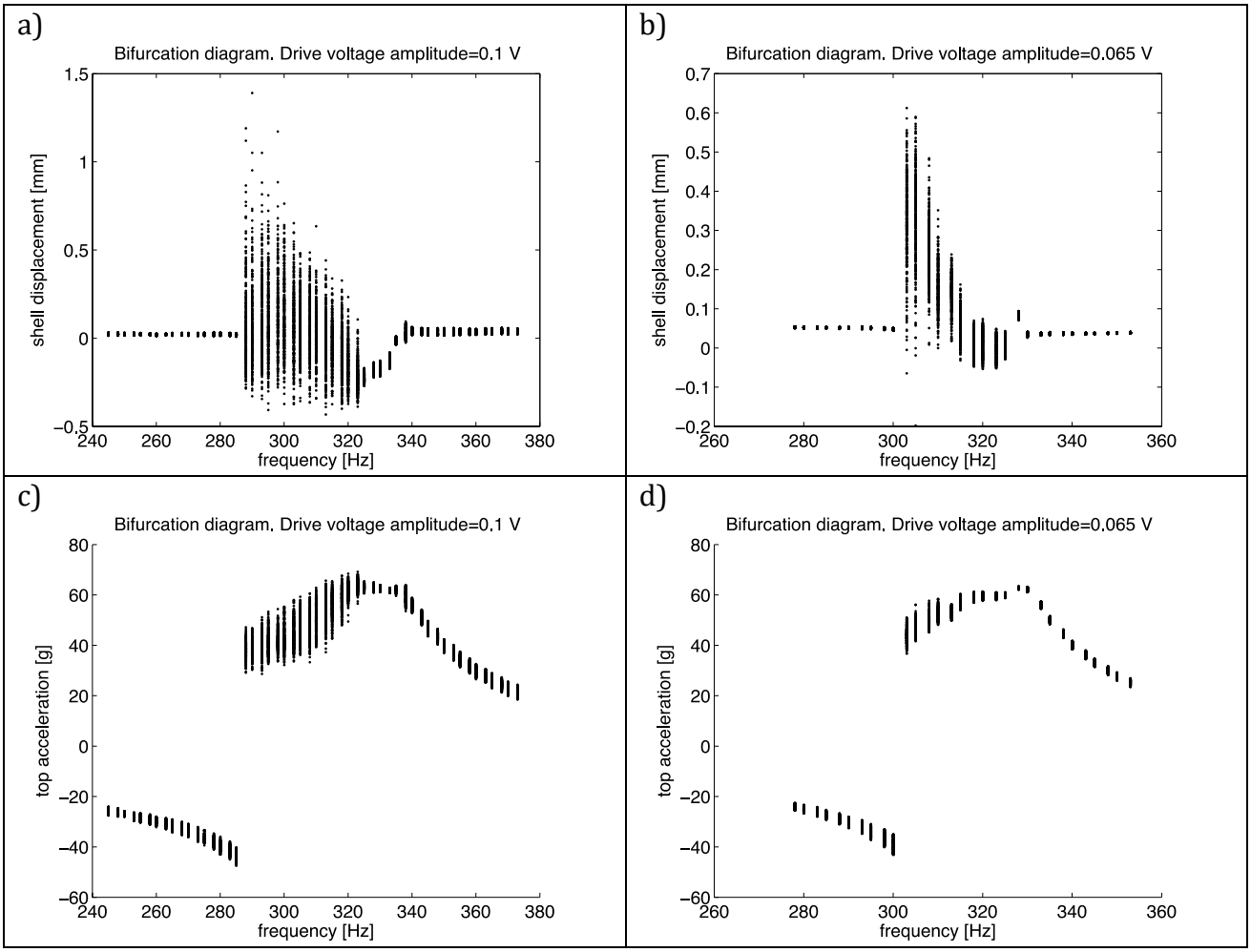

Figure 17 Bifurcation diagrams of Poincaré maps, long shell: a,b) shell vibration; c,d) top disk vibration

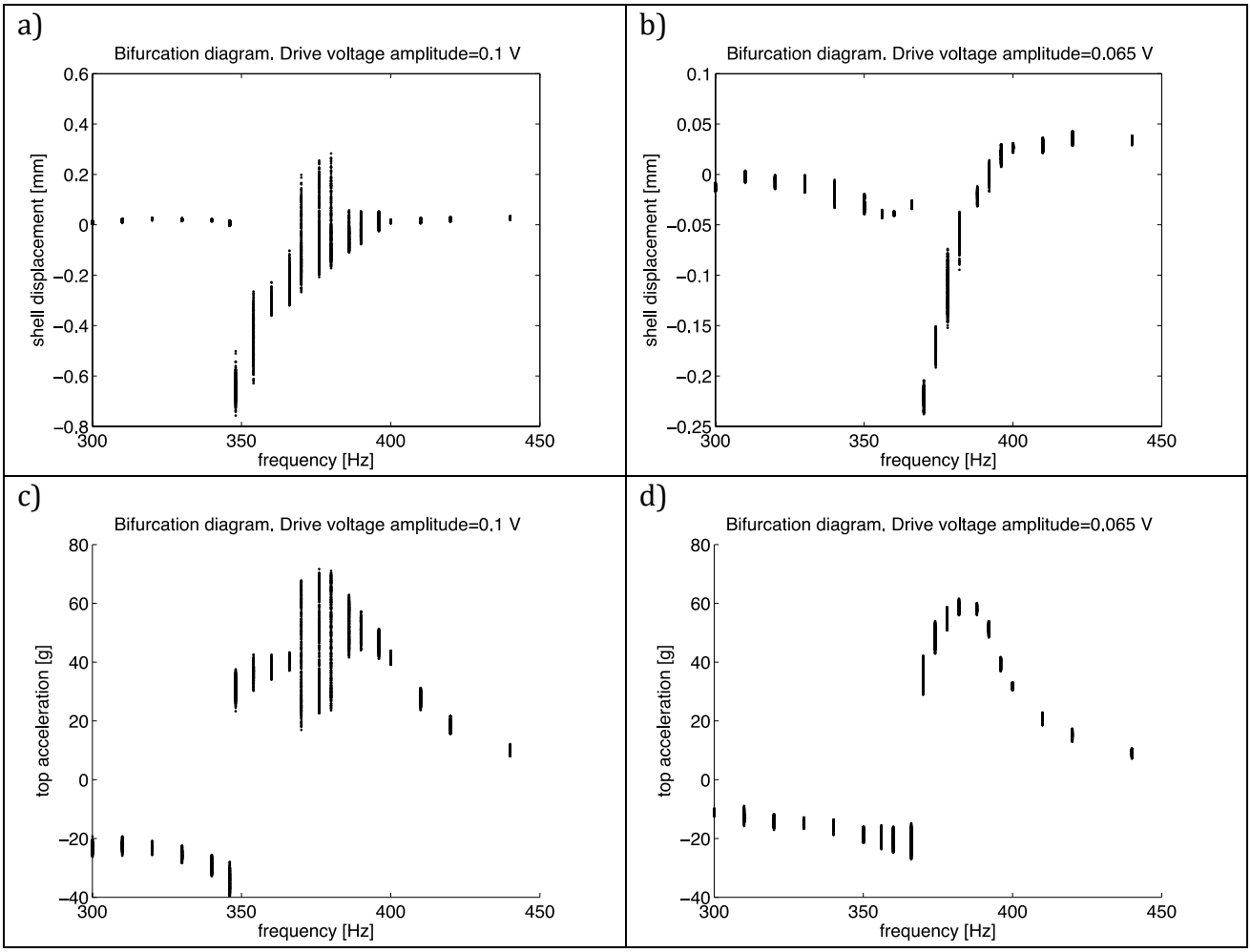

Figure 18 Bifurcation diagrams of Poincaré maps, short shell: a,b) shell vibration; c,d) top disk vibration

Dynamic character of the response

In this section, a detailed analysis of the nonlinear dynamics is carried out; by taking advantage from the knowledge of the amplitude-frequency and bifurcation diagrams, some specific regimes are investigated in detail: time histories, spectra, phase portraits and Poincaré maps are shown.

Let us consider first the long shell, for which the amplitude-frequency diagrams evidenced two regimes within the instability region.

In Figure 19 the response of the system excited at 280Hz and 0.1V drive amplitude is shown; this regime corresponds to the lower part (frequency) of the response of Figure 12, where the instability appears; such regime is in the flat region having the higher amplitude, it is also characterized by a big scattering in the bifurcation diagram, see Figure 17. The top disk time history shows an irregular response, strongly driven by the excitation frequency, this is confirmed by the frequency spectrum, where a spike located at the excitation frequency ($w/w_{\text{drive}}=1$) is dominant, this spike is accompanied by super-harmonics, typical of nonlinear responses. The shell shows a completely different response with respect to the top disk, the vibration is irregular and nonstationary, characterized by irregularly spaced spikes in the time domain (actually the zoom clarifies that these are not spikes but sudden amplitude increments and decrements, like bursts); the frequency spectrum does not present dominant spikes, it is broadband with energy accumulation close to the driver frequency ($w/w_{\text{drive}}=1$)

and its super-harmonics; such kind of response explains the big difference among the MAX and RMS diagrams of Figure 13. The Poincaré map is represented through the projection on the (shell-top disk) plane, it confirms the irregular motion as it is completely shapeless; the phase portrait, as a counterpart of the Poincaré map, shows a completely filled region of the phase-space.

Now a higher excitation frequency is considered, 335Hz, at 0.1V; the system is inside the instability region, but the response is much more regular, see the bifurcation diagram of Figure 17. This is confirmed by the analysis of this regime, indeed the time history of Figure 20 (regarding the top disk) appears almost sinusoidal; consequently, the frequency spectrum is dominated by fundamental harmonic (drive frequency) with minor super-harmonics of order two, three and four; moreover, the spectrum shows also minor spikes, in particular several sidebands with distance $\Delta\omega/\omega_{drive}$ equal to 0.045 or 0.25 or 0.5 from the drive frequency; the minor sidebands are two or more orders of magnitude smaller than the principal, for such reason the time history appears almost sinusoidal. The shell vibration presents a different property but it appears more regular than the case at 290Hz, it seems "steady"; the spectrum presents an evident spike at the drive frequency; however, this peak is not the main one, the dominant harmonic is located at the frequency $4\times\omega_{drive}$, other important harmonics are located at $0.5\times\omega_{drive}$, $1.5\times\omega_{drive}$, $2\times\omega_{drive}$, $2.5\times\omega_{drive}$, $3\times\omega_{drive}$, $3.5\times\omega_{drive}$. In the case of the shell vibration, a spike at ω/ω_{drive}=0.1493 is visible. The phase portrait shows a trajectory that fills a portion of the phase space, this portion is however narrower than the case of full chaotic motion (290Hz); the Poincaré map is "almost" a point (the pollution always introduces some little scattering), confirming the regularity of the dynamics.

Eventually, it is of interest to analyse briefly the response outside the resonance, with 0.1V excitation amplitude the instability disappears below 285Hz and over 340 Hz; in Figure 21 the case of 275Hz is considered. The top disk vibration is almost sinusoidal with spectrum having a spike at 275Hz and other negligible peaks. The shell vibration is negligible and essentially dominated by noise, there is an offset of about 20μm. The presence of noise is unavoidable for several reasons: the shaker is a big machine with several disturbances such as the cooling system; the Laser telemeter in principle measures the vibration in direction orthogonal to the shell surface; however, the shaker induces a vertical motion to the shell, so the Laser is actually measuring a "moving surface"; therefore, small irregularities of the surface, combined with the axial motion, result in polluted radial vibration measurement. These issues are well known, on the other hand this part of the analysis is useful to identify the intervals of confidence in the experimental data presented here.

Let us consider now the short shell, that presents apparently an overall dynamic behaviour quite similar to that of the long shell, e.g., violent vibrations appearing suddenly close to the resonance of the first axisymmetric mode.

In Figure 22 an excitation frequency of 380Hz with drive amplitude of 0.11V is analysed. The top disk vibration is strongly modulated, this is also visible in the spectrum characterized by strong sidebands having $\Delta\omega/\omega_{drive}$ of 0.139 and 0.3421. The shell vibration perfectly follows the top disk, one can see an amplitude modulation accompanied by sidebands; moreover, the shell spectrum exhibits strong super-harmonics of order two and three. The phase portrait shows a regular trajectory that fills a portion of the phase space. However, the regularity of the time history and phase portrait is just apparent; indeed, the Poincaré map shows a fractal structure having a defined shape; probably the fractal attractor is the consequence of the breakdown of a regular quasi-periodic attractor.

Figure 23 shows the response obtained at 350Hz and 0.095V of drive excitation; similarly to the previous case, the shell vibration is huge, about 1mm maximum amplitude, however the response is now regular, no modulations are present and sidebands are negligible. Also phase portraits and Poincaré maps confirm the regularity of the motion.

The previous analyses prove the intrinsic complexity of the dynamic scenario this dynamical system: small variations of the external excitation cause dramatic changes in the shell response.

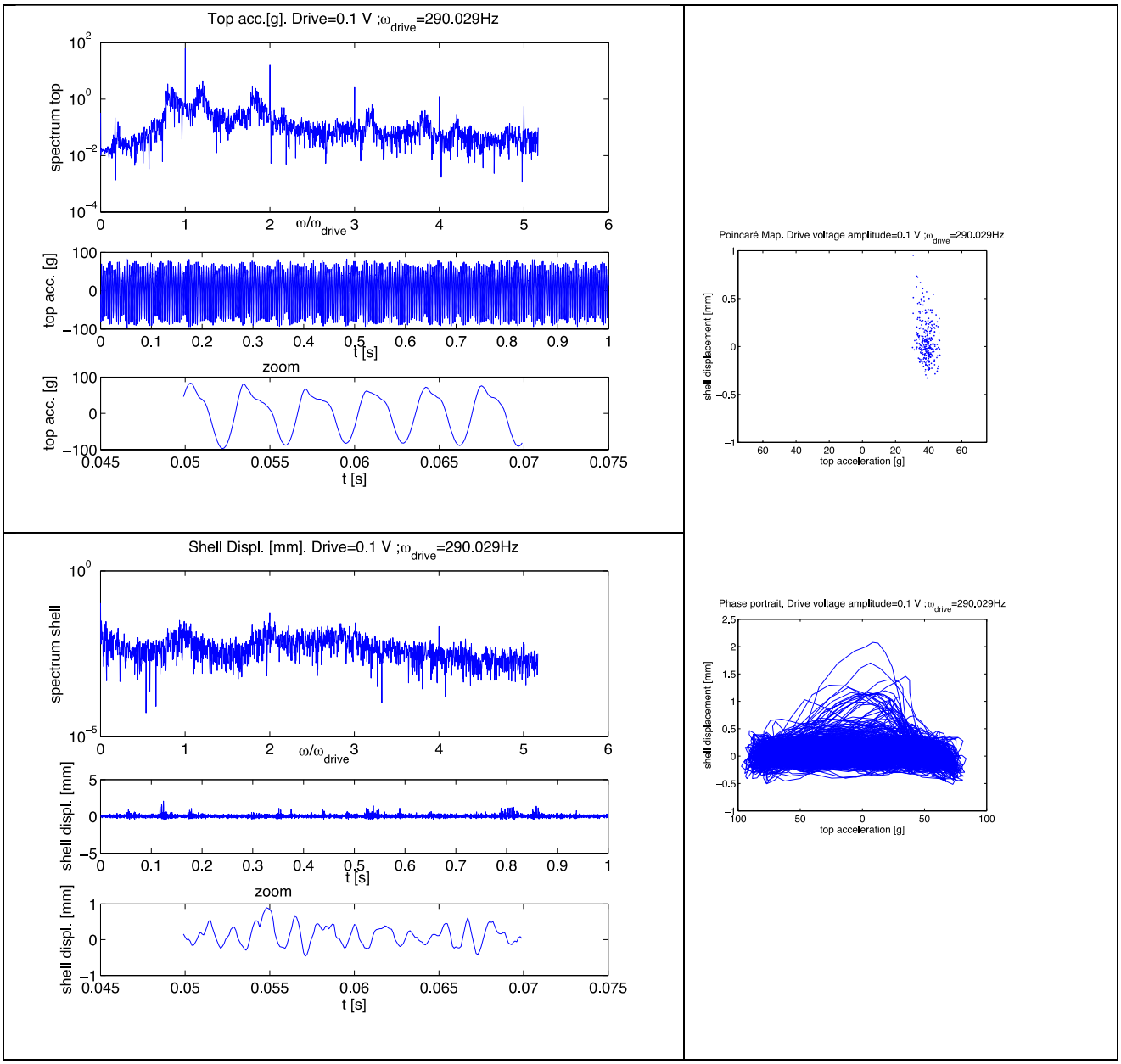

Figure 19 Dynamic properties, first instability region, 290Hz, drive 0.1V. Long shell.

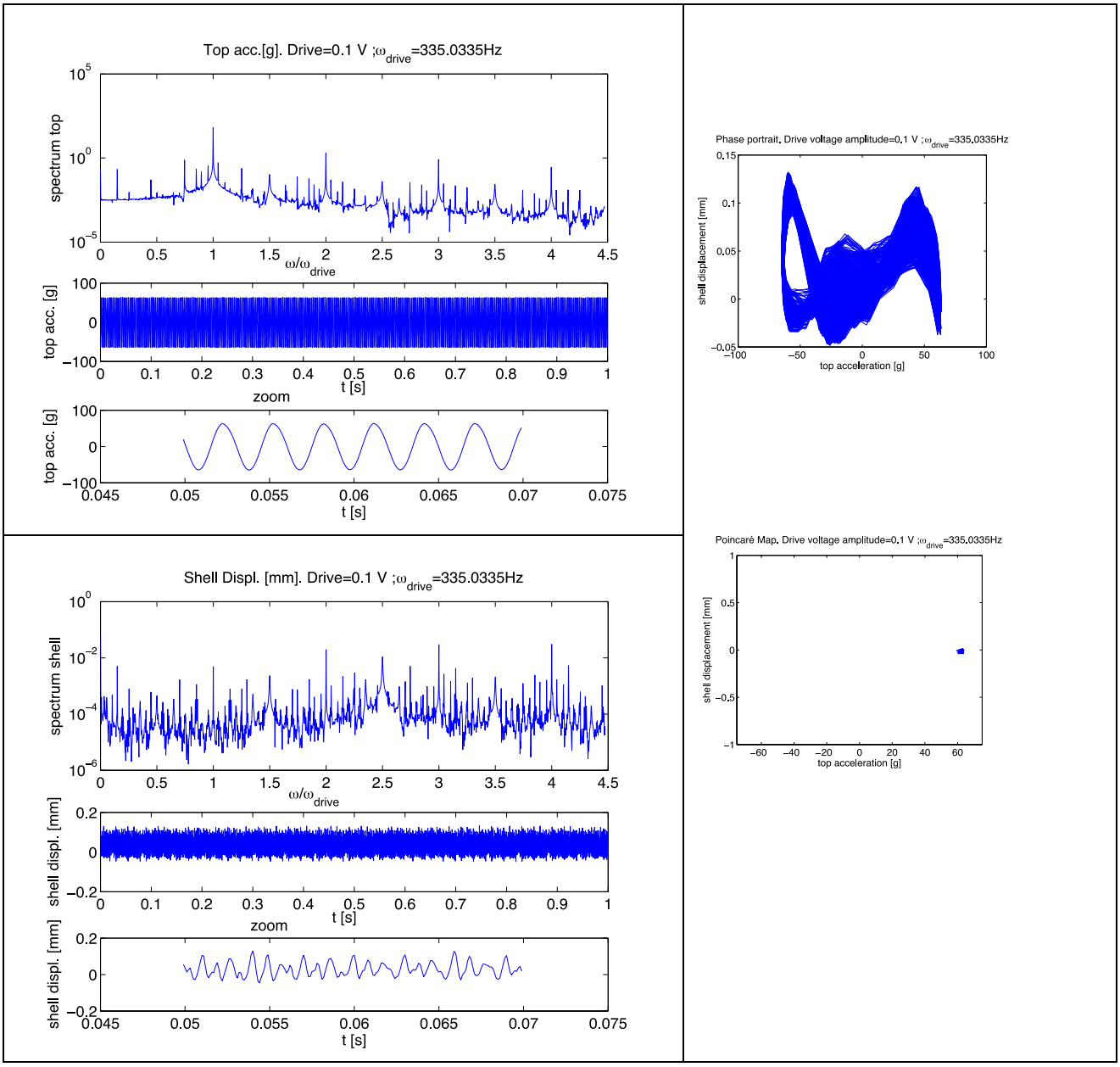

Figure 20 Dynamic properties, second instability region, 335Hz, drive 0.1V. Long shell.

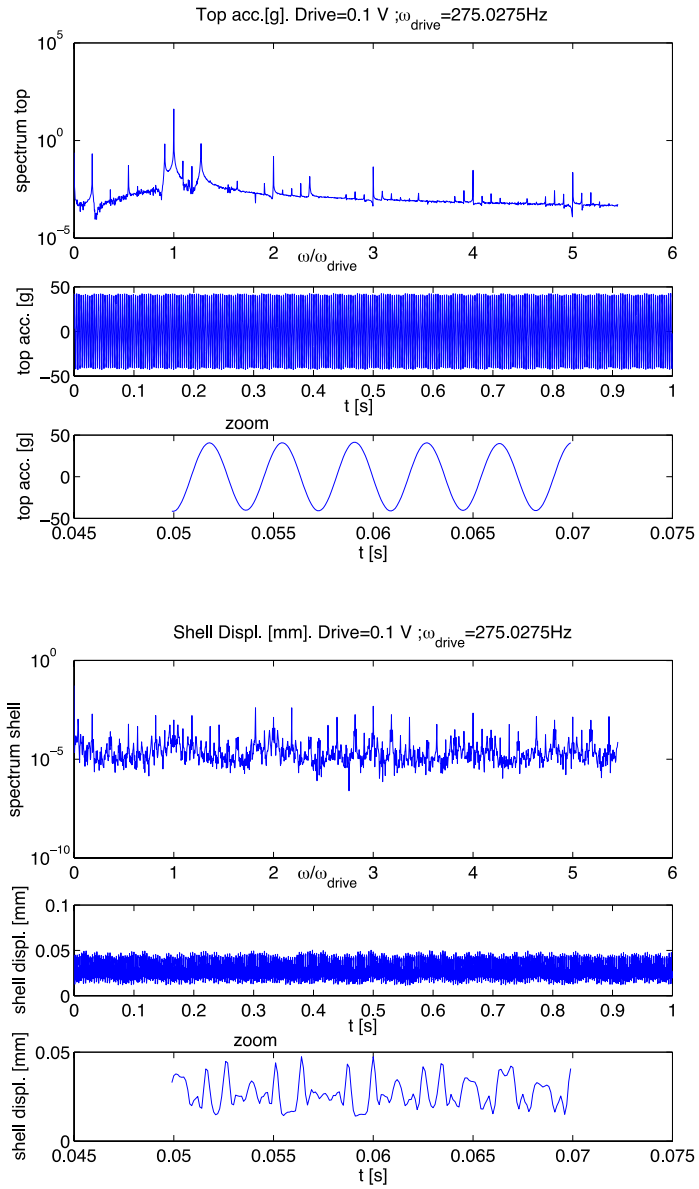

Figure 21 Dynamic properties, out of resonance, 275Hz, drive 0.1V. Long shell.

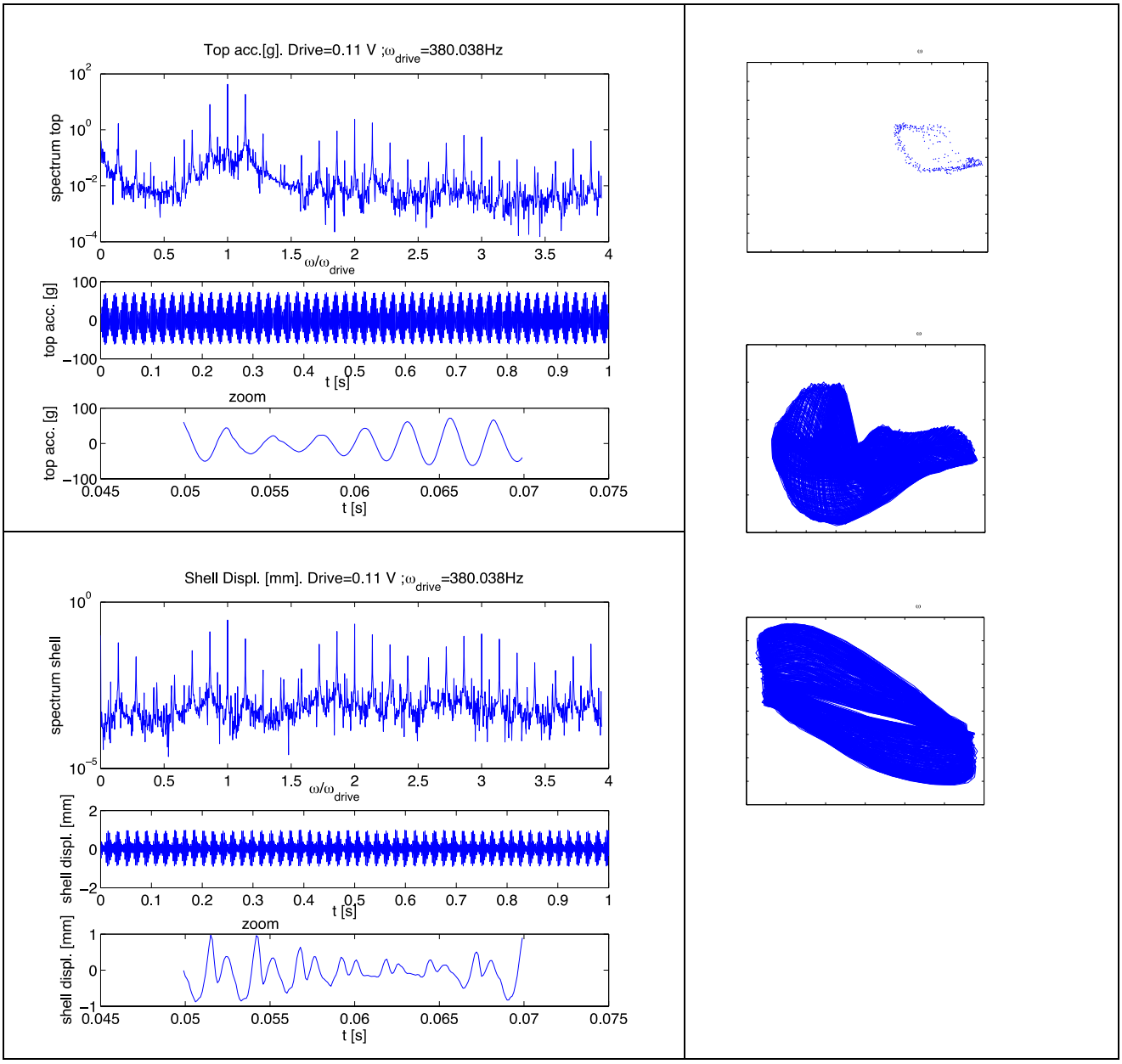

Figure 22 Dynamic properties, instability region, 380Hz, drive 0.11V. Short shell

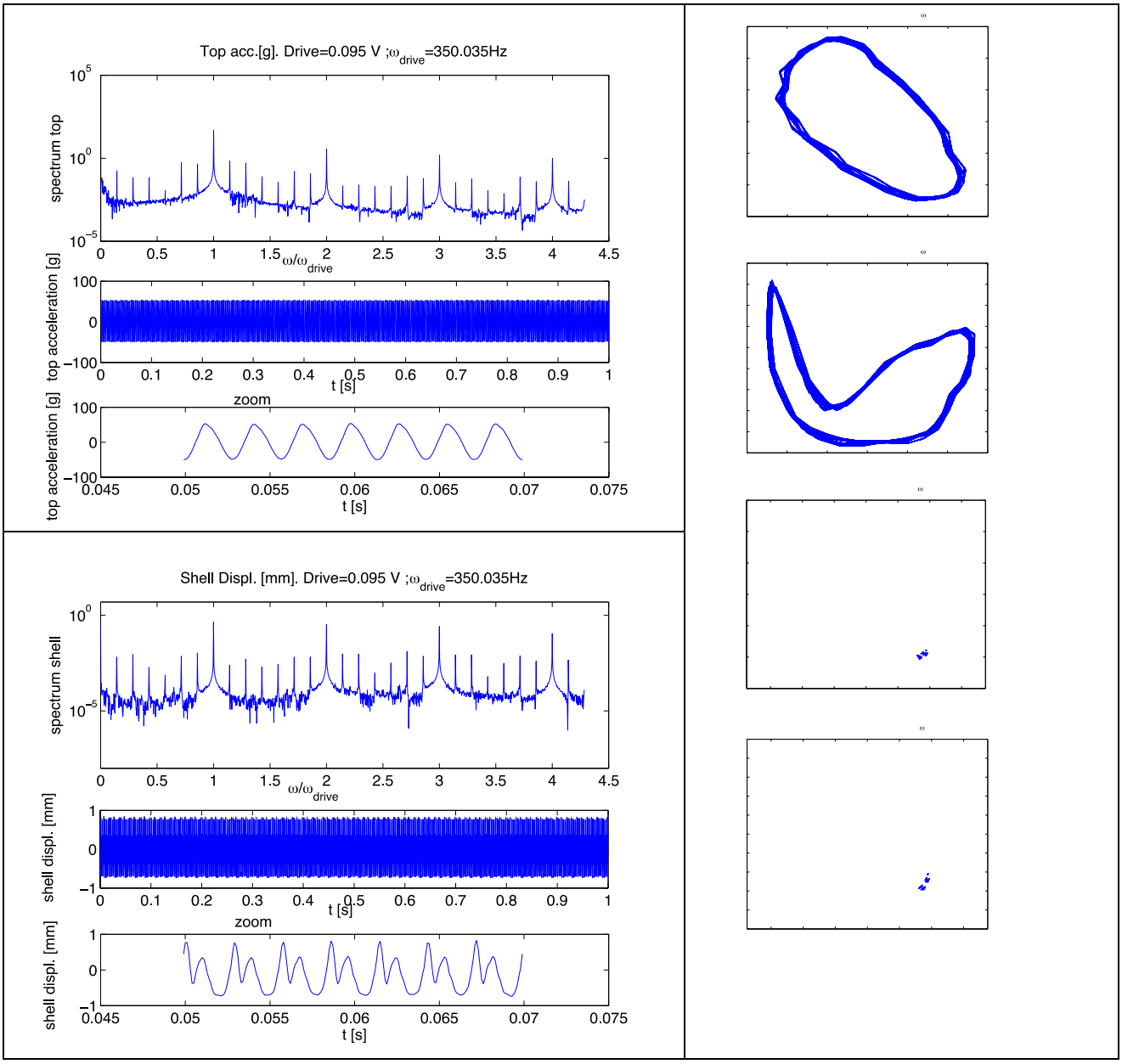

Figure 23 Dynamic properties, instability region, 350Hz, drive 0.095V, hyper-subharmonic. Short shell

Conclusions

The nonlinear dynamics of shells under base excitation, interacting with a rigid body and a finite source of excitation, is analysed experimentally. This paper fills an important gap about the knowledge of the complex dynamics of such systems; moreover, the experimental data are useful for further analytical and numerical development of the phenomena observed, and they will serve as benchmark for future theoretical speculations.

The experimental analysis reveals the extreme complexity of the dynamic response of such structures when they are excited by a sinusoidal resonant excitation. A saturation phenomenon is observed on the top disk vibration; when the excitation frequency is close to the first axisymmetric mode, it should induce a regular increment of the top disk vibration amplitude; conversely, within a large frequency region, the response appears flat and the vibration energy flows towards shell-like modes (asymmetric), this transfer causes unexpected large amplitude lateral vibrations. The saturation phenomenon is accompanied by a strong noise, an energy transfer from low to high frequency, with a frequency spreading due to non-stationary and chaotic response. The reason of such phenomenon is mainly given by a dynamic instability due to membrane stresses, which excites parametrically the high frequency asymmetric modes; on the other hand, previous analytical studies proved that an important role is due to the interaction with the electro-dynamic shaker.

The onset of the instability and its nonlinear character do not change significantly when the excitation level is changed or when the shell geometry is varied. Therefore, the important novelties of the present work are: i) a complete experimental scenario is given, with several quantitative analyses; ii) the instability is proven to be quite robust as different parameters are varied (geometry and excitation level).

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