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Temperature gradient effect on dynamic properties of a polymeric circular cylindrical shell / Zippo, Antonio; Barbieri, Marco; Pellicano, Francesco. - In: COMPOSITE STRUCTURES. - ISSN 0263-8223. - 216:(2019), pp. 301-314. [10.1016/j.compstruct.2019.02.098]

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## Accepted Manuscript

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PII: S0263-8223(19)30113-8  
DOI: <https://doi.org/10.1016/j.compstruct.2019.02.098>  
Reference: COST 10723

To appear in: *Composite Structures*

Received Date: 16 January 2019  
Revised Date: 22 February 2019  
Accepted Date: 25 February 2019

Please cite this article as: Zippo, A., Barbieri, M., Pellicano, F., Temperature gradient effect on dynamic properties of a polymeric circular cylindrical shell, *Composite Structures* (2019), doi: <https://doi.org/10.1016/j.compstruct.2019.02.098>

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# TEMPERATURE GRADIENT EFFECT ON DYNAMIC PROPERTIES OF A POLYMERIC CIRCULAR CYLINDRICAL SHELL

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**Keywords:** vibration, shells, thermal effects, FGM

**Abstract**

In this paper, an experimental study on the dynamic of cylindrical shells made of Polyethylene terephthalate (PET) is presented; a thermic gradient has been applied on a specimen of the present work to obtain a functionally gradient material (FGM) equivalent properties: the PET shell had been exposed at a thermal temperature gradient in the range of its glass transition temperature of 79°C. A complex setup has been specifically designed and built to characterise, with dynamic tests, the structural properties of the specimen on temperature change from -10°C up to about 90°C and under thermic gradient with different forcing load. Predicting the mechanical properties of shells, panels and plates is one of the main concerns of structural engineers; since shell elements present complicated stability behaviours, rich linear vibration spectra (high modal density), high sensitivity to perturbations and strong interactions with surrounding elements. The linear and dynamic behaviour have been investigated. The shell behaviour is also investigated by means of a finite element model, in order to enhance the comprehension of experimental results.

## 1. Introduction

It seems almost obvious to recall here that shells, panels and plates represent the most important structural element in many applications, just few examples can be useful: fuselages, wings and any aerodynamic element in Aerospace industry; pipes and heat exchangers in Nuclear and conventional Energy production industry; structural and style elements in Automotive.

In most of the aforementioned examples we can find common problems, in particular the environment is quite aggressive with the presence vibrations, extreme temperatures, chemical agents and ionizing radiations. We must point out that the latter two points are beyond the purposes of the present research; indeed, we were mainly concerned with vibrations and the effect of the temperature gradients.

Many dangerous and catastrophic phenomena can take place in shells due to static or dynamic loads. Shells, plates and panels are very efficient structural elements, the designers generally keep the level of local stresses far from plasticity; however, such structures are very likely to undergo instability and sudden collapse. The interaction with surrounding fluids induces a virtual added mass, reducing the natural frequencies, increases the damping; on the other hand, when the fluid is flowing it can induce divergence (static instability), flutter (dynamic instability) or simply steady vibrations (turbulence). In many applications the fluid can reach extreme temperatures (very high in heat exchangers, very low in high altitude flights); in some cases, the temperatures are uniform, in other cases (e.g. heat exchangers) there are strong gradients across the shell thickness.

Predicting the mechanical properties of shells, panels and plates is, therefore, one of the main concerns of structural engineers; since shell elements present complicated stability behaviours, rich linear vibration spectra (high modal density), high sensitivity to perturbations and strong interactions with surrounding elements. There is need of both theoretical and experimental research.

The following analysis of the literature shows that in the past the topic was object of an intense research activity.

It is worth to start the analysis of the literature with the description of four interesting review papers, which are strongly suggested to the reader to have a wide overview of the state of the art.

In 1992, Noor and Burton [1] published a review paper focused on computational models for composite plates and shells. The review focused the attention to thermal problems and the following application areas: heat transfer; thermal stresses; curing, processing and residual stresses; buckling; vibrations; large deflection and post-buckling.

In 1995 Burton and Noor [2] published a comprehensive analysis of models suitable for analysing the static response of curved sandwich panels, including thermal effects; they compared nine different

approaches based on two-dimensional shell theories; they stated: “the accurate determination of detailed response characteristics, such as the through-the-thickness distributions of transverse stresses, requires the use of either higher-order discrete three-layer models or predictor- corrector approaches”.

In 1996 Noor, Burton and Bert [3] published an important review paper focused on computational methods for sandwich plates and shells with applications on: heat transfer; thermal and mechanical stresses; free vibrations and damping; transient dynamic response; buckling; large deflection and post-buckling; discontinuities; geometric changes; damage and failure; experiments; optimization and design. They concluded that “accurate predictions of transverse stresses require the use of three-dimensional equations”.

In 2014 Alijani and Amabili [4] published a comprehensive review on nonlinear vibrations of shells. The review was dedicated to shells and curved panels, isotropic, composite, piezoelectric, FGM and hyperelastic materials. Applications to parametric vibrations, stability, dynamic buckling, non-stationary vibrations and chaotic vibrations were considered as well as specific topics such as: fluid–structure interactions, geometric imperfections, effect of geometry and boundary conditions, thermal loads and electrical loads. Modelling approaches were considered in this review as well: reduced-order models, perturbation techniques, proper orthogonal decomposition, non-linear normal modes and meshless methods. More than 300 papers published in the decade 2003-2013 were referenced.

In addition to the previously commented review papers, some other papers, strictly related to the object of the present paper, are described.

In 1994 Tzou and Howard [6] published a theory for studying piezothermoelastic structures with applications to shells, rings and beams.

Librescu et al. [7] investigated the interaction between a temperature field, external load on the fundamental frequency associated with small vibrations of panels, both in pre-buckling and post-buckling conditions. The study was carried out through the use of a higher-order transverse-shear-deformation theory of shallow shells, including geometric nonlinearities and initial geometric imperfections.

Dawe [8] published in 2002 a paper concerning the use of finite strip method for the buckling and vibration analysis of shells; thermal buckling was considered as well. In the same year, Sharma and Sharma [9] used the Lord-Shulman, Green-Lindsay, and Green Nagdhi theories of thermoelasticity to investigate the effect of thermal gradients on the free vibrations.

Free vibration and dynamic parametric instabilities of cylindrical FGM (functionally graded material) were studied in Ref. [10] using the Reddy’s higher order shear deformation shell theory. A semi-analytical approach, based on one-dimensional differential quadrature approximation, the

Galerkin technique and the Bolotin's method, are used to obtain the natural frequencies and the instability conditions of the panel. The influence of different materials, temperature, geometry, and boundary conditions was analysed. The temperature was considered uniform in evaluating the thermal effects.

In 2004 Carrera [11] published a study on the use of the Murakami's zig-zag function in the two-dimensional modeling of multi-layered plates and shells, using different shell theories. Bending, vibration and thermal stress response were separately treated. In the same year Catellani et al. [12] investigated the role of imperfections in the dynamic instabilities of shells, the pre-buckling conditions were investigated.

In 2004 Shih, Smith and Tzou [13] presented a study on the use of photo-restrictive materials in vibration control, the actuator control effectiveness was evaluated with respect to actuator placements". The authors concluded that: modal resonances in shells can be suppressed using segmented photostrictive actuator patches, the use of an array of patches can improve the control, the bending control effect is relatively insignificant as compared with the membrane one, the actuator along the circumferential axis is much more effective than the longitudinal actuator.

In 2006, R. Kadoli and N. Ganesan [14] presented a study on linear buckling a free vibration analysis of functionally graded cylindrical shells, made of stainless steel and zirconia; the thermal effects due to a temperature variation across the shell thickness were analysed numerically. In the same year a similar study, focused on conical shells, was published in Ref. [15].

Free vibrations of circular cylindrical shells were studied in Ref. [16], considering a distribution of temperature across the shell thickness; a theoretical model based on Love's shell theory and the von Karman–Donnell-type of kinematic nonlinearity was used for developing a model. They found that: "The effect of temperature rise on decreasing the natural frequencies of FG cylinders is also apparent but the rate of decrease changes suddenly for a certain value of temperature".

In 2007 Matsunaga [17] studied pre-buckling deformations of the shells subjected to a temperature change. They found that global higher-order theories can provide accurate results for critical temperatures of general cross-ply laminated composite shallow shells.

In 2007 Pellicano and Avramov [18] presented an analytical approach for studying shells vibrations in linear and nonlinear regimes: the approach was based on the Sanders-Koiter theory, a series expansion for reducing the initial PDE to ODE and a perturbation approach, based on the multiple-scale method, for solving the ODE. In addition to the pure modelling, this paper presented results of a new series of experiments.

In 2008 Sheng and Wang [19] published a paper focused on functionally graded cylindrical shells; they investigated vibration, buckling and dynamic stability problems under the influence of thermal loads; the study was based on a first order shear deformation theory including rotary inertia and

transverse shear strains. In their numerical study Sheng and Wang found that the instability regions are magnified by the temperature gradient across the shell thickness.

The effect of thermal shocks was investigated by Santos et al. [20] in 2008, in the framework of linear elasticity theory, in this study a semi-analytical finite element approach was proposed. FGM shells made of Zirconia or Stainless Steel were investigated, the role of the transient temperature variation in determining deformations and stresses.

Sheng and Wang [21] proposed in 2008 a model for analysing fluid conveying shells considering the effect of temperature. The model, based on the first order shear deformation theory, included the heat conduction equation along the thickness to determine the temperature distribution. They investigated the effects of material composition, thermal loading, static axial loading, flow velocity, medium stiffness and shell geometry parameters on the dynamic properties.

Sheng and Wang [22], 2009, proposed an active vibration control method for FGM shells in thermal environment. A constant-gain negative velocity feedback approach was used for the vibration control.

In 2009 a further publication [23] was focused on the effect of imperfections on the parametric instability of axially loaded circular cylindrical shells. It was found that the sensitivity to imperfection is really important when the shell is subjected to a high static compression.

In Ref. [24] a model for FGM piezoelectric materials was presented, with applications to the linear buckling and free vibration analysis of cylindrical shells. The authors investigated numerically the combined effects of thermal loading and constant voltage.

In 2010 Vel [25] presented analytical solutions for the vibration of anisotropic FGM shells; e presented an interesting analysis of mode shapes.

New experimental evidence of complex vibrations in circular cylindrical shells under axial loading was published in [26]; the paper presented also a nonlinear shell model including electromechanical interaction, i.e. the shaker generally used in lab experiments for applying forces.

A study on the nonlinear vibrations of composite cylindrical shells, reinforced through carbon nanotubes, having FGM properties, was published in 2012, Ref. [27]. A higher-order shear deformation theory, based on von Kármán kinematics, was used; a Lindstedt-Poincaré-type perturbation approach was used to determine the dependence of the natural frequencies with the amplitude of oscillation. An unusual hardening type nonlinearity is found for all case studies. The authors concluded that: “The results show that the natural frequencies are reduced but the nonlinear to linear frequency ratios are increased by increasing the temperature”. In Ref. [28], Shen published a study on large amplitude of vibration of FGM cylindrical shell embedded in a outer elastic medium, considering thermal effects of the surrounding environment; the surrounding elastic medium is modelled through a Pasternak model.



Malekzadeh, Fiouz and Sobhrouyan [29], in 2012, investigated the natural frequencies of FGM conical shells with thermal stresses using the differential quadrature method. The authors clarified the importance in considering the effect of the temperature on the material properties and consequently on the natural frequencies. Malekzadeh and Heydarpour [30] investigated the free vibrations of rotating cylindrical shells using a linear shell theory and the Differential Quadrature method.

Lang and Xuewu [31] developed in 2013 a method for analysing buckling and vibrations of FGM magneto-electro-thermo-elastic circular cylindrical shells. They investigated the effect external forces, temperature, surface electric voltage and magnetic field, on the buckling response of circular cylindrical shells.

Shen and Wang [32] published in 2014 a paper focused on shear deformable FGM cylindrical panels, subjected to large amplitude of vibration. A higher order shear deformation shell theory, including panel-foundation interaction, was used. Thermal effects on the vibrations were investigated.

Chaotic vibrations of shells were analysed in Ref. [33] both from experimental and numerical points of view. Inertial loads were used to produce axial excitation during experiments. The complex nature of the shell dynamics was revealed and at the same time it was proven the possibility of modelling such complexity.

A fully experimental paper devoted to cylindrical shells vibrations caused by combined axial static and dynamic loads was published by Zippo et al. [36] in 2016; then in 2017 Zippo et al. [37][38] presented further results on polymeric shells under different thermal conditions.

The analysis of the literature confirms the great interest in the object of the present study; at the same time, it is clear that experimental studies are infrequent and, therefore, there is lack of data.

The motivations of the present work are based in the previous literature analysis: the actual state of the art shows an interesting scientific production of models and an extreme need of experiments having the purpose of: identifying structural parameters, giving benchmark data for validation purposes, discovering new and unexpected phenomena.

The characterization of the dynamic properties of a circular cylindrical shell made of Polyethylene terephthalate (PET) is the object of the present work, this is a continuation of a more complex work regarding Functionally gradient materials (FGMs) models applied to shell dynamics. The dynamic of shell under base excitation is part of a long research activity covering theoretical and experimental aspects of the shell behaviour. More specifically, here a campaign of experiments is presented to reveal the extreme complexity of simple structures such as circular cylindrical shells made with PET. An experimental setup has been designed and built in order to characterize, through dynamic tests, the mechanical properties of the specimen in a temperature change from  $-10^{\circ}\text{C}$  up to  $90^{\circ}\text{C}$ .

An accurate experimental modal analysis was used for setting up and validate a FEM (Nastran based) modelling capable of handling complex boundary conditions, new experimental data were presented with shell properties.

The experimental activity is focused here on linear regimes, with the specific goal of identifying the modal parameters and their dependence from the temperature.

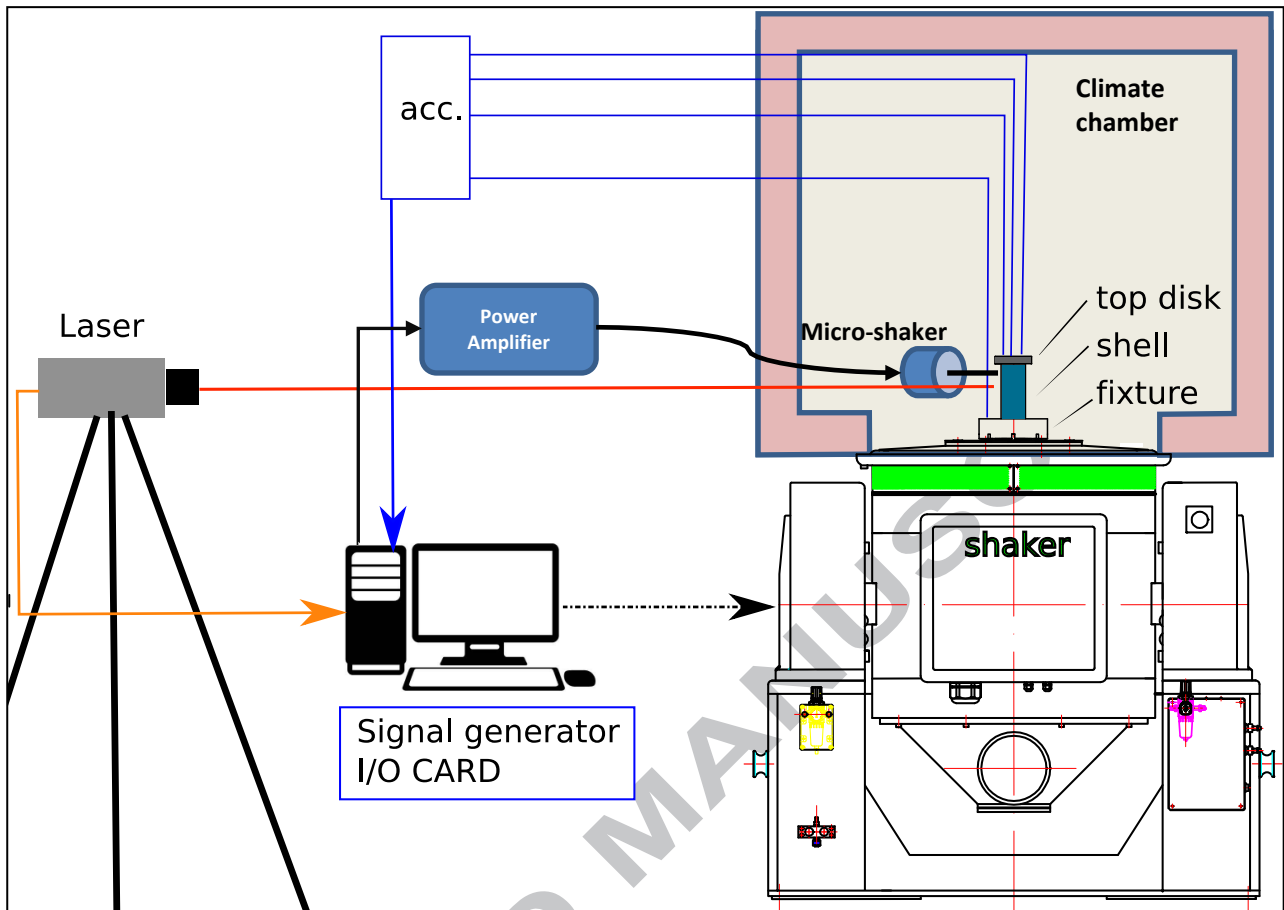
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## 2. Experimental setup

First of all, it is to point out that the experiments are carried out in controlled environment, i.e. the specimen is inside a climate chamber (about  $\frac{1}{2}$  m<sup>3</sup>), where the temperature can have extreme values (between -70°C and +180°C). For such reason the standard modal testing, carried out manually through an impact hammer, cannot be considered, because the inner of the climate chamber is not accessible by the operator.

In order to circumvent the inaccessibility of the climate chamber, two kinds of excitation have been considered: a) seismic like excitation from the base, Figure 1; b) lateral excitation of the shell by means of a micro-shaker. The two excitations are needed because we need to provide energy to any shell mode: the seismic excitation forces axisymmetric modes; the lateral excitation forces the asymmetric modes. In the test rig of Figure 1 one can clearly see that the specimen (the circular shell) is mounted vertically on an electrodynamic shaker, on the top of the shell a rigid mass is connected. A very stiff fixture connects the shell to the shaker, the fixture is designed in order to have the first natural frequency much higher than any expected vibration arising during the tests, it is bolted on the shaker in order to assure a pure rigid translation; on its upper part the fixture is connected to the bottom of the shell through a steel shaft collar represented in

Figure 2 (see also in Figure 3 the black ring).



**Figure 1. Experimental setup: acquisition and control chain**

The circular cylindrical shell under investigation is made of Polyethylene terephthalate (P.E.T.) a thermoplastic polymer, its base is clamped to the fixture by means the shaft collar; the top end of the shell is glued to a rigid thick disk (the top disk) with epoxide glue (see

Figure 2 and Figure 3). The collar ensures a uniform high pressure on the outer shell surface, which perfectly sticks the inner part to the fixture; the collar allow also an easy dismounting of the system. On the top of the shell the collar is not needed, indeed the access to the inner part is obtained by dismounting the bottom.

The geometry of the shell is represented in

Figure 2, where all dimensions are reported for the shell, the fixture and the top disk.

The system description is completed by Table 1, where the material characteristics are reported. The fixture, i.e. the sub-structure connecting the shell to the shaker, is an aluminium alloy thick circular disk, it is rigidly bolted to the shaker. The top disk is connected to the shell by means of a special epoxy glue, resistant to high temperature; the role of the disk is: i) to impose a rigid body displacement at the top end of the shell; ii) exert an inertial axial load to the shell when a seismic motion is imposed.

An accelerometer is located on the fixture, such accelerometer is used by the control system of the shaker when a closed loop strategy is selected.

Three triaxial accelerometers are located on the top disk, they are used to measure all the six dofs of the rigid body, this allows to detect axisymmetric modes, asymmetric modes, beam-like modes and torsional modes.

A laser vibrometer is used to measure the lateral vibration of the shell without contact.

Inside the shell a cartridge heater is present, see Figure 3, the cylindrical shaped, industrial Joule heating element; the heater is used to modify the temperature inside the shell and to create a thermal gradient across the shell wall.

Two thermocouples are used to measure the temperature, inside and outside the shell.

The outer temperature of the shell is controlled by the climate chamber, which controls both temperature and humidity.

The temperature ranges of the climate chamber are  $-70^{\circ}\text{C}$  up to  $+180^{\circ}\text{C}$ .

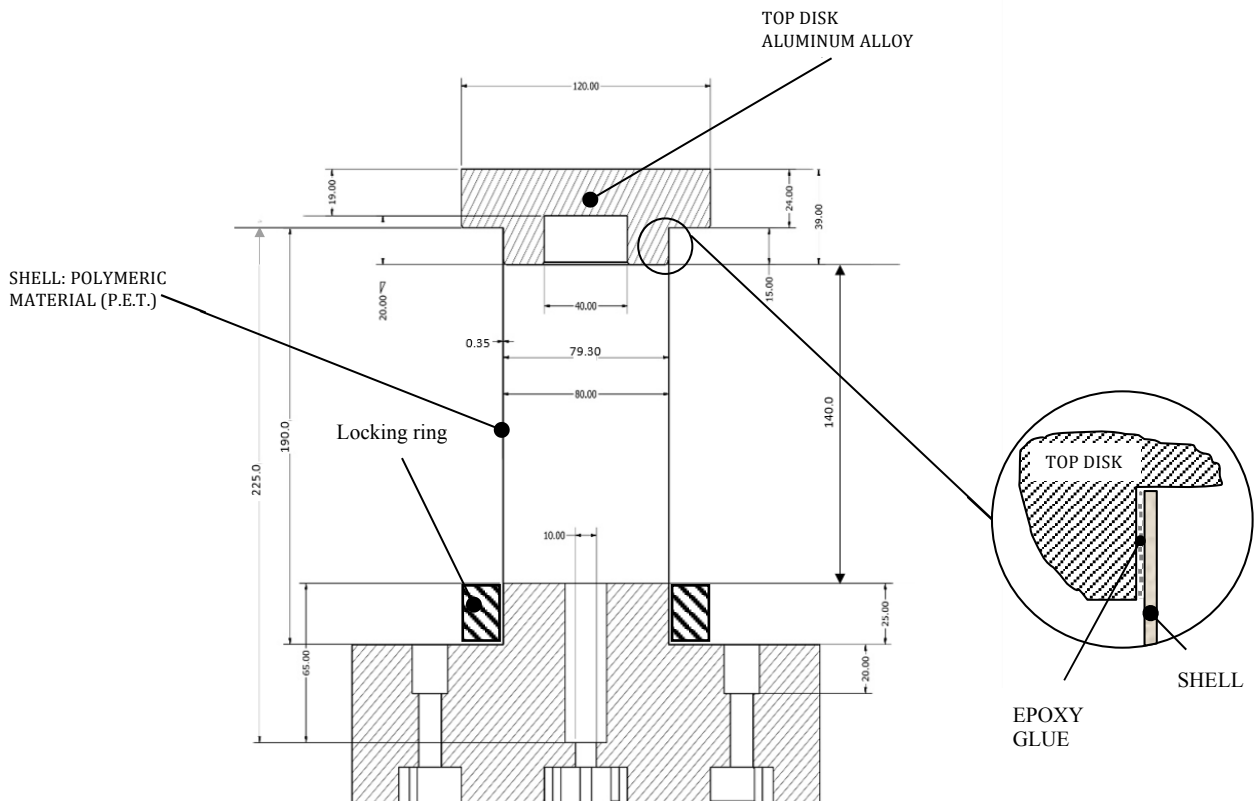


Figure 2. Geometrical data of the specimen and the fixture

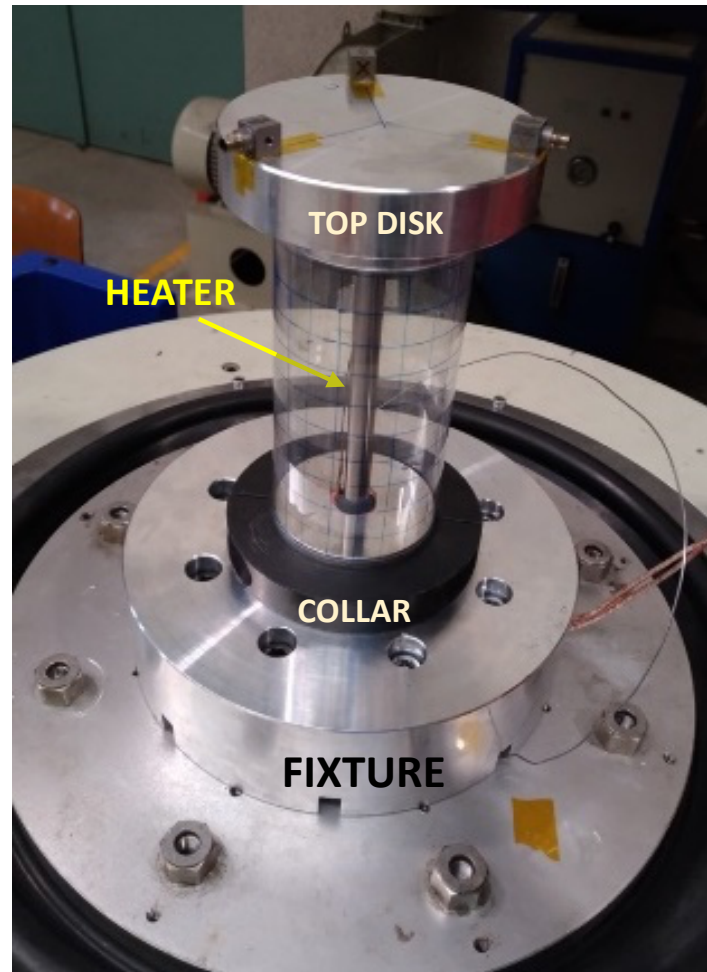


Figure 3. Experimental setup: Shell, locking ring, top disk, inner heater, sensors.

Table 1. Material parameters of the shell, the fixture and top disk

Fixture and Top disk		
Material	Aluminium Alloy	
General	Mass Density	2700 kg/m <sup>3</sup>
Stress	Young's Modulus	68.9 GPa
	Poisson's Ratio	0.33
Shell		
Material	P.E.T.	
General	Mass Density	1366 kg/m <sup>3</sup>

Stress	Young's Modulus	3.2 GPa (first approximation)
	Poisson's Ratio	0.417

### 3. Experimental and Numerical Modal Analysis: room temperature

In this section the vibrational properties of the system, i.e. natural frequencies and mode shapes, are analysed both numerically and experimentally at room temperature. This analysis is needed in order to understand the shell behaviour and for identifying the natural frequencies, damping ratios and modal shapes. This step is crucial for the choice of the mode that will be used to identify the dynamic property of the PET shell when the temperature is varied.

It is to note that, the full modal testing was possible at room temperature only; indeed, test at different temperature must be carried out with the specimen located inside the climate chamber, where the accessibility is extremely limited and hammer tests cannot be performed.

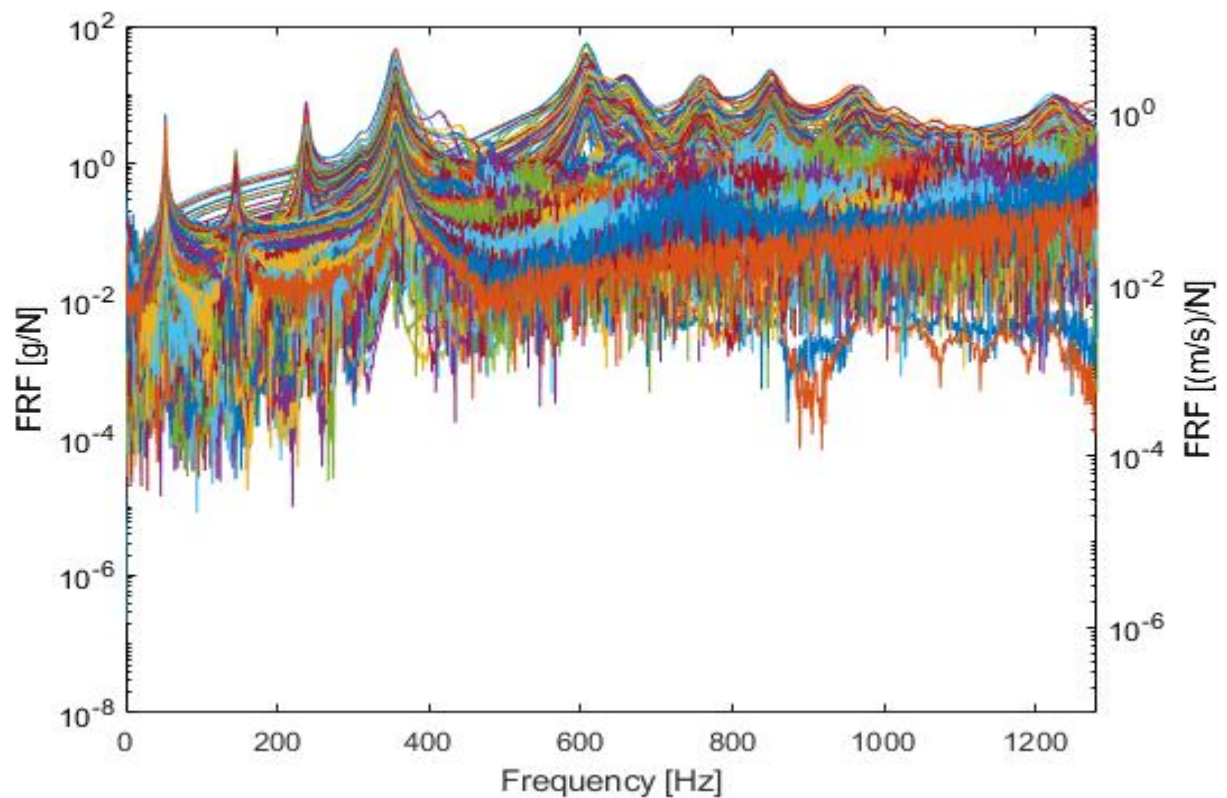
Tests are carried out on the shell of Figure 3 using an impact hammer for modal analysis as excitation, vibration response is measured through a Laser Doppler vibrometer on the shell and three tri-axial accelerometers on the top, this allows to identify both the shell response and the top disk motion, including the tilt. A grid of excitation points is considered: 11 points along the axis direction and 12 points along the circumferential direction, for a total of 132 points. One half of the shell is tested in circumferential direction in order to reduce the testing time; indeed, mode shapes can be identified without testing the entire shell. Transfer functions between excitation and response are obtained using the LMS TestLab, for each FRF at least three measurements are considered in order to calculate the transfer function through power and cross spectral densities. Once the FRF are obtained, data is processed in order to identify natural frequencies, damping ratios and modal shapes. Figure 4 show the absolute value of the summation of all FRF, this representation is commonly used because it allows to find the main resonances through a quick visual check.

Table 2 shows the natural frequencies identified through experimental FRF and the corresponding frequencies calculated through a commercial finite element software; this table contains: i) a description of the mode shape; ii) the number of axial half waves  $m$  and the number of nodal diameters  $n$  for shell like modes; iii) experimental frequencies; iv) simulated frequencies; v) percent difference. For the sake of brevity, the analysis is limited to the first two beam like modes, the first torsional mode, the first axisymmetric mode, the first five shell-like modes. The natural frequencies of the FEM model are obtained after selecting suitable values of the Young modulus and Poisson ratio, which give the best match with experimental data.



Table 3 shows the mode shapes corresponding to the natural frequencies listed in Table 2, both experimental and numerical mode shapes are shown; the experimental mode shapes are identified through the post-processing of all FRF (curve fitting) that returns both mode shape and damping ratio for each mode. The shape correspondence is excellent for the most of modes; in some cases the experimental identification gives partial results: the 1<sup>st</sup> torsional mode is correctly measured on the top disk only, because the Laser Doppler cannot measure tangential vibration of the shell; the 1<sup>st</sup> axisymmetric mode is correctly identified on the top disk, conversely the measurement of the later vibration of the shell was affected by noise because the amplitude of vibration was extremely small.

It is to note that four modes only are characterized by the displacement/rotation of the top disk: 1<sup>st</sup> and 2<sup>nd</sup> beam like mode, 1<sup>st</sup> torsional mode, 1<sup>st</sup> axisymmetric mode. This means that, in the case of base motion the 1<sup>st</sup> axisymmetric mode is directly excited, the beam like modes could be excited indirectly due to the presence of imperfections, the 1<sup>st</sup> torsional mode should not be excited.



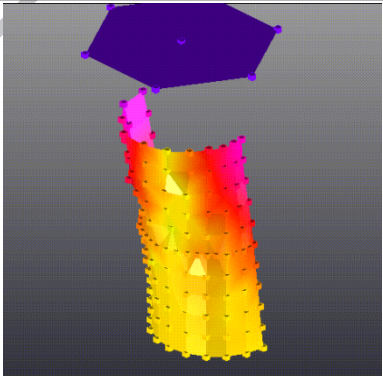
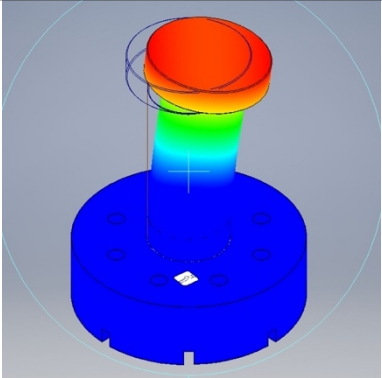
**Figure 4. FRF between 10 responses (3 triaxial accelerometer and 1 vibrometer) and 131 references (impact hammer)**

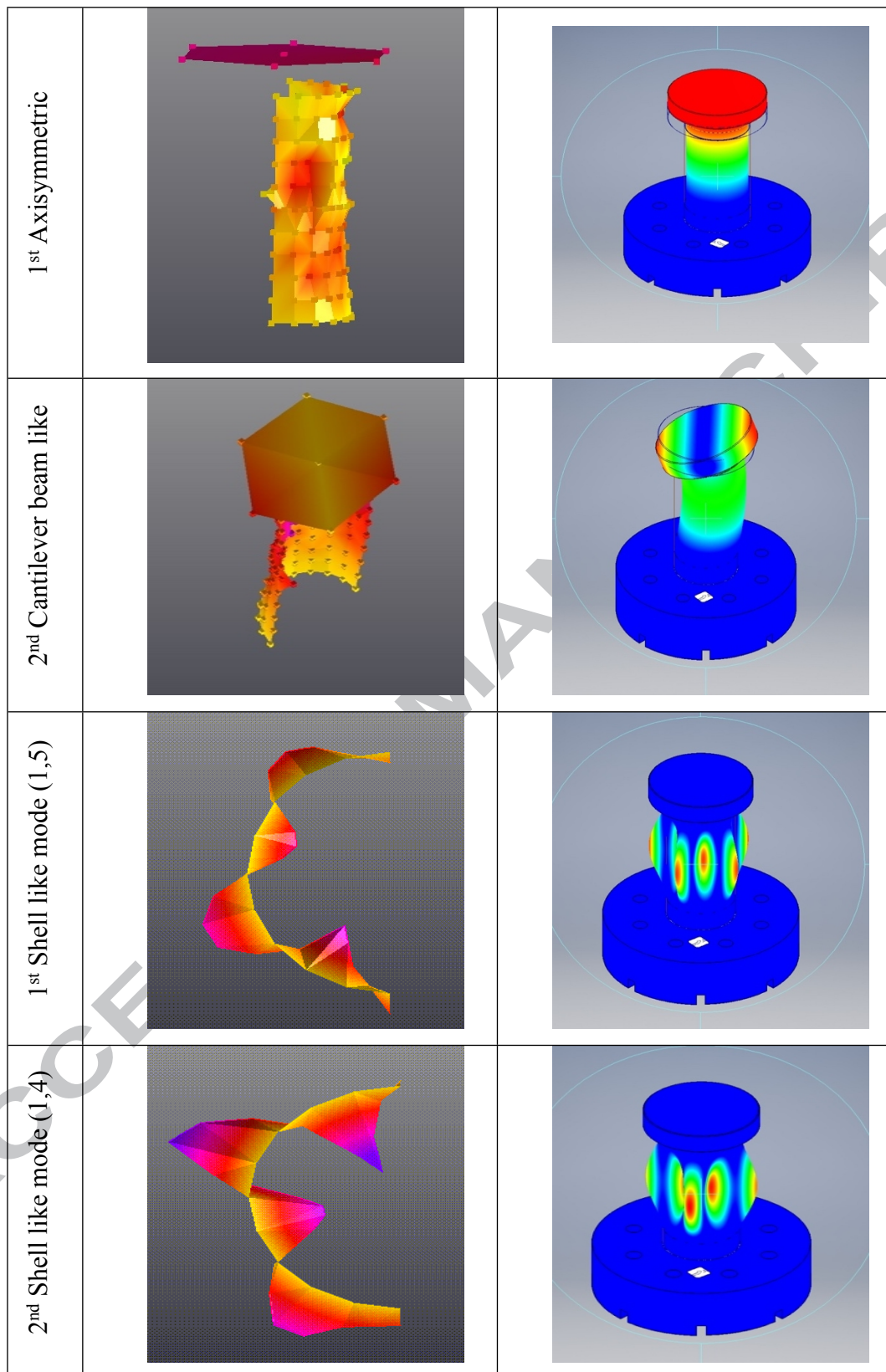


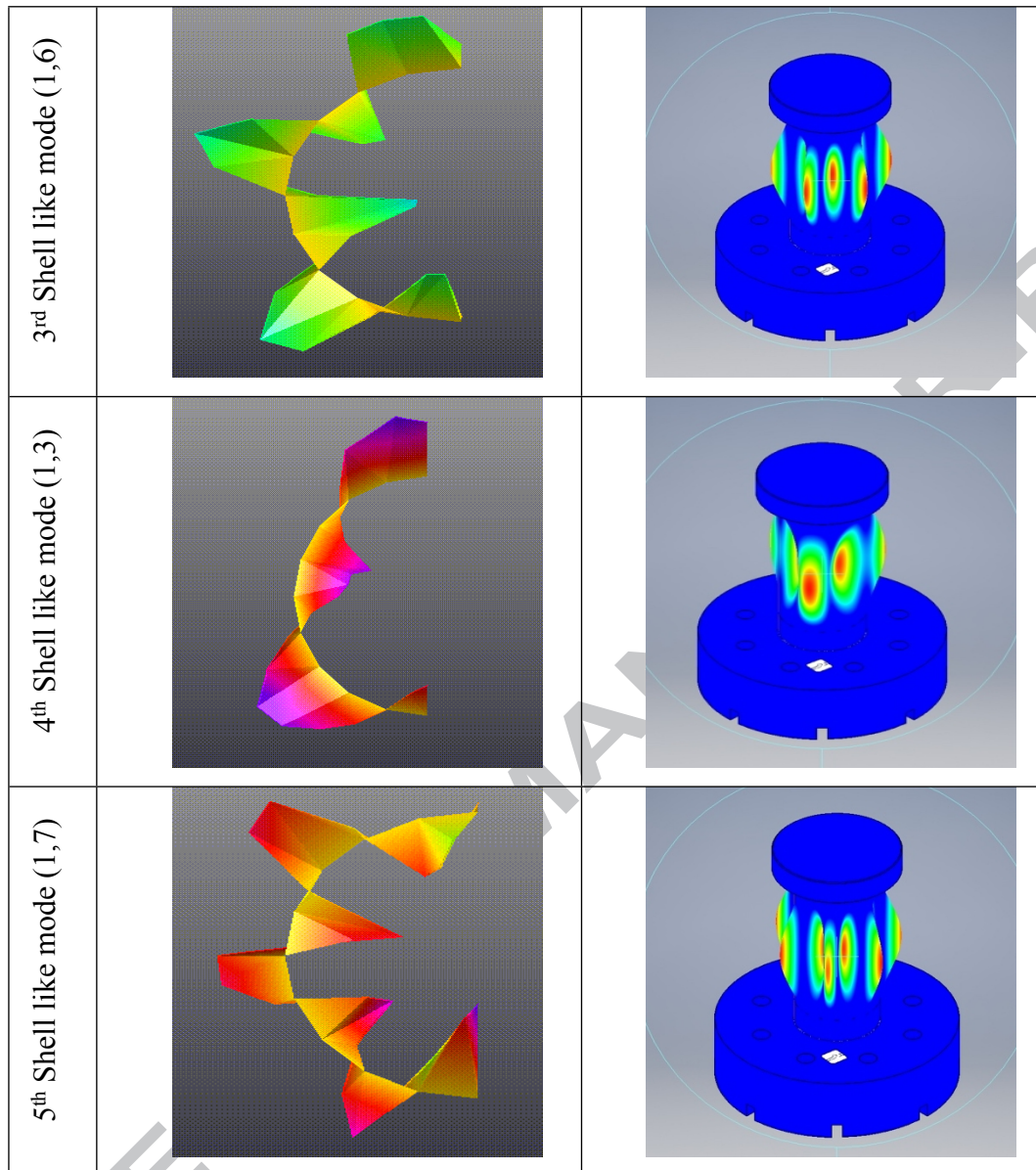
**Table 2. Experimental and FEM frequencies**

Mode	m	n	Experimental [Hz]	FEM [Hz]	% difference
1 <sup>st</sup> Cantilever beam like	-	-	52.73	52.94	0.40%
1 <sup>st</sup> Torsional	-	-	144.92	134.28	7.34%
1 <sup>st</sup> Axisymmetric	-	-	237.5	234.72	1.17%
2 <sup>nd</sup> Cantilever beam like	-	-	356.3	337.5	5.28%
1 <sup>st</sup> Shell like mode	1	5	608	543.94	10.54%
2 <sup>nd</sup> Shell like mode	1	4	664	566.43	14.69%
3 <sup>rd</sup> Shell like mode	1	6	764	654.82	14.29%
4 <sup>th</sup> Shell like mode	1	3	850	771.45	9.24%
5 <sup>th</sup> Shell like mode	1	7	963	846.22	12.13%

**Table 3. Experimental and FEM modal shapes**

N	Experimental	FEM
1 <sup>st</sup> Cantilever beam like		





#### 4. Property identification: effect of uniform temperature

In this section the effect of a uniform temperature on the shell behaviour is analysed. The specimen is inside a climate chamber, see Figure 1, the excitation is given by the base motion induced by the shaking table. Such kind of excitation furnishes energy to the 1<sup>st</sup> axisymmetric mode only.

A series of tests is carried out from  $-10^{\circ}\text{C}$  to  $92^{\circ}\text{C}$ ; Figure 5 shows the FRFs (input base acceleration, output top disk acceleration) as the temperature changes: 22 temperatures are analysed. This result proves that the amplification depends on the temperature, the extrema of the interval (lowest and higher temperature) show the minimum magnification in resonance, the medium-high temperature ( $60^{\circ}\text{C}$ ) induces the maximum amplification.

Table 4 contains the resonance frequency and the corresponding quality factor at different temperatures: the resonance frequency increases regularly when the temperature decreases; the quality factor is maximum at 60°C. The quality factor is evaluated through the half band method.

Results of Table 4 are graphically represented in Figure 6 (resonance frequency vs. temperature) and Figure 7 (quality factor vs. temperature): the resonance frequency monotonically decreases with the temperature; the quality factor increases with the temperature up to the maximum at 60°C, then decreases regularly.

Not that the quality factor does not correspond to the maximum amplification because the system under investigation is not a simple one degree of freedom system and the influence of other modes changes the amplification in resonance. Here the quality factor is evaluated by considering the half power points formula  $Q = \frac{1}{2\zeta} = \omega_{res}/\Delta\omega_{3dB}$ , where  $\omega_{res}$  is the resonance frequency and  $\Delta\omega_{3dB}$  is the difference of the frequencies when the amplitude is one half of the maximum.

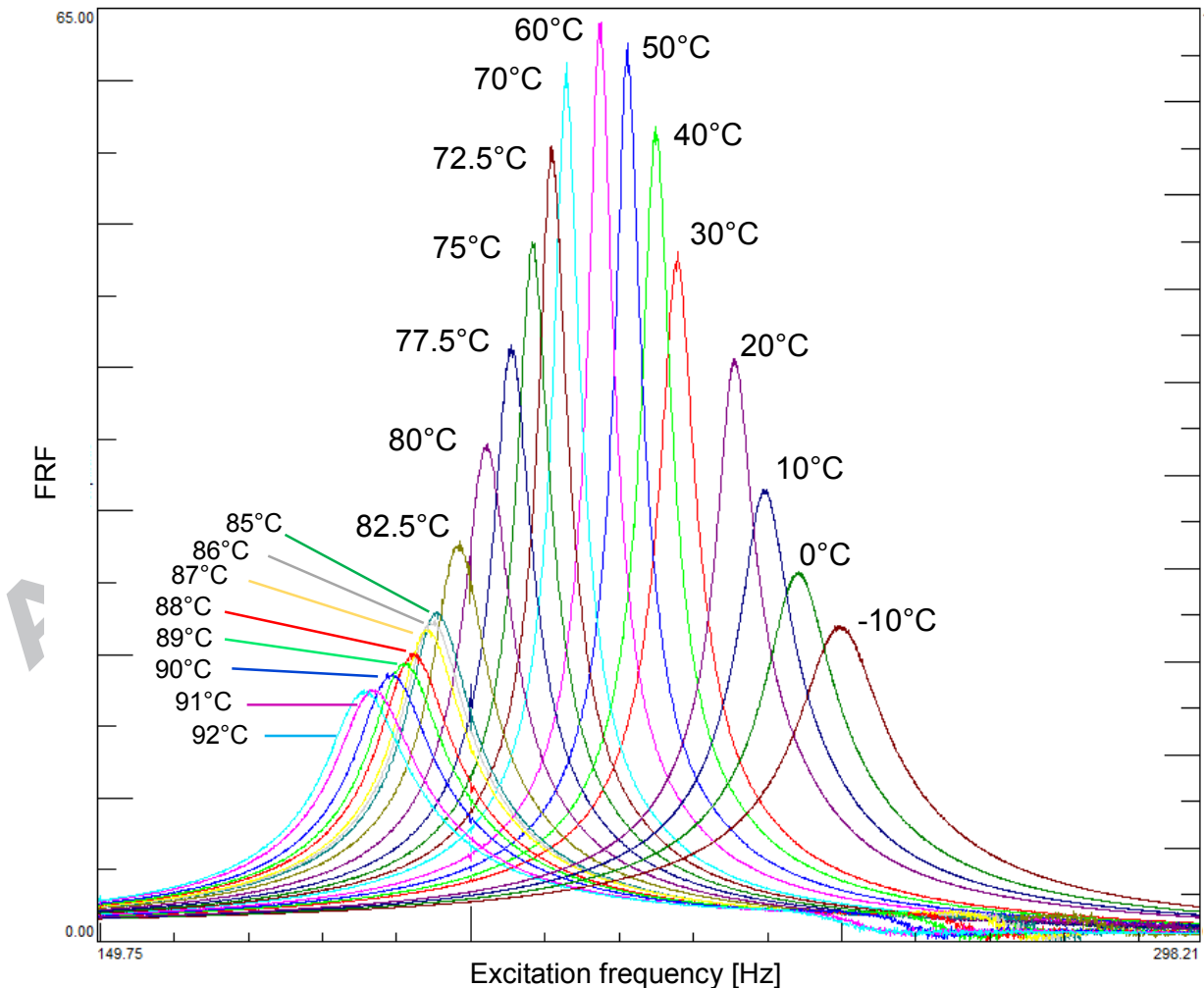


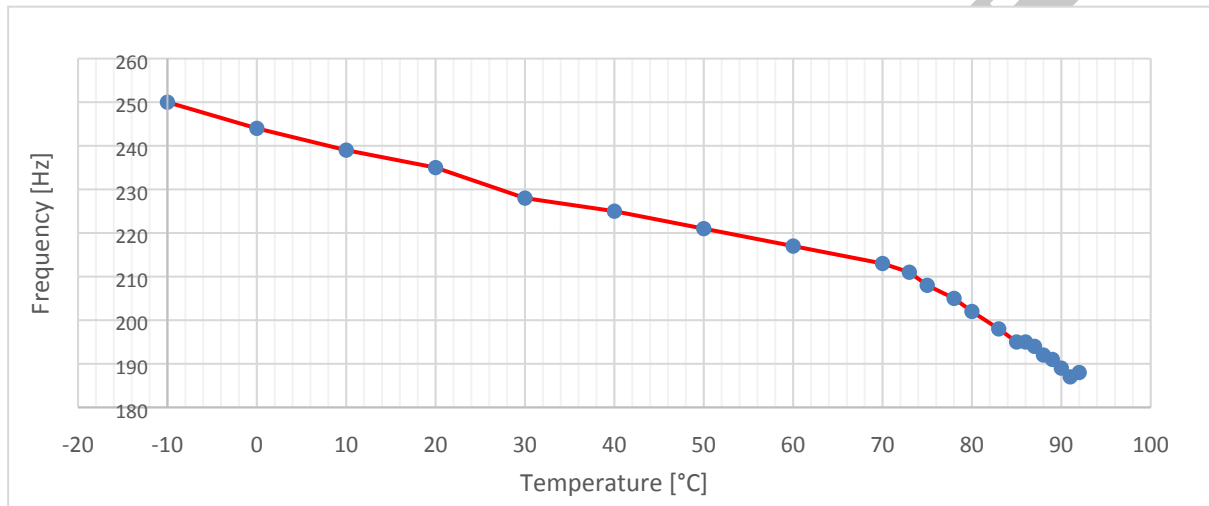
Figure 5. Resonance of the 1<sup>st</sup> axisymmetric mode v.s temperature: FRF top/base acceleration

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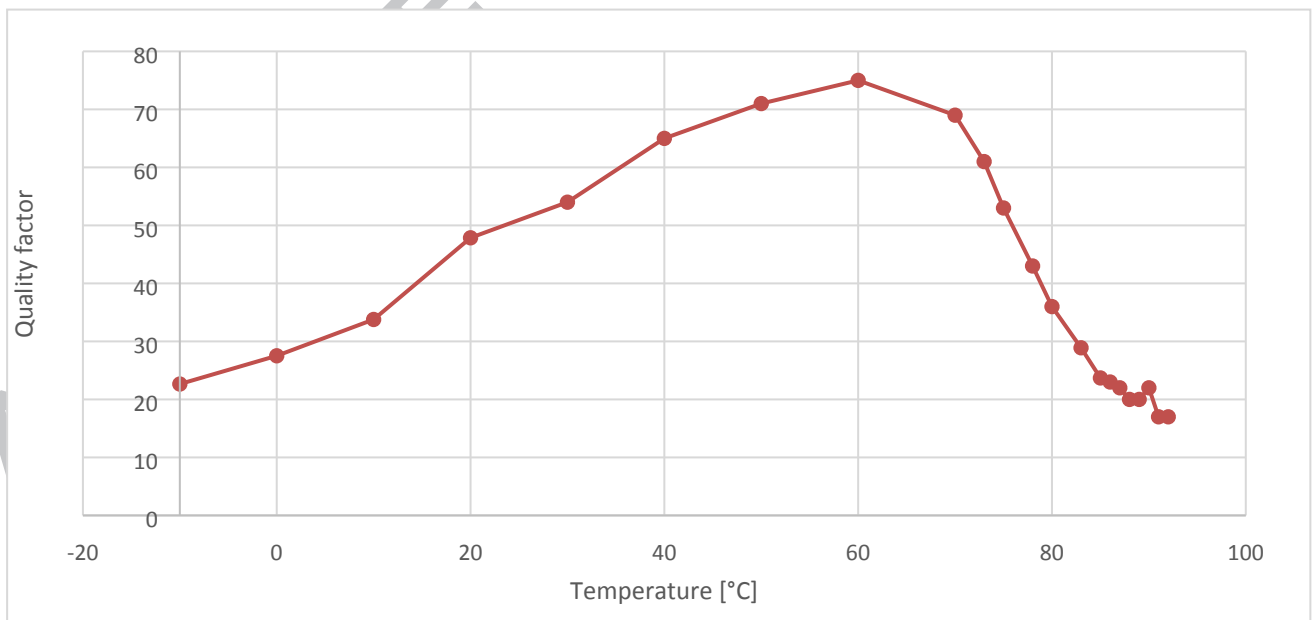
Table 4. frequency and quality factor on temperature change

$\tau$ [° C]	f [Hz]	Quality factor
-10	250	23
0	244	28
10	239	34
20	235	48
30	228	54
40	225	65
50	221	71
60	217	75
70	213	69
73	211	61
75	208	53
78	205	43
80	202	36
83	198	29
85	195	24
86	195	23
87	194	22
88	192	20
89	191	20
90	189	22
91	188	17
92	187	17

The fundamental frequencies of the first axisymmetric mode reported in Table 4 are now used in order to set the correct Young's modulus of the FEM analysis; this procedure is repeated for each temperature. Figure 8 shows the dependence of the Young's modulus with the temperature, it monotonically decreases as the temperature increases. At high temperature ( $>70^{\circ}\text{C}$ ) the curve increase its negative slope; this is not surprising because the PET presents the glass transition temperature at about  $79^{\circ}\text{C}$ .



**Figure 6. Frequency of third mode on temperature change**



**Figure 7. Quality factor on temperature change**



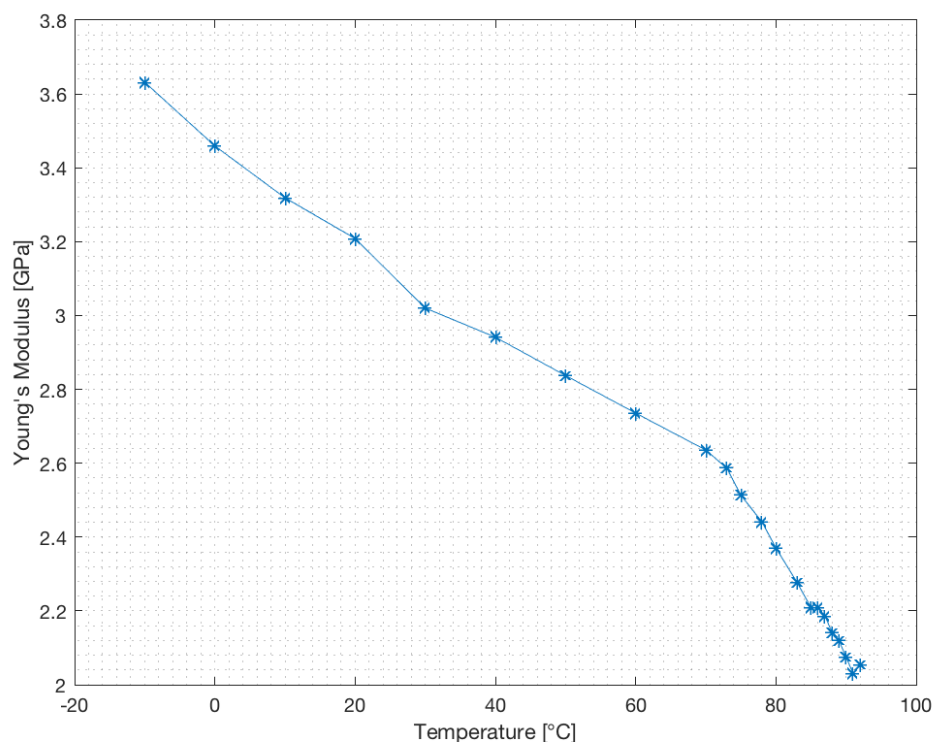


Figure 8. Young's Modulus on temperature change

## 5. Property identification: effect of thermal gradients

### 5.1 Test procedure

The experimental setup is now modified in order to obtain a thermal gradient across the shell thickness; such gradient will induce a continuous variation of elastic and dissipative properties through the thickness of the shell. This is achieved by imposing a different temperature inside and outside the cylindrical shell.

Inside the shell a heater is mounted (Figure 3), such device has been improved by adding an aluminium foil tape and creating four finned surfaces, this increases the heat transfer rate and allows to reach a higher inner temperature, see Figure 9.

In this series of tests, a different top mass is used, the new cap is about four times lighter than the one used in the previous section. The new cap has requested changes (inner reaming) in anticipation of future test on non-linear behaviour with large amplitude forcing loads, to avoid contacts between internal heater and the cap. The new mass weight 205 grams respect to 850 grams of the previous.

Three thermocouples have been used to control: the heater temperature, the internal and the external shell wall temperature.

Two triaxial accelerometer has been placed, one on the base of the shell and one on the top disk.



Several preliminary tests were carried out, in order to ensure that steady thermic conditions were reached; such test were used to fix the time needed to stabilize the temperature on the whole surface of the shell after a temperature variation in the climatic chamber.

The test campaigns were of two types: i) high internal and external homogeneous temperature (50 °C) as starting condition, then the external wall temperature was reduced by controlling the climate chamber; ii) low homogenous internal and external temperature (-10 ° C) as starting condition, then the internal temperature was increased by controlling the climate chamber.



**Figure 9 – Set-up for axial forcing load tests with thermic gradient**

## 5.2 FEM

Using data identified in the previous section and assuming a linear temperature variation across the shell thickness a finite model has been developed. Standard shell cards of the software do not allow to model a graduation of elastic properties across the shell. In order to circumvent this limitation, from Table 5 values of the Young modulus corresponding to the internal and external temperature are calculated for each condition, the average is used in FEM simulations.

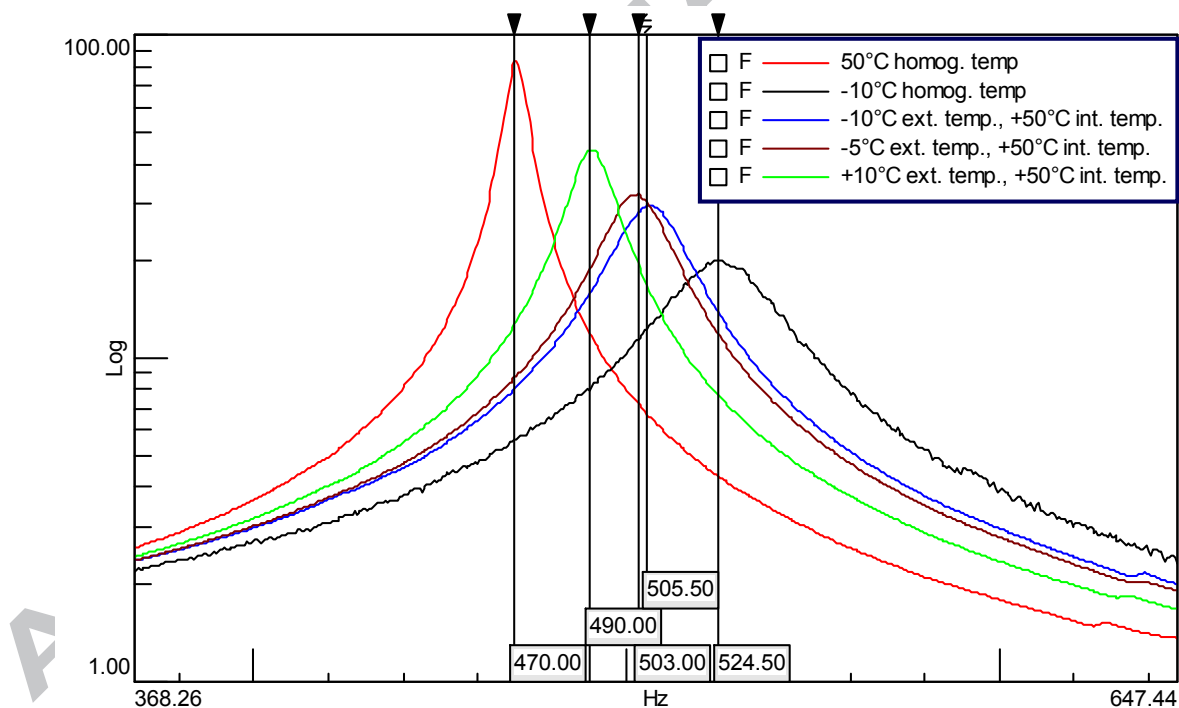
### **Axial excitation**

For the sake of brevity only one test will be shown. Figure 10 shows an FRF of the 1<sup>st</sup> axisymmetric mode for different temperatures and gradients: i) the red line represents the FRF at 50°C homogeneous temperature; ii) the black line represents the FRF at -10°C homogeneous

temperature; iii) the green line represents the FRF with a thermal gradient, the external temperature is  $10^{\circ}\text{C}$  and the internal temperature is  $50^{\circ}\text{C}$ ; iv) the blue line represents the FRF with a thermal gradient, the external temperature is  $-10^{\circ}\text{C}$  and the internal temperature is  $50^{\circ}\text{C}$ ; v) the brown line represents the FRF with a thermal gradient, the external temperature is  $-5^{\circ}\text{C}$  and the internal temperature is  $50^{\circ}\text{C}$ .

For each thermal condition, the natural frequency of the 1<sup>st</sup> axisymmetric mode is experimentally measured, see Table 5, five thermic gradients applied to the shell; equivalent young's Modulus has been calculated with comparison of experimental data to FEM modelling.

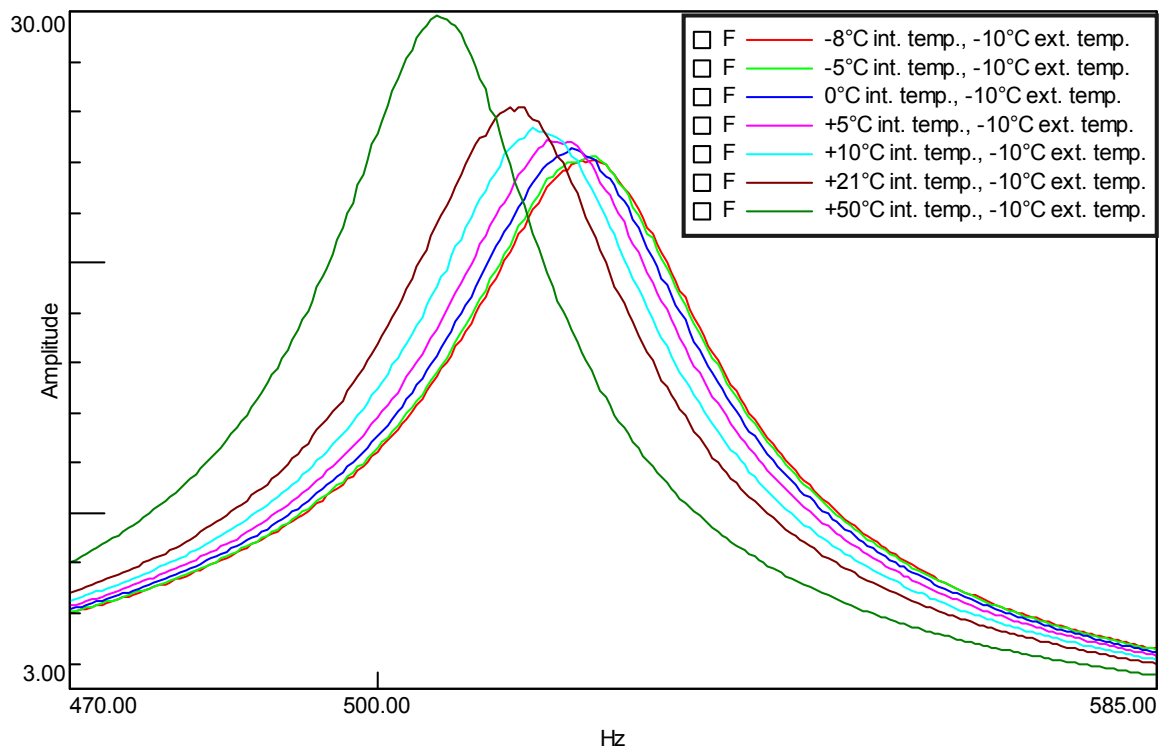
Note that, from Figure 10 one can verify that the amplification in resonance condition regularly increases with the internal temperature, such behaviour is coherent with results obtained with uniform temperature. Similar behavior is observed when the external temperature is constant and the internal one is varied, Figure 11, the increment of internal temperature causes a reduction of the natural frequency and an increment of the quality factor.



**Figure 10. 1<sup>st</sup> axisymmetric mode (FRF). Red line  $50^{\circ}\text{C}$  (inside and outside), black line:  $-10^{\circ}\text{C}$  (inside and outside), blue line  $-10^{\circ}\text{C}$  outside  $50^{\circ}\text{C}$  inside, brown line  $-5^{\circ}\text{C}$  outside  $50^{\circ}\text{C}$  inside, green line  $+10^{\circ}\text{C}$  outside  $50^{\circ}\text{C}$  inside**

**Table 5. Experimental frequency and Young's Modulus on temperature gradient**

Line color Figure 10	Temperature gradient	Experimental Frequency	Young's Modulus (FEM modelling)
Red	50°C external – internal 50°C	470 Hz	2,850 GPa
Green	10°C external – internal 50°C	490 Hz	2,950 GPa
Brown	-5°C external – internal 50°C	503 Hz	3,100 GPa
Blue	-10°C external – internal 50°C	505.5 Hz	3,110 GPa
--	-20°C external – internal 50°C	518 Hz	3,300 GPa



**Figure 11. 1<sup>st</sup> axisymmetric mode resonance (FRF) with thermal gradient, external temperature -10°C, internal temperature: red -8°C, green -5°C, blue 0°C, magenta +5°C, light blue +10°C, brown +21°C, dark green +50°C**

### 5.3 Radial excitation

The limitation of the approach followed in the previous section is the impossibility of exciting some modes, in particular shaking the system from the base, the forcing is inertial and it is mainly due to the top mass inertia; on the other hand the shell-like modes do not present any top disk motion, therefore, a base excitation cannot excite shell modes.

In order to circumvent the previously mentioned problems a new test configuration has been developed, in order to radially excite shell modes. A mini-shaker is connected to the lateral surface of the shell, see Figure 12; a load cell was placed between the shaker and the shell in order to measure the force applied to the structure, a laser vibrometer was used to measure the lateral vibration of the shell.

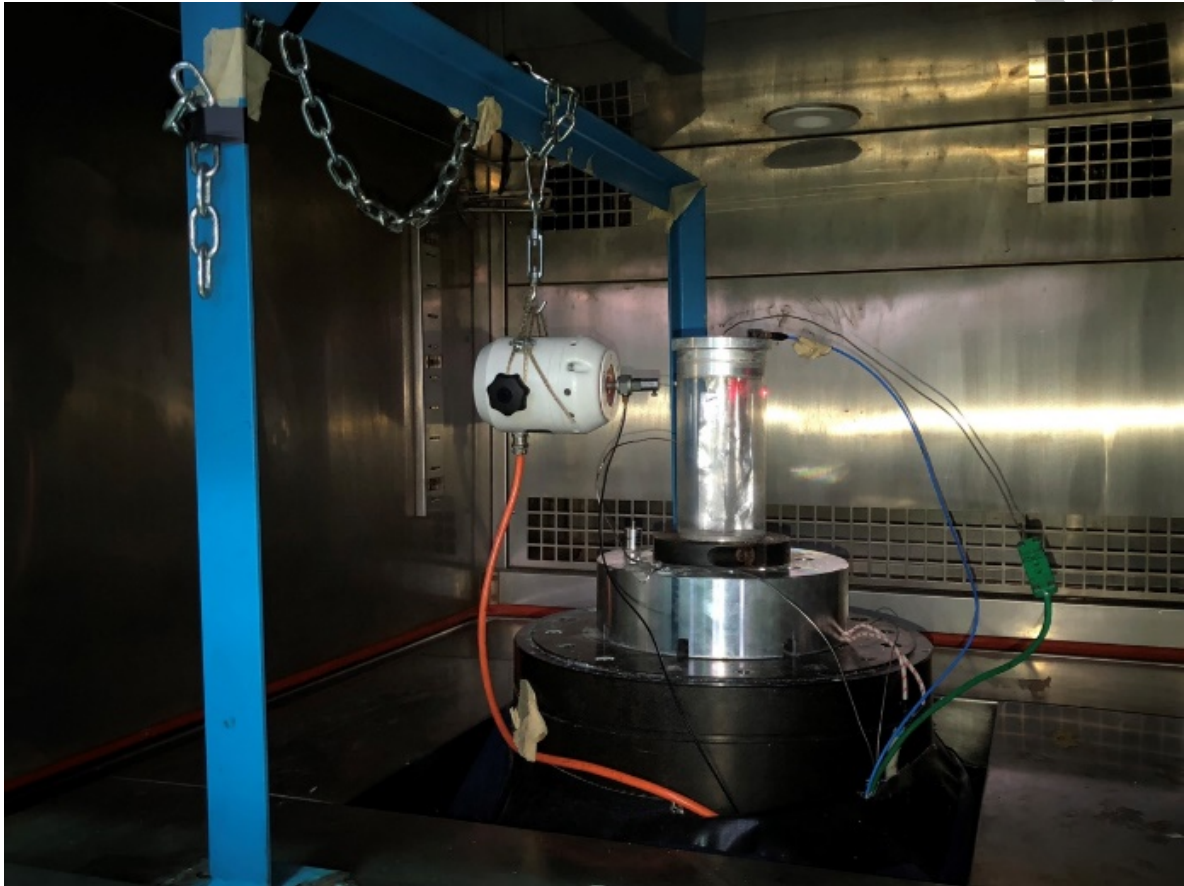


Figure 12. New setup configuration to identify shell like modes frequencies

The tests campaign consists of four tests at homogeneous temperature respectively at  $-10^{\circ}\text{C}$ ,  $0^{\circ}\text{C}$ ,  $20^{\circ}\text{C}$  and  $50^{\circ}\text{C}$ ; and 3 test with thermic gradient with internal temperature of  $50^{\circ}\text{C}$  and external temperature at  $-10^{\circ}\text{C}$ ,  $0^{\circ}\text{C}$  and  $20^{\circ}\text{C}$ . Figure 13 shows frequency response of lateral velocity respect to load cell for the homogeneous temperature cases, Figure 14 shows the results in presence of thermal gradient between  $-10^{\circ}\text{C}$  and  $50^{\circ}\text{C}$  (green line) respect to the two homogeneous temperatures at  $-10^{\circ}\text{C}$  (blue line) and  $50^{\circ}\text{C}$  (red line), the three shell-like modes excited in the frequency band shows a frequency shift and a different damping due to the temperature; in particular damping increase at lower temperatures and resonance frequency increase at lower temperatures.

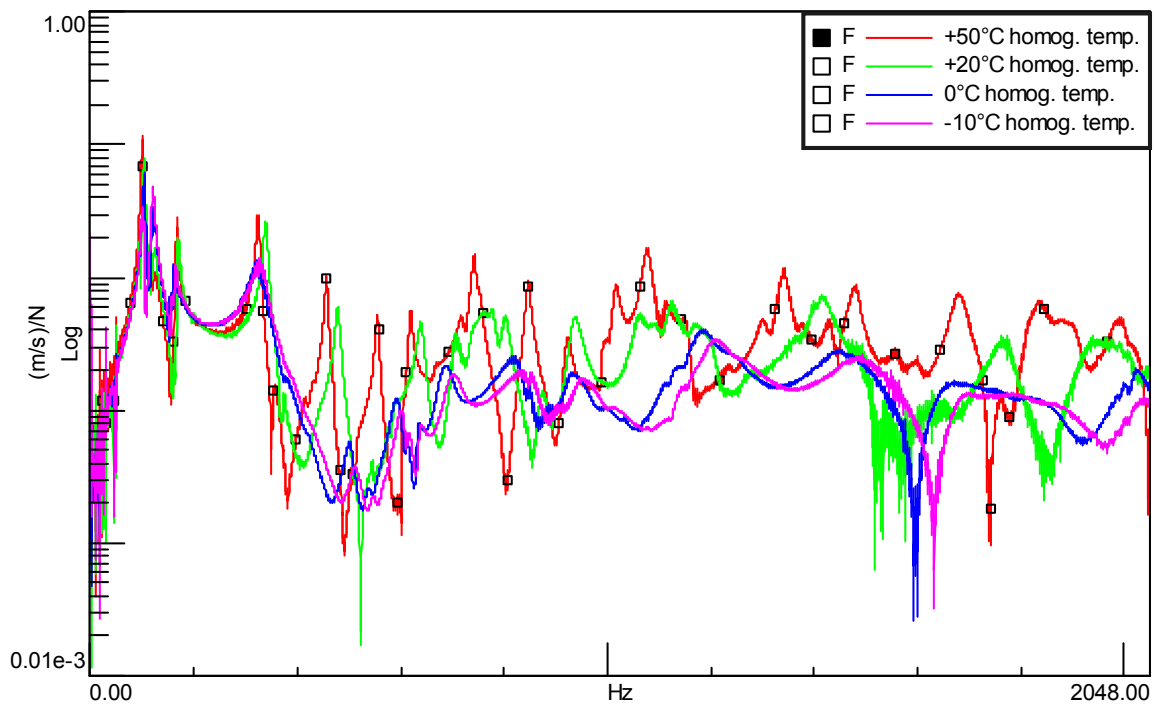
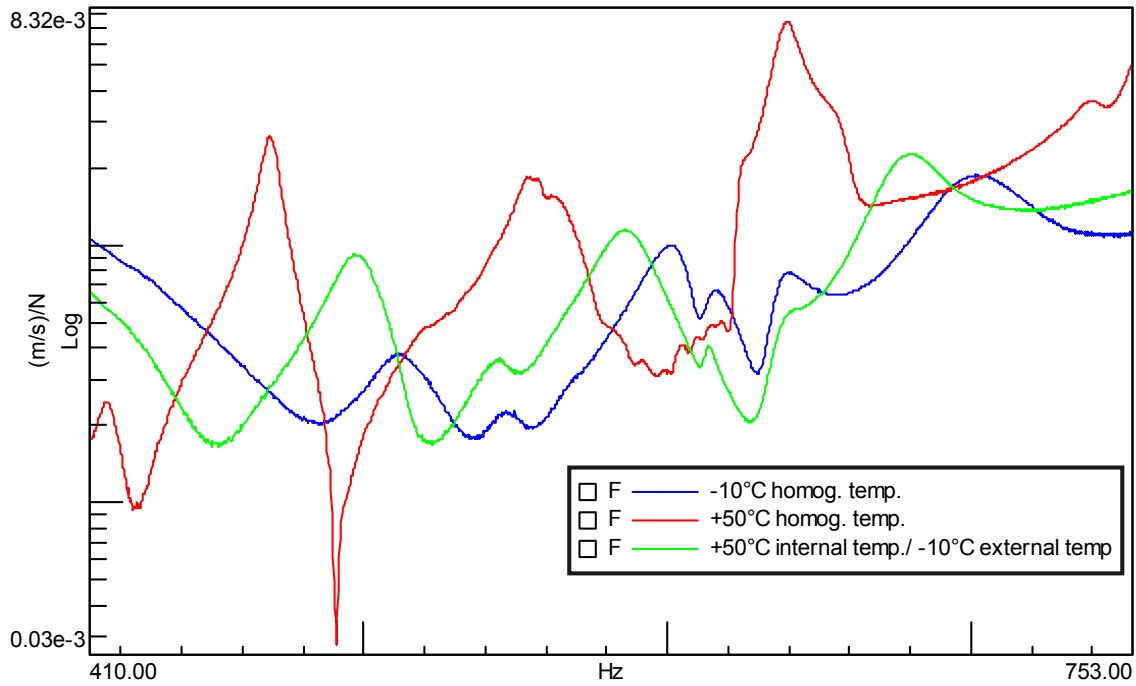
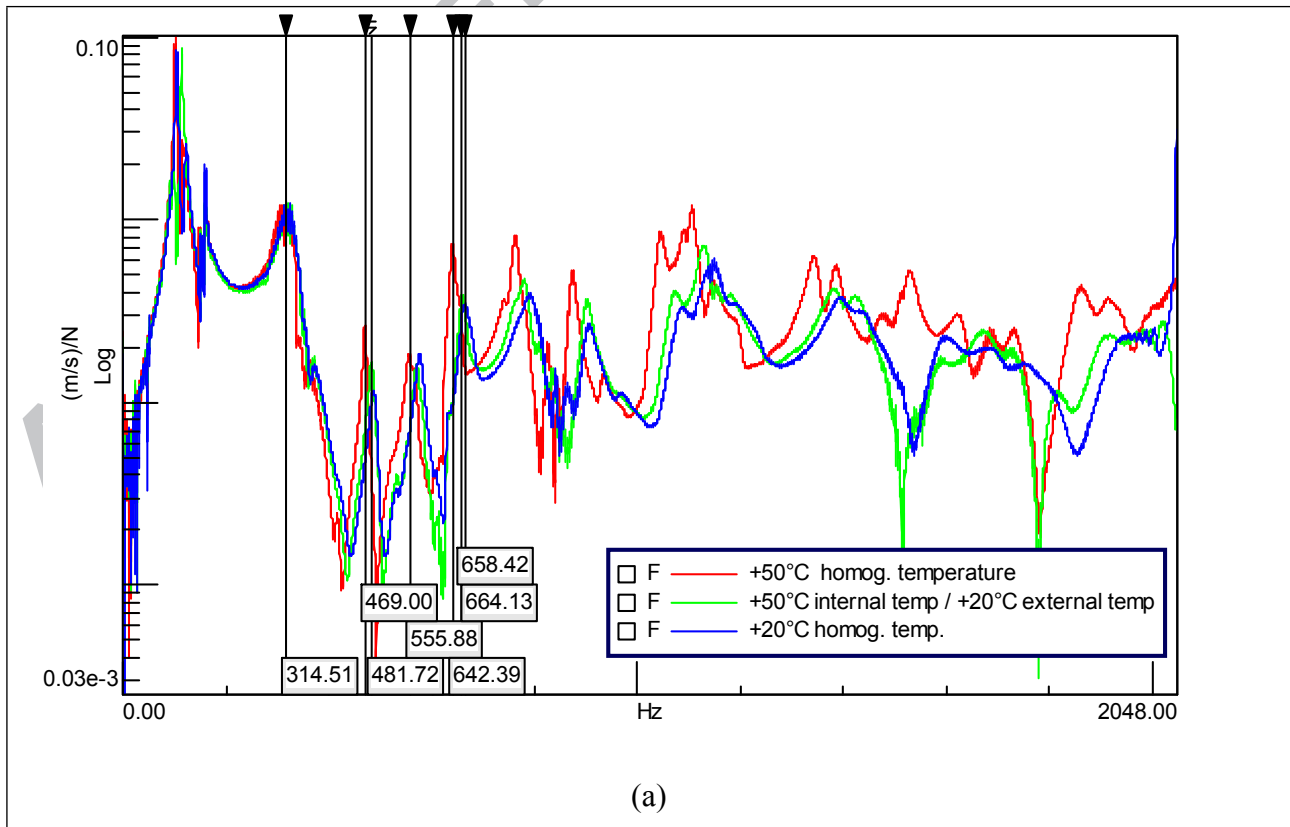
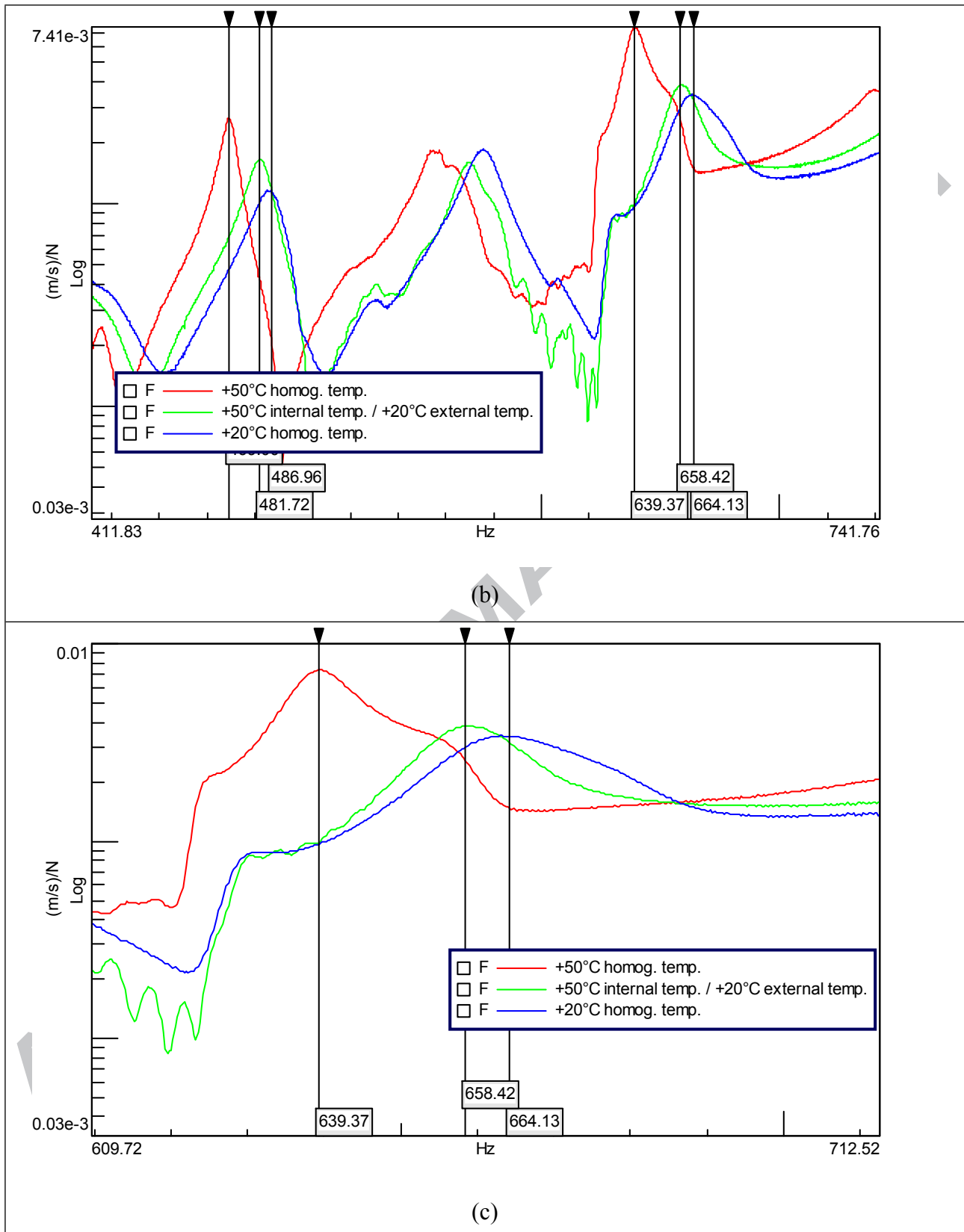


Figure 13. FRF between laser vibrometer (lateral velocity) and load cell. Homogeneous temperature



**Figure 14. FRF between laser vibrometer and load cell. Red line: homogeneous temperature of  $50^\circ\text{C}$ , blue line: homogeneous temperature of  $-10^\circ\text{C}$ , green line: gradient between  $50^\circ\text{C}$  (inner) and  $-10^\circ\text{C}$  (outside)**





**Figure 15.** FRF between laser vibrometer and load cell. Red line: homogeneous temperature of +50°C, blue line: homogeneous temperature of +20°C, green line: gradient between 50°C (inner) and 20°C (outside)

## 6. Conclusion

In the present work an extensive test campaign were carried out, different experimental setups to identify the dynamic property of a PET circular cylindrical shell was explained and the results of experimental and FEM modal analysis were shown. Different shell configuration was tested with two top mass of different weight to enhance the experimental analysis; moreover, a characterization of the properties was carried out in a wide temperature range with several tests, both at homogeneous temperature and with a thermic gradient between inner and outer shell's wall. The damping and stiffness properties of the shell shown a complex behavior that imply that using linear modelling could underestimate the dynamic response of the structure under thermal gradients.



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