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Highlights

- The role of investment and financing in value creation is measured.
- A unique project rate of return is found, combining financing rate and investment rate.
- The role of ROA and WACC as well as equity and debt in value creation is studied.
- Generalization is provided for varying rates and varying costs of capital.
- The NPV is decomposed into equityholders' NPV and debtholders' NPV.

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Carlo Alberto Magni*

Abstract

Evaluating an industrial opportunity often means to engage in financial modelling which results in estimation of a large amount of economic and accounting data, which are then gathered in an economically rational framework: the pro forma financial statements. While the standard net present value (NPV) condenses all the available pieces of information into a single metric, we make full use of the crucial information supplied in the pro forma financial statements and give a more detailed account of how economic value is created. In particular, we construct a general model, allowing for varying interest rates, which decomposes the project into investment side and financing side and quantifies the value created by either side; an equity/debt decomposition is also accomplished, which enables to appreciate the role of debt in adding or subtracting value to equityholders. Further, the major role of accounting rates of return as value drivers is highlighted, and new relative measures of worth are introduced: the project ROA and the project WACC, which aggregate information deriving from the period rates of return. To achieve these results, we make use of the Average-Internal-Rate-of-Return (AIRR) approach, recently introduced, which rests on capital-weighted arithmetic means and sets a direct relation between holding period rates and NPV.

JEL Codes. G11, G12, G31, G32, C0, D4, D92, M41.
Keywords. Value creation, net present value, Return On Assets, WACC, weighted mean, equity, debt.

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1 Introduction

The analysis of economic performance of capital asset investments is a matter of central importance in corporate finance, engineering economy and, in general, managerial science. The valuation of new industrial opportunities is often associated with estimation of economic data which are used to draw up pro forma financial statements which aim at assembling, in an economically rational way, a massive amount of information. Pro forma financial statements consist of (i) income statements, where incremental revenues and costs associated with the project are collected, (ii) balance sheets, where sources of funds (equity, debt) are recorded as well as uses of funds (fixed assets and working capital), (iii) cash flow statements, which convert the estimated accounting and economic data into a stream of free cash flows (Titman and Martin 2011). The use of such financial modelling is rather common in corporate projects and in private equity investments, and it is an indispensable tool in project finance transactions. Project finance is a no-recourse form of financing, whereby a new legal entity is created, named Special Purpose Vehicle (SPV), with the explicit aim of undertaking a project with limited life (Gatti, 2012). Originated in the energy generation sector, project finance is now widely used for several kinds of engineering projects, such as oil & gas, power and telecom projects, and, more recently, Internet and e-commerce projects (Borgonovo, Gatti and Peccati 2010)

The abundant quantity of economic, accounting, and financial data which are recorded in the pro forma financial statements is usually condensed into one single metric, expressing the project's economic profitability, which is either an absolute measure of economic profitability, such as the Net Present Value (NPV), or a relative measure of worth, such as a rate of return (most notably, the Internal Rate of Return, IRR).

As for the NPV, its use of in industry for project valuation is commonplace (Gallo and Peccati, 1993; Herroelen et al. 1997; Giri and Dohi, 2004; Borgonovo and Peccati, 2004, 2006, Herroelen and Leus 2005; Wiesemann et al. 2010) and is endorsed as a theoretically correct decision criterion in corporate financial theory (see Brealey et al. 2011, Berk and DeMarzo 2011). The IRR, albeit subject to several drawbacks (see Magni 2013 for a compendium of eighteen flaws) is often used in place or even in conjunction with the NPV for investment evaluation, as well as other criteria such as payback or residual income (Remer et al., 1993; Sandahl and Sjögren, 2003; Lindblom and Sjögren, 2009; Magni, 2009).

While the standard NPV does detect value creation, it does not identify the projects' value drivers and is not capable of explaining, in a detailed way, how the economic referents underlying the project contribute to generating (or subtracting) value. In other words, the NPV alone cannot disentangle the constituents of a project: for example, given that the NPV does not distinguish investment from borrowing, it does not tell us whether value is created because funds are invested at a return rate greater than the minimum required rate of return or value is created because funds are borrowed at a borrowing rate which is smaller than the maximum acceptable borrowing rate. Also, the standard NPV cannot separate the contribution of equityholders from the contributions of debtholders in value creation or value destruction. Nor is it available, in the literature, a sufficiently general model able to establish a direct link between the accounting data estimated in the pro forma financial statements and the project's NPV. Paradoxically, while cash flows necessarily arise from (pro forma) accounting data, it is usually believed that accounting rates of return such as Return On Equity (ROE) or Return On Assets (ROA) have no financial meaning and are not reliable for economic analysis (Kay, 1976; Peasnell, 1982a,b; Whittington, 1988, Stark, 2004).

The aim of this paper is just to provide a methodological framework capable of exploiting, to a full extent, the information provided by the financial modelling underlying a capital asset investment. In particular, it aims at detecting the value drivers of a project and investigating their formal and conceptual relations; it aims at showing how value is created and, in particular, (i) whether such a value is made out of investment or out of financing (ii) what the role of equityholders and debtholders is in generating value, (iii) how accounting variables can be aggregate in metrics that are economically significant and that enable one to establish a direct link between the project's ROE and ROA and the project's NPV.

To achieve the required results, we build upon Magni's (2010, 2013) approach, which uncovers the existing relations between a project NPV and its period rates of return. This approach, named Average Internal Rate of Return (AIRR), also enables to compute, from the financial statements, a unique NPV-consistent project rate of return which is devoid of the flaws which mar the IRR. Owing to the flexibility of the AIRR approach, we also allow for varying rates, and define a new return metric, named the *project ROA*, which aggregates all the estimated ROAs, and a new cost of capital, named the *project WACC* (Weighted Average Cost of Capital), which aggregates all the project's period WACCs.

A twofold decomposition will be finally supplied, which decomposes the value created by source of funds (debt vs. equity) and by the nature of capital (investment vs. financing).

The remainder of the paper is summarized as follows.

- Section 2 summarizes the results of Teichroew, Robichek and Montalbano (1965a, b) (TRM) which allow for a project to have financing periods as well as investment periods. Investment periods generate returns for the firm at a (constant) investment rate, financing periods generate borrowing costs at a (constant) financing rate. TRM devise two NPV-consistent decisions rules that assume that the cost of capital is constant and equal to either the investment rate or the financing rate.

- Section 3 supplies the missing link among investment rate, financing rate, cost of capital and Net Present Value (NPV). The AIRR approach is used for dividing the economic value created into investment NPV and financing NPV and for combining investment rate and financing rate into an economically significant project rate of return.

- Section 4 generalizes the results of the previous section removing the restrictive assumptions of constant rates: varying investment rates and varying financing rates are allowed, as well as varying costs of capital. Using again the AIRR approach, the project investment rate and project financing rate are obtained and combined into a project rate of return. Also, a project cost of capital is obtained, which is splitted up into an investment cost of capital and a financing cost of capital, which act as benchmark return rate and benchmark financing rate in the investment and financing periods, respectively.

- Section 5 takes into consideration the role of equity and debt in value creation and shows the relations among the various rates (ROE, ROD, ROA) and the various project-specific costs of capital (cost of equity, cost of debt, WACC). The NPV is decomposed into equity and value component and, using the results of the previous sections, each component is in turn decomposed in investment NPV and financing NPV and a project ROA is obtained, which, compared with the project WACC, signals value creation or destruction.

- Section 6 illustrates a simple example of a levered project, that is, a project which is partly financed with debt, where it is assumed that some periods are financing periods.

Some concluding remarks end the paper. An Appendix is devoted to highlighting the differences with the well-known Modified Internal Rate of Return.

2 Investment side and financing side of a project

While many industrial opportunities are pure projects (i.e., either investment or financing), some other opportunities are *mixed* projects. It may occur, in some periods, that the invested capital is negative: this means that the project acts as a financing rather than as an investment; more specifically, in these periods, the assets used by the firm for undertaking the project serve the scope of *financing* the stakeholders (equityholders and debtholders), who take on the unusual role of capital *borrowers*, instead of being capital *providers*. In mixed projects, the identification of a period as an investment period or a financing period is essential to better disentangle the way value is created or destroyed by the project: in an investment period, the return on capital is a rate of return, and the cost of capital is the minimum return rate required by the capital providers. However, in a financing period, the capital is a borrowed amount, so the "return" on capital is not a rate of *return* at all: it is to be interpreted as a borrowing rate, and the cost of capital expresses the maximum financing rate acceptable by the stakeholders.

Whether a project is pure or mixed depends on whether the capital committed is positive or negative. For example, consider a bank account whose interest rate is 5% if the account balance is positive and 10% if the account balance is negative. Suppose a client of the bank deposits €100 in the account, then withdraws €215 at the end of the period, then deposits €110 at the end of the second period and closes off the account. The cash-flow vector of this transaction is $(-100, 215, -121)$: in the first period, the customer invests €100 in the account. At the end of the period, before the withdrawal, the account balance is positive and equal to $100(1+0.05) = 105$; by withdrawing €205, the account balance turns negative and equal to €-110, which means that, at the beginning of the second period, the client borrows €110 from the bank. At the end of the second period, the customer repays debt plus interest and closes off the account with a payment of €121: $-110(1+0.1) + 121 = 0$. This simple transaction is a mixed project: the first period is an investment period (a €100 account balance represents invested capital), the second period is a financing period (a €-110 account balance represents borrowed capital).[1]

Therefore, in general, a project can be described as having two sides: an investment side, consisting of periods where capital is invested, and a financing side, consisting of periods where capital is borrowed. A pure project can be seen as a particular case of mixed project where all periods are either investment periods or financing periods.

Consider an economic agent (e.g., a firm) facing the opportunity of investing in a project whose cash-flow stream is $\vec{a} = (a_0, a_1, \ldots, a_n)$. We assume that the project-specific cost of capital is ϱ, which represents the expected rate of return of an alternative opportunity that investors forego which is equivalent in risk to the project.

In general, the Net Present Value (NPV) of a project, computed at the discount

[1] From the bank's perspective, it is the other way around: investment in the second period, financing in the first period.

rates $x_1, x_2, \dots, x_n$, is the discounted value

$$NPV(x_1, x_2, \dots, x_n) = a_0 + \sum_{t=1}^{n} a_t \prod_{h=1}^{t} (1 + x_h)^{-1}.$$

An internal vector $\vec{r} = (r_1, \dots, r_n)$ is a vector of interest rates that make the NPV equal to zero:

$$NPV(r_1, r_2, \dots, r_n) = a_0 + \sum_{t=1}^{n} a_t \cdot \prod_{h=1}^{t} (1 + r_h)^{-1} = 0$$

(see Weingartner 1966). If $r_t = \imath$ for all $t \in T = \{1, 2, \dots, n\}$, then, the common value is called internal rate of return (IRR): $NPV(\imath) = a_0 + \sum_{t \in T} a_t (1 + \imath)^{-t} = 0$. Acceptance or rejection of a project is determined by picking $x_t = \varrho$ for all t. The project creates value (and therefore it is worth undertaking) if and only if $NPV(\varrho) = a_0 + \sum_{t \in T} a_t (1 + \varrho)^{-t} > 0$.

Teichroew, Robichek and Montalbano (TRM) (1965a, 1965b) just proposed a model of economic profitability capable of managing both pure and mixed projects, and derived two rate-of-return-based decision rules consistent with the NPV criterion. We summarize TRM model as follows.

Any project, just like the bank-account example illustrated above, may be interpreted as an economic relation between two parties, the project and the investor (e.g., the firm) which exchange monetary amounts a_t at the various dates. This situation is described by TRM (1965a, b) in terms of *project balance*, denoted as F_t:

$$F_t = F_t(r_B, r_I) = \begin{cases} F_{t-1}(r_B, r_I)(1 + r_B) + a_t & \text{if } F_{t-1} > 0 \\ F_{t-1}(r_B, r_I)(1 + r_I) + a_t & \text{otherwise} \end{cases} \tag{1}$$

where $F_0 = a_0$ and a_t denotes cash flow at time t (inflow if $a_t > 0$, outflow if $a_t < 0$). The terminal boundary condition for a project is $F_n(r_B, r_I) = 0$ (see TRM 1965a, p. 401; TRM, 1965b, p. 169). When $F_t(\cdot) < 0$, the firm loans to the project, so it is in a lending position; when $F_t(\cdot) > 0$, the firm loans from the project, that is, it is in a borrowing position. Therefore, generally speaking, the investor can be a lender in some periods and a borrower in some other periods. The rate r_I is the rate at which a firm injects funds in the project whenever it is in a lending position, while the rate r_B at which a firm borrows from a project whenever it is in a borrowing position. If the pair (r_B, r_I) fulfills the terminal condition, then r_B is said to be a *project financing (or borrowing) rate* (PFR),[2] and r_I is said to be a *project investment rate*. It is worth

[2] We will henceforth use the terms "borrowing" and "financing" interchangeably.

noting that the notion of project balance is equivalent to the notion of *capital* (invested or borrowed); for example, if $F_t = -100$, it means that the investor invests €100 in the project at the beginning of interval $[t, t+1]$. In other words, €100 is the firm's capital *invested* in the project. If $F_t = 100$, then €100 is the firm's capital *borrowed* from the project. We use the symbol $c_t := -F_t$ to denote the capital:[3]

$$c_t = c_t(r_B, r_I) = \begin{cases} c_{t-1}(r_B, r_I)(1 + r_B) - a_t & \text{if } c_{t-1} < 0 \\ c_{t-1}(r_B, r_I)(1 + r_I) - a_t & \text{otherwise.} \end{cases} \tag{2}$$

The rate r_B is active in the borrowing periods ($c_t < 0, F_t > 0$), the rate r_I is active in the investment periods ($c_t > 0, F_t < 0$).

Mathematically, r_B and r_I generate an internal return vector $\vec{r} = (r_1, \dots, r_n)$ such that

$$r_t = \begin{cases} r_B & \text{if } c_{t-1} < 0 \\ r_I & \text{otherwise} \end{cases}$$

so that $NPV(r_B, r_I) = \sum_{j=0}^{n} a_j (1 + r_B)^{-\alpha_j} (1 + r_I)^{-\beta_j} = 0$, where α_j represents the number of financing periods and β_j represents the number of investment periods between time 0 and time j, so that $\alpha_j + \beta_j = j$, $j = 1, 2, \dots, n$, and $\alpha_0 = \beta_0 = 0$.

TRM showed the following result connecting r_B and r_I.

Proposition 1. *The boundary condition $F_n(r_B, r_I) = NPV(r_B, r_I) = 0$ generates an implicit function $r_B = r_B(r_I)$ and an implicit function $r_I = r_I(r_B)$, which is the inverse function of the former.*

(See TRM 1965a, Theorem IV, Corollary IVB; TRM 1965b, p. 169).
Using Proposition 1, TRM proved the following result.

Proposition 2. *For any acceptable interest rate i (i.e., belonging to the domain of the implicit functions),*

$$NPV(i) > 0 \qquad iff \quad r_I(i) > i \tag{3a}$$

$$NPV(i) > 0 \qquad iff \quad r_B(i) < i. \tag{3b}$$

(See TMR 1965a, Theorem V, TRM 1965b, p. 176).
Therefore, considering that economic value is created if and only if $NPV(\varrho) > 0$, the following accept/reject decision rule can be stated.

[3] The account balance in the above bank-account example is just equal to c_t, with $r_B = 0.1$ and $r_I = 0.05$.

Proposition 3. *Given the project cost of capital ϱ,*

$$\textit{accept project if} \quad r_I(\varrho) > \varrho \tag{4a}$$

$$\textit{accept project if} \quad r_B(\varrho) < \varrho \tag{4b}$$

(TRM 1965a, p. 403; TRM 1965b, section VI and p. 177).

From a graphical point of view, Proposition 3 informs that TRM suggest to move along the locus of points (r_B, r_I) which fulfill $F_n(r_B, r_I) = 0$ and consider the points $(\varrho, r_I(\varrho))$ and $(r_B(\varrho), \varrho)$. The comparison of abscissa and ordinate in either pair determines project acceptability.

Example 1. Consider $\vec{a} = (55, -50, -48, -50, 100)$ and assume the cost of capital is $\varrho = 0.07$. If one sets $r_B = \varrho = 0.07$, then $F_n(r_B, r_I) = 0$ becomes $F_4(0.07, r_I(0.07)) = 0$ whose solution is $r_I(0.07) = 0.088$. Therefore, in the borrowing periods, the firm borrows at 7%, while investing at 8.8% in the investment periods. The project is accepted, since $r_I(\varrho) = 0.088 > 0.07 = \varrho$. If, alternatively, one sets $r_I = \varrho = 0.07$, then $F_n(r_B, r_I) = 0$ becomes $F_4(r_B(0.07), 0.07) = 0$ whose solution is $r_B(0.07) = 0.038$. Under this assumption, the firm pays interest equal at 3.8% in the borrowing periods, while investing funds at 7% in the investment periods. The answers is the same: accept project, because $r_B(\varrho) = 0.0384\% < 0.07 = \varrho$.

3 Investment NPV, financing NPV and project rate of return

TRM did not provide any functional relation between $NPV(\varrho)$ and the two-rate model presented. We now supply the missing functional relation, explicitly linking, r_B, r_I and $NPV(\varrho)$. This will enable us to (i) understand the implicit assumption of TRM's rules, (ii) grasp the role played by the cost of capital in value creation and its relations with r_B and r_I, (iii) appreciate the role of investment periods and financing periods in creating value, and (iv) supply a unique project rate of return.

Consider the disjoint subsets $T_I = \{t \in T : c_{t-1} \geq 0\}$, $T_B = \{t \in T : c_{t-1} < 0\}$: if $t \in T_I$, then $[t-1, t]$ is an investment period, if $t \in T_B$, then $[t-1, t]$ is a borrowing (financing) period. One can manipulate the NPV in the following way:

$$\begin{aligned} NPV(\varrho) &= a_0 + \sum_{t \in T} a_t v^t \\ &= a_0 + \sum_{t \in T} (a_t - c_t + c_t) v^t \\ &= \sum_{t \in T} (-c_{t-1} v^{t-1} + (a_t + c_t) v^t). \end{aligned} \tag{5}$$

Using (2) and the equality $F_n = c_n = 0$, and manipulating, one may write

$$NPV(\varrho) = \sum_{t \in T} c_{t-1} \cdot (r_t - \varrho) \cdot v^t \tag{6}$$

where

$$r_t = \begin{cases} r_B & \text{if } t \in T_B \\ r_I & \text{otherwise.} \end{cases} \tag{7}$$

Equation (6) breaks down the NPV into n summands, each of which is the product of an excess rate and the capital committed at the beginning of the periods. In a borrowing period ($t \in T_B$) the term $c_{t-1}(r_B - \varrho)v^t$ positively contributes to value creation if and only if $r_B < \varrho$, whereas in an investment period ($t \in T_I$) the term $c_{t-1}(r_I - \varrho)v^t$ positively contributes to value creation if and only if $r_I > \varrho$. In such a way, NPV is partitioned into two shares: an *investment NPV* and a *financing NPV*:

$$NPV(\varrho) = \sum_{t \in T_I} c_{t-1}(r_I - \varrho)v^t + \sum_{t \in T_B} c_{t-1}(r_B - \varrho)v^t. \tag{8}$$

The first addend in the sum measures the value which is created in the investment periods, the second addend measures the value which is created in the financing periods. We have then proved the following proposition.

Proposition 4. *Assume the project balance depends on two rates r_B and r_I, as expressed in (2), not necessarily equal to ϱ. Then, the economic value created can be partitioned into two shares: an* investment *NPV*

$$NPV_I = I \cdot (r_I - \varrho) \tag{9}$$

and a financing *(or* borrowing*) NPV*

$$NPV_B = B \cdot (r_B - \varrho) \tag{10}$$

where $I := \sum_{t \in T_I} c_{t-1} \cdot v^t, B := \sum_{t \in T_B} c_{t-1} \cdot v^t$ and $v^t := (1+\varrho)^{-t}$.

The proposition provides a functional relation among the rates and the NPV. Also, the NPV is decomposed and, as such, it enables the evaluator to obtain information on the way value is created: value is created (destroyed) either by investing capital I at a greater (smaller) rate than ϱ in the investment periods or by borrowing capital B at a smaller (greater) rate than ϱ in the borrowing periods. Equation (8) and the associated Proposition 4 makes it clear that the net effect depends on the relation among three rates: r_B, r_I and ϱ (as well as on the capital bases I and B). It is also clear that the market rate ϱ has a twofold nature: it acts as a benchmark lending rate in the investment periods (i.e., it expresses the minimum acceptable rate of return) and as a benchmark borrowing rate in the borrowing periods (i.e., it expresses the maximum acceptable borrowing rate). The comparison between r_B and ϱ only tells us whether economic value is created in the borrowing periods, while the comparison between r_I and ϱ only tells us whether value is created in the investment periods. The direct comparison of r_I and r_B is not informative.

Proposition 4 also sheds light on the meaning of TRM's rules. Rule (4a) can be derived from (8) by assuming that the PFR rate is equal to the cost of capital ($r_B = \varrho$), which means to assuming that the borrowing periods are value-neutral (i.e., $NPV_B = 0$) so that (8) becomes

$$NPV(\varrho) = I(r_I - \varrho) = NPV_I \tag{11}$$

where $r_I = r_I(\varrho)$. Value creation is then shifted upon the lending periods and the comparison between ϱ and the related rate $r_I(\varrho)$ signals value creation or destruction. Similarly, rule (4b) can be derived from (8) by assuming that the PIR rate is equal to the cost of capital ($r_I = \varrho$), which is equivalent to assuming that the lending periods are value-neutral (i.e., $NPV_I = 0$) so that (8) becomes

$$NPV(\varrho) = B(r_B - \varrho) = NPV_B \tag{12}$$

where $r_B = r_B(\varrho)$. Value creation is then shifted upon the borrowing periods: the comparison between rate ϱ and $r_B(\varrho)$ signals value creation or destruction. Proposition 4 implies that the same NPV can be obtained by different (infinite) combinations of NPV_I and NPV_B. TRM's rules are the result of two extreme combinations: $NPV_I = NPV$ and $NPV_B = 0$ or $NPV_B = NPV$ and $NPV_I = 0$. But TRM did not commit themselves to the choice of either combination. They left the choice to the evaluator, without providing clues as to when either alternative should be more appropriate. Furthermore, both assumptions $r_B = \varrho$ and $r_I = \varrho$, are unrealistic and practically unhelpful: in real-life applications (and, in particular, in industrial projects and project finance transactions), firms do not usually borrow funds at the cost of capital nor invest funds at the cost of capital. Both r_B and r_I are different from ϱ, which means that, notwithstanding its important theoretical contribution, TRM rules are only applicable to exceptional economic transactions.

A third feature of the TRM model is that neither $r_I(\varrho)$ nor $r_B(\varrho)$ refer to the whole project; they refer to the investment side and the financing side of the project, respectively. In other words, $r_I(\varrho)$ represents the rate of return of the project in the investment periods (under the assumption $r_B = \varrho$), and $r_B(\varrho)$ represents the rate of cost in the borrowing periods (under the assumption $r_I = \varrho$). TRM did not supply a *project* rate of return, capable of measuring the project's economic profitability, i.e., capable of combining the performances of the investment side and the financing side.

Recently, a new approach to economic profitability has been introduced and developed, named Average Internal Rate of Return (AIRR) (Magni 2010, 2013) which enables us to combine the PFR and the PIR in order to supply an overall project rate of return. It suffices to consider (8) and impose the invariance requirement

$$NPV(\varrho) = I(r_I - \varrho) + B(r_B - \varrho) = (I + B)(\overline{r} - \varrho). \tag{13}$$

Solving for $\overline{r}$, the following result obtains.

Proposition 5. *A project's rate of return is the capital-weighted average of the PFR and the PIR:*

$$\overline{r} = \frac{r_I \cdot I + r_B \cdot B}{I + B}. \tag{14}$$

The following rule holds:

$$\textit{accept project if } \overline{r} > \varrho. \tag{15}$$

Proposition 5 fills the gap between TRM's PIR and PFR and the notion of project rate of return: PIR and PFR, which measure value creation in their own specific setting (investment periods and financing periods, respectively) are naturally combined into a unique metric which summarizes the value created in relative terms (i.e., percentage), so constituting a counterpart of the NPV, which measures value creation in absolute terms (i.e., euros). The amount $I+B$ is the net capital committed, which is invested at an overall return rate $\overline{r}$: compared with the benchmark ϱ, value creation is determined. Equation (13) then informs one that the investor invests I at a rate r_I and borrows B at a rate r_B, which is equivalent to investing a net capital $I + B$ at a return rate equal to $\overline{r}$.[4]

Proposition 5 enables one to free the evaluator from TRM's restrictive assumptions ($r_B = \varrho$ or $r_I = \varrho$) and allow for a selection of the PIR and the PFR which more properly represents the economic transactions underlying the project.

Remark 1. It is worth noting that the invariance condition (13) we have used to derive the project rate of return is a particular case of the invariance condition Magni (2010, p. 159) used to derive an Average Internal Rate of Return (AIRR). In general, an AIRR

[4] If $I + B < 0$, then r is a rate of *cost* and ϱ acts a a benchmark borrowing rate, so value is created if and only if $r < \varrho$. The sign in (15) is then reversed.

is defined as any capital-weighted mean of period rates k_t:[5]

$$\overline{k} = \frac{\sum_{t \in T} k_t \cdot c_{t-1} \cdot v^t}{\sum_{t \in T} c_{t-1} \cdot v^t}$$

(Magni 2010, eq. (5); Magni 2013, eq. (18)). It is evident that $\overline{k}$ equals $\overline{r}$ if one assumes $k_t = r_B$ for $t \in T_B$ and $k_t = r_I$ for $t \in T_I$, and that (15) is just an instantiation of Magni's (2010) Theorem 2 under this assumption. Therefore, Proposition 5 is a direct derivation of the AIRR approach. This means that (14) is just a particular case of AIRR, lying on the iso-value line, which describes the infinitely many combinations of capital and rate leading to the same NPV (see Magni 2013).

Example 2. Consider again $\vec{a} = (55, -50, -48, -50, 100)$ and suppose that, other things unvaried, the actual financing rate and investment rate are, respectively, $r_B = 20\%$ and $r_I = 18.76\%$. In such a situation, we are able to understand how value is affected by two contrasting forces: investment NPV is positive, for value is created in the investment periods ($18.76\% > 7\%$) whereas financing NPV is negative, for value is destroyed in the financing periods ($20\% > 7\%$). In other words, the firm borrows funds at a greater rate than the benchmark borrowing rate, but also invests at a greater rate than the benchmark lending rate. The net effect is determined by the capital base to which the excess rates are applied. In particular, Table 1 reports the capital amounts, the cash flows and the project rate of return, obtained as an average of the PFR and the PIR. It is worth noting that the first two periods are borrowing periods. In these periods value is destroyed for financing occurs at a greater rate than the cost of capital. The overall borrowed capital is 65.38, and the excess financing rate is $r_B - \varrho = 20\% - 7\% = 13\%$. Applied to the borrowed amount one gets the value destroyed in the first two periods: $NPV_B = -65.38 \cdot 0.13 = -8.499$. The last two periods are investment periods. In these periods, value is created since the excess investment rate is positive: $r_I - \varrho = 18.76\% - 7\% = 11.76\%$. Applied to the invested capital $I = 87.75$, one gets $NPV_I = 87.75 \cdot 0.1176 = 10.319$, which more than compensates the value destruction occurred in the financing periods. As a result, the project's financing side is a value-destroying one, whereas the project's investment side creates value to such an extent that the net effect is positive: $NPV = NPV_I + NPV_B = 1.82$. Note that value is created even though the PIR is smaller than the PFR; actually, there is no point in comparing r_B and r_I for determining value creation. Rather, the two rates can be conveniently combined via the AIRR approach into a significant project rate of return, which can be compared with the cost of capital. In our case, the project rate of return

[5] More properly, given that the weights can be negative, the aggregations consist of *affine combinations*. For this reason, the resulting mean can be greater than the greatest period rate or smaller than the smallest period rate.

turns out to be $\overline{r} = 15.14\%$. This is obtained as the capital-weighted average 18.76% and 20%, or, which is the same, as the ratio of the project return divided by the net invested capital: the return, net of borrowing costs, is $0.1876 \cdot 87.75 - 0.2 \cdot 65.38 = 3.386$ and the net invested capital is $87.75 - 65.38 = 22.37$. Therefore, the firm overall invests a net capital of €22.37 earning a return of €3.386, which just means a 15.14% (=3.386/22.37) rate of return.

Table 1: The project rate of return as an AIRR

Time	Cash flows	Capital		Rate
t	a_t	$c_t < 0$	$c_t > 0$	r
0	55	−55		20%
1	−50	−16.0		20%
2	−48		28.8	18.76%
3	−50		84.2	18.76%
4	100			
Total	$NPV = 1.82$	$B = -65.38$	$I = 87.75$	$\overline{r} = 15.14\%$

As noted, in the TRM world the project rate of return is not supplied. In principle, it is possible to use the AIRR approach to compute the project rate of return under TRM's assumption of $r_B = \varrho$ or $r_I = \varrho$.[6] However, TRM model cannot be used for practical purposes, just because it artificially forces either the investment side or the financing side to be value-neutral, so distorting the economic analysis of the project. The AIRR approach enables the evaluator to free from TRM's restrictive assumptions and properly rest on the *actual* economic data and, in particular, to combine the actual PIR and PFR in a significant project rate of return.[7].

In the following sections, we will make extensive use of the AIRR approach and aggregate non-constant rates via weighted means in order to derive the investment rate, the financing rate, and the project rate of return, as well as the project cost of equity, the project cost of debt, and the project WACC.

[6]It can be checked that, if one picked $r_B = \varrho = 7\%$, the project rate of return would be $\overline{r} = 11.29\%$; conversely, if one picked $r_I = \varrho = 7\%$, the project rate of return would be $\overline{r} = 10.89\%$.

[7]In section 5 we will show how to derive the actual PIR and the actual PFR from the project's pro forma financial statements.

4 Varying rates and costs of capital

In section 3 we have removed the restrictive assumption according to which either investment rate or financing rate is equal to the cost of capital. In this section, we further generalize the approach by allowing varying investment and financing rates and varying costs of capital.

Let $\vec{\varrho} = (\varrho_1, \varrho_2, \ldots, \varrho_n)$ be the vectors collecting the varying costs of capital holding in the various periods. Equation (2) generalizes to

$$c_t(\vec{r}) = c_{t-1}(\vec{r}) \cdot (1 + r_t) - a_t \tag{16}$$

where $\vec{r}$ is, as seen, the vector of internal rates of return, with

$$r_t = \begin{cases} r_{t,B} & \text{if } t \in T_B \\ r_{t,I} & \text{otherwise.} \end{cases} \tag{17}$$

The rates $r_{t,B}$ are financing rates, the rates $r_{t,I}$ are investment rate. The boundary condition with varying rates can be expressed as $c_n(\vec{r}) = 0$. The NPV is

$$NPV(\vec{\varrho}) = \sum_{t=0}^{n} a_t \cdot v_{t,0} \tag{18}$$

where $v_{t,0} := \prod_{h=1}^{t}(1 + \varrho_h)^{-1}$, $v_{0,0} := 1$. Using (16) and (18), after some algebraic manipulations one gets

$$NPV(\vec{\varrho}) = \sum_{t \in T} c_{t-1}(\vec{r}) \cdot v_{t,0} \cdot (r_t - \varrho_t). \tag{19}$$

Analogously to the previous section, we exploit the linearity of (19) and impose invariance conditions in order to obtain the PIR and the PFR and and link them to the project NPV:

$$NPV(\vec{\varrho}) = I(\overline{r}_I - \overline{\varrho}_I) + B(\overline{r}_B - \overline{\varrho}_B) \tag{20}$$

where I and B are now generalized as $I := \sum_{t \in T_I}^{n} c_{t-1} v_{t,0}$ and $B := \sum_{t \in T_B}^{n} c_{t-1} v_{t,0}$, and

$$\overline{r}_I = \frac{\sum_{t \in T_I} r_{t,I} \cdot c_{t-1} v_{t,0}}{I} \tag{21a}$$

$$\overline{r}_B = \frac{\sum_{t \in T_B} r_{t,B} \cdot c_{t-1} v_{t,0}}{B} \tag{21b}$$

$$\overline{\varrho}_I = \frac{\sum_{t \in T_I} \varrho_t \cdot c_{t-1} v_{t,0}}{I} \tag{21c}$$

$$\overline{\varrho}_B = \frac{\sum_{t \in T_B} \varrho_t \cdot c_{t-1} v_{t,0}}{B} \tag{21d}$$

are, respectively, the PIR, the PFR, the investment cost of capital, the financing cost of capital. A project rate of return $\overline{r}$ is obtained by combining $\overline{r}_I$ and $\overline{r}_B$ as in the previous section, and the NPV becomes

$$NPV(\overline{\varrho}) = (I+B)(\overline{r}-\overline{\varrho}) \tag{22}$$

where

$$\overline{r} = \frac{\overline{r}_I \cdot I + \overline{r}_B \cdot B}{I+B} \tag{23a}$$

$$\overline{\varrho} = \frac{\overline{\varrho}_I \cdot I + \overline{\varrho}_B \cdot B}{I+B}. \tag{23b}$$

Then, Propositions 4-5 are generalized as follows.

Proposition 6. *Suppose the capital growth rate is not constant, so that* (16) *holds. Then, the economic value created can be partitioned into two shares: an* investment *NPV*

$$NPV_I = I \cdot (\overline{r}_I - \overline{\varrho}_I) \tag{24}$$

and a financing *NPV*

$$NPV_B = B \cdot (\overline{r}_B - \overline{\varrho}_B). \tag{25}$$

Equations (23a)-(23b) *supply the project rate of return and the cost of capital, and the following rule holds:*

$$\textit{accept project if } \overline{r} > \overline{\varrho} \tag{26}$$

(the sign is reversed if $I+B<0$).

Proposition 6 provides a full generalization of the previous section. Whatever the pattern of investment rates, financing rates, and costs of capital, the PIR ($\overline{r}_I$) is a capital-weighted average of investment rates and the PFR ($\overline{r}_B$) is a capital-weighted average of borrowing rates. In turn, the project rate of return is a capital-weighted average of the PIR and the PFR. Likewise, the cost of capital is decomposed into an *investment* cost of capital (capital-weighted average of the period costs of capital in the investment periods) and a *borrowing* cost of capital (capital-weighted average of the period costs of capital in the borrowing periods). To better appreciate the result, one should bear in mind that the costs of capital ϱ_t can be considered investment rates, when the investor invests capital, or borrowing rates, when the investor borrows capital. If $c_{t-1} > 0$, then r_t and ϱ_t are investment rates of return and the product $c_{t-1}(r_t - \varrho_t)$ says that the firm invests c_{t-1} euros at the rate r_t while renouncing to investing the same monetary amount at the rate ϱ_t: the difference between these two alternative investments supplies the economic value created in the interval $[t-1, t]$. Symmetrically, if $c_{t-1} < 0$, then r_t and ϱ_t are financing rates and the product $c_{t-1}(r_t - \varrho_t)$ says that the firm borrows $|c_{t-1}|$ euros the rate r_t while renouncing to borrowing the same monetary amount at the interest rate ϱ_t: the difference between these two alternative financings supplies the economic value created in the given period. We split the project's lifespan into investment side, which includes the periods where the firm invests, and financing side, which includes the periods where the firm borrows. In other words, we reframe the project as a portfolio consisting of two assets, an investment and a financing, and aim to capture value creation (or destruction) for each of them. As for the investment side, value creation is determined by the comparison of a sequence of project investment rates $r_{t,I}, t \in T_I$, and a sequence of investment costs of capital $\varrho_t, t \in T_I$. To accomplish the comparison, we aggregate the investment costs of capital as well as the project investment rates into weighted arithmetic means, where the capital amounts represent the weights. This results in the project investment rate, $\overline{r}_I$, and project investment cost of capital, $\overline{\varrho}_I$. The latter is a benchmark investment rate which aggregates the various period benchmark rates, and thus expresses the minimum attractive (average) rate of return. If $\overline{r}_I > \overline{\varrho}_I$, value is created in the investment periods. Likewise, for the financing side, value creation is determined by the comparison of a sequence of project financing rates $r_{t,B}, t \in T_B$, and a sequence of financing costs of capital $\varrho_t, t \in T_B$. To accomplish the comparison, we aggregate the rates into capital-weighted arithmetic means, which results in the rates $\overline{r}_B$ and $\overline{\varrho}_B$, the latter representing the maximum acceptable (average) financing rate. If $\overline{r}_B < \overline{\varrho}_B$, value is created in the financing periods. It is worth noting that $\overline{\varrho}_I$ and $\overline{\varrho}_B$ are not discount rates for cash flows; rather, they *aggregate* the discount rates into suitable means which express average benchmark rates for investment and financing, respectively.

Example 3. An investor has the opportunity of depositing and withdrawing cash flows from an account balance with prefixed borrowing rates and lending rates which change period by period. The borrowing rates are activated when the account balance is negative and the lending rates are activated when the account balance is positive.

period	borrowing rate	lending rate
1	23%	16%
2	13%	10%
3	8%	6%
4	20%	19%

Suppose the investor deposits €2 in the account, withdraws €20 after one period, deposits €5 and €75 after two and three periods, respectively, and, finally, withdraws €70 at the end of the fourth period. The sequence of cash flows is then

$\vec{a} = (-2, 20, -5 - 75, 70)$. It can be checked that the investment periods are the first one and the fourth one (i.e., $T_I = \{1, 4\}$), so the lending rates 16% and 19% are applied to the (positive) account balances $c_0 = 2$ and $c_3 = 58.82$. The financing periods are the second one and the third one (i.e., $T_B = \{2, 3\}$), so the borrowing rates 13% and 8% are applied to the (negative) account balances $c_1 = -17.68$ and $c_3 = -14.98$. The internal vector is then $\vec{r} = (0.16, 0.13, 0.08, 0.19)$.[8] Assuming that the vector of costs of capital is $\vec{\varrho} = (0.21, 0.1, 0.16, 0.12)$, the investment side consists of two periods: a wealth-creating period, the fourth one, where investor renounce to investing funds at 12% while receiving a 19% from the project (so earning an excess 7%); a wealth-destroying period, where investors receive a 16% but forego a 21% (so losing an excess 5%). To assess the net effect of these two conflicting results, the 19% and 16% investment rates are aggregated into a unique metric which summarizes the overall performance in the investment periods: from (21a), $\overline{r}_I = 18.86\%$; analogously, the 12% and 21% costs of capital are aggregated into a suitable average expressing the benchmark rate of return: from (21c) $\overline{\varrho}_I = 12.42\%$. On average, the investor invests funds in two periods at 18.86%, so foregoing the opportunity of investing funds at 12.42%. The net effect is positive, so the investment side of this transaction creates value. As for the financing side, the second period destroys value, for funds are borrowed at 13% while the market only requires 10%. In the third period, value is created, for funds are borrowed at 8% while the market requires a 16% interest rate. To assess the net effect, one aggregates the project borrowing rates and the costs of capital by applying (21b) and (21d). The result is $r_B = 10.89\%$ and $\varrho_B = 12.53\%$, which means that, overall, the financing periods create value, since, on average, funds are borrowed at 10.89% while the market requires 12.53%. By Proposition 6, the NPV of the entire operation is

$$\begin{aligned} NPV &= NPV_I + NPV_B \\ &= 35.67 \cdot (0.1886 - 0.1242) + (-22.98) \cdot (0.1089 - 0.1253) \\ &= 2.3 + 0.38 = 2.68. \end{aligned} \tag{27}$$

In turn, aggregating the investment rate and the borrowing rate, as well as the investment and financing costs of capital, the project rate of return and the project cost of capital are obtained: from (23a) and (23b), $\overline{r} = 33.3\%$ and $\overline{\varrho} = 12.2\%$. As $I + B = 12.69 > 0$, the project is a net investment of €12.69 at an (average) return rate of 33.3% with a cost of capital of 12.2%. (Obviously, $12.69 \cdot (0.333 - 0.122) = 2.68$ and the NPV is found back again).

[8]Therefore,

$$-2 + \frac{20}{1.16} - \frac{5}{1.16 \cdot 1.13} - \frac{75}{1.16 \cdot 1.13 \cdot 1.08} + \frac{5}{1.16 \cdot 1.13 \cdot 1.08 \cdot 1.19} = 0.$$

While this generalization does enrich the economic analysis of the project, nothing is said about the way the project is financed, and the way equity and debt interact in the investment and financing periods. The next section is just devoted to showing how the economic information collected in the pro forma financial statements can be used for investigating the role of equity and debt in value creation, as well as the role of Return On Assets (ROA) and Weighted Average Cost of Capital (WACC).

5 ROA, WACC and the role of equity and debt in creating value

When a project is undertaken, equity and/or debt is involved. Let E_t be the equity invested in the project at time t, and let D_t denote the amount of outstanding debt at time t which finances the project, net of short-term financial assets such as cash, bank accounts, etc.,[9] $t \in T_0 = T \cup \{0\}$. Let $\vec{e} = (e_0, e_1, \dots, e_n) \in \mathbb{R}^{n+1}$ be the vector of cash flows to equity generated by the project. Analogously, let $\vec{d} = (d_0, d_1, \dots, d_n) \in \mathbb{R}^{n+1}$ be the vector of cash flows to debt. If we denote as C_t the entire capital committed in the project and as f_t the free cash flow of the project, then $C_t = E_t + D_t$ and $f_t = e_t + d_t$. Denoting with I_t^e the net income and I_t^d the interest payment, the following relation for the free cash flow holds:

$$f_t = I_t^e + I_t^d - (\Delta E_t + \Delta D_t) = I_t^e + I_t^d - \Delta C_t \tag{28}$$

where $\Delta y_t := y_t - y_{t-1}, y := D, E, C$ is the difference operator. The ratio $r_t^e := I_t^d / E_{t-1}$ is the Return On Equity (ROE) and the ratio $r_t^d := I_t^d / D_{t-1}$ is the Return On Debt (ROD). Equation (28) can be rewritten as

$$E_t + D_t = C_t = C_{t-1}(1 + ROA_t) - f_t \tag{29}$$

where

$$ROA_t = \frac{r_t^e \cdot E_{t-1} + r_t^d \cdot D_{t-1}}{E_{t-1} + D_{t-1}}. \tag{30}$$

is the so-called Return On Assets (ROA), obtained as a weighted average of the ROE and the ROD.

To compute the project value, one needs compute the project-specific Weighted Average Cost of Capital (WACC), which must reflect the specifics of the individual project and thus it may differ from the firm's WACC (see Titman and Martin 2011, ch. 5). If the project is financed with nonrecourse debt, a specific amount of debt is

[9] D_t represents the net financial obligations, that is, financial liabilities minus financial assets.

attached to the project, which is helpful for computing the weights on the investment's debt and equity financing. In this case, the project is very similar to an independent firm and the project is the sole source of collateral. In project financing transactions, a new legal entity is indeed created, called Special Purpose Vehicle (SPV) or *project company*: the capital invested in the project by the sponsoring firm is the SPV's equity and the SPV's debtholders have no recourse to the sponsoring firm's assets. If, conversely, the project is financed on-balance sheet, one must first estimate the debt and equity that can be attributed to the project and then estimate the cost of capital (this is a more complex task, which involves managerial judgment, for the financing of the project is intermingled with the financing of the firm's other investments). Let k_t^e denote the project's *cost of equity* (i.e., the required return to equity) and k_t^d denote the project's *cost of debt*, (i.e., the required return to debt). Denote as V_t^e and V_t^d the economic (i.e., market) value of the equity and the debt, respectively. Then, by definition, $k_t^e = (V_t^e + e_t)/V_{t-1}^e - 1$ and $k_t^d := (V_t^d + d_t)/V_{t-1}^d - 1$. Hence,

$$V_t^e + V_t^d = (V_{t-1}^e + V_{t-1}^d)(1 + WACC_t) - (e_t + d_t) \tag{31}$$

where

$$WACC_t = \frac{k_t^e \cdot V_{t-1}^e + k_t^d \cdot V_{t-1}^d}{V_{t-1}^e + V_{t-1}^d}. \tag{32}$$

It is worth noting that the project WACC is time-variant. Even if the cost of equity and the cost of debt are assumed to be constant, the weights in (32) change, for V_{t-1}^e and V_{t-1}^d change.[10] Let k_t^u be the project-specific unlevered cost of assets, and let V_t denote the economic value of the project. Then, $V_t = V_{t-1}(1 + k_t^u) - f_t$.[11] Value additivity implies $V_t = V_t^e + V_t^d$, which in turn implies $k_t^u = WACC_t$; that is, the unlevered cost of assets is equal to the WACC.[12]

The project NPV is $NPV = \sum_{t\in T_0} f_t \cdot v_{t,0}$, where $v_{t,0} := \prod_{h=1}^t (1 + WACC_h)^{-1}$. The equityholders' NPV is $NPV^e = \sum_{t=0}^n e_t \cdot v_{t,0}^e$ with $v_{t,0}^e := \prod_{h=1}^t (1 + k_h^e)^{-1}$; the debtholders' NPV is $NPV^d = \sum_{t=0}^n d_t \cdot v_{t,0}^d$ with $v_{t,0}^d := \prod_{h=1}^t (1 + k_h^d)^{-1}$.

We now apply the results found in the previous section, separately, to the equity cash-flow stream $\vec{e}$ and to the debt cash-flow stream $\vec{d}$; this will directly result in a twofold decomposition of the project value created. Let us then apply (19) with

[10] While a firm can adjust debt in such a way as to keep a constant target debt/equity ratio, in project-financed investments the amortization schedule is prefixed and debt cannot be targeted so as to keep the weights constant.

[11] Note that this means $V_t = \sum_{h=t+1}^n f_t \cdot v_{h,t+1}$ for every $t \in T_0$, $v_{h,t+1} := (1 + k_{t+1}^u)^{-1}(1 + k_{t+2}^u)^{-1} \dots (1 + k_n^u)^{-1}$.

[12] This result implicitly assumes a no-tax world and is just a reframing of Modigliani and Miller's (1958) Proposition I.

$c_{t-1} = E_{t-1}\ r_t = r^e_t,\ \varrho_t = k^e_t$, so that

$$NPV^e = \sum_{t\in T} E_{t-1}(r^e_t - k^e_t)\cdot v^e_{t,0}. \tag{33}$$

The same reasoning applies to debtholders' NPV: picking $c_{t-1} = D_{t-1}$, $r_t = r^d_t$, $\varrho_t = k^d_t$ in (19) one gets

$$NPV^d = \sum_{t\in T} D_{t-1}(r^d_t - k^d_t)\cdot v^d_{t,0}. \tag{34}$$

By value additivity, $NPV = NPV^e + NPV^d$, which implies that the project's NPV is

$$NPV = \sum_{t\in T} E_{t-1}(r^e_t - k^e_t)\cdot v^e_{t,0} + \sum_{t\in T} D_{t-1}(r^d_t - k^d_t)\cdot v^d_{t,0}. \tag{35}$$

Let $r^e_{t,B}, t\in T_B = \{t\in T : C_{t-1} < 0\}$ denote the ROE in a financing period and $r^e_{t,I}, t\in T_I = \{t\in T : C_{t-1} \geq 0\}$ denote the ROE in an investment period. Let $E_I := \sum_{t\in T_I} E_{t-1}v^e_{t,0}$, $E_B := \sum_{t\in T_B} E_{t-1}v^e_{t,0}$ denote the part of the equity committed in the investment periods and in the financing periods, respectively, and let

$$\bar{r}^e_I = \frac{\sum_{t\in T_I} r^e_{t,I}E_{t-1}\cdot v^e_{t,0}}{E_I} \tag{36a}$$

$$\bar{r}^e_B = \frac{\sum_{t\in T_B} r^e_{t,B}E_{t-1}\cdot v^e_{t,0}}{E_B}; \tag{36b}$$

be the average ROE of the project's investment side and the average ROE of the project's financing side. Analogously,

$$\bar{k}^e_I = \frac{\sum_{t\in T_I} k^e_{t,I}\cdot E_{t-1}\cdot v^e_{t,0}}{E_I} \tag{37a}$$

$$\bar{k}^e_B = \frac{\sum_{t\in T_B} k^e_{t,B}\cdot E_{t-1}\cdot v^e_{t,0}}{E_B}. \tag{37b}$$

denote the average cost of equity for the investment side and the financing side of the project, respectively. Then, (33) is reframed as

$$NPV^e = E_I(\bar{r}^e_I - \bar{k}^e_I) + E_B(\bar{r}^e_B - \bar{k}^e_B). \tag{38}$$

A symmetric reasoning and analogous notations can be used for (34), which becomes

$$NPV^d = D_I(\bar{r}^d_I - \bar{k}^d_I) + D_B(\bar{r}^d_B - \bar{k}^d_B) \tag{39}$$

where D_I (D_B) denotes the part of the net financial obligations committed in the project in the investment periods and financing periods, respectively; $\bar{r}^d_I$ ($\bar{r}^d_B$) is the

ROD for the investment (financing) periods, and $\overline{k}_I^d$ ($\overline{k}_B^d$) is the cost of debt for the investment (financing) periods.[13]

The weighted means

$$ROA_I = \frac{\overline{r}_I^e \cdot E_I + \overline{r}_I^d \cdot D_I}{E_I + D_I} \tag{40a}$$

$$ROA_B = \frac{\overline{r}_B^e \cdot E_B + \overline{r}_B^d \cdot D_B}{E_B + D_B} \tag{40b}$$

express, respectively, the return on assets for the investment side of the project (henceforth *investment* ROA) and the return on assets for the financing side of it (henceforth *financing* ROA). Letting $E := E_I + E_B = \sum_{t \in T} E_{t-1} v_{t,0}^e$ and $D := D_I + D_B = \sum_{t \in T} D_{t-1} v_{t,0}^d$ be the overall commited equity and debt, respectively, we can now define the *project* ROA as

$$ROA = \frac{ROA_I \cdot (E_I + D_I) + ROA_B \cdot (E_B + D_B)}{E + D}, \tag{41}$$

the *project* ROE as

$$\overline{r}^e = \frac{\overline{r}_I^e \cdot E_I + \overline{r}_B^e \cdot E_B}{E}, \tag{42}$$

and the *project* ROD as

$$\overline{r}^d = \frac{\overline{r}_I^e \cdot D_I + \overline{r}_B^d \cdot D_B}{D}. \tag{43}$$

Owing to (40a)-(40b), the project ROA can be framed as the weighted average of the project ROE and the project ROD:

$$ROA = \frac{\overline{r}^e \cdot E + \overline{r}^d \cdot D}{E + D}. \tag{44}$$

Analogously,

$$WACC_I = \frac{\sum_{t \in T_I} WACC_t (E_{t-1} + D_{t-1})}{E_I + D_I} \tag{45a}$$

$$WACC_B = \frac{\sum_{t \in T_B} WACC_t (E_{t-1} + D_{t-1})}{E_B + D_B} \tag{45b}$$

represent the *investment* (*financing*) WACC and

$$WACC = \frac{WACC_I \cdot (E_I + D_I) + WACC_B \cdot (E_I + D_I)}{E + D} \tag{46}$$

is the *project WACC*, while the *project cost of equity* is

$$\overline{k}^e = \frac{\overline{k}_I^e \cdot E_I + \overline{k}_B^e \cdot E_B}{E} \tag{47}$$

[13] All these variables are defined like the equity counterparts, with the symbols D and d replacing the symbols E and e, respectively.

and the *project cost of debt* is

$$\overline{k}^d = \frac{\overline{k}_I^d \cdot D_I + \overline{k}_B^d \cdot D_B}{D}. \tag{48}$$

Owing to (45a)-(45b), (46) can be framed, more intuitively, as the weighted average of the project cost of equity and the project cost of debt

$$WACC = \frac{\overline{k}^e \cdot E + \overline{k}^d \cdot D}{E + D} \tag{49}$$

The following result is then straightforward.

Proposition 7. *The economic value created by a project, $NPV = \sum_{t \in T_0} f_t \cdot v_{t,0}$, can be decomposed into four shares: (i) value created by equity in the investment periods, (ii) value created by debt in the investment periods, (iii) value created by equity in the financing periods,(iv) value created by debt in the financing periods*

$$NPV = E_I(\overline{r}_I^e - \overline{k}_I^e) + D_I(\overline{r}_I^d - \overline{k}_I^d) + E_B(\overline{r}_B^e - \overline{k}_B^e) + D_B(\overline{r}_B^d - \overline{r}_B^d). \tag{50}$$

Also,

$$NPV = C_I(ROA_I - WACC_I) + C_B(ROA_B - WACC_B) \tag{51}$$

where $C_I := E_I + D_I$ and $C_B := E_B + D_B$ denote the capital committed in the investment periods and in the borrowing periods, respectively. Furthemore,

$$NPV = E(\overline{r}^e - \overline{k}^e) + D(\overline{r}^d - \overline{k}^d) \tag{52}$$

From the above proposition, a straightforward corollary follows.

Corollary 1. *The economic value created can be obtained as the product of the net committed capital $C := C_I + C_B = E + D$ and the difference between the overall ROA and the overall WACC:*

$$NPV = C \cdot (ROA - WACC). \tag{53}$$

In terms of rates of return, economic value is created if and only if

$$ROA > WACC.$$

Proposition 7 highlights the role of the two dualities existing in a project: the duality investment/financing and the duality equity/debt. Equation (51) divides the project NPV into value created by *investing* capital and value created by *borrowing* capital; equation (52) distinguishes the value created by equityholders from the value generated by debtholders. Corollary 1 condenses the four souls of the project into a succint, economically significant, relation informing that value creation is measured by an (overall) excess return whose sign and magnitude creation depends on the net capital committed C and the difference between the overall ROA and the overall WACC. (See Table 2).

Table 2: Decomposition of economic created valued

	Equity	Debt	Total
Investment	$E_I \cdot (\overline{r}_I^e - \overline{k}_I^e)$	$D_I \cdot (\overline{r}_I^d - \overline{k}_I^d)$	$C_I \cdot (ROA_I - WACC_I)$
Borrowing	$E_B \cdot (\overline{r}_B^e - \overline{k}_B^e)$	$D_B \cdot (\overline{r}_B^d - \overline{k}_B^d)$	$C_B \cdot (ROA_B - WACC_B)$
Total	$E \cdot (\overline{r}^e - \overline{k}^e)$	$D \cdot (\overline{r}^d - \overline{k}^d)$	$C \cdot (ROA - WACC)$

Remark 2. It is worth noting that the ROAs constitute an internal return vector: given $\overrightarrow{ROA} = (ROA_1, ROA_2, \ldots, ROA_n)$, the project NPV, discounted at the ROAs, is zero: $NPV(\overrightarrow{ROA}) = \sum_{t\in T_0} f_t \cdot \prod_{h=1}^{t}(1+ROA_h)^{-1} = 0$. The investment ROA and financing ROA are the actual PIR and PFR of the project we searched for: $\overline{r}_I = ROA_I$ and $\overline{r}_B = ROA_B$, which are unambiguously drawn from the financial statements. Therefore, (51) splits the NPV into investment NPV and financing NPV in an unambiguous way.

Remark 3. It is worth noting that, in many real-life applications, $T_B = \emptyset$, that is, all periods are investment periods. However, the case in which $C_t < 0$, is economically significant and less uncommon than one might think. To better appreciate the economic interpretation of this case, consider that C_t can be divided into two main asset classes: net fixed assets and working capital, such that, for every $t \in T_0$,

$$NFA_t + WC_t = E_t + D_t$$

where WC_t is the working capital (inventories plus accounts receivables minus accounts payable) and NFA_t denotes the fixed assets, net of depreciation. Consider also that a change in sign means that the financial role of a balance sheet item is reversed: a negative asset becomes a borrowing, and a negative liability becomes an investment. More specifically, when the two sides of the equalities are positive, as usual, the relation tells us that the funds raised from capital providers (debtholders and equityholders) are invested in working capital and net fixed assets; when the two sides are negative, it means that the assets are used to finance debtholders and equityholders. In other words, capital providers do not provide economic resources at all; rather, they absorb resources from the assets. To see how this is possible, just consider that the capital C_t is negative whenever both NFA and WC are negative or, alternatively, when either $NFA_t < -WC_t < 0$ or $WC_t < -NFA_t < 0$. The first case is less common but not impossible (for example, net fixed assets can be negative when there are disposal costs associated with them which exceed their residual value and working capital is sufficiently small in value); the second case is more common: working capital can be negative if there are considerable upfront payments from customers, who then "coin" money for the firm. More generally, every time accounts receivables and inventories are sufficiently low as opposed to accounts payables (e.g., whenever the company raises cash quickly from customers, even before purchasing materials from suppliers), the working capital is negative.[14] If total capital $C_t = NFA_t + WC_t$ is negative, then $E_t + D_t$ is negative as well, which means that C_t is the amount (borrowed) to finance the capital providers, who absorb (rather than inject) funds from the firm. This means that both E_t and D_t are negative, or, alternatively, that either $E_t < -D_t < 0$ or $D_t < -E_t < 0$. The latter case is not economically meaningless: we have defined D_t as financial obligations net of financial assets, and it may well occur that debt is very small (or even zero) compared to cash and bank accounts, resulting in a negative financial liability.[15]

Remark 4. Equityholder value creation is given by NPV^e. Thanks to the results found, it is now easy to appreciate the role of capital structure in creating equity value. From (52) and (53),

$$NPV^e = C \cdot (ROA - WACC) + D \cdot (\overline{k}^d - \overline{r}^d). \qquad (54)$$

This equality explains the equity value created as the result of the operating activity and the financing policy. In particular, the first addend expresses the project's economic profitability: if the ROA exceeds the costs of assets, the operating assets involved in the project create value to equityholders. The second addend discloses the effect of financial position on equity value creation; depending on whether the cost of debt is greater or smaller than ROD, debt adds or subtracts value to equityholders (as long as $D > 0$) Whenever $\overline{k}^d = \overline{r}^d$, the project NPV is entirely grasped by equityholders.

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6 An illustrative example

To better interpret the results, we remind that, to a firm, a positive asset and a negative liability represent uses of funds (i.e., an investment), so the corresponding rate is a rate of return (i.e., lending rate), whereas a negative asset and a positive liability represent sources of funds (i.e., financing), so the related rate is a rate of cost (i.e., a financing rate). Also, a positive return rate for an investment means that capital

[14] A negative working capital has been skillfully and successfully used, in the recent past, by many companies such as McDonald, Microsoft and Amazon.

[15] A particular case is when the project is financed with a loan granted by a bank, and the firm has a current account by the same bank: when the outstanding debt is smaller than the account balance, then the net financial obligations are negative (i.e., the bank is borrowing money from the firm).

invested increases (income is positive), while a negative return rate for an investment means that the capital invested decreases (income is negative); viceversa, a positive financing rate means that capital borrowed increases (interest expense is positive), while a negative financing rate means that capital borrowed decreases (interest expense is negative). Consider a project finance transaction and suppose a Special Purpose Vehicle (SPV) is created to undertake a capital asset project with estimated life equal to five years. At time 0, the sponsoring firms (the SPV's equityholders) contribute $1,800$ and a group of banks (the SPV's debtholders) contribute $1,200$, for a total of 3,000 investment. Pro forma financial statements are drawn on the basis of estimated revenues, costs, depreciation and on the amortization plan of the loans. Table 3 collects the input data (in boldface) and the pro forma balance sheets and income statements.

Table 3: Input data and pro forma financial statements

Time	0	1	2	3	4	5
BALANCE SHEET						
Gross fixed assets	**1 000**	**1 000**	**1 000**	**1 000**	**1 000**	**1 000**
−cumulative depreciation	0	−200	−400	−600	−800	−1 000
Net fixed assets (NFA_t)	1 000	800	600	400	200	0
Working capital (WC_t)	**2 000**	**1 000**	**−1 200**	**−700**	**100**	**0**
NET ASSETS (C_t)	3 000	1 800	− 600	− 300	300	0
Debt (D_t)	**1 200**	**800**	**500**	**200**	**100**	**0**
Equity (E_t)	1 800	1 000	−1 100	− 500	200	0
TOTAL LIABILITIES (C_t)	3 000	1 800	− 600	− 300	300	0
INCOME STATEMENT						
Revenues		**5 700**	**5 100**	**5 500**	**5 100**	**5 300**
Operating costs		**5 000**	**5 000**	**5 000**	**5 000**	**5 000**
Depreciation ($-\Delta NFA_t$)		**200**	**200**	**200**	**200**	**200**
ROD (r_t^d)		**8%**	**7%**	**5%**	**3%**	**4%**
Interest (I_t^d)		96	56	25	6	4
Net Income (I_t^e)		404	−156	275	−106	96

The cost of asset is assumed to be variable and equal to $k_1^u = 8\%$, $k_2^u = 9\%$, $k_3^u = 10\%$, $k_4^u = 10\%$, $k_5^u = 11\%$. Table 4 collects the various cash-flow streams, the ROEs, the RODs and the ROAs, as well as the costs of equity, the costs of debt, and the weighted average costs of capital. The economic values are also provided. Tables 5-8 accomplish the twofold decomposition for capital, rate, cost of capital, and, finally, economic value created.

As can be gleaned from Table 5, the SPV invests €4,515.6 which, net of the borrowed €602.1, results in a net invested capital equal to €3,913.5: €1,408.2 of it is supplied by equityholders, €2,505.3 is supplied by debtholders. The project ROE is 30.92% as opposed to a 12.06% cost of equity: equity value is increased by $NPV^e = 1,408.2(30.92\% - 12.06\%) = 265.6$. The project ROD is 6.77%, as opposed to a cost of debt of 5.82%: debt value is increased by $NPV^d = 2,505.3(6.77\% - 5.82\%) = 23.9$. Overall, the project economic value created is $NPV = 289.4$, 71.7 of which is created in the investment periods and 217.7 is generated in the financing periods. It is worth noting that debtholders destroy value in the borrowing periods, that is, in those periods where net assets are negative, they lend money at a financing rate (4.45%), which is smaller than the 6.73% required return to debt. From the shareholders' point of view, in the borrowing periods, the equity value created (230.9) is greater than the project NPV: in other words, the shareholders' financing policy is a value-creating policy: debtholders are paid less than the market would require by an amount of €13.1. In the lending periods, the reverse obtains and the debtholders are paid at an interest rate (7.47%) which is greater than the interest rate required by the market (5.55%): this implies that equityholders give up part of the project NPV to debtholders. The additional equity value created by the financing policy in the borrowing periods is more than compensated by the loss in equity value in the investment periods, which is just the reason why the equity NPV is smaller than the project NPV.

The project rate of return is $ROA = 15.46\%$, which is greater than the project WACC: $WACC = 8.07\%$. The excess return rate is then 7.39% (15.46% − 8.07%), which, multiplied by the capital base $C = 3913.5$, supplies the project NPV (289.4).

Note that part of the cake, so to say, is grasped by debtholders: the cake is 289.4 and only 265.6 is gained by equityholders. In many real-life applications the assumption $r^d_t = k^d_t$ for every t is appropriate, so the project economic value created coincides with the equity value created. In particular, if one assumes $r^d_1 = k^d_1 = 8\%$, $r^d_2 = k^d_2 = 7\%$, $r^d_3 = k^d_3 = 5\%$, $r^d_4 = k^d_4 = 3\%$, , $r^d_5 = k^d_5 = 4\%$, then the 289.42 created value is entirely grasped by equityholders (i.e., debt is value-neutral); the major part of it is generated in the borrowing periods ($NPV^e_I = 223.4$) and the remaining part is created in the investment periods ($NPV^e_B = 66.1$).

Overall, owing to the negative WCs at time 2 and 3, whose absolute values exceed the the book values of NFAs, there are three investment periods and two financing periods: $T_B = \{3, 4\}$ and $T_I = \{1, 2, 5\}$. In the first period, an investment one, the ROA is

Table 4: Cash flows, rates and economic values

Time	0	1	2	3	4	5
Cash flow to equity (e_t)	−1 800	1 204	1 944	− 325	− 806	296
Cash flow to debt (d_t)	−1 200	496	356	325	106	104
Free cash flow (f_t)	−3 000	1 700	2 300	0	− 700	400
ROE (r_t^e)		22.44%	−15.60%	−25.00%	21.20%	48.00%
ROD (r_t^d)		**8.00%**	**7.00%**	**5.00%**	**3.00%**	**4.00%**
ROA (ROA_t)		16.67%	−5.56%	−50.00%	33.33%	33.33%
Cost of assets (k_t^u)†		**8%**	**9%**	**10%**	**10%**	**11%**
Cost of debt (k_t^d)		**6%**	**5%**	**7%**	**6%**	**4%**
Project value (V_t)	3 289	1 853	− 281	− 309	360	0
Value of debt (V_t^d)	1 224	801	485	194	100	0
Value of equity (V_t^e)	2 066	1 051	− 766	− 503	260	0
Cost of equity (k_t^e)		9.19%	12.05%	8.10%	8.46%	13.69%

† $k_t^u = WACC_t$.

Table 5: Decomposition of capital

	Equity	Debt	Total
Investment	$E_I = 2\,588.6$	$D_I = 1\,927$	$C_I = 4\,515.6$
Borrowing	$E_B = -1\,180.4$	$D_B = 578.3$	$C_B = -602.1$
Total	$E = 1\,408.2$	$D = 2\,505.3$	$C = 3\,913.5$

Table 6: Decomposition of return rates

	Return On Equity	Return On Debt	Mean
Investment	$\bar{r}_I^e = 11.64\%$	$\bar{r}_I^d = 7.47\%$	$ROA_I = 9.86\%$
Borrowing	$\bar{r}_B^e = -11.36\%$	$\bar{r}_B^d = 4.45\%$	$ROA_B = -26.54\%$
Mean	$\bar{r}^e = 30.92\%$	$\bar{r}^d = 6.77\%$	$ROA = 15.46\%$

Table 7: Decomposition of cost of capital

	Cost of Equity	Cost of Debt	Mean
Investment	$\overline{k}^e_I = 10.38\%$	$\overline{k}^d_I = 5.55\%$	$WACC_I = 8.27\%$
Borrowing	$\overline{k}^e_B = 8.20\%$	$\overline{k}^d_B = 6.73\%$	$WACC_B = 9.62\%$
Mean	$\overline{k}^e = 12.06\%$	$\overline{k}^d = 5.82\%$	$WACC = 8.07\%$

Table 8: Decomposition of economic value created (see also Table 2)

	Equity	Debt	Total
Investment	$NPV^e_I = 34.7$	$NPV^d_I = 37$	$NPV_I = 71.7$
Borrowing	$NPV^e_B = 230.9$	$NPV^d_B = -13.1$	$NPV_B = 217.7$
Total	$NPV^e = 265.6$	$NPV^d = 23.9$	$NPV = 289.4$

positive ($ROA_1 = 16.67\%$) and greater than the cost of asset ($k^u_1 = WACC_1 = 8\%$), so economic performance is positive and value is created (in particular, value is created for both equityholders and debtholders, since ROE is greater than the cost of equity and ROD is greater than the cost of debt). The second period, an investment period as well, income is negative, and this is signalled by a negative ROE: value is destroyed for equityholders ($r^e_2 < k^e_2$), whereas it is created for debtholders ($r^d_2 > k^d_2$). At time 2, the SPV uses the net assets ($C_2 = -600$) as a source of funds for stakeholders; and, precisely, given that net financial obligations are positive ($D_2 = 500$), equityholders owe money to the enterprise by an amount of 1,100. This means that, in the second period, ROE is a borrowing rate. Given that net income is positive at time 3, the ROE is negative ($r^e_3 = -25\%$), which means that equityholders borrow money at time 2 from customers and are able to make money out of it in the third period. The equity value created is positive, for, if equityholders borrowed 1,100 in the market, they would have to pay a positive $k^e_3 = 8.1\%$ interest rate. On the other hand, debtholders receive a $ROD_3 = 5\%$ on the outstanding debt ($D_2 = 500$), while they might earn a $k^d_3 = 7\%$ on the same amount: the value created for debtholders is negative; overall, in the third period, stakeholders are in a net borrowing position by an amount of 600; the project performance is positive, for 600 are financed at a negative rate of cost equal to $ROA_3 = -50\%$ (i.e., for every euro borrowed, stakeholders earn, overall, 0.5 euros). At time 3, net assets (and liabilities) are still negative, but project performance is not satisfying: overall, the stakeholders pay 33.33% on that borrowed amount, while the market would charge them only $WACC_4 = 10\%$. In the last period, the net assets are positive as well, and the investment of the net assets creates value for equityholders, ($r^e_5 > k^e_5$), whereas this period is a value-neutral periods as regards debt ($r^d_5 = k^d_5$).

Consider now the same project, but assume that, other things unvaried, the working capital at time 1 and 2, is positive and equal to $WC_1 = 300, WC_2 = 100$.[16] With respect to the base case analyzed above, equity becomes positive: $E_2 = 400, E_3 = 300$ so that $T_B = \emptyset$. There are no borrowing periods, so the project is a *pure* investment. Net income changes with respect to the base case, so that ROEs and cash flows change as well.[17] This implies that the economic values V^e_t are different, which in turn implies that costs of equity are different. It can be checked that the the resulting project ROE is $\overline{r}^e = 14.02\%$ and the related project cost of equity is 10.95%. The excess return is then $14.02\% - 10.95\% = 3.07\%$ which, multiplied by the overall equity invested $E = E_I = 3,048.9$, supplies the equity NPV: $NPV^e = 93.6$. As we have assumed no other changes in the estimated data, the debt NPV is not changed ($NPV^d = 23.9$), so the project NPV is $NPV = 117.42$, with a ROA equal to 10.75% and a project WACC equal to 8.63%.

7 Concluding remarks

Economic assessment of industrial projects are often accompanied by a thorough work of estimation for several economic, accounting, and financial variables underlying the project. Estimated data are then gathered, in an economically meaningful way, in sophisticated models consisting of a series of *financial statements* (i.e., balance sheets, income statements, cash flow statements). The financial experts then condense the forecasts into a single metric expressing economic value created by the project: Net Present Value (NPV), if an absolute amount is required, or a rate of return, if a relative measure of worth is needed. This paper shows that the considerable amount of information gathered by pro forma financial statements can be used for accomplishing an economic analysis and for reconciling accounting variables and financial metrics, which are often considered conflicting. In particular, we use a recent approach, named AIRR approach (Magni 2010, 2013) to show that such accounting metrics as the Return On

[16] We also assume no change in the project risk, which implies that $WACC_t$ ($= k^u_t$) remains the same.

[17] In particular, the equity cash flow vector becomes $\vec{e} = (-1800, 1204, 444, 375, -6,\ 296)$.

Equity (ROE), Return on Debt (ROD) and Return on Assets (ROA) bear significant relations to the project NPV. They supply additional information as to how value is created and how equity and debt interact in generating value. We also show how to gather varying WACCs to obtain a single *project WACC*, which combined with the *project ROA*, determines value creation or destruction.

Our framework includes, beside standard projects, more complex environments, such as nonconventional (mixed) projects: in these projects investment periods alternate with financing periods, so that the roles of the rates and of the costs of capital alternate: in investment periods, rates are investment (i.e., lending) rates, whereas in financing periods, rates are financing (i.e., borrowing) rates.

Making full use of the information collected in the pro forma financial statements, this analysis enables the evaluator to appreciate the role of the various value drivers for generating economic value. We achieve a detailed decomposition of rates, costs of capital, capital amounts and NPV which enables the evaluator to accomplish a richer economic analysis which is impossible to accomplish with the only information provided by the NPV or the IRR.

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Appendix. *Project ROA and Modified Internal Rate of Return*

A Modified Internal Rate of Return (MIRR) is a variant of the IRR which consists of modifying the stream of the free cash flows in such a way that the IRR of the modified cash-flow stream exists and is unique. The project ROA we have introduced in this paper differentiates itself from a MIRR in several ways.

In first place, while the project ROA is unambiguously computed from the pro forma financial statements, it is not clear how the original cash-flow stream should be modified for computing a MIRR. Ross et al.(2011) classify the procedure into three classes: the discounting approach, which consists in discounting back the negative cash flows; the reinvestment approach, which consists in compounding all cash flows except the first out to the end of the project's life; the combination approach, where negative cash flows are discounted back and positive cash flows are compounded to the end of the project. Formally, let $T^+ = \{t \in T : f_t > 0\}$ and $T^- = \{t \in T : f_t < 0\}$. In the discounting approach, the MIRR is the rate y such that

$$\sum_{t \in T^-} \frac{f_t^-}{(1+\varrho)^t} + \sum_{t \in T^+} \frac{f_t^+}{(1+y)^t} = 0 \tag{55}$$

In the reinvestment approach, the MIRR is the rate such that

$$f_0 + \frac{\sum_{t \in T} f_t \cdot (1+\varrho)^{n-t}}{(1+y)^n} = 0 \tag{56}$$

In the combination approach, the MIRR is the rate such that

$$\sum_{t \in T^-} \frac{f_t^-}{(1+\varrho)^t} + \frac{\sum_{t \in T^+} f_t^+ \cdot (1+\varrho)^{n-t}}{(1+y)^n} = 0 \tag{57}$$

However, (55)-(57) are three out of many other ways of adjusting the original cash-flow stream into a modified one which supplies a unique IRR (for example, the so-called Sinking Fund Methods represent other ways of obtaining MIRRs. See Herbst 2002, ch. 11). Furthermore, a two-rate MIRR is sometimes considered, where negative cash flows are discounted at a discount rate j which is different from the reinvestment rate, k, and each of them may differ from the cost of capital. In this case, the MIRR is the rate y such that

$$\sum_{t \in T^-} \frac{f_t^-}{(1+j)^t} + \frac{\sum_{t \in T^+} f_t^+ \cdot (1+k)^{n-t}}{(1+y)^n} = 0 \tag{58}$$

(see Hartman 2007, p. 397).[18]

As a result, while the project ROA presented in this paper is a well-defined unambiguous metric, the MIRR is a methodology comprising a vast class of metrics: there are many different ways of adjusting the cash-flow stream, so there are many different MIRRs, and "there is no clear reason to say one of [the] methods is better than any other" (Ross et al. 2011, p. 250). So, MIRR is not really unique: there are as many MIRRs as are the ways of modifying the cash-flow stream.[19]

A second feature which differentiates MIRR from project ROA is that, in contrast with the latter, the MIRR cannot be considered the *project*'s rate of return, for "it's a rate of return on a modified set of cash flows, not the project's actual cash flows" (Ross, Westerfield and Jordan 2011, p. 250). Indeed, to take reinvestment of interim cash flows into consideration means to include other future investments that should not affect the decision process. As Brealey, Myers and Allen (2011, p. 141) put it: "The prospective return on another independent investment should never be allowed to influence the investment decision".

The latter consideration leads to the third difference: project ROA is consistent with the NPV whereas MIRR is not. To understand this statement, first note that the two-rate MIRR may signal value creation when the NPV is negative and viceversa. Therefore, coherence with the NPV rule is not guaranteed. As a simple counterexample, consider the cash-flow stream $(-100, 390, -503, 214.5)$ which has multiple IRRs equal to 10%, 30%, 50%. Suppose $j = \varrho = 15\%$ and $k = 8\%$. Value is destroyed by the project, for $NPV = -1.73$, whereas MIRR signals value creation (applying (58), the solution is $y = 39.36\%$, which is greater than the cost of capital, $\varrho = 15\%$). As for the other versions of the MIRR, while it is true that, formally, $y > \varrho$ if and only if $NPV > 0$, the MIRR approach makes explicit use of reinvestment (barring the discounting approach), so it summarises the performance of a course of action which includes the project *and* the reinvestments of the interim cash flows. Conversely, NPV does not assume reinvestment of interim cash flows: NPV is the difference between the project value and the project cost, and the project value does not depend on reinvestment of cash flows (and, in particular, on the riskiness of such reinvestments) but on the risk-adjusted cost of capital, that is, the expected rate of return of an equal-risk asset traded in the market. This implies that the MIRR is based on assumptions which are different from

[18]Note that (57) is a particular case of (58) where $k = j = \varrho$. Note also that, except (55), the other versions imply that MIRR is a geometric mean, as opposed to the project ROA, which is an arithmetic mean.

[19]Evidently, this non-uniqueness is different from the non-uniqueness of IRR: "multiple IRRs" means "multiple solutions of a polynomial equation", whereas "multiple MIRRs" means "multiple ways of modifying the project's cash flow-stream".

the NPV and provides a piece of information which is not equivalent to that provided by the NPV. The project ROA does not depend on reinvestments of cash flows, so providing a piece of information which is consistent with that provided by the NPV.

A fourth difference lies in the fact that the use of the MIRR approach rules out the possibility of decomposing economic value created into debt component and equity component. The reason is that the MIRR has no direct relations with the economic referents of the project. More precisely, the invested capital c_t used for computing the project ROA consists of recognisable resources (property, plant and equipment, inventories, receivables, etc.) which stem from actual economic transactions and which are estimated through careful deliberations about how benefits are expected to be distributed through time and the uncertainties associated with their realization; in contrast, MIRR have no such empirical referents, being it simply an outcome of solving a polynomial equation.[20]

Finally, the average-based approach introduced enables to manage time-variant costs of capital in an easy way: the project ROA is a weighted average of the project's ROAs and the project WACC is the weighted average of the various period WACCs, so the comparison between project ROA and project WACC signals value creation. Conversely, it is not clear whether and how an economically significant cutoff rate can be derived such that its comparison with MIRR correctly signals value creation.

[20]And considering that the equation derives from a distortion of the project's cash flows, the relation with the empirical referents is diminished further still.