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# A Location-Allocation model for Fog Computing Infrastructures



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# New challenges

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- New **paradigm**: Smart cities large scale sensing applications
- Several fields of application:
  - Urban applications
  - Industrial
  - Automotive
  - Healthcare
  - ...
- New **scenarios**: Cyber-physical systems
  - Geographically distributed sensors
  - Huge amount of information produced

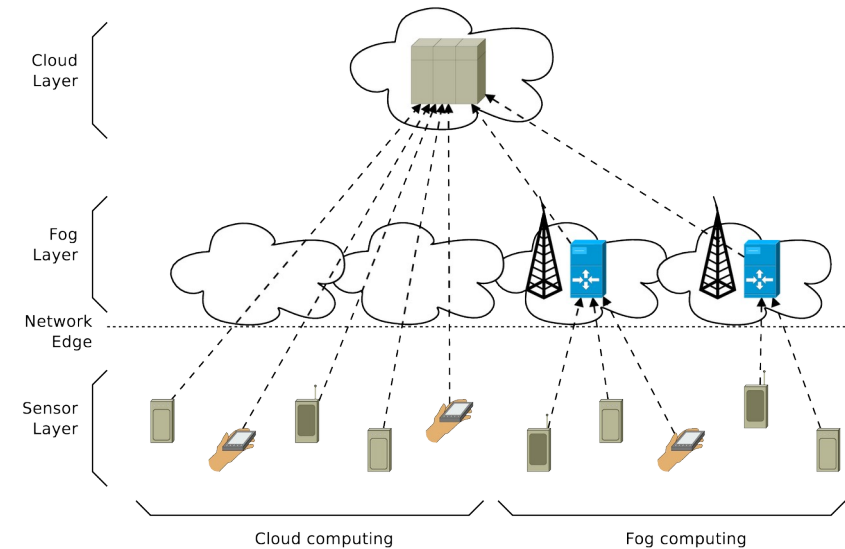
# New challenges

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- New requirements for the infrastructure
- **Scalability** challenge
  - Huge amount of data to transfer and process
  - Geographically distributed systems
  - Example: CPU- and bandwidth-bound applications
- **Low latency** challenge
  - Support for real time applications
  - Example: latency-bound applications
- Cloud computing is not enough
- *(5G alone is not an answer)*

# Pros and Cons of Fog

- Benefits of Fog computing
- Scalability:
  - Pre-processing offloaded to fog nodes
  - Less strain on Cloud network links
- Latency:
  - Latency-critical tasks offloaded to Fog
  - Fog nodes are closer to the edge



- New open issues:
  - new Fog infrastructure
  - Fog node deployment
  - Sensors-to fog mapping
- Joint problem

# Our contribution

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- Model for the design of Fog infrastructures
  - Based on **location-allocation optimization** problem
- Model decisions:
  - **How many** fog nodes do we need?
  - **Which Fog** nodes (among a set) turn on?
  - How to **map sensors over fog** nodes?
- **Double optimization goal**
  - Reduce infrastructure cost
  - Optimize performance
- Use of **SLA constraints**

# Notation

## Model parameters

$\mathcal{S}$	Set of sensors
$\mathcal{F}$	Set of fog nodes
$\mathcal{C}$	Set of cloud data centers
$\lambda_i$	Outgoing data rate from sensor $i$
$\lambda_j$	Incoming data rate at fog node $j$
$1/\mu_j$	Processing time at fog node $j$
$\delta_{ij}$	Communication latency between sensor $i$ and fog $j$
$\delta_{jk}$	Communication latency between fog $j$ and cloud $k$
$c_j$	Cost for locating a fog node at position $j$ (or for keeping the fog node turned on)

## Model indices

$i$	Index for a sensor
$j$	Index for a fog node
$k$	Index for a cloud data center

## Decision variables

$E_j$	Location of fog node $j$
$x_{ij}$	Allocation of sensor $i$ to fog $j$
$y_{jk}$	Allocation of fog node $j$ to cloud $k$

# Optimization problem

- Objective function
  - → Cost for fog nodes
  - → Response time
- Contributions to response time:
  - Sensor → Fog avg net delay
  - Fog → Cloud avg net delay
  - Fog processing time
- Caveat: definition of  $\lambda_j$
- Main constraints:
  - Response time < SLA
  - Load on nodes

$$C = \sum_{j \in \mathcal{F}} c_j E_j$$

$$T_R = T_{netSF} + T_{netFC} + T_{proc}$$

$$T_{netsf} = \frac{1}{\sum_{i \in \mathcal{S}} \lambda_i} \sum_{i \in \mathcal{S}} \sum_{j \in \mathcal{F}} \lambda_i x_{i,j} \delta_{i,j}$$

$$T_{netfc} = \frac{1}{\sum_{j \in \mathcal{F}} \lambda_j} \sum_{j \in \mathcal{F}} \sum_{k \in \mathcal{C}} \lambda_j y_{j,k} \delta_{j,k}$$

$$T_{proc} = \frac{1}{\sum_{j \in \mathcal{F}} \lambda_j} \sum_{j \in \mathcal{F}} \frac{\lambda_j}{\mu_j - \lambda_j}$$

$$\lambda_j = \sum_{i \in \mathcal{S}} x_{i,j} \cdot \lambda_i$$

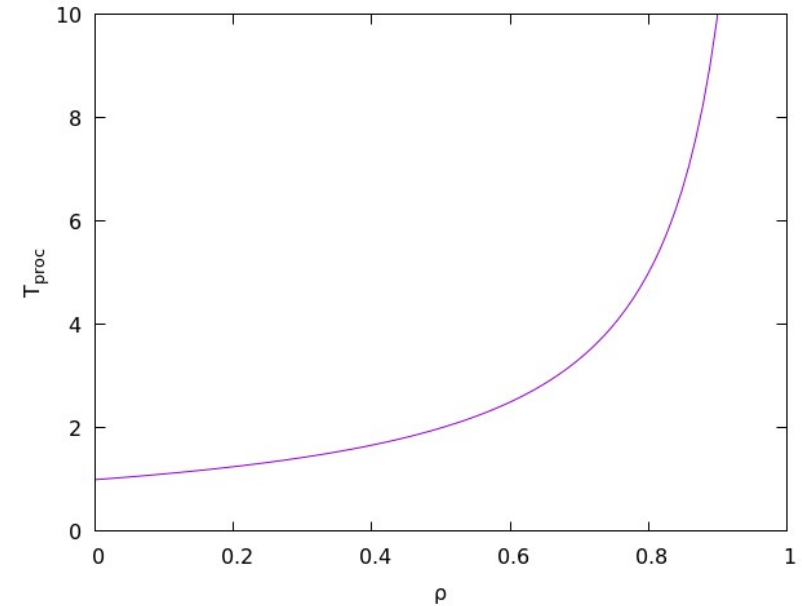
$$T_R \leq T_{SLA}$$

$$\lambda_j < E_j \mu_j, \quad \forall j \in \mathcal{F}$$

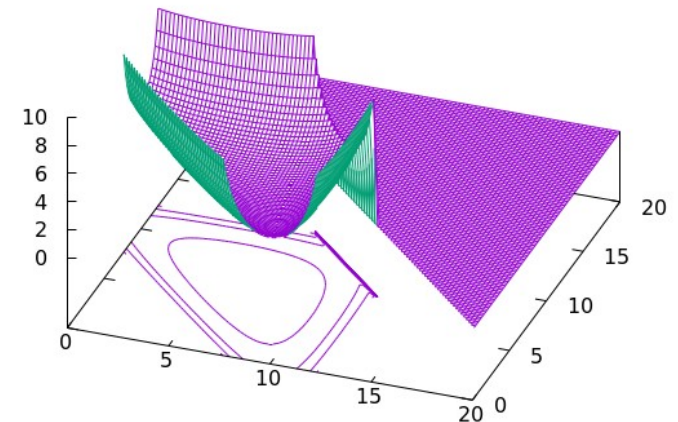
# Processing time

- Based on **queuing theory**
  - M/G/1 models
  - Consistent with PASTA theorem
- Non linear model
- Response time as a function of system load

$$T_{proc} = \frac{1}{\mu - \lambda} = \frac{1}{\mu} \cdot \frac{1}{1 - \rho}$$



Processing time —





# Scenario definition

- **Parameters** to describe scenarios
- Average **network delay**  $\delta$ 
  - Typically set to  $\sim 10\text{ms}$
- **Network delay / Processing time balance**  $\delta \mu$ 
  - Scenario CPU bound or Network bound


- **System load**  $\rho$ 
  - Average load of fog nodes

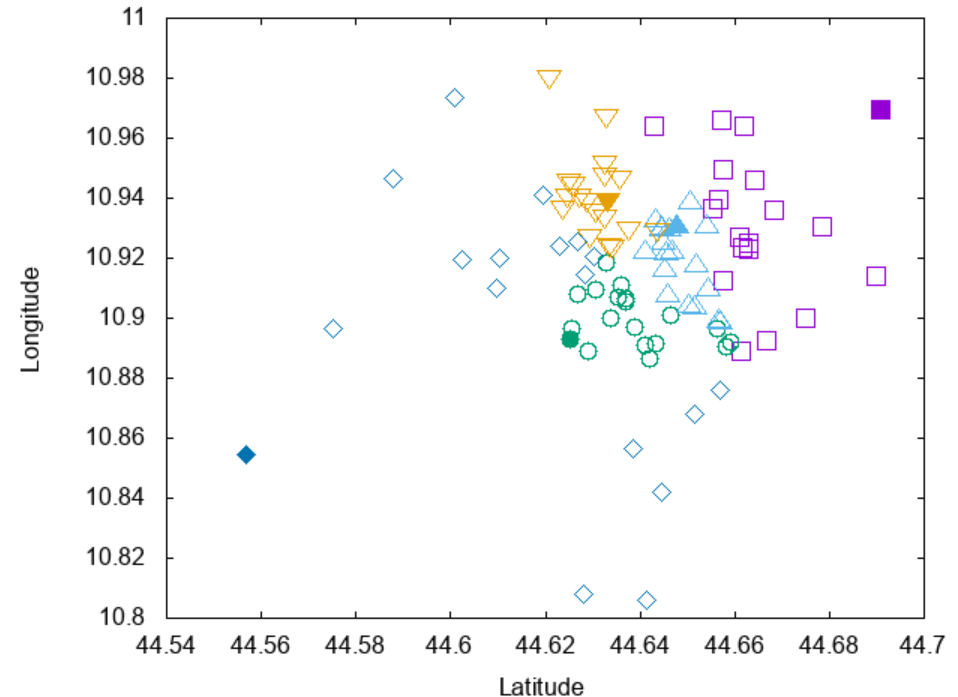
$$\delta = \frac{\sum_{i \in \mathcal{S}} \sum_{j \in \mathcal{F}} \delta_{i,j} + \sum_{j \in \mathcal{F}} \sum_{k \in \mathcal{C}} \delta_{j,k}}{|\mathcal{S}| \cdot |\mathcal{F}| + |\mathcal{F}| \cdot |\mathcal{C}|}$$

$$\delta \mu = \delta \cdot \frac{\sum_{j \in \mathcal{F}} \mu_j}{|\mathcal{F}|}$$

$$\rho = \frac{\sum_{i \in \mathcal{S}} \lambda_i}{\sum_{j \in \mathcal{F}} \mu_j}$$

# Experimental scenario

- Smart City scenario based on **real example**
  - Italian city (Modena),
  - ~180,000 inhabitants
- **Traffic monitoring** case
  - Sensors on streets
  - Fog nodes in public buildings
  - LoRa connections 
- Evaluation using solver
- Comparison with:
  - **Continuous** model (no bool) ←
  - **Simplified** model ( $E_i = 1$ )



 **LocalSolver**

(Ideal lower bound,  
used as baseline comparison)

# Experimental results

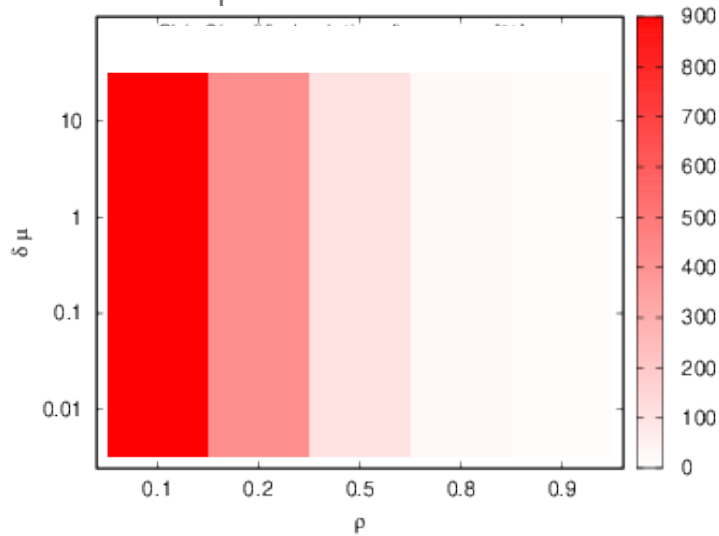
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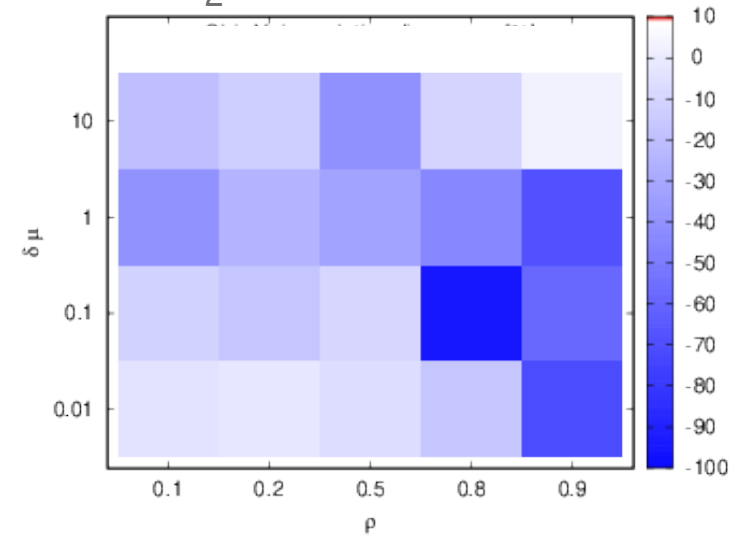


Simplified Model

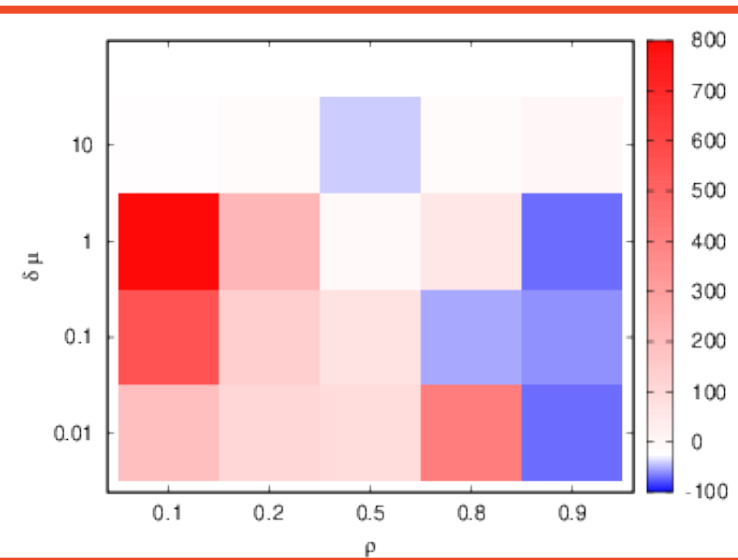
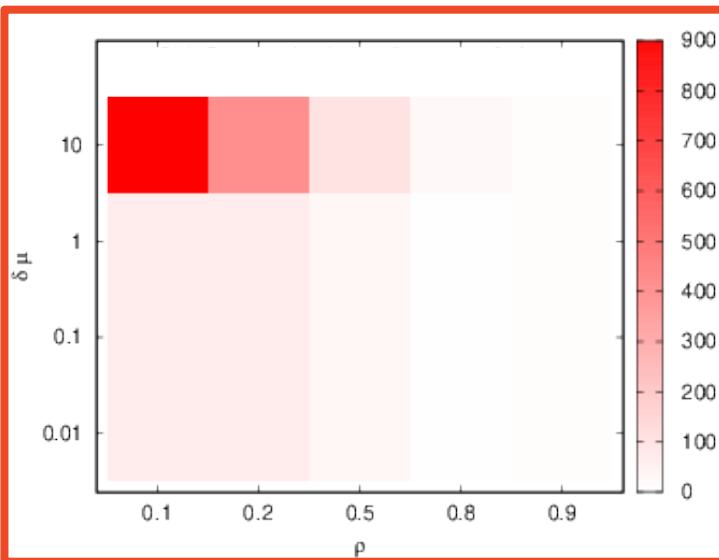
Obj<sub>1</sub> divergence [%]



Obj<sub>2</sub> divergence [%]



Proposed Model



# Conclusions

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- **Challenges** of Fog computing
  - Selection of fog nodes and mapping of sensors
- Contribution: **proposal of a model**
  - Based on **location-allocation** optimization problem
  - Dual objective function
  - **Non linear** problem
- **Validation** of the model
  - Focus on a realistic scenario
  - Wide range of parameters considered
- **Open issues**
  - Heuristics (GA, Variable Neighborhood Search)
  - Dynamic scenarios