24th International Conference on Production Research (ICPR 2017)
ISBN: 978-1-60595-507-0

FMECA-BASED OPTIMIZATION APPROACHES UNDER AN EVIDENTIAL REASONING FRAMEWORK

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Abstract

One of the major shortcomings of traditional failure modes, effects and criticality analysis is the absence of any interconnection between failure ranking and a procedure for selecting the most critical maintenance/improvement tasks to be carried out. This limits the potential of FMECA for implementation in real environments. In order to bridge this gap, three different 0-1 knapsack models have been formulated. The first aims to select the failures in order to maximise cost savings. The second enriches the selection problem by also taking into account the probabilities of solving the failures with a set of maintenance tasks. The third aims to select the maintenance tasks to maximise the expected profit. In particular, the last two models make use of an evidential reasoning framework to deal with the epistemic uncertainty related to these probabilities.

A dataset from a manufacturer of lift winches has been used to validate this proposal, as well as to comment on the need for group decision support systems that are capable of converting the FMECA ranking into maintenance tasks in real environments.

Keywords:

FMECA, maintenance, theory of evidence, optimisation.

1 INTRODUCTION

Failure modes, effects and critically analysis (FMECA) represents a well-established approach to achieving a ranking of the failures of products and processes, and can be applied both at design and at production stages. The core of the standard FMECA process lies in calculating risk priority numbers (RPNs) associated with the failures. These are given by the product of the occurrences (O), severities (S), and detectabilities (D) of the failures, on a 1-10 scale. The failures are thus prioritised on the basis of their RPNs on a scale from 1 to 1000. Despite the practical advantages of traditional RPN calculation, it demonstrates several weaknesses. For a review, the reader can refer to [1]. First, the three aforementioned risk factors are equally weighted in the standard multiplicative form, and different sets of risk factors may produce the same RPNs even if the hidden risks of failure modes are totally different. Moreover, the multiplicative form is questionable because it produces RPNs between 1 and 1000 that are not uniformly distributed, with only 6% of the values lying between 500 and 1000. Furthermore, the values given to the failure modes on the aforementioned risk factors are often affected by the subjectivity of the decision-makers (DMs) when assessing uncertain and vague scores for O, S, and D. Thereby, several contributions are devoted to solving these drawbacks by modelling FMECA as a multi-criteria decision making (MCDM) problem, eventually coupled with approaches that are able to deal with uncertainty. [2] introduced multi-attribute failure mode analysis (MAFMA), which uses the analytic hierarchy process (AHP) to calculate weights for the risk factors. The analytic network process (ANP) has been applied by [3], who decomposed the risk factors into subcriteria. Several MCDM methods have been applied to FMECA, with a trend towards incorporating them with fuzzy logic [4], e.g. fuzzy TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) ([5]; [6]; [7]; [8]); VIKOR (VIsekriterijumska optimizacija i KOmpromisno Resenje) with fuzzy logic [9]; fuzzy AHP ([10]; [11]); fuzzy logic with grey theory [12]; or simply fuzzy logic applied to the risk factors [13]. The adoption of subjective criteria to either rank or sort a set of alternatives results in a group decision problem arising. Early on, [14] proposed a group-based evidential reasoning approach for dealing with the epistemic uncertainty and diversity of the assessment information in FMECA when a group of DMs are asked to score the failure modes. A group-decision FMEA approach was also proposed by [15], where grey relational projection and D numbers are merged to represent the uncertain information used to rank the failure modes. [16] adopted interval type-2 fuzzy sets for dealing with both the variation in one expert's understanding (intra-personal uncertainty) and the variations in understanding between experts (inter-personal uncertainty). [17] proposed a Promethee-based group sorting approach, an extension of FlowSort [18] to group sorting problems, with the aim of clustering the failure modes into ordered classes by taking into account the divergent opinions of DMs. [19] adopted the Dempster-Shafer theory of evidence to achieve an RPN ranking that is able to deal with the epistemic uncertainty of DMs in a more robust fashion, and [1] combined the Dempster-Shafer theory of evidence with fuzzy assessment of the risk factors.

A further drawback of traditional FMECA that drew the attention of researchers is that economic aspects are ignored if only O, S, and D are adopted as risk factors. Two examples of cost models are described in [20] and [21], with the goal of determining an estimate of the failure costs affecting the customer.

Despite the plethora of contributions devoted to overcoming the main drawbacks of standard RPN calculation, it may be argued that a further practical issue should be considered, i.e. the relationship between the prioritisation of failure modes and the maintenance tasks to be carried out in order to ensure continuous improvement. [22] investigated the adequacy of preventive maintenance tasks on failure modes prioritised on the basis of RPNs. [23] proposed a complete maintenance scheme by integrating fuzzy FMECA and fault propagation graphs to calculate a composite risk measure for failures. Finally, a binary decision tree is used to determine the failure ascertainment order. [24] introduced a 0-1 matrix for visualising the maintenance tasks that could potentially solve a set of failure modes, while a clustering algorithm aims to select the most critical ones. In the authors' opinion, the adoption of an FMECA-based prioritisation approach for failure modes, coupled with a robust selection approach for maintenance tasks, deserves to be investigated. Moreover, a further issue related to epistemic uncertainty arises. Suppose that more sources (i.e. datasets) contain successful completions of a maintenance task carried out to solve a failure. This task might be undertaken either alone or along with a set of other tasks. The epistemic uncertainty lies in the probability of solving a failure by means of one specific task, since the effects of multiple simultaneous tasks overlap. In the event that a dataset is unavailable, more DMs (i.e. sources) could provide different interval-valued probabilities for each failure-task couple. In both cases, the epistemic uncertainty about the probabilities of solving the failures by means of maintenance tasks (also in relation to the ignorance of a process-system) should be taken into account, and the basic concepts of the Dempster-Shafer theory of evidence provide valid support in dealing with this issue. In particular, this contribution refers to a group decision support system, where the DMs are the multiple evidence sources. This approach could nevertheless also be extended to more pieces of evidence, deriving from multiple datasets. The combination of the evidence related to these probabilities is used as a fourth risk factor for the failures, and three different 0-1 knapsack models are proposed for dealing with selection problems (failures and/or maintenance tasks). To the best of the authors' knowledge, this is the first application of evidential reasoning to the uncertain probabilities of solving the failure modes through a group decision support system where the epistemic uncertainty of probabilities is elicited by the DMs. It should be noted that despite its aforementioned drawbacks, RPN calculation is not the focus of this contribution, and thus the failures are associated with generic RPNs without specifying how to calculate them.

The remainder of the paper is organised as follows. Section 2 contains an overview of the basic concepts of the Dempster-Shafer theory of evidence applied to this specific case. Section 3 introduces three 0-1 knapsack models, while Section 4 reports a case study relating to a manufacturer of lift winches. Section 5 contains the conclusion and some suggestions for the further research agenda.

2 BASIC CONCEPTS

Dempster [25] and later Shafer [26] introduced the theory of evidence, generally named the Dempster-Shafer theory of evidence (DSTE). The novelty of this theory lies in the ability to deal with the epistemic uncertainty inherent in the system/process due to the lack of knowledge. Consider a stochastic variable X with two states x and $\bar{x}$. Under a probabilistic framework $P(x)+P(\bar{x})=1$, in DSTE a probability is assigned not only to each state, but also to each proper subset of the domain (i.e. the power set). It follows that $m(x)+m(\bar{x})+m(\{x,\bar{x}\})=1$, where $m(\{x,\bar{x}\})$ represents the partial ignorance of X. The $m-values$, also known as the basic probability assignment function, will be defined in the following. In the specific case under consideration in the present work, the binary variable subjected to epistemic uncertainty is the resolution of a failure mode $i=1,\dots I$ by means of a maintenance task $j=1,\dots J$. Without losses of generality, $X_{i,j}=1$ and $X_{i,j}=0$ if the failure is solved and unsolved, respectively. The *Frame of Discernment* Ω therefore contains two exhaustive and mutually exclusive states, which provide the power set $2^{\Omega}=\{\emptyset,0,1,(0,1)\}$ composed of the focal elements of Ω. The likelihood of resolution $p_{i,j}$ of a failure is not completely known by the experts (or sources of evidence), and thus it could be elicited through DSTE-based evidential reasoning on the continuum between 0 and 1. Each DM $k=1,\dots,K$ is asked to provide the $m-values$: $m_{i,j,k}(1)$ as the evidence supporting the resolution of failure $i=1,\dots I$ by maintenance task $j=1,\dots J$; $m_{i,j,k}(0)$ as the evidence supporting the non-resolution; and $m_{i,j,k}(\{0,1\})$ as the partial ignorance on the resolution. It may be argued that $m_{i,j,k}(1)$ and $[m_{i,j,k}(1)+m_{i,j,k}(\{0,1\})]$ give the lower and the upper bounds of the resolution probability respectively, which are expressed as the interval-valued $p_{i,j,k}=[\underline{p}_{i,j,k},\overline{p}_{i,j,k}]$ with $\underline{p}_{i,j,k}=m_{i,j,k}(1)$ and $\overline{p}_{i,j,k}=[m_{i,j,k}(1)+m_{i,j,k}(\{0,1\})]$. In crisp form, $p_{i,j,k}=\left[\overline{p}_{i,j,k},\overline{p}_{i,j,k}\right]=\left[\underline{p}_{i,j,k},\underline{p}_{i,j,k}\right]$. These likelihoods may be intervals overlapping one another, nested or disjoint in case of conflicting evidence. Actually, $(\overline{p}_{i,j,k}-\underline{p}_{i,j,k})$ represents the partial ignorance of DM k on $X_{i,j}$, i.e. $m_{i,j,k}(\{0,1\})$.

Three basic concepts of DSTE are given in a more general fashion below.

Definition 2.1: *Basic Probability Assignment* (BPA).

The BPA, named $m(E)$, is the amount of knowledge associated with every subset E in the power set, providing the degree of the evidence supporting E, and is defined as follows:

$$m(E)\colon 2^{\Omega}\rightarrow[0,1] \quad (2)$$

$$m(\emptyset)=0 \quad (3)$$

$$\textstyle\sum_{E\in 2^{\Omega}} m(E)=1 \quad (4)$$

BPAs are analogous to probability mass functions in probability theory, but the focal elements of DSTE may be overlapped intervals.

Given a failure i, a maintenance task j, and a set A included in Ω, two basic concepts of DSTE are introduced below.

Definition 2.2: Belief

The belief of I is obtained from $m(E)$ as:

$$Bel(A)=\textstyle\sum_{E\subseteq A} m(E) \quad (5)$$

The belief function quantifies the sum of the probability masses of all the focal elements into I, and thus the amount of belief supporting the fact that E lies in A. For the variable $X_{i,j}$ defined before, a belief function is defined for each DM k, such that $Bel_{i,j,k}(1)=\underline{p}_{i,j,k}=m_{i,j,k}(1)$.

Definition 2.3: Plausibility

The plausibility of A is obtained, again from $m(E)$, as:

$$Pl(A)=\textstyle\sum_{E\cap A\neq\emptyset} m(E) \quad (6)$$

The plausibility function is given by the sum of the probability masses assigned to all the focal elements whose intersections with A are not empty. This indicates the possibility that E lies in I. For the variable $X_{i,j}$ defined before, a plausibility function is defined for each DM k, such that $Pl_{i,j,k}(1)=\overline{p}_{i,j,k}=[m_{i,j,k}(1)+m_{i,j,k}(\{0,1\})$. That is to say, each DM provides the interval-valued $p_{i,j,k}=\left[\underline{p}_{i,j,k},\overline{p}_{i,j,k}\right]=\left[Bel_{i,j,k}(1),Pl_{i,j,k}(1)\right]$ for all the failures and maintenance tasks. The underlying assumption is that $p_{i,j,k}$ is uniformly distributed between $\underline{p}_{i,j,k}$ and $\overline{p}_{i,j,k}$.

In of the event that there are more independent sources of fully reliable evidence, a combination rule has to be applied to obtain the resulting belief and plausibility functions. The kernel of DSTE is Dempster's rule of combination of evidence deriving from the sources. This is a non-compensatory combination approach that, however, is not applicable in the event of conflicting evidence. A multitude of combination rules have therefore been proposed, especially for addressing the limitations of the standard Dempster's rule of combination. The reader can refer to [27]. The rule of combination of evidence adopted here is based on the expected value of $p_{i,j}$, i.e. the resulting probability of solving i by j after having established its probability density function (see Section 3). The subsequent issue addressed by DSTE is the propagation of epistemic uncertainty to the system/process, which is defined through a dependent variable whose epistemic uncertainty derives from the epistemic uncertainty propagation of its independent variables. In this paper, the propagation of uncertainty to the selection problems has not been considered, but this topic could be investigated as a part of the further research agenda.

3 FRAMEWORK OF EVIDENTIAL REASONING

The set of the I failure modes has been already scored via RPN calculation, but this is not the focus of this work. Each failure mode $i = 1, \dots I$ may be solved by means of multiple maintenance tasks $j = 1, \dots J$, while a maintenance task might solve multiple failure modes. In order to avoid any consideration of conditional probabilities related to the maintenance tasks, which would make analytical modelling increasingly complex, only one task must be selected per failure. Firstly, a tri-dimensional matrix $(I \times J \times K)$ is compiled, whose elements are $p_{i,j,k} = \left[\underline{p}_{i,j,k}, \overline{p}_{i,j,k}\right]$ as defined in Section 2. Each cell therefore contains the range of the probability that j solves i for DM k. It should be noted that some tasks could be not combined with any failure for some DMs, and vice versa. Given a couple (i, j), this is equal to imposing for all these DMs that $p_{i,j,k} = [0,0]$. For a single DM k, the resulting $(I \times J)$ matrix is as follows:

$$\begin{pmatrix} p_{1,1,k} & \cdots & p_{1,J,k} \\ \cdots & \cdots & \cdots \\ p_{I,1,k} & \cdots & p_{I,J,k} \end{pmatrix} \quad (7)$$

For the aforementioned reason, only one non-null $p_{i,j,k}$ must be selected for each line, and it must be the same for all the DMs. This should refer to the most effective j for solving i, where the effectiveness must be globally evaluated, because each DM provides a different piece of evidence on $p_{i,j}$. Given a failure i, the most effective j may be obtained simply by selecting the task j_i between 1 and J that maximises the expected value of $p_{i,j}$ as follows:

$$j_i = argmax_{j=1,\dots,J} \frac{\sum_{k=1}^{K}\left(\underline{p}_{i,j,k} + \overline{p}_{i,j,k}\right)/2}{K}, \ \forall i = 1, \dots, I \quad (8)$$

Equation (8) is obtained by considering equally reliable and credible DMs, which justifies the denominator K, and uniformly distributed $p_{i,j}$ between the lower and the upper bounds associated with the DMs, which justifies the denominator equal to 2. It follows that, $\forall i = 1, \dots, I$, only one j_i is selected.

The first step of the proposed approach consists of defining the step-wise probability density function of p_{i,j_i} as explained in the sequel. Given a couple (i, j_i), the vector of all the $\underline{p}_{i,j_i,k}$ and $\overline{p}_{i,j_i,k}$ is ordered, which therefore contains $2K$ elements. For clarity of notation, this ordered vector is indicated as $\boldsymbol{p}_{i,j_i} = \left(p_{i,j_i}^{(1)}, p_{i,j_i}^{(2)}, \dots, p_{i,j_i}^{(2K)}\right)$, with $p_{i,j_i}^{(1)} \le p_{i,j_i}^{(2)} \le \dots \le p_{i,j_i}^{(2K)}$. Thereby, $(2K - 1)$ intervals between 0 and 1 are defined at most, because some of them might be null in case of two coincident elements. This eventuality might occur in two cases:

i) $\overline{p}_{i,j,k1} = \underline{p}_{i,j,k2}$ for some couple $k1 \neq k2$ of DMs;

ii) $\underline{p}_{i,j,k} = \overline{p}_{i,j,k}$ for some k.

The uniform probability density function of p_{i,j_i} between two consecutive and distinct elements of $\boldsymbol{p}_{i,j_i}$, named $p_{i,j_i}^{(l)}$ and $p_{i,j_i}^{(l+1)}$, is as follows:

$$f_{(l),(l+1)} = \frac{1}{K}\sum_{k \in K^*} \frac{1}{\left(\overline{p}_{i,j_i,k} - \underline{p}_{i,j_i,k}\right)} \quad (9)$$

where K^* is the set of all the DMs that provide interval-valued $\left[\underline{p}_{i,j_i,k}, \overline{p}_{i,j_i,k}\right]$ including $\left[p_{i,j_i}^{(l)}, p_{i,j_i}^{(l+1)}\right]$, i.e.

$$\left[p_{i,j_i}^{(l)}, p_{i,j_i}^{(l+1)}\right] \subseteq \left[\underline{p}_{i,j_i,k}, \overline{p}_{i,j_i,k}\right] \quad (10)$$

Through Equation (9), the cumulative probability is thus given by:

$$Pr\left\{p_{i,j_i}: p_{i,j_i} \ge p_{i,j_i}^{(l)} \, and p_{i,j_i} \le p_{i,j_i}^{(l+1)}\right\} = = f_{(l),(l+1)}\left(p_{i,j_i}^{(l+1)} - p_{i,j_i}^{(l)}\right) \quad (11)$$

It follows that the expected value of p_{i,j_i} is:

$$E(p_{i,j_i}) = \frac{1}{2}\sum_{i=1}^{2K-1} f_{(l),(l+1)}\left(p_{i,j_i}^{(l+1)} - p_{i,j_i}^{(l)}\right)\left\{\left[p_{i,j_i}^{(l+1)}\right]^2 - \left[p_{i,j_i}^{(l)}\right]^2\right\} \quad (12)$$

Actually, any null-interval makes Equation (9) impossible. However, any null-interval due to case i) does not contribute to $E(p_{i,j_i})$ because $p_{i,j_i}^{(l)} = p_{i,j_i}^{(l+1)}$, and thus it may be neglected in Equation (12). Conversely, Equation (9) has to be rewritten in case ii) as $f_{(l),(l+1)} = \frac{1}{K}$, and this contributes to Equation (12) separately from the sum.

In this way, all the failures are associated with a risk priority number RPN_i, and a probability $E(p_{i,j_i})$ of being solved by means of the most effective maintenance task j_i. In the following sections, three 0-1 knapsack problems, named P1, P2 and P3, are formulated.

3.1 P1: Savings maximisation

Each failure i is associated with a saving s_i occurring in the event of resolution. Given an upper bound R_{max} of cumulative RPNs to undertake, the problem of failure selection might be formulated as follows:

$$f = max\sum_{i=1}^{I} s_i x_i \quad (13)$$

s.t.

$$\sum_{i=1}^{I} RPN_i x_i \le R_{max} \quad (14)$$

$$x_i \in \{0; 1\}, \forall i = 1, \dots, I \quad (15)$$

where the decision variables x_i are:

$$x_i \quad \begin{cases} 1 \, if \, the \, failure \, i \, is \, selected \\ 0 \qquad otherwise \end{cases} \quad (16)$$

3.2 P2: Savings maximisation with uncertainty

Since the resolution of a failure i is affected by p_{i,j_i} (see Section 3), the problem P1 might be reformulated as follows:

$$f = max\sum_{i=1}^{I} E(p_{i,j_i}) s_i x_i \quad (17)$$

s.t.

$\sum_{i=1}^{I} E(p_{i,j_i}) RPN_i x_i \leq R_{max}$ (18)

$x_i \in \{0; 1\}, \forall i = 1, \dots, I$ (19)

The values of decision variables x_i assume the same meaning as reported in Equation (16). In this case, the upper bound R_{max} has a probabilistic meaning due to $E(p_{i,j_i})$.

3.3 P3: Profit maximisation with uncertainty

The selection is now focused on the maintenance tasks with the objective of maximising the profit, calculated as the balance between maintenance costs and expected savings. The resolution uncertainty is taken into account as in problem P2. Moreover, a lower bound represented by R_{min}, i.e. the minimum uncertain risk that should be solved overall, is added as a further constraint.

Given a maintenance cost of c_j for the task j, the model is formulated as follows:

$f = max[\sum_{i=1}^{I} E(p_{i,j_i}) s_i x_i - \sum_{j=1}^{J} c_j z_j]$ (20)

s.t.

$\sum_i E(p_{i,j_i}) RPN_i\, x_i \geq R_{min}$ (21)

$\sum_{i=1}^{I} x_i m_{i,j} \leq z_j M_j,\ \forall j = 1, \dots, J$ (22)

$z_j \in \{0; 1\},\ \forall j = 1, \dots, J$ (23)

$x_i \in \{0; 1\},\ \forall i = 1, \dots, I$ (24)

The decision variables z_j and x_i assume these meanings respectively:

$z_j \begin{cases} 1\ if\ the\ maintenance\ task\ j\ is\ selected \\ 0 \qquad otherwise \end{cases}$ (25)

$x_i \begin{cases} 1\ if\ the\ failure\ i\ is\ selected \\ 0 \qquad otherwise \end{cases}$ (26)

A maintenance task could solve more failures, but not vice versa (see Equation (8)), and such a selection is driven by the objective function. Equation (21) aims to select the failures in order to reach at least the lower bound R_{min}. Equation (22) expresses the relationship between z_j and x_i. In particular:

$m_{i,j} \begin{cases} 1 \quad if\ j = j_i \\ 0\ otherwise \end{cases}$ (27)

Note that, from Equation (8), only one j is selected for each i and this is named j_i, i.e. $\sum_{j=1}^{J} m_{i,j} = 1$. Given a task j, if all the failures solvable by j are selected, i.e. all the failures such that $m_{i,j} = 1$, then $\sum_{i=1}^{I} x_i m_{i,j}$ reaches the maximum value $\sum_{i=1}^{I} m_{i,j}$. Thereby, z_j must be 1, i.e. j is selected, and the big M_j may be fixed to $\sum_{i=1}^{I} m_{i,j}$. Nevertheless, even if only one failure is selected with $m_{i,j} = 1$, z_j is again forced to be equal to one. In other words, just one failure is enough to activate the maintenance task able to solve it.

4 NUMERICAL EXAMPLE

A dataset coming from a manufacturer of lift winches has been used to validate the proposed approach. Two years (2015-2016) of failures of a specific winch have been analysed, and a traditional FMECA ranking has been obtained. In particular, one hundred and sixty-two failures have been extrapolated and their corresponding RPNs have been calculated by adopting the traditional 1-10 scoring method for the three risk factors. The first forty-nine failures have been selected ($I = 49$) for problems P2 and P3, and thirteen ($J = 13$) maintenance tasks have been collected by three DMs ($K = 3$). They were asked to provide the interval-valued $p_{i,j,k} = [\underline{p}_{i,j,k}, \overline{p}_{i,j,k}] = [Bel_{i,j,k}(1), Pl_{i,j,k}(1)]$ for each failure-task couple. Three matrixes (see Equation (7)) have therefore been compiled, one per DM. Table 1 shows a sample of these values per DM, i.e. DM1, DM2 and DM3, where the rows and the columns refer to the failures (F) and the maintenance tasks (M), respectively.

Table 1. Interval-valued probabilities provided by DMs.

	M_1			…	M_{13}		
	DM1	DM2	DM3	…	DM 1	DM2	DM 3
F_1	[0.5,0.7]	[0.4,0.7]	[0.55,0.8]	…	…	…	…
…	…	…	…	…	…	…	…
F_{49}	…	…	…	…	…	…	…

This table might contain multiple maintenance tasks per failure. In order to select the most effective task per failure, Equation (8) is applied. In this way, Table 1 is simplified by eliding the interval-valued probabilities referring to the less effective maintenance tasks, and maintaining only those referring to the most effective ones. For instance, suppose that Table 1 contains this row (F_1) with more than one M (M_1 and M_{12}):

Table 2. Example of row to simplify.

	M_1			M_{12}		
	DM1	DM2	DM3	DM1	DM2	DM3
F_1	[0.5,0.7]	[0.4,0.7]	[0.55,0.8]	[0.35,0.55]	[0.3,0.6]	[0.4,0.6]

$$j_1 = argmax_{j=1,12} \frac{\sum_{k=1}^{3}(\underline{p}_{1,j,k} + \overline{p}_{1,j,k})/2}{3} = 1$$

That is to say, only M_1 is selected for F_1 and used for the subsequent steps. Table 1 is simplified in this way for each failure, i.e. for all the rows of Table 1.

The interval-valued probabilities provided by the DMs on the couple (F_1, M_1) are plotted in Figure 1.

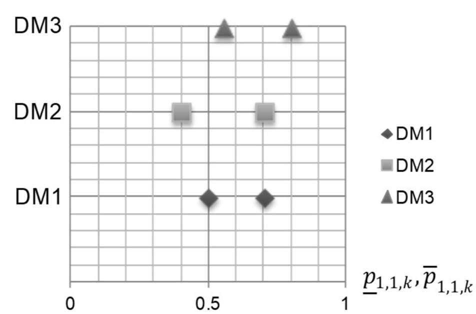

Figure 1. Interval-valued provided by the DMs.

The ordered vector is $\boldsymbol{p_{1,1}} = (0.4, 0.5, 0.55, 0.7, 0.7, 0.8)$, which provides 5 intervals overall. However, the interval $[0.7, 0.7]$ is not considered because it does not contribute to the expected value (see Equation (12)). The uniform probability density function of $p_{1,1}$ is the step-wise function given by (see Equations (9) and (10)):

$\frac{1}{3}\frac{1}{(0.7-0.4)} = 1{,}11\ \ if\ p_{1,1} \in [0.4; 0.5]$

$\frac{1}{3}\frac{1}{(0.7-0{,}4)} + \frac{1}{3}\frac{1}{(0.7-0.5)} = 2{,}78\ \ if\ p_{1,1} \in [0.5; 0.55]$

$\frac{1}{3}\frac{1}{(0.7-0{,}4)}+\frac{1}{3}\frac{1}{(0.7-0.5)}+\frac{1}{3}\frac{1}{(0.8-0.55)}=4.11\ if\ p_{1,1}\in[0.55;0.7]$

$\frac{1}{3}\frac{1}{(0.8-0.55)}=1{,}33\ if\ p_{1,1}\in[0.7;0.8]$

For Equations (11) and (12), the expected value of $p_{1,1}$ is:

$E(p_{1,1})=\frac{1}{2}1.11(0.5^2-0.4^2)+\frac{1}{2}2.78(0.55^2-0.5^2)+$
$+\frac{1}{2}4.11(0.7^2-0.55^2)+\frac{1}{2}1.33(0.8^2-0.7^2)=0.61$

These calculations are repeated for all the forty nine failures.

4.1 Selection problems: P1, P2, and P3

The problem P1 is applied to the whole set of one hundred and sixty-two failures, and requires the definition of the upper bound R_{max}, which is fixed to 3000, 4000, 5000 and 6000, for a sum of RPNs equal to 25894. The results (i.e. the percentage of selected failures, the objective function, and the sum of the RPNs of the selected failures) are reported in Table 3.

Table 3. Results of P1.

R_{max}	Selected failures	Savings [€/Month]	Sum of RPNs
3000	6.8%	13944	2983
4000	9.3%	14564	3963
5000	13.6%	15078	4997
6000	16.7%	15298	5998

Actually, the objective function does not consider the cost of the actions planned for solving the selected failures. Nevertheless, it could be argued that $R_{max}=6000$ does not allow a relevant increment of the savings with respect to $R_{max}=5000$. This is due to the low savings related to the failures selected for $R_{max}=6000$ and not for $R_{max}=5000$.

The problems P2 and P3 deal with the uncertainty related to the probabilities of solving the failures by means of the set of maintenance tasks, whose representation has been explained before. In both problems, the most critical forty-nine failures are selected.

In particular, the results achieved through P1 are reported in Table 4, where the upper bound R_{max} is fixed again to 3000, 4000, 5000 and 6000, for a sum of RPNs equal to 8520.

Table 4. Results of P2.

R_{max}	Selected failures	Savings [€/Month]	Sum of RPNs
3000	32.7%	14760	2935
4000	44.9%	15053	3971
5000	57.1%	15207	4996
6000	69.4%	15284	5993

As is predictable, the adoption of $E(p_{i,j_i})$ as a multiplicative factor of the RPNs leads to the selection of more failures than in P1 under the R_{max} constraint. Nevertheless, similar savings are achieved, which is due to the fact that in P1 the selection involves one hundred and sixty-two failures.

The problem P3 is initially launched after relaxing the constraint (21) on the minimum risk R_{min} to be achieved overall. Table 5 contains the results of P3 expressed in terms of the percentage of failures and maintenance tasks selected, the profit, and the sum of the RPNs of the selected failures.

Table 5. Results of P3 without the R_{min} constraint.

Selected failures	Selected maintenance tasks	Profit [€/Month]	Sum of RPNs
71.4%	53.8%	8561	5864

If the R_{min} constraint is restored with $R_{min}>5864$, a suboptimal profit is achieved. In particular, four values of R_{min} are tested, i.e. 6000, 7000, 8000, and the fourth must be equal to 8520, i.e. the sum of all the RPNs. In fact, the solution corresponding to the selection of all the failures and maintenance tasks would be unfeasible for $R_{min}>8520$. The achieved results are reported in Table 6.

Table 6. Results of P3 with the R_{min} constraint.

R_{min}	Selected failures	Selected maintenance task	Profit [€/Month]	Sum of RPNs
6000	73.4%	61.5%	8524	6136
7000	83.7%	69.2%	8427	7060
8000	93.9%	92.3%	8055	8059
9000	100%	100%	7880	8520

5 CONCLUSIONS AND FURTHER RESEARCH AGENDA

FMECA is a well-established approach for ranking the failures from the most to the less critical in terms of their risk priority numbers, which are derived in the standard FMECA process from their scores on three risk factors. One of the major shortcomings of traditional FMECA is the absence of a procedure for using such a raking to select the maintenance/improvement tasks to be carried out. In particular, the probability that a maintenance/improvement task will solve a failure is typically subject to epistemic uncertainty. In fact, more sources of evidence, e.g. decision-makers, might provide different interval-valued probabilities, and a combination procedure is required in order to obtain their expected values. In this paper, the basic concepts of the Dempster-Shafer theory of evidence are adopted to deal with the epistemic uncertainty of these probabilities, enriching the traditional FMECA approach by also taking into account the relationship between failures and maintenance tasks. Three 0-1 knapsack problems are defined with different objective functions, i.e. savings maximisation for failure selection with and without uncertainty related to the solving probabilities, and profit maximisation with uncertainty for the selection of the maintenance tasks. In synthesis, FMECA-based optimisation approaches that are able to deal with epistemic uncertainty are introduced, allowing more decision-makers to be involved in the decision support system by providing interval-valued probabilities. Finally, a case study is used to validate these proposals.

Despite the novelty of addressing the epistemic uncertainty related to the solving probabilities, some weaknesses of this contribution need underlining. First, the propagation of the uncertainty to the solutions of the optimisation models deserves to be investigated further. Moreover, the correlation between the maintenance tasks is avoided here by selecting only one task per failure, but this issue could be addressed in a more robust fashion. Finally, the procedure adopted for RPN calculation is the standard one, since although it exhibits some weaknesses, the focus of this contribution lies elsewhere. However, epistemic uncertainty also involves the standard risk factors, which might be addressed under an evidential reasoning framework as well.

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