

INVENTORY CONTROL SYSTEM FOR INTERMITTENT ITEMS WITH PERISHABILITY

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Abstract

Perishable items, with a limited lifespan and a known expiration date, are found in a variety of industrial settings. From the food to the pharmaceutical industries, the supply chains specialize their inventory control systems to handle the added complexity. These efforts are enhanced when the items present also an intermittent consumption, characterized by frequent periods without demand mixed to highly variable positive demand events. In this paper, a novel periodic inventory control system aims at bridging the gap between these two product features, managing intermittent items with expiration dates. The proposed system performs a combinatorial analysis evaluating all the demand scenarios before and after an expiration date to measure the expected fill rate. An optimization algorithm then sets the order quantity, using mathematical properties of the system to define efficient search boundaries.

Keywords:

Intermittent demand, perishability, inventory control, periodic review, optimization

1 INTRODUCTION AND RESEARCH BACKGROUND

Intermittent demand plays a relevant role in several industrial settings. Spare parts warehouses are the typical example of industrial context in which the demand occurs sporadically with highly variable sizes. Intermittency represents a critical issue to address both for achieving accurate forecasts and for designing effective inventory control systems.

The pioneering work on the accuracy of intermittent demand estimators has been proposed by Croston [1], who demonstrated that the simple exponential smoothing is biased in case of intermittent demand. He introduced an original estimator founded on the decomposition of the demand generation process into its constitutive elements, i.e. the demand size and the inter-demand interval. Two independent simple exponential smoothing are adopted, as estimators of demand size and inter-demand interval, whose ratio provides the estimator of mean demand per period. From the work of Croston, a wide plethora of authors have addressed the topic of intermittent demand forecasting by modifying the original Croston's method. Reader can refer to Rao [2], Johnston and Boylan [3], Syntetos and Boylan [4], [5]. Other forecasting techniques have been proposed for dealing with intermittent demand. See for instance the ARIMA (Auto Regressive Integrated Moving Average) modelling ([6], [7], [8], [9]) and artificial neural networks (e.g. [10], [11], [12]). For a comparison between different intermittent demand estimators, the reader can refer to [13] and [14].

The other issue related with an effective intermittent demand management refers to the replenishment policies. In case of stochastic demand, the relationship between forecasts and inventory control system is strict, as experimentally verified by [15] for intermittent demand. On the inventory control of intermittent demand, several contributions focused on compound distributions for setting the inventory control parameters, e.g. [16] and [17]. However, as the data become more erratic, the demand generator process may not comply with any standard theoretical distribution [18]. Theoretical distributions have been hypothesized also for the pseudo-random generation in case of unavailability of a large data-set (see for example [19] and [20]). Generally, the order-up-to-level

inventory control is the most commonly used policy for items with intermittent demand. In this work, the inventory system proposed in [16] is adopted, but some relevant modifications are introduced enabling it to deal with perishable items.

Food and pharmaceutical warehouses are typical examples of industrial settings subjected to perishability, perishable items are also often subjected to intermittency as well, due to the sources of intermittency reported in [21]. It is worth remarking that the perishability should not be confused with the obsolescence: the former implies that the items have an expiration date and thus a limited shelf life, the latter leads to a progressive decrease of the quantities in order. Intermittent demand with obsolescence has already been addressed in [22] with the aim of increasing the demand estimators accuracy. Conversely, to the best of the authors' knowledge, the perishability of intermittent demand has never been addressed even if this kind of demand generation process is relevant in several real settings. A recent approach focusing on non-stationary demand subjected to perishability is reported in [23], where the well-known Silver and Meal heuristic for time-varying demand is adapted to the case of perishable items. However, the intermittency is not taken into account specifically.

This paper is organized as follows:

- Chapter 2 recalls some theoretical basis, defines the model and a technique for its practical implementation.
- Chapter 3 is devoted to the model experimental validation in different scenarios. Both an exact and an approximated use is proposed.
- Chapter 4 contains the conclusions obtained from the experimental validation.
- Chapter 5 contains the references.

2 METHODOLOGY

2.1 SBA approximation

As referred in section, 1 the standard intermittent demand model was proposed in [1]. The demand d_i of a period t_i is defined as:

$$d_i = \begin{cases} \phi(d_i, 1) \text{ prob. } p & \\ 0 & \text{prob. } 1 - p \end{cases} \quad (1)$$

where p is the probability a positive demand takes place in a period and $\phi(d_i, 1)$ is the probability a positive demand has the magnitude d_i if a positive demand takes place.

Given the model expressed in Equation (1), an effective (e.g. [15], [24]) forecasting method was proposed in [4]:

$$\hat{z}_i = (1 - \alpha) \cdot z_{i-1} + \alpha \cdot \hat{z}_{i-1} \quad (2)$$

$$\hat{m}_i = (1 - \alpha) \cdot m_{i-1} + \alpha \cdot \hat{m}_{i-1} \quad (3)$$

$$\hat{d}_i = \left(1 - \frac{\alpha}{2}\right) \cdot \frac{\hat{z}_i}{\hat{m}_i} \quad (4)$$

where \hat{z}_i is the estimated magnitude of a positive demand, after the positive demand z_{i-1} in period $i - 1$, and \hat{m}_i is the estimate inter-arrival between positive demands, calculated from the last inter-arrival m_{i-1} between positive demands. Both these forecasts are used in Equation (4) to produce an estimate of the demand \hat{d}_i . Equations (2), (3) and (4) are updated only after a positive demand using the smoothing parameter $\alpha \in (0,1)$ that defines how much new data impact on the previous estimates.

From Equation (3), an estimate of the probability p can be obtained as well:

$$\hat{p}_i = \frac{1}{\hat{m}_i} \quad (5)$$

In order to compute an expected variance $\widehat{Var}(z_i)$ for the positive demand, an exponential smoother is applied to the squared error, as in [16]:

$$\widehat{Var}(z_i) = (1 - \beta) \cdot \widehat{Var}(z_{i-1}) + \beta \cdot (z_{i-1} - \hat{z}_{i-1})^2 \quad (6)$$

where the smoothing parameter $\beta \in (0,1)$ does not necessarily equal the smoothing parameter α .

2.2 The Teunter-Syntetos-Babai model

The proposed replenishment method modifies the one described in [16], from now on referred to as the TSB model, adapting it to perishable items.

In the TSB model the underlying intermittent demand is assumed to follow the structure described in section 2.1. From this pattern an order-up-to (T, S) policy is defined, assuming the positive demand random variable follows a known distribution whose parameters are unknown.

An order can be placed every t periods, collectively defining the constant review time, and requires a fixed lead time l to arrive. In this scenario, at the beginning of a review time, a stock $s \geq 0$ is available and an order o can be placed. The total amount $s + o$ is expected to cover the demand of t periods after the lead time, a new order can in fact be placed only after $t - l$ periods and requires l periods to arrive. The performance measure associated to this model is the fill rate over the specified t periods, defined as the probability a positive demand taking place in one of those periods is satisfied by $o + s$ and thus generates no stock out.

Given a stock s and order quantity o , the fill-rate is:

$$fr = \frac{1}{t} \sum_{i=1}^t \sum_{k=0}^{t_i+l-1} \binom{t_i+l-1}{k} p^k (1-p)^{t_i+l-1-k} \cdot \Phi(o+s, k+1) \quad (7)$$

where $\Phi(x, y)$ is the cumulative probability y positive demand periods yield a total demand inferior to x while p is the probability a period yields a positive demand.

The main component of Equation (7) is:

$$fr_i = \sum_{k=0}^{t_i+l-1} \binom{t_i+l-1}{k} p^k (1-p)^{t_i+l-1-k} \cdot \Phi(o+s, k+1) \quad (8)$$

The probability a positive demand taking place in t_i is satisfied with $o + s$.

2.3 Inventory replenishment model

The proposed replenishment model divides the stock s into two separate stocks:

- s_e the amount of goods that will expire at the end of one of the t periods after the lead time l .
- s_{ne} the amount of goods that will not expire in said time frame.

These quantities are updated as in the TBS model at the beginning of each lead time, before the order o is placed. The expired stocks are discarded and the expiring stock is moved from s_{ne} to s_e . The stock s_e is assumed to expire at the end of period t_e , calculated from the update period before the lead time, while the ordered quantity o is assumed not to expire in the time frame.

Given a hypothetical positive demand d_i at period t_i , two mutually exclusive cases can arise:

- The period t_i occurs before the expiration date.
- The period t_i occurs after the expiration date.

In the first case the perishability has no effect, thus Equation (7) is used. In the second case s_e has expired and a different Equation is required. As in section 2.2, given a positive demand d_i in period t_i after the lead time l , all the possible demands in the previous periods $\sum_{k=0}^{t_i+l-1} d_k$ must be considered. This leads to two scenarios:

- The demands before the expiration date partially or totally consumed the expiring stock, i.e. $\sum_{k=0}^{t_e+l-1} d_k \leq s_e$.
- The demands before the expiration date consumed more than the expiring stock, i.e. $s_e < \sum_{k=0}^{t_e+l-1} d_k \leq s_e + s_{ne} + o$.

The fill rate fr_{i1} of the first scenario is the probability that the demands before t_e are satisfied by s_e and the demands after t_e including d are satisfied by $s_{ne} + o$:

$$fr_{i1} = \sum_{k=0}^{t_e} \sum_{h=0}^{t_i+l-t_e-1} \binom{t_e}{k} p(k, t_e) \Phi(s_e, k) \cdot \binom{t_i+l-t_e-1}{h} p(h, t_i + l - t_e - 1) \Phi(o + s_{ne}, h + 1) \quad (9)$$

where:

$$p(x, y) = p^x (1 - p)^{y-x} \quad (10)$$

is a notation shortcut for the probability that x periods over y present a positive demand, and:

$$\Phi(x, 0) = 1 \quad \forall x \geq 0 \quad (11)$$

since in absence of positive demands no stock out can occur.

The fill rate fr_{i2} of the second scenario is the probability the demands d_{be} before t_e are satisfied by $o + s$ and the demands after t_e including d_i are satisfied by the remaining stock $o + s - d_{be}$ with $d_{be} > s_e$:

$$fr_{i2} = \sum_{k=1}^{t_e} \sum_{h=0}^{t_i+l-t_e-1} \sum_{d_{be}=s_e+1}^{o+s} \binom{t_e}{k} p(k, t_e) \Phi(d_{be}, k) \cdot \binom{t_i+l-t_e-1}{h} p(h, t_i + l - t_e - 1) \Phi(o + s - d_{be}, h + 1) \quad (12)$$

These scenarios are mutually exclusive, thus the fill rate fr_i of period t_i is:

$$fr_i = fr_{i1} + fr_{i2} \quad (13)$$

The overhaul fill rate fr accounts for the individual fill rates of t periods after the lead time, as in Equation (7):

$$fr = \frac{1}{t} \sum_{i=1}^t fr_i \quad (14)$$

This methodology expands the one defined in section 2.2 considering a portion s_e of the stock as perishable. The calculations above refer to a single expiration date but similar considerations can be applied to address multiple expiration dates in the frame of analysis.

From a computational perspective the proposed methodology is more demanding than the original one, and the calculation of fr_{i2} requires an analysis of $o + s_{ne}$ demands before t_e . This calculation is necessary since the last component of Equation fr_{i2} , defining the probability a demand after t_e does not produce a stock out, requires the number of units $o + s - d_{be}$ left in stock.

2.4 Modified inventory replenishment model

As stated at the end of section 2.3, the proposed methodology is quite expensive from a computational standpoint, in particular when $o + s_{ne}$ is high.

In order to reduce the computational effort in those cases a different Equation for fr_{i2} is proposed:

$$fr_{i2} = \sum_{k=1}^{t_e} \sum_{h=0}^{t_i+l-t_e-1} \binom{t_e}{k} p(k, t_e) \binom{t_i+l-t_e-1}{h} \cdot p(h, t_i + l - t_e - 1) \cdot \left[\Phi(o + s, k + h + 1) - \sum_{d_{be}=0}^{s_e} \phi(d_{be}, k) \cdot \Phi(o + s - d_{be}, h + 1) \right] \quad (15)$$

This alternative Equation requires the analysis of $s_e + 1$ values of d_{be} instead of $o + s_{ne}$, and thus the following solving procedure can be applied:

- If $o + s_{ne} \leq s_e + 1$ compute Equation (12).
- If $o + s_{ne} > s_e + 1$ compute the Equation (15).

This procedure aims at mitigating the computational costs of fr_{i2} calculation but does not reduce the effort when $o + s_{ne} \cong s_e$ or when both $o + s_{ne}$ and s_e are high.

2.5 Inventory replenishment model solution

The model presented in section 2.4 aims at defining the order quantity o_{min} at the beginning of lead time l . o_{min} is the minimum order capable of achieving a target fill rate fr_{target} for t periods after the lead time l . In contrast, Equation (14) calculates the fill rate fr of t periods after the lead time l given a predefined order quantity o . Equation (14) is not easy to invert, thus no direct equation is available to solve the problem at hand. A common solution in the relevant literature involves a stepwise search:

- Start assuming $o = 0$
- Calculate fr for the value of o under analysis.
- If $fr \geq fr_{target}$ then stop, $o_{min} = o$.
- If $fr < fr_{target}$ then increase o by one unit and go back to the second bullet point.

This procedure is feasible if the computational cost for the fill rate calculation is limited. In the case at hand such cost is significant and increases with o , thus the algorithm reactivity decreases as it goes on.

An alternative procedure, based on the secant method, is proposed to decrease the amount of calculations involved. The optimum is formally defined as:

$$o_{min} = \inf\{o: fr(o, s_e, s_{ne}) \geq fr_{target}\} \quad (16)$$

Where $fr(o, s_e, s_{ne})$ is the fill rate relative to the order quantity o and the stocks s_e and s_{ne} .

Since, fixed the stocks s_e and s_{ne} , the fill rate can grow only if o increases, Equation (16) can be rewritten as:

$$o_{min}: fr(o_{min}, s_e, s_{ne}) = \inf\{fr(o, s_e, s_{ne}): fr(o, s_e, s_{ne}) \geq fr_{target}\} \quad (17)$$

Two properties of the fill rate, as calculated in Equation (14), provide two extremes o_{sup} and o_{inf} to initialize the secant method. This initialization requires no initial calculation of Equation (14) itself:

- Ceteris paribus a decrease in s_e reduces fr .
- Ceteris paribus substituting part of s_e with stock not expiring in t increases fr .

From these properties two quantities can be defined:

$$o_{sup}: fr(o_{sup}, 0, s_{ne}) = \inf\{fr(o, 0, s_{ne}): fr(o, 0, s_{ne}) \geq fr_{target}\} \quad (18)$$

$$o_{inf}: fr(o_{inf}, 0, s) = \inf\{fr(o, 0, s): fr(o, 0, s) \geq fr_{target}\} \quad (19)$$

with the property:

$$o_{inf} \leq o_{min} \leq o_{sup} \quad (20)$$

In Equation (18), starting from the optimum order quantity as defined in Equation (17), the elimination of s_e reduces fr . From this point, in order to achieve $fr(o, 0, s_{ne}) \geq fr_{target}$ fixed s_{ne} , the order quantity now defined o_{sup} increases. A similar effect takes place in Equation (19) where the expiring stock is fully substituted by stock non expiring. The substitution increases the fill rate and, for this new configuration, the initial order quantity is no longer the minimum required to achieve $fr(o, 0, s_{ne}) \geq fr_{target}$. The order quantity now defined o_{inf} decreases to reach the minimum fill rate required.

Equation (18) and Equation (19) contain no expiring stock, and thus the computationally expensive calculations of sections 2.3 and 2.4 are not required. Equation (7) is iteratively applied to define both o_{sup} and o_{inf} .

In order to apply the bisection method, the fill rate expressed in Equation (14) is shifted by fr_{target} :

$$fr_{shifted} = fr - fr_{target} \quad (21)$$

The fill rate function is strictly increasing fixed s_e and s_{ne} . If Equation (21) has roots in the interval $[o_{inf}, o_{sup}]$ it has a single root, if Equation (21) has no roots in the interval then $fr(o_{inf}, s_e, s_{ne}) > fr_{target}$. In this last scenario $o_{min} = o_{inf}$ and the algorithm terminates during the calculation of $fr(o_{inf}, s_e, s_{ne})$ in the first step as described below.

Given o_{sup} and o_{inf} the calculation of their fill rate using Equation (14) is required at the beginning of the bisection algorithm. This defines the extreme values of fr_{shift} and makes possible the initial secant calculation:

$$fr_{sup} = fr(o_{sup}, s_e, s_{ne}) \quad (22)$$

$$fr_{inf} = fr(o_{inf}, s_e, s_{ne}) \quad (23)$$

During the generation of new fr_{sup} and fr_{inf} , and the respective o_{inf} and o_{sup} , the algorithm operates only on integer values of o . The new quantity o identified by the secant must be approximated by the nearest integer. If it falls over the current o_{sup} or under the current o_{inf} , the value is respectively approximated by $\lfloor o \rfloor$ and $\lceil o \rceil$. The algorithm terminates when $fr(o, s_e, s_{ne}) = fr_{target}$, when a floor approximation reaches o_{min} or a ceil approximation reaches o_{sup} . In the last two scenarios the interval of analysis has unitary length and by construction $o_{inf} < o_{min}$, thus $o_{min} = o_{sup}$.

3 EXPERIMENTAL ANALYSIS

3.1 Probability distribution and estimations

In order to compute Equation (14), both the distribution function $\phi(x, y)$ and the cumulative distribution function $\Phi(x, y)$ of a positive demand x during y periods must be known. It is also required the knowledge of the probability p a positive demand occurs during a period. These three components of Equation (14) vary across time and must be indirectly forecasted from the item time series. As suggested in [16], the positive demand distribution (both cumulative and not cumulative) is hard to determine over an arbitrary number of periods, unless the multiple periods distribution can be defined from the single period distribution. This experimental analysis assumes that the positive demand during a single period is negative binomial distributed. The sum of independent negative binomial distributions is a negative binomial distribution itself, with different parameters depending on the number of random variables added. In the case at hand, the number of random variables is the number of periods. The use of a discrete random variable, instead of a continuous one as in [16], is coherent with Equation (12) and Equation (15) where d_{be} moves through integer values.

In order to estimate the single period parameters of the negative binomial distribution, a time series analysis is required. The methodology used for this experimental analysis is the same applied in [16]; the forecasting technique described in section 2.1 is applied to define z_i and $\widehat{Var}(z_i)$ for the positive demand. From these values the negative binomial distribution parameters are derived, while the forecasting technique estimates the probability p directly.

3.2 Experiment settings

The experimental analysis consists in a set of simulations, carried over with different parameters, where the proposed methodology is tested on both generated and historical series. Each simulation outputs a set of performance measures that are compared to each other and evaluated against a benchmark.

The model effectiveness is assessed in three different scenarios:

- Ideal scenario.
- Approximate model scenario.
- Real scenario.

The first scenario refers to an ideal case in which time invariant intermittent distributions following Equation (1) are generated. As defined in section 3.1 their positive demand magnitude follows a negative binomial distribution with known parameters:

- p_b success probability.
- r required number of successes before the first failure.

The probability p a positive demand occurs in a period is also known and time invariant. In this first scenario the estimations of section 3.1 are not used since the simulation is fed from the beginning with the correct parameters.

The second scenario measures the effectiveness of the proposed methodology in a context where components of stock s_e expire in two different dates during the $t + l$ periods following the update period. Equation (14) is not modified to admit a multiple number of expiration dates since this experiment measures the method effectiveness while it is implemented as an approximation. In the case at hand Equation (14) is implemented considering the first expiration date only and attributing to said period all the expiring stock s_e . Like the previous one this scenario does

not utilize the estimation methodology proposed in section 3.1.

The third scenario applies the proposed methodology to historical demand series. In this case the expiration date is single, as in the first scenario, while the positive demand parameters p_b and r are unknown as the positive demand occurrence probability p . These parameters are obtained using the forecasting techniques reported in section 2, and recalled in section 3.1.

For each scenario a set of fixed parameters is defined while a second set of parameters varies among the simulations. All the possible combinations of the second set of parameters are tested in order to assess the method performance in different contexts. Table 1 and Table 2 summarize both the fixed and the variable parameters for the first, second and third scenarios.

Table 1. Parameters (first and second scenario).

Parameter	Description	Category	Values
n_c	Number of cycles	Fixed	100
t	Periods in a cycle	Fixed	10
n_e	Periods before expiration	Fixed	12 first sc., 19 sec. sc.
l	Lead time	Variable	3, 5
fr_{target}	Fill rate target	Variable	0.8, 0.9
p	Demand probability	Variable	0.1, 0.3
p_b	Success probability	Variable	0.3, 0.5, 0.7
r	Required number of successes	Variable	1, 3, 5, 7

Table 2. Fixed and variable parameters (third scenario).

Parameter	Description	Category	Values
n	Number of periods	Fixed	120
n_w	Warm up periods	Fixed	30
n_s	Simulation periods	Fixed	90
t	Periods in a cycle	Fixed	10
n_e	Periods before expiration	Fixed	12
l	Lead time	Fixed	3
fr_{target}	Fill rate target	Variable	0.8, 0.9

The only fixed simulation parameter that changes between the first and the second scenario is the number of periods before expiration n_e . When a product is ordered it arrives after l periods and then lasts for n_e periods before expiring, if the value of n_e follows the relation:

$$\exists x \in N: x \cdot t < n_e \leq x \cdot (2t - l) \quad (24)$$

The generated system presents only a single expiration date during the $t + l$ periods of analysis. Otherwise n_e can generate two different expiration dates to be managed in each analysis.

The results are collected for each simulation period after the first lead time, when the first order has already arrived. In this context no initial level of backorders and stock is required to make the first measurements fair. In the third scenario 30 initial warm up periods are also required to initialize the intermittent demand forecast outlined in section 2. These periods are for initialization purposes only and the actual simulation starts from period 31.

3.3 Performance metrics

For each experiment outlined in section 3.2 a performance measurement takes place in each simulation period. If the period presents a positive demand, then the total number of positive demand in the simulation is updated. At the same time, the performance record keeps track of the number of positive demand that have been satisfied from the stock, not adding to the backlog. The ratio between these two raw measurements is the fill-rate of the simulation fr_{sim} .

In each simulation the procedure reported in section 2.5 is repeated for each cycle to define the optimal order quantity o_{min} . The parameters s_e or s_{ne} cannot be changed and only positive values of o_{min} are produced. The fill rate is thus set to achieve $fr(o_{min}, s_e, s_{ne}) \geq fr_{obj}$. This goal setting leads to difficulties while comparing fr_{sim} and fr_{obj} since, by construction, on average $fr_{sim} \geq fr_{obj}$ thus the difference $fr_{sim} - fr_{obj}$ is designed to be ≥ 0 . To avoid this unfair comparison the values of $fr(o_{min}, s_e, s_{ne})$ are collected in each simulation as they are generated and their average is compared with fr_{sim} instead of fr_{obj} :

$$\Delta fr = fr_{sim} - avg(fr(o_{min}, s_e, s_{ne})) \quad (25)$$

3.4 Results

Table 3 contains the expected value and sample standard deviation of Δfr for each scenario.

Table 3. Performance obtained.

Scenario	Expected value	Sample STD
Scenario 1	-0.01	0.036
Scenario 2	-0.09	0.04
Scenario 3	0.02	0.13

In the first scenario the expected value of Δfr is low, and the proposed model presents a small negative bias in its calculation of the optimal fill rate. The standard deviation is quite low as well, revealing the good performance of the model under ideal conditions.

An Anderson-darling test is used in the first scenario on the Δfr data to identify if the performance follows a Normal distribution. The test yields a p-value of 0.56, thus such hypothesis cannot be rejected.

In the second scenario the expected value of Δfr increases, and this increased bias reveals that the approximation underestimates the expected value of the fill rate. At the contrary the standard deviation remains quite stable and thus, except for the bias, the approximation retains a performance close to the ideal one.

An Anderson-darling test is used in the second scenario on the Δfr data to identify if the performance follows a Normal distribution. The test yields a p-value of 0.002, and thus the normality hypothesis must be rejected.

In the third scenario the expected value of Δfr is comparable in magnitude to the one in scenario 1 while the increased standard deviation identifies a sharp performance decrease. The performance reduction can be attributed both to the use of approximate values for the negative binomial parameters and to the lack of fit between the negative binomial itself and the positive demand values.

4 CONCLUSIONS AND FURTHER RESEARCH

The results presented in the experimental analysis validate the effectiveness of the proposed method for the management of intermittent items characterized by perishability. The best performance, achieved when the

methodology is applied in ideal conditions, proves the correctness of the underlying theory and acts as a benchmark for both the other analysis presented and for future applications.

A critical aspect emerged during the solution technique discussion in relation to the calculations computational costs. In case multiple expiration dates arise, the use of approximations to reduce this computational cost presents significant bias, as outlined in the experimental analysis. Nevertheless, the standard deviation of the measured error remains close to the ideal scenario. Future research can thus focus on the isolation and correction of this bias term in order to uniform the performance of the approximate case with that of the ideal one.

The validation with real items presented an error standard deviation far higher than in the ideal case. This phenomenon can relate to a mismatch between the expected positive demand distribution and the proposed one. Future research could focus on the closure of this gap, with the application of sampling techniques to adapt the model to the positive demand distribution in real cases.

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