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Spare Parts Replacement Policy Based on Chaotic Models

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Abstract: Poisson point processes are widely used to model the consumption of spare parts. However, when the items have very low consumption rates, the historical sample sizes are too small. This paper presents a modelling technique for spare parts policies in the case of items with a low consumption rate. We propose the use of chaotic models derived from the well-known chaotic processes logistic map and Hénon attractor to assess the behaviour of a set of five medium voltage motors supplying four drives in the rolling mill of a steelmaking plant. Supported by the chaotic models, we conclude that the company needs an additional motor to ensure full protection against shortages.

Keywords: Maintenance; Reliability; Safety; Complex Adaptive Systems; Maintenance Strategy, Spare Parts; Poisson Processes; Chaos; Steelmaking; Rolling Mills.

1. INTRODUCTION

Manufacturing operations management includes the management of the materials needed for machine maintenance (Marquez and Gupta, 2006). Common maintenance materials used by various maintenance crews, such as paints, grinding wheels, or welding, usually follow predictable consumption patterns. Crews can also share technical materials, such as bearings, circuit breakers, and lamps, whose consumption, although lower, is still predictable. Finally, spare parts are specific to certain machines and usually have erratic, unpredictable consumption patterns (Huiskonen, 2001; Lengu et al., 2014). Common maintenance and technical materials have regular consumption patterns modelled by exponential, normal, lognormal distribution or other well-fitted distributions (Rego and Mesquita, 2015). Logistics techniques, such as demand forecasting, reorder point (i.e. the level of inventory that triggers the replenishment) and the management of the replenishment time can increase the likelihood of success in their purchasing and replenishment operations (Guajardo et al., 2015; Garg and Deshmukh, 2006).

Maintenance procedures and the management of spare parts require a specific type of human-machine interaction. Maintenance crews interact with industrial machinery by breakdown interventions, when the production runs into unscheduled stoppages, solved as quickly as possible by replacing the damaged parts. Maintenance crews also interact by means of preventive procedures, scheduled by probabilistic methods, to be implemented before the next stoppage. Finally, by predictive procedures, maintenance crews monitor the evolution of the main failure modes and gather data to forecast, the time up to the next stoppage. Spare parts policies play an important role in these three types of human-machine interaction (Tsang, 2002; Sherwin, 2000).

Regarding spare parts, due to the erratic behavior characterized by the low consumption rate (Lengu et al., 2014), the reorder point and management of the replenishment time may not suffice. The low consumption of parts leads to major difficulties in the management process (Cavalieri et al., 2008). In fact, sometimes, the consumption is zero, because the spare part may be stored for long periods, without being required, as the original part remains fully functioning. In advanced manufacturing, the obsolescence of spare parts, even before their employment, is not uncommon (Luxhøj et al., 1997).

The application of logistic techniques such as Poisson processes, appropriate for predictable, high consumption rates, may fail with erratically performing items. Due to small samples, confidence intervals are excessively large, weakening the power of the technique. Moreover, manufacturing has recently incorporated features from complex adaptive systems (CAS), with non-linearities and mutual interactions among components leading to unpredictability. This complexity justifies using machine learning tools for predicting spare parts consumption (e.g. Lolli et al., 2017). Logistic techniques relying on complex or chaotic models, rather than on predictable and high consumption patterns, are also effective in such cases (Efthymiou et al., 2014).

This paper presents a modelling technique for the analysis of spare parts policies of items with a low consumption rate. The research method relies on chaotic models. The research object is a set of four 1,500 HP AC motors that drive rolling mills in a steelmaking plant. The modelled variable is the time between failures that require changing one of the four

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motors. The research technique relies on two chaotic models, i.e. a logistic map with a single dimension, and a Hénon map with two dimensions. The specific objectives are to: (i) analyse the spare parts situation of the set of motors by nonhomogeneous Poisson processes; (ii) repeat the analysis supported by chaotic models; and (iii) compare the results.

2. POISSON POINT PROCESSES AND SPARE PARTS REPLACEMENT POLICIES

Spare parts consumption behaves usually like counting processes (Louit et al., 2009). A counting process is a type of Poisson point process, or simply a Poisson process (PP). A PP consists of points randomly located in a numerical space, and can model random events in stochastic processes. PP has an intensity parameter λ related to the expected number of points in a given bounded region of the numerical space (Leemis, 1991). Four types of counting processes are useful for analysing spare parts policies: homogeneous Poisson processes (HPP), renewal processes, nonhomogeneous Poisson processes (NHPP), and imperfect repair processes. Renewal and imperfect processes involve, respectively, a complete overhaul ("as good as new" repair) and partial modifications in machines along with the spare part replacement. We focus on HPP and NHPP, which more realistically describe the "as bad as old" repair policy prevalent in manufacturing activities. In manufacturing, a system consisting of a large number of sub-systems fails when a single part fails and the crew replaces it after a negligible downtime (Rausand and Hoyland, 2004). Therefore, it is reasonable to assume that the overall reliability remains the same after the repair, thus characterizing the minimal repair policy (Kahle, 2007).

For our purposes, the numerical space of a PP is a unidimensional timeline, where individual numbers correspond to time intervals between zero and infinity. HPP has a constant intensity λ and NHPP has a power-law intensity $\lambda(t)$. Hereafter, we will refer to $\lambda(t)$ as the failure rate function. In spare part consumption processes, an HPP acts as a counting process and models times to failure according to a constant failure rate λ. From the properties of HPP, we highlight that $N(0) = 0$, times to failure are independent, and the number of failures in any interval T is a Poisson random variable with mean λT. $E[N(T)] = \lambda T$ and that the probability of $N(T)$ being equal to x is given by the Poisson model (Rausand and Hoyland, 2004). The main implication is that if the consumption of a spare part follows a certain constant failure rate λ, the probability of x events in the time interval T is given by:

$$P(N = x) = \frac{(\lambda T)^x e^{-\lambda T}}{x!}. \quad (1)$$

If the failure rate of a certain part is not constant, which means that the part has improved or deteriorated, then a NHPP should model the failure sequence and consequently the spare part consumption. Such an NHPP is a generalization of an HPP, relaxing the stationary pattern of failures and assuming a power-law pattern, which can be positive or negative. If the power-law has a unitary exponent, the NHPP turns into an HPP (Rausand and Hoyland, 2004). The main implication of an NHPP is that if the consumption of a spare part follows a power-law failure rate function $\lambda(t)$ with shape factor γ and scale factor θ, the probability of x events in the time interval T is as follows:

$$P(N = x) = \frac{(T/\theta)^{\gamma x} e^{-(T/\theta)^{\gamma}}}{x!}. \quad (2)$$

Modelling the failure data by a maximum likelihood estimation (MLE) and fitting a Weibull distribution provide the shape and scale factors. On occasions, there may also be a shift parameter t_0, the failure free time. The cumulative intensity function provides the expected number of failures by time T (3). Equations (4) and (5) provide the mean time between failures (MTBF) and the reliable lifetime (T_R) with a confidence level of 95%, respectively.

$$\Lambda(T) = \int_0^T \lambda(t)dt \quad (3)$$

$$MTBF = t_0 + \theta\Gamma[\frac{1}{\gamma} + 1] \quad (4)$$

$$T_R|_{95\%} = t_0 + \theta[-ln(0.95)]^{\frac{1}{\gamma}} \quad (5)$$

If a maintenance crew knows about the failure rate function and can estimate the time to the next failure, they can provide a spare part replacement policy (Park, 1979). The simplest policy is to predict the probabilities of the next failure over time, or calculate the reliable life and then define a certain number of spare parts. However, a problem arises in the case of a very low failure rate. As the MTBF is very large, the sample size is low, thus producing large confidence intervals for the parameters. Moreover, even if a distribution fits a small sample size, it is not possible to ensure that the assumptions that characterize HPP and NHPP remain over time (Efthymiou et al., 2014). For example, it is not possible to ensure that there has been no change in the requirement of the manufacturing process or the required workload. To help solve this problem, we propose a method to manage spare parts with a low consumption rate based on chaotic models.

3. CHAOTIC MODELS

Chaotic models describe phenomena with deterministic formation laws, but which, at first glance, seem to be random. Such behaviour originates from the interactions among internal parts of complex and dynamic systems, with a fundamental instability, i.e. the sensitivity to the initial conditions. Although they originate from deterministic rules, the recurrence of the application of the rule, under certain circumstances, makes chaotic phenomena unpredictable in the long term. The extreme dependence on the initial conditions of the parameters determines that the output of a chaotic phenomenon will become unstable over time (Thietart and Forgues, 1995). The consequence of this instability is that the results of deterministic systems, even with definite evolution laws, are extremely sensitive to disturbances and noise, thus making them unpredictable. Even in the absence of noise, non-linearities and interactions among components amplify minimal errors in parameters, generating deterministic chaos (Capeáns et al., 2017).

Positive feedback generates chaotic situations, which lead to points of inflection and rupture, i.e. bifurcation points. The simplest chaotic model is possibly the logistic map, which is a positive feedback process involving a quadratic function of itself. The logistic map is a recurrence relation that exemplifies how complex and chaotic behaviours arise from the application of a simple and deterministic rule (Yang and Cheng, 2007). The transition from order to chaos justifies the use of the expression 'deterministic chaos' for such models (Phatak and Rao, 1995). The logistic map model is given by:

$$x_{(t+1)} = ax_t(1 - x_t) \tag{6}$$

with $4 > a > 0$.

If $a < 3$, the process converges to a fixed value. If $a < 1$, the process converges to zero. If $1 < a < 3$, the process converges to $s = (a-1)/a$, i.e. a fixed attractor. If $3 < a < 3.57$, the process becomes cyclic, with bifurcations that lead to multiple values, a cyclic attractor. If $a > 3.57$, the outcome is nearly unpredictable with oscillating behaviour, a strange attractor. This region is the chaos edge. In the chaos edge, at first glance, the time series of the logistic map does not differ from a random time series. Instead, by plotting $[x_{(t+1);}\ x_t]$, a regular pattern arises, which reinforces the notion of chaos out of the order or deterministic chaos. Finally, if $a > 4$, the outcome is chaotic (Capeáns et al., 2016; Yang and Cheng, 2007).

Figure 1 illustrates fifty executions of the logistic map under different parameters, as well as the difference between the subjacent relationships in random execution and chaotic execution, which illustrates the notion of deterministic chaos. The figure shows executions of the logistic map with $s(0) = 0.5$ and (a) $a = 1$; (b) $a = 3$; and (c) $a = 3.95$. The figure also shows a series generated by a random generator (d), a dispersion graph of a random execution [n x (n-1)] (e), without a noticeable pattern, and a dispersion graph of a logistic map execution [n x (n-1)] (f), with an almost linear pattern. The two last windows of the figure illustrate the difference of a random process, without a subjacent formation law, and a chaotic process, with a subjacent, deterministic formation law.

The other chaotic model of interest is the Hénon map. This map takes a point (x, y) and maps it to a new point (x´, y´). Equations (7) and (8) support the mapping process as follows:

$$x_{(t+1)} = y_{(t-1)} + 1 - a\left[x_{(t)}\right]^2, \tag{7}$$

$$y_{(t+1)} = bx_{(t).} \tag{8}$$

For $[a, b] = [1.4; 0.3]$, the map approaches the Hénon attractor. For other values, the map may be also chaotic, but it may also be intermittent, or converge to a periodic orbit (Capeáns et al., 2016). Figure 2 illustrates one thousand executions of the Hénon map, forming the Hénon attractor, with $x_0 = 1.2$ and $y_0 = 1.4$.

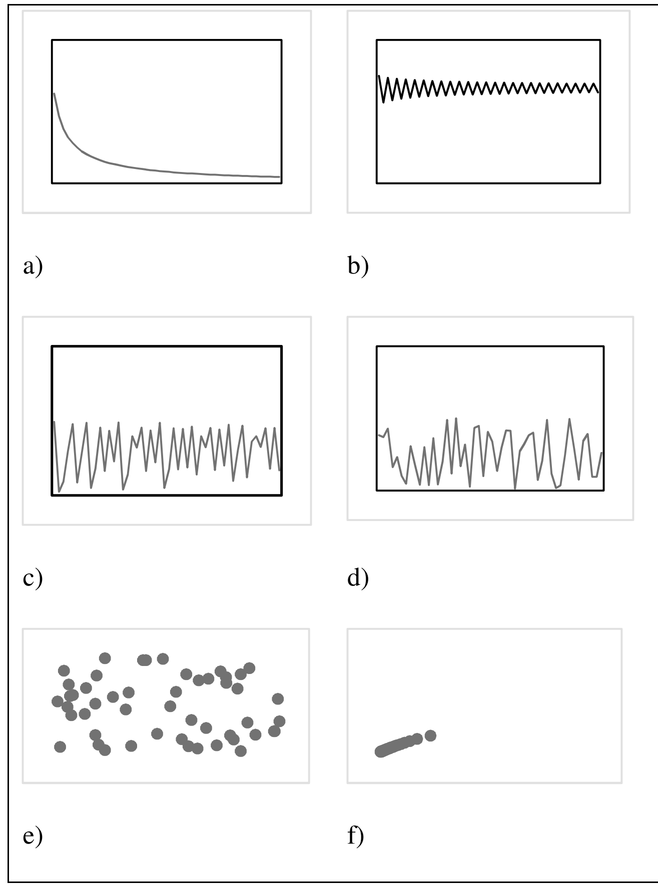

Figure 1 – Elements of the logistic map model

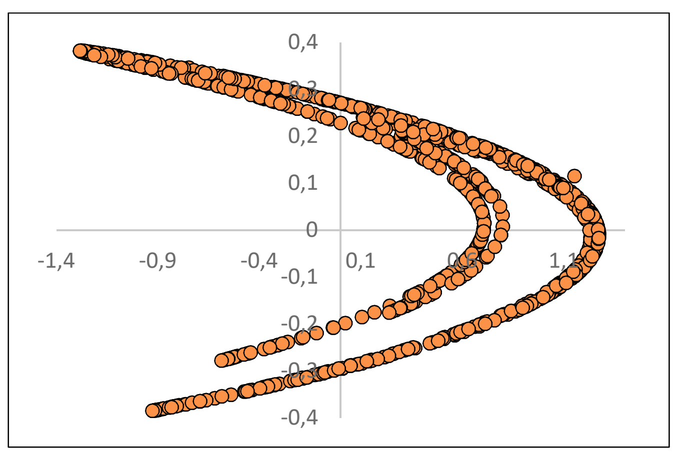

Figure 2 – The Hénon attractor

4. THE SUBJECT AND THE RESEARCH

The focus of our research is the rolling mill plant of a semi-integrated steelmaking plant. There are two technological routes for steel production, with iron ore (integrated plants) and with metallic scrap and pig iron (semi-integrated plants). Semi-integrated plants operate the steel refining stage in melt shops, and the conformation stage in rolling mill shops.

Usually, the main drivers of rolling mills are medium voltage alternate current squirrel-cage induction motors. The project studied a set of five 1,500 HP motors that drive four hot rolling mills plus a spare motor. Each time a motor fails, the spare motor replaces it. Once repaired, the damaged motor returns to the warehouse until a new failure occurs, and so on. As a typical capital good, the motors have a long lifespan, overcoming several decades (Hekimoglu et al., 2018).

Large equipment failures follow a degradation path that might take up to several months or even years until the occurrence of a breakdown event (Gebraeel et al., 2009). Two components represent the time to failure (TTF). The first component is the time elapsing between the last repair and the beginning of a potential failure, the initial event of the degradation process that will cause the failure. The second is the time between the beginning of the potential failure and the functional failure, the breakdown event that requires maintenance actions (Jardine et al., 2006). Christer (1999), Ahmadi et al. (2009), and Bazovski (2004) state that the TTF follows an exponential model for random failures and a normal distribution can approach TTF for wear-out failures. As the focus is in the wear-out phase, the normal distribution is used in the calculation of the reliable life.

The research has limitations. The optimization of the chaotic models relied on the excel software solver optimizer. A more powerful package would lead to more robust models, including other chaotic models, such as the Lorenz attractor. Although a stochastic variable, the time to repair (*TTR*) considered is deterministic, which requires the use of simulation techniques. The research assumed normality for the determination of the variability in forecasting. Although this last assumption is supported in the literature, future research could include methodologies to bridge both gaps.

4. RESULTS AND DISCUSSION

The typical times to repair (TTR) are eight and twelve months if the damage requires major repairs and the construction of new subsystems, respectively. Table 1 shows the cumulative time of the process, the time between failures (*tbf*) of the motors in months and the normalized *tbf*, a more useful form for the analysis.

Table 1 – TBF (in months)

#	Cumulative time	*tbf*	Normalized *tbf*
1	-	11	0.275
2	36	25	0.625
3	76	40	1
4	99	23	0.575
5	119	20	0.5
6	152	33	0.825
7	178	26	0.65
8	210	32	0.8
9	228	18	0.45

An MLE fitted a Weibull distribution with $t_0 = 3.01$, $\gamma = 2.51$, and $\theta = 24.51$, with a significance level of 23% (χ^2 test) and 14% (Kolmogorov-Smirnoff test), while $MTBF = 24.76$ and $T_R|_{95\%} = 10.52$ months ((4) and (5)). Confidence intervals (95%) for γ and θ are respectively [1.20 - 3.69] and [17.50 - 34.14]. Applying (3) to the MLE values of γ and θ, and considering eight and twelve months for TTR, the model leads to the results reported in Table 2.

Table 2 - NHPP of the replacement of motors

x	Probability of *x* failures in 12 months	Cumulative probability	Probability of *x* failures in 8 months	Cumulative probability
0	0.923	0.923	0.95599	0.956
1	0.074	0.997	0.04302	0.999
2	0.003	1.000	0.00097	1.000

As $T_R|_{95\%} = 10.52$ months, maintaining no spare parts other than the five motors is a safe policy for major repair, but is not for subsystem manufacturing. Maintaining one spare part, the probabilities of a shortage are [1 – 0.997] and [1 – 0.999] respectively, which is a safe policy. In any case, it is necessary to consider that $T_R|_{95\%} = 10.52$ months results from the adoption of MLE parameters. Considering uncertainty, the extreme situations are: (i) $\gamma, \theta = [3.69, 34.14]$, $T_R|_{95\%} = 18.27$ months; (ii) $\gamma, \theta = [1.20, 17.50]$, $T_R|_{95\%} = 4.48$ months.

The observations lasted for more than twenty years, in which the manufacturing evolved, approaching CAS. Among other transformations, there was a learning period and the technological development of maintenance techniques, with the introduction of remote sensing, predictive analysis, and corrective maintenance procedures. In the first instance, such evolutions reduced the downtime, which encouraged the company to increase production, which increased downtime further, in a feedback loop. Another example is the use of multiple fuels in the reheating furnace, which changed the billet temperature and reduced the load on the drives. The company then increased production, increasing the load on the same drives. Such feedback loops, characteristic of CAS, may violate a premise of PP, i.e. the independence between failure events, opening up the possibility of using complex, alternative methods.

We propose an alternative method to analyse the spare part policy. Table 3 shows the time evolution, with a step of five positions, of the logistic map with $s(0) = 0.896316$ and $a = 3.698554$. A commercial solver finds the values minimizing the minimum square error (MSE) of the regression formed by the map and the normalized *tbf*.

Table 3 – Time evolution for the logistic map

#	s(#)	*tbf* norm	SE
1	0.344	0.275	0.005
6	0.710	0.625	0.007
11	0.912	1.000	0.008
16	0.505	0.575	0.005
21	0.666	0.500	0.027
26	0.736	0.825	0.008
31	0.626	0.650	0.001
36	0.797	0.800	0.000
41	0.450	0.450	0.000
forecast	0.802	MSE =	0.087

The model provides the most likely normalized time to the next failure, that is $s(48) = 0.802$. As $\sigma = 0.219$ and assuming a normal distribution (one-tailed test), the new $TR|_{95\%}$ = 40*(0.802-1.64*0.219) = 17.71 months.

Table 4 shows the time evolution, with a step of three positions, of the Hénon map with $x(0) = 0.027266$ and $y(0) = 0.281145$ for outcome x and $x(0) = -0.33122$ and $y(0) = -1.48856$ for outcome y. The solver finds these values minimizing the minimum square error (MSE) of the regression formed by the map evolution and the normalized *tbf*.

Table 4 – Time evolution for the Hénon map

#	x(#) norm	y(#) norm	*tbf* norm	SE $x(t)$	SE $y(t)$
2	0.000	0.000	0.28	0,076	0,076
5	0.624	0.596	0.63	0,000	0,001
8	1.000	1.000	1.00	0,000	0,000
11	0.601	0.497	0.58	0,001	0,006
14	0.674	0.526	0.50	0,030	0,001
17	0.796	0.905	0.83	0,001	0,006
20	0.788	0.502	0.65	0,019	0,022
23	0.846	0.897	0.80	0,002	0,009
26	0.376	0.443	0.45	0,005	0,000
forecast	0.682	0.610	MSE =	0.129	0.123

The two models provide the normalized time to the next failure, $x(48) = 0.682$ and $y(26) = 0.610$, which are then used to calculate $TR|_{95\%}$ = 40*(0.682-1.64*0.219) = 12.71 months and $TR|_{95\%}$ = 40*(0.610-1.64*0.219) = 10.03 months.

Table 5 summarizes the results of the study and evaluates the two spare parts policies.

Table 5 – Analysis of spare parts policies

Process and policy	NHPP	Logistic Map	Hénon x	Hénon y
$TR|_{95\%}$ (months) =	10.52	17.71	12.71	10.03
No spare part				
Major repairs (10 months)	safe	safe	safe	safe
New part (12 months)	unsafe	safe	safe	unsafe
One spare part				
Major repairs (10 months)	safe	safe	safe	safe
New part (12 months)	safe	safe	safe	safe

The first policy is to have no spare part, meaning that the company rotates five motors around four positions. The second policy is with one spare part, meaning that the company maintains an additional motor in the warehouse, thus being able to rely on six motors to provide four drives. The table shows that only the second policy is entirely safe. This conclusion does not necessarily imply the purchase of new equipment. For very short periods, the company can manufacture with only three drives. In this case, one of the two rolling mills must operate with only one drive, producing only heavy, rough sections.

Finally, regarding accuracy, Figure 3 shows the relationship between the models (logistic map, Hénon x, and Hénon y respectively) and the set of life data. All R^2 are near to 1, which means that the forecasting provided by the models can be reasonably considered for decisions on spare parts.

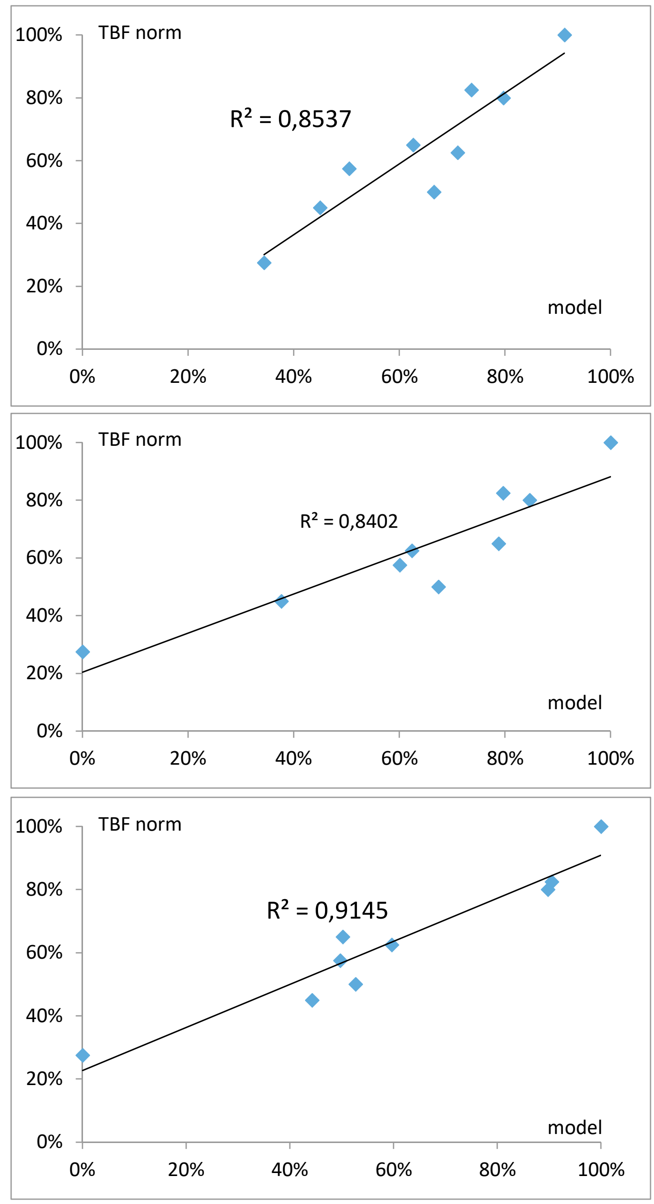

Figure 3 – Relationship between lifedata and the three models (logistic, Hènon x, Hènon y)

5. CONCLUSIONS

We have presented a modelling technique for the analysis of spare parts policies of items with a low consumption rate, specifically a set of five medium voltage motors that drive four rolling mills in a steelmaking plant. We found that, according to all the tested models, in addition to the five motors appropriated for the four drives, to ensure full safety the company needs an additional spare part. Alternatively, the company could reduce production for short periods with only three drives.

The study focused on the evaluation of a few policies in a system involving a few large parts that fail with low frequency. Larger and complex systems with imbricated mutual relationships usually provide much more lifedata. Therefore, the solution for the spare part problems by stochastic methods usually satisfies. Anyway, even a spare part replacement policy for larger, imbricated systems could benefit with chaotic methods, when the lifedata amount is not enough to produce suitable confidence intervals. It would be necessary to identify the main relationships among parts within the system and to produce a simulation model to verify the efficacy of different policies.

Further research shall bridge the gaps of this study, the use of a more robust optimizer, the use of more chaotic model, other models than the normal for the forecasting, and taking into account the stochastic nature of the *TTR* variable.

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