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Modelling production cost with the effects of learning and forgetting

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Abstract: Defining a dynamic model for calculating production cost is a challenging goal that requires a good fitting ability with real data over time. A novel cost curve is proposed here with the aim of incorporating both the learning and the forgetting phenomenon during both the production phases and the reworking operations. A single-product cost model is thus obtained, and a procedure for fitting the curve with real data is also introduced. Finally, this proposal is validated on a benchmark dataset in terms of mean square error.

Keywords: learning curves; production cost; autonomous learning; induced learning; curve fitting

1. INTRODUCTION

Experience is a concept that extends across a large pool of disciplines. For centuries, people have known that *repetita iuvant* ("repeating does good"), but only in recent decades have researchers attempted to translate this aphorism into mathematical models. Several experience curves have therefore been proposed in scientific and humanistic branches and nowadays these play a crucial role in industrial and social fields.

In the industrial field, learning curves are certainly correlated with the strategic dimension. Several authors have highlighted that learning curves represent sources of competitive advantage (e.g. Hatch et al., 2004), which means that a firm with a steeper learning curve than its competitors may gain a competitive advantage in the long term. Furthermore, learning curves may also drive medium to short-term decisions along tactical and operative dimensions respectively. Ergonomics (e.g. Anzanello and Fogliatto, 2011), assembly and production lines (e.g. Anzanello and Fogliatto, 2007; Anzanello and Fogliatto, 2010; Dolgui et al., 2012; Jaber and Glock, 2013; Otto and Otto, 2014; Pan et al., 2014), inventory control (e.g. Jaber and Guiffrida, 2008; Jaber et al., 2010; Teng et al., 2014; Khan et al., 2014), production planning (e.g. Glock et al., 2012) and quality improvement (e.g. Lolli et al., 2016) are some examples of the application of learning curves in different research fields.

Indeed, many learning phenomena may also simultaneously concur with a single dependent variable, as is evident in the case of total costs accounting such as total production cost. A complete, reliable and (ideally) easy-to-use total production cost model is certainly a challenging goal, and is necessarily related to model and fit with more simultaneous learning processes. The core of the present paper is thus to propose a total cost production model which embraces autonomous, induced and forgetting components, both in production and in reworking activities. In particular, reworking activities are considered here for the first time as affected by a learning phenomenon. Moreover, the dual source of experience, i.e. autonomous and induced, enriches the accounting model with the aim of making it both more flexible and suitable for application to several optimisation problems involving the proactive intervention of management. Induced learning activities such as training and investment strategies for improved productivity and quality should in fact be supported by a cost model that takes into account both sources of experience.

A four-parameter curve is achieved and a fitting procedure is proposed for establishing these parameters. To the best of the authors' knowledge, this is new in the field of learning curves and is assumed to have promising applications.

The remainder of the paper is organized as follows. Section 2 contains the notation adopted throughout the manuscript. Section 3 focuses on some relevant contributions in the field of learning curves. Section 4 presents the cost model, while Section 5 is devoted to the procedure for fitting the curve parameters. Section 6 reports the experimental analysis, and Section 7 closes the paper with some conclusions and the further research agenda.

2. NOTATION

c_{in}^{p} = initial unitary cost of production.
$\bar{c}_i^{f.int}$ = initial unitary cost of internal failure of type i.
$c_i^{f.ext}$ = unitary cost of external failure of type i.
c_{min}^{p} = minimum initial unitary cost of production.
$c_s^{p}(t)$ = minimum unitary cost of production in period t.
$\hat{c}_i^{f.int}(t)$ = minimum unitary cost of internal failure of type i in period t.
$Q_{cum}(t)$ = cumulative volume of production in period t.
$q_{cum}(t)$ = cumulative volume of failures in period t.

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$Q(t)$ = production volume in period t.
$q(t)$ = number of failures in period t.
Φ_i = likelihood of failure of type i.
$q_i(t) = q(t) \cdot \Phi_i$ = number of failures of type i in period t.
p_i = likelihood that a failure of type i is internal.
$(1 - p_i)$ = likelihood that a failure of type i is external.
$\hat{Q}(t) = Q_{cum}(t) - q_{cum}(t)$ = number of conforming products produced until period t.
$c^p(t)$ = unitary production cost in period t.
$c_i^{f.int}(t)$ = unitary cost of failure of type i in period t.
K = form factor of autonomous learning.
$\beta(t)$ = improvement factor in period t.
C_{TOT} = total cost (production cost + failure cost + prevention cost + appraisal cost).
α = parameter of forgetting in production.
γ = parameter of forgetting in reworking.
$t_{stop}^p(t)$ = number of consecutive periods without production until period t.
$t_{stop}^f(t)$ = number of consecutive periods without reworks for failure of type i until period t.
$P(t)$ = prevention cost in period t.
$A(t)$ = appraisal costs in period t.
$h(t)$ = number of training hours in period t.
c_{tr} = hourly training cost.
$C_m(t)$ = preventive maintenance cost in period t.
c_a = unitary appraisal cost.
$C^f(t)$ = failure cost in period t.

3. RESEARCH BACKGROUND

An early attempt to define a relationship between production volume and performance increase was made by Wright (1936) who introduced a mathematical model describing how an increase in performance is related to an increase in the production rate. The Wright learning curve is represented by the following log-linear model:

$$C_{TOT} = c_{in}^p \cdot x^b \quad (1),$$

where x is the number of units to produce and b represents the slope of the learning curve.

In subsequent years, a lot of alternative models were presented which may be clustered into two different categories depending on the number of independent variables, i.e. univariate and multivariate. The univariate models can in turn be clustered into three categories, i.e. log-linear, exponential and hyperbolic. For an in-depth explanation of these categories of learning models, the reader is referred to Anzanello and Fogliatto (2011) and Grosse et al. (2015).

Wright's model contains some significant drawbacks:

- If cumulative production goes to infinity, Wright's model is unreliable because it does not show any plateau effect, i.e. the total cost goes to zero if cumulative production tends to infinity. This is not possible because of fixed costs (Jaber and Glock, 2013);
- It supposes that production is defect-free, which is unrealistic (Jaber and Glock, 2013);
- It does not consider the forgetting component. In many processes the forgetting evaluation may be as important as the learning one;
- It deals only with autonomous learning, i.e. learning-by-doing. The induced component is not taken into account despite its relevance as competitive leverage;
- The prior experience in a task is neglected.

Many models have therefore been proposed for improving Wright's model. In particular, many models aim to solve the problem of prior experience, the most famous of which is the Stanford-B model:

$$C_{TOT} = c_{in}^p \cdot (x + B)^b \quad (2),$$

where B is the number of units previously produced. Although this model improves Wright's model, it still retains all the other drawbacks.

Another model has been specifically proposed with the goal of introducing the plateau effect; this consists of adding a constant (lower bound) to Wright's model as follows:

$$C_{TOT} = c_{min}^p + c_{in}^p \cdot x^b \quad (3)$$

In this case, if cumulative production goes to infinity, the total cost tends to c_{min}^p. However, this model still has all the other drawbacks of the original approach.

One of the first attempts to model the forgetting phenomenon was made by Carlson and Rowe (1976), who created a forgetting curve similar to Wright's learning curve. This approach was validated some years later by Globerson et al. (1989), whose empirical finding is that the log-linear model describes better than others both the workers' forgetting and the learning phenomenon. Carlson's forgetting model is as follows:

$$\hat{C}_{TOT} = \hat{c}_{in}^p \cdot x^f \quad (4),$$

where $\hat{C}_{TOT}$ is the cost for the xth unit of lost experience of the forgetting curve, $\hat{c}_{in}^p$ is the cost for the first unit of the forgetting curve, x is the number of units that would have been produced if production had not stopped, and f is the slope of the forgetting curve. Alternative forgetting curves have been proposed by Jaber and Bonney (1996) and Tarakci et al. (2013). In particular, the former integrates Wright's learning curve with Carlson's forgetting curve, leading to the first learn-forget curve, but the other drawbacks still remain.

For a long time most of the proposed models retained the strong hypothesis of defect-free processes. The quality-based element in the learning curves was firstly introduced by Jaber and Guiffrida (2004), who proposed two different cases. The first one extends Wright's law with the hypothesis that the process is not defect-free, but the workers do not learn by reworking:

$$C_{TOT} = c_{in}^p \cdot x^b + c_i^f \cdot \rho \cdot x \quad (5),$$

where ρ is the likelihood that a process goes out of control and c_i^f is the unitary failure cost. Conversely, the second case allows the workers to learn by reworking as follows:

$$C_{TOT} = c_{in}^p \cdot x^b + 2c_i^f \cdot \left(\frac{\rho}{2}\right)^{1-\varepsilon} \cdot x^{1-2\varepsilon} \quad (6),$$

where ε is the learning exponent for reworks. However, this model does not distinguish between internal and external failures. Furthermore, an item may show different kinds of failures, and thus requires different reworks.
A topic strictly related with modelling is the curve fitting. Bailey and McIntyre (1997) explored the relationship between the form of the learning curve and the quality of fit. They demonstrated that a well-fitting learning curve does not necessarily provide the best predictions. Additionally, they discovered that the so-called Log-Log model shows the best predictive ability with respect to other learning curves.
A lot of fitting approaches have been proposed over the decades. The reader is referred to Daneman (1988) for mono-parameter curves, and Wang and Yu (2011) and Motlagh et al. (2013) for multi-parameter curves. Nevertheless, the most common approach is Mean Squared Error (MSE) minimization, which has been adopted, among others, by Jaber and Glock (2013) to set the parameters of two learning curve models, i.e. Wright's model (1936) and that provided by Dar-El et al. (1995).
In this paper, a comprehensive cost model is introduced with the aim of overcoming the aforementioned drawbacks of Wright's curve, and a procedure for fitting this curve is also proposed.

4. THE COST MODEL

Before explaining the model, it is mandatory to specify that it is built on a single-item by supposing a negligible correlation among the items. Moreover, the likelihood of a failure occurring is supposed to be independent of another one occurring.
The cost model is as follows:

$$C_{TOT} = \sum_t (Q(t) \cdot c^p(t) + C^f(t) + P(t) + A(t)) \qquad (7)$$

The terms included in (7) are subsequently defined.
$c^p(t)$ is given by:

$$c^p(t) = c_{in}^p - \left(c_{in}^p - c_s^p(t)\right) \cdot log_K(\hat{Q}(t) + 1) \qquad (8),$$

where c_{in}^p is the initial unitary cost of production, and $c_s^p(t)$ is the minimum unitary cost of production in period t, which represents the plateau of the learning curve and may be defined as follows:

$$c_s^p(t) = c_s^p(t-1) \cdot (1 - \beta(t)) \cdot (1 + \alpha \cdot t_{stop}^p(t)) \qquad (9),$$

This is time-dependent, increases in a disruption period, i.e. without production leading to the forgetting phenomenon, and decreases if an improvement action is undertaken. The term $\left(1 - \beta(t)\right)$ represents in fact the induced learning; if an improvement action is undertaken in period t, the improvement parameter $\beta(t)$ is non-zero, $\left(1 - \beta(t)\right)$ assumes a value between 0 and 1, and thus the minimum unitary cost of production of period t will be smaller than that of period $(t-1)$. On the contrary, the term $(1 + \alpha \cdot t_{stop}^p(t))$ represents the forgetting phenomenon. In particular it is assumed that the workers' forgetting increases proportionally (parameter α) with the number of disruption periods. This is supported by literature (e.g. Globerson et al., 1989) in which it has been found that human forgetting depends both on the break length, and on the level of experience gained before the break. This postulate on human forgetting is ensured by (9), where the forgetting term depends both on the break length (t_{stop}^p) and on the minimum unitary cost of production of the previous period ($c_s^p(t-1)$), which in turn is also affected by the experience gained before the break. To return to $c^p(t)$, the difference between the initial and the minimum unitary cost of production provides the maximum improvement that may be gained in period t. In fact, if the amount of conforming units produced until period t (i.e. $\hat{Q}(t)$) rises up to K, the minimum production cost is reached. The logarithmic base is represented by the form factor K, which is set according to the learning ability of the firm. Every individual in fact shows a different learning skill in relation to all other individuals, as well as every team (being made up of individuals). The bigger this form factor is, the slower the cost reduction, and vice versa.
$C^f(t)$ is the failure cost in period t and is defined as follows:

$$C^f(t) = \sum_i \left(c_i{}^{f.int}(t) \cdot q(t) \cdot \Phi_i \cdot p_i + c_i^{f.ext} \cdot q(t) \cdot \Phi_i \cdot \cdot (1 - p_i)\right) \qquad (10),$$

where $c_i{}^{f.int}(t)$ is defined as:

$$c_i{}^{f.int}(t) = \bar{c}_i^{f.int} - \left(\bar{c}_i^{f.int} - \hat{c}_i^{f.int}(t)\right) \cdot \log_K(q_{cum}(t) \cdot \Phi_i \cdot \cdot p_i + 1) \qquad (11)$$

As well as (9), $\hat{c}_i^{f.int}(t)$ may be defined as a dynamic plateau:

$$\hat{c}_i^{f.int}(t) = \hat{c}_i^{f.int}(t-1) \cdot (1 - \tfrac{\beta(t)}{2}) \cdot (1 + \gamma \cdot t_{stop}^f(t)) \qquad (12)$$

The difference between (9) and (12) is that in (12) only the half of the improvement factor has been used in the induced learning term. This choice is driven by the fact that there are some improvement actions which affect both the production and the reworking activities (e.g. machinery changes, reduction of product complexity and so on), while others affect only the production (e.g. training activities focused on production). It seems therefore reasonable to apply only half of the improvement parameter for reworking.
Similarly, $c_i^{f.ext}$ is the unitary cost of an external failure of type i but, for the sake of simplicity, it is not expected to be time-varying. It is worth underling the complexity of defining the external cost due to certain non-quantitative factors, e.g. customer and reputation losses.
The breakdown between internal and external costs allows us to generate an accounting model that complies with the cost of quality model proposed by Feigenbaum (1956).
$P(t)$ is the prevention cost:

$$P(t) = h(t) \cdot c_{tr} + C_m(t) \qquad (13),$$

where $(h(t) \cdot c_{tr})$ represents the amount paid for training activities in period t, and $C_m(t)$ is the cost of preventive maintenance. Other cost items related to preventive activities could be introduced into (13).
Finally, $A(t)$ is the appraisal cost:

$$A(t) = \left(\frac{q_{cum}(t)}{Q_{cum}(t)} \cdot Q(t)\right) \cdot c_a \quad (14)$$

The appraisal cost is thus expressed as the product of the unitary appraisal cost (i.e. c_a) with the number of items to inspect. This quantity is given by the amount of conforming units produced in period t multiplied for a percentage (i.e. $\frac{q_{cum}(t)}{Q_{cum}(t)}$), which is in turn estimated as the ratio between the cumulated number of non-conforming units and the cumulated number of produced items. In this way, if the process is improved, the number of non-conforming units decreases, as does this ratio. Consequently, the number of products to inspect decreases, as would naturally be expected. Equation (7) may therefore be rewritten as follows:

$$C_{TOT} = \sum_t \left(Q(t) \cdot \left[c_{in}^p - \left(c_{in}^p - \left(c_s^p(t-1) \cdot (1-\beta(t)) \cdot \cdot \left(1 + \alpha \cdot t_{stop}^p(t)\right)\right)\right) \cdot \log_K\left(\hat{Q}(t)+1\right) \right] + + \sum_i \left(\left[\bar{c}_i^{f.int} - \left(\bar{c}_i^{f.int} - \left(\hat{c}_i^{f.int}(t-1) \cdot \left(1 - \frac{\beta(t)}{2}\right) \cdot \cdot \left(1 + \gamma \cdot t_{stop}^f(t)\right)\right)\right) \cdot \log_K(q_{cum}(t) \cdot \Phi_i \cdot p_i + 1) \right] \cdot \cdot q(t) \cdot \Phi_i \cdot p_i + c_i^{f.ext} \cdot q(t) \cdot \Phi_i \cdot (1-p_i) \right) + + \left(h(t) \cdot c_{tr} + C_m(t)\right) + \left(\frac{q_{cum}(t)}{Q_{cum}(t)} \cdot Q(t)\right) \cdot c_a \right) \quad (15)$$

5. THE CURVE FITTING

Equation (15) is a four parameter curve, and the best fit for a series of data points has to be searched. These parameters are:

- The form factor K;
- The parameter of forgetting in production α.
- The parameter of forgetting in reworking γ;
- The improvement factor $\beta(t)$;

An empirical procedure of curve fitting is now presented, which consists in applying the Mean Square Error (MSE) once for every unknown parameter.

Starting with the K estimation, the procedure consists of these steps:

1) From $t = 0$, find the first period t' in which:
   a) $Q(t') \neq 0$;
   b) $q(t') \neq 0$;
   c) No improvement action has been undertaken ($\beta(t') = 0$).

2) Put $t_0 = t'$.

3) Determines the first time period t'' subsequent to t_0 in which almost one of conditions at point 1 is not respected.

4) Put $t_n = t'' - 1$.

5) Solve the following MSE problem:

$$Minimize \frac{1}{t_n - t_0} \cdot \sum_{t=t_0}^{t_n} MSE(t) \quad (16)$$

where:

$$MSE(t) = \left[c_{true}^p(t) - \left(c_{in}^p - \left[c_{in}^p - c_s^p(t)\right] \cdot \log_K\left(\hat{Q}(t) + +1\right)\right)\right]^2 \quad (17)$$

In order to solve this non-linear optimization problem, the Generalized Reduced Gradient Method (Lasdon et al., 1974) is adopted.

To estimate α, these steps are followed:

1) From $t = 0$, find the first period t' in which:
   a) $Q(t') = 0$;
   b) $q(t') = 0$;
   c) No improvement action has been undertaken ($\beta(t') = 0$).

2) Put $t_0 = t'$.

3) The unitary production cost in t_0 is as follows:

$$c^p(t_0) = c_{in}^p - \left(c_{in}^p - c_s^p(t_0 - 1) \cdot (1 - \beta(t_0)) \cdot (1 + \alpha \cdot \cdot t_{stop}^p(t_0))\right) \cdot \log_K\left(\hat{Q}(t_0) + 1\right) \quad (18)$$

The term $\beta(t_0)$ is null, therefore (19) may be simplified:

$$c^p(t_0) = c_{in}^p - \left(c_{in}^p - c_s^p(t_0 - 1) \cdot (1 + \alpha \cdot t_{stop}^p(t_0))\right) \cdot \cdot \log_K\left(\hat{Q}(t_0) + 1\right) \quad (19)$$

All the terms into (19) are known with the exception of $c_s^p(t-1)$, which can be calculated by using the expression of unitary production cost in period $(t_0 - 1)$:

$$c^p(t_0 - 1) = c_{in}^p - \left(c_{in}^p - c_s^p(t_0 - 1)\right) \cdot \log_K\left(\hat{Q}(t_0 - 1) + +1\right) \quad (20)$$

Hence:

$$c_s^p(t_0 - 1) = \frac{c^p(t_0-1) - c_{in}^p}{\log_K(\hat{Q}(t_0-1)+1)} + c_{in}^p \quad (21)$$

By replacing (21) into (18):

$$c^p(t_0) = c_{in}^p - \left(c_{in}^p - \left(\frac{c^p(t_0-1) - c_{in}^p}{\log_K(\hat{Q}(t_0-1)+1)} + c_{in}^p\right) \cdot (1 + \alpha \cdot \cdot t_{stop}^p(t_0))\right) \cdot \log_K\left(\hat{Q}(t_0) + 1\right) \quad (22),$$

and the unknown parameter α is given by:

$$\alpha = \frac{\dfrac{\frac{c^p(t_0) - c_{in}^p}{\log_K(\hat{Q}(t_0)+1)} + c_{in}^p}{\frac{c^p(t_0-1) - c_{in}^p}{\log_K(\hat{Q}(t_0-1)+1)} + c_{in}^p} - 1}{t_{stop}^p(t_0)} \quad (23)$$

The parameter of forgetting in reworking, i.e. γ, may be calculated in the same way, with the difference that the equation of the unitary cost of internal failure (11) is used instead of the equation of the unitary cost of production (8). The parameter γ is therefore calculated as follows:

$$\gamma = \frac{\dfrac{\frac{c_i^{f.int}(t_0) - \bar{c}_i^{f.int}}{\log_K(q_{cum}(t_0) \cdot \Phi_i \cdot p_i + 1)} + \bar{c}_i^{f.int}}{\frac{c_i^{f.int}(t_0-1) - \bar{c}_i^{f.int}}{\log_K(q_{cum}(t_0-1) \cdot \Phi_i \cdot p_i + 1)} - \bar{c}_i^{f.int}} - 1}{t_{stop}^f(t_0)} \quad (24)$$

Finally, in order to estimate $\beta(t)$, for each period t in which an improvement action has been undertaken, $\beta(t)$ is not null and thus has to be estimated. For each of these periods the following method is adopted.
Let t' be a period in which there is an improvement. Equation (8) in this period is:

$$c^p(t') = c_{in}^p - \left(c_{in}^p - c_s^p(t'-1)\cdot(1-\beta(t'))\cdot(1+\alpha\cdot t_{stop}^p)\right)\cdot \log_K(\hat{Q}(t')+1) \quad (25),$$

thereby:

$$\beta(t') = 1 - \frac{\frac{c^p(t')-c_{in}^p}{\log_K(\hat{Q}(t')+1)}+c_{in}^p}{c_s^p(t'-1)\cdot(1+\alpha\cdot t_{stop}^p)} \quad (26)$$

From (8) in $(t'-1)$, it follows that:

$$c_s^p(t'-1) = \frac{c^p(t'-1)-c_{in}^p}{\log_K(\hat{Q}(t'-1)+1)} + c_{in}^p \quad (27)$$

By replacing (28) into (27):

$$\beta(t') = 1 - \frac{\frac{c^p(t')-c_{in}^p}{\log_K(\hat{Q}(t')+1)}+c_{in}^p}{\left(\frac{c^p(t'-1)-c_{in}^p}{\log_K(\hat{Q}(t'-1)+1)}+c_{in}^p\right)\cdot(1+\alpha\cdot t_{stop}^p)} \quad (28)$$

6. EXPERIMENTAL ANALYSIS

In order to validate our proposal, the laboratory study performed by Bailey (1989) was used. In this laboratory study, the unitary production times of an Erector Set toy were collected from a cluster of 35 operators involved for four or eight consecutive hours in assembly and disassembly tasks. In order to smooth the outliers, the mean times spent by operators for a unit are used instead of the times of single operators.
However, due to the unavailability of data on disruptions, reworks, allocation of training hours and unitary costs, only (8) has been validated in terms of assembly task times. For the purposes of benchmarking, the fitting ability of (8) in terms of MSE was then compared with that achieved by means of Wright's curve, being nowadays one of the most widely used approaches for estimating the cost (and time) reduction with the volume increase.
Table 1 shows the real data, along with the times obtained by applying both the new and Wright's model. In particular, an initial unitary time of production is required, which has been set to 25 minutes/unit for both models. Hence, it is possible to apply the fitting approach proposed in Session 5 for estimating the form factor K.
Table 2 shows the MSE reached by the two models under comparison, along with their best-fitting form factors, K and b respectively. The MSE reduction confirms the satisfactory fitting ability of (8).

Table 1. Times calculated using the new model and Wright's model

Unit	Real time [min]	New model [min]	Wright's model [min]
1	23.421	25	25
2	15.078	16.391	17.152
3	12.698	14.137	13.759
4	11.746	12.388	11.768
5	10.826	10.959	10.423
6	10.583	9.751	9.440
7	10.010	8.705	8.681

Table 2. Form factors and MSE achieved by the models

	Form factor	MSE
New model	25.301 (K)	1.336
Wright's model	-0.544 (b)	1.594

Further conclusions would have been drawn if all data into (15) were available. This could be part of a further research agenda.

7. CONCLUSIONS

There are different motivations behind the introduction of new accounting models for production cost. First, to the best of our knowledge, the literature lacks comprehensive models able to deal with learning and forgetting both in production and in reworking operations. Moreover, the most adopted learning curve is that proposed by Wright (1936). While on the one hand this suffers from several drawbacks, on the other hand alternative models do not show the plateau effect.
In this paper, a new learn-forget model for production cost accounting has been proposed whose main features are: i) the presence of the plateau; ii) the forgetting part depending on both the length of the disruption and the experience previously gained, and iii) the dual sources of experience, i.e. autonomous and induced.
However, it is worth noting that this model requires a lot of data, and thus it is expected to be suitable for firms with highly structured processes for data collection. In fact, the unavailability of data meant that we were unable to fully validate the model. A laboratory study on the fitting ability of the model is thus encouraged. Future research might also be directed at expanding the model to the multi-item case, as well as applying it to some strategic, tactical, and operative decisions.

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