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Nonlinear Dynamics of Pre-Compressed Circular Cylindrical Shell Under Axial Harmonic Load: Experiments

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Abstract
The nonlinear dynamics of circular cylindrical shells under combined axial static and periodic forces are experimentally investigated. The goal is to study the dynamic scenario and to analyze non- linear regimes. The linear behavior under static preload is analyzed by means of the usual modal testing techniques. The complex dynamics arising when a periodic axial load excites the first axisymmetric mode are analyzed by means of amplitude frequency diagrams, waterfall spectrum diagrams, bifurcation diagrams of Poincaré maps, time histories, spectra, phase portraits.

Keywords
Circular cylindrical shell, experiments, non-linear dynamics, chaos

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Introduction

Shells usually exhibit a complicated dynamic behavior because the curvature strongly couples together the flexural and in-plane deformations, and the three displacement and three rotation fields simultaneously appear in each of the governing partial differential equations and boundary conditions (for thin shells rotations are dependent on the displacements, so they are not real unknowns in the governing equations). Therefore, it is clear that the axial constraints can have direct effects on a predominantly radial modes. For instance, it has been shown that the natural frequencies for the circumferential modes of a simply supported circular cylindrical shell can be noticeably modified by the constraints applied in the axial direction.

The high modal density of shell structures can cause resonances and consequently high amplitude vibrations. It is well known that moderately high amplitudes of vibration in shells give rise to strong nonlinear phenomena such as: super and sub harmonic response, quasi-periodic motion, chaos. This is well documented by a large scientific literature, mostly focused on modeling and much less on experimentation. For a comprehensive literature analysis the reader is referred to Refs. [1–8] where very interesting reviews of the western and eastern literature can be found, as well as details regarding theories and the most representative experimental findings.

In the following some works strictly related to the present paper are commented.

In 2013 Strozzi and Pellicano [9] analyzed the nonlinear vibrations of functionally graded (FGM) circular cylindrical shells. Pellicano and Avramov [10] published a paper concerning the nonlinear dynamics of a shell carrying a top disk with base excitation. In 2012 Pellicano [11] presented an early experimental study on circular cylindrical shells under base excitation, more recent experimental results are presented in [12]. A theoretical model is developed to reproduce the experimental evidence and provide an explanation of the complex dynamics observed experimentally [13–15].

In the present paper, a setup for testing a circular cylindrical shell under static axial (compressive) load is described. The linear dynamics of the shell under preload is investigated by means of impact testing, while the non-linear behaviors under combined static and dynamic axial loads are pointed out.

1. The experiment

The system under investigation consists of a circular cylindrical shell, made of aluminium, clamped at both base and top ends by means of two rings at two rigid supports (see Figure 1a.1).The geometric and physical parameters of the circular cylindrical shell are reported in tab. 1. The bottom support is an aluminum alloy thick circular disk rigidly bolted to the shaker. The top disk is connected to the frame by means of a dynamic load cell, a stinger, and a static load cell. The stinger is introduced in order to reduce the effects of misalignments. A laser vibrometer is used to measure the lateral vibration of the side of the shell and its output is routed both to the spectrum analyser and to the shaker controller. The control system is open- loop in order to avoid control instabilities induced by the nonlinear behavior of the system.

Shell length	L	$0.117m$
Shell thickness	h	$0.15 \cdot 10^{-3}m$
Mean radius	R	$32.9 \cdot 10^{-3}m$
Density	ρ	$2796\frac{kg}{m^3}$
Young's modulus	E	$71.02 \cdot 10^9\frac{N}{m^2}$
Poisson's ratio	ν	0.31
Static preload	P_0	$0N - 250N$

Table 1. Physical data

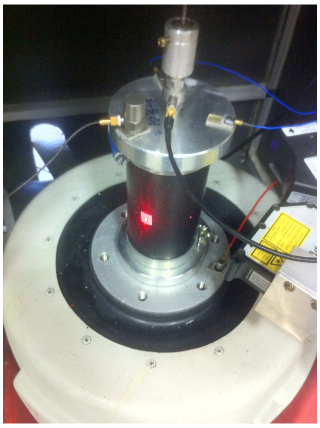


Fig. 1 The experiment

The specimen consists of a thin walled circular cylindrical aluminum shell. The specimen is crucial to have repeatable test: for this reason a standard procedure for preparing new specimens in case of damage has been established. The production of the specimen starts from a standard beverage aluminum canister, which must be suitably cut to obtain the desired dimension and eliminate the end diaphragms. The first step consists of refilling the empty can of distilled water and put it in a climatic chamber at −10oC for two hours. After that, the frozen can is cut at the required length without introducing any deformation. At the end, the specimen is rigidly connected to the two aluminum disks by means of two hose clamps interjected with a metallic band.

2. Modal analysis

A preliminary experimental modal analysis has been carried out for better understanding the shell behaviour and for identifying the natural frequencies, damping ratios and modal shapes of the modes. Modal analyses have been performed in the frequency band between 1100 and 2000Hz, with three different levels of preload: 0N, 50N and 100N. In the case of 0N preload, a vertical series of 11 points is considered along the length of the specimen as well as a horizontal series of 20 point along the circumference, exactly at one half of the length. For 50N and 100N static compressive load analyses, two circumferential series of 20 points of measurement has been added at one-fourth and three-quarters of the shell length. In tab. 2 the measured natural frequencies are reported together with the

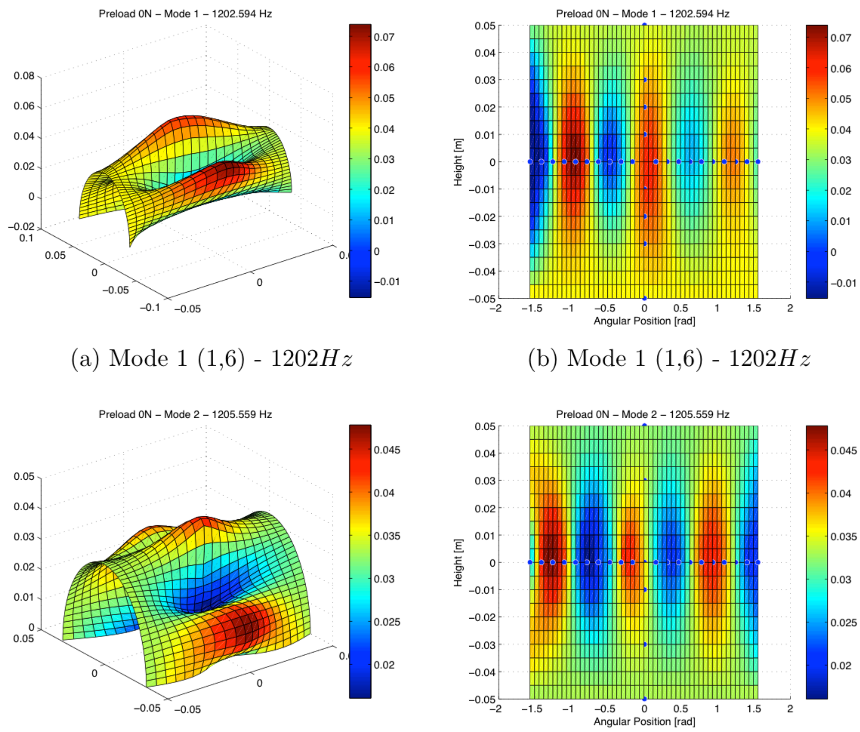
description of the corresponding modal shapes. In this table m represents the number of axial half waves, while n is the number of nodal diameters.

It can be noted the presence of double modes, identified by the same pair of numbers (m, n); these modes should have theoretically the same frequency. In the real case, due to the presence of imperfections, double modes generally split into two modes having the same shape (shifted of a quarter of period circumferentially) and similar frequency. For some modes it was impossible to find experimentally the two conjugate (double) modes, this happens typically when the frequency splitting is not

m	n	0N	50N	100N
1	6	1202	1198	1192
1	6	1205	1203	1196
1	7	1244	1238	1230
1	7		1249	1242
1	5	1391	1390	1384
1	5			
1	8	1456	1450	1443
1	8	1471	1456	1453
1	9	1754	1747	1741
1	9	1760	1755	1749
1	4	1866	1865	1861
1	4	1896	1893	1888

Table 2. Natural frequencies

enough to distinguish the conjugate modes.



(a) Mode 1 (1,6) - 1202Hz (b) Mode 1 (1,6) - 1202Hz

(c) Mode 2 (1,6) - 1205Hz (d) Mode 2 (1,6) - 1205Hz

Figure 2. Mode shapes

3. Nonlinear Dynamics

Since the occurrence of nonlinear effects due to the high lateral displacement of the shell is expected, it is important to choose a suitable test strategy. The peculiar feature of the proposed approach is that a smooth sine step test is applied. In order to study the behavior of the system at varying excitation frequency, the following procedure is used:

- test is started at the initial frequency f0 and a certain number of excitation periods at f0 are sent to shaker;
- only the last periods at f0 are stored for performing analyses (regime solution);
- frequency is varied from f0 to f1 = f0 + Δf using a sine sweep;
- the signal is smoothly connected with a new signal at f1 frequency and a certain number of periods are sent to the shaker at f1;
- only the last periods at f1 are stored for performing analyses (regime solution);
- frequency is varied again at f2 = f1 + 2Δf using a sine sweep, and this procedure is iterated up to the last frequency.

The whole procedure is performed with an iterative control on the output signal (in Volts), in order to ensure that output signal is synchronous with respect to the measured signals. In the proposed experiments, the sine step excitation range goes from 1100Hz to 2000Hz, with a step frequency of 1.0 Hz. For some special region, a finer step of 0.1Hz is used. Each run starts at the highest frequency (2000Hz) towards the lower frequency (1100Hz) and then it goes back to the highest frequency. Several excitation levels are considered in terms of voltage amplitude applied to the shaker amplifier. Amplitude range is between 0.1 and 1V, with 0.1 V increment.

Different axial pre-loads are considered: P0 = 0N, P0 = 100N and P0 = 250N (max shaker load capacity), for the sake of brevity, here only the latter tests are reported.

1.1 Tests at 250N preload

At the maximum preload, a lot of violent phenomena occur: for better describing the most interesting nonlinear behaviors of the shell under this condition, several tests have been performed.

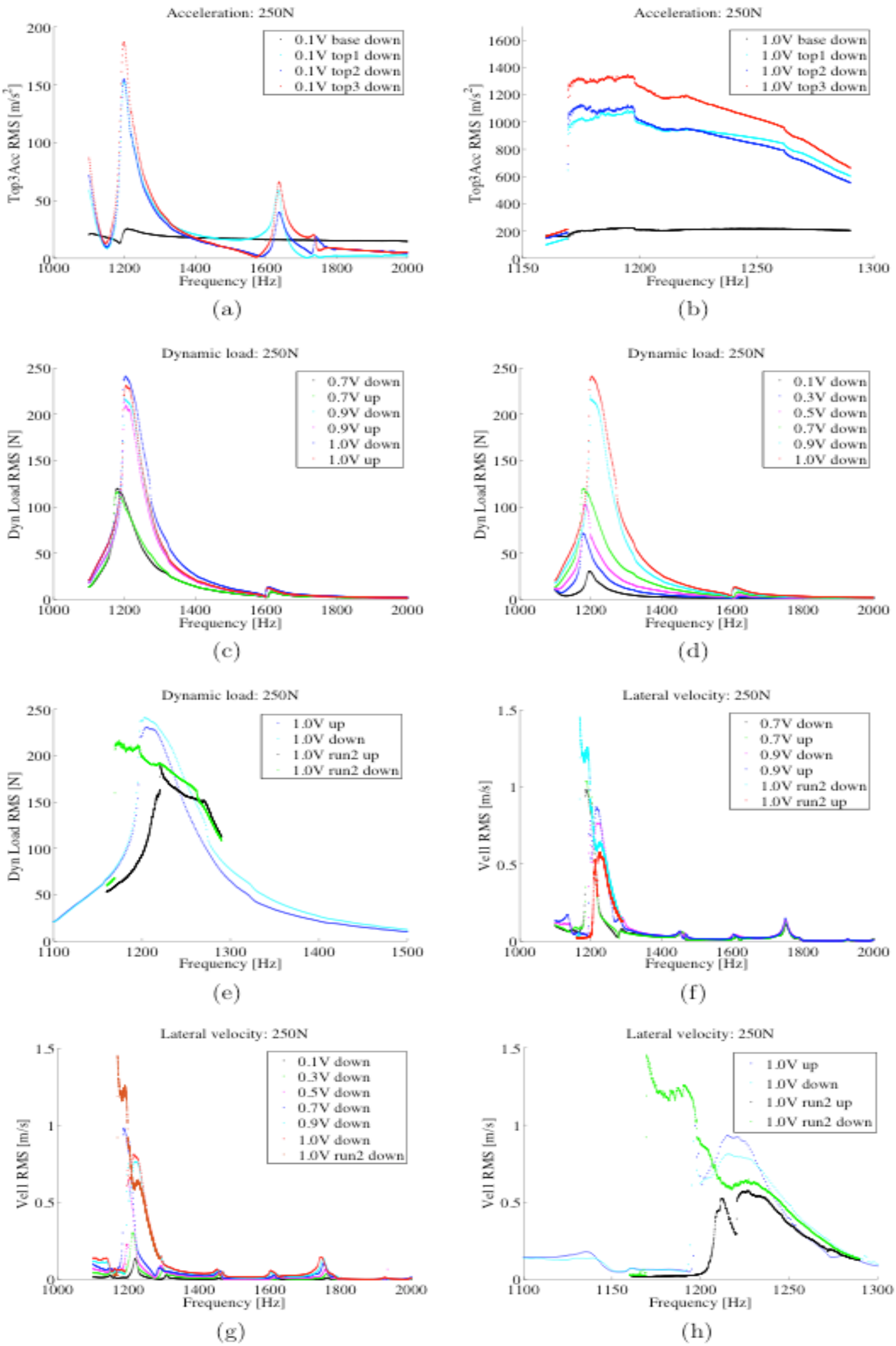


Figure 3. Amplitude-frequency diagrams at 250N preload: (a,b) acceleration; (c-e) dy- namic load; (f-h) lateral velocity

In particular, downwards and upwards tests have been repeated increasing the frequency resolutions up to 0.1Hz and time of acquisition up to 1500 periods for each frequency tested (in the range 1160-1290Hz), these refined tests are identified as run 2 (both upwards and downwards). The top acceleration, shows regular resonances at 0.1V drive (fig. 3a), whereas for the drive voltage of 1V (fig. 3b) close to 1200Hz a saturation appears and the amplitude remains almost constant up to 1170Hz, then suddenly the top vibration drops down to the level of the base vibration. This is a clear symptom of a strong nonlinear modal interaction.

Looking the dynamic axial load (Figure 3c), the previous conjectures are confirmed; both up and down tests give similar results and no jumps are observed. Close to 1200Hz one finds the main resonance, it is to note that the maximum amplitude of axial load is not proportional to the drive, this is unexpected; comparing also Figure 3d) one can see that far from the resonance at 1200Hz the axial load amplitude is proportional to the drive, but close to such resonance up to 0.7V drive one see a regular grows, then a huge increment is observed when the drive is increased to 0.9V.

The second test (run 2) is carried out with a reduced drive frequency step (0.1Hz), thus allowing for a better following of complex phenomena (see Figure 3e run2); indeed, in a narrow frequency band close to 1200Hz a saturation phenomenon is visible on the dynamic load. The two tests at 1Hz and 0.1Hz give different pictures of the dynamics as two stable attractors coexist.

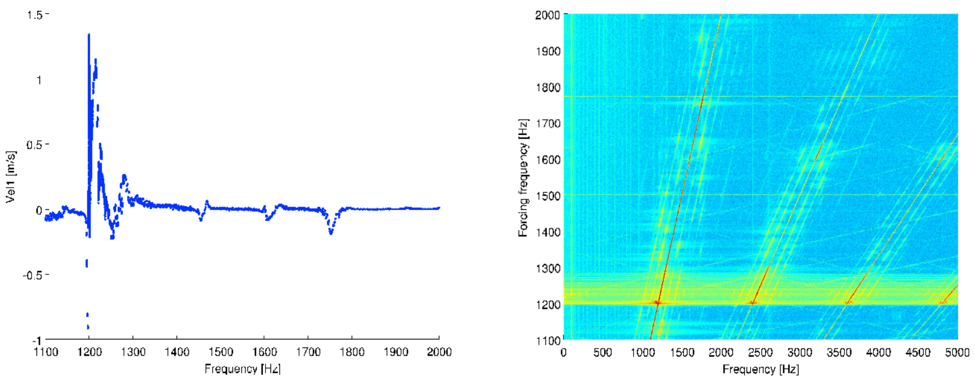


Figure 4. Bifurcation diagram and Waterfall diagram at 0.9V downwards

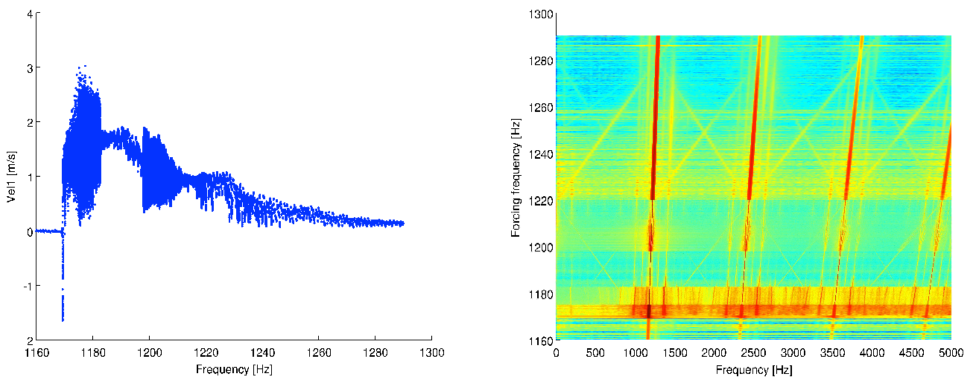
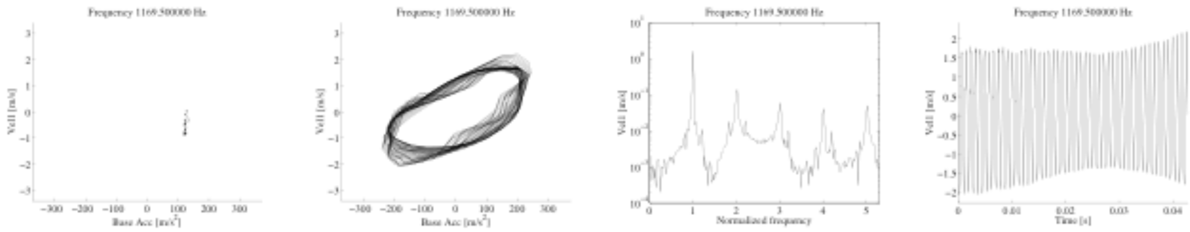


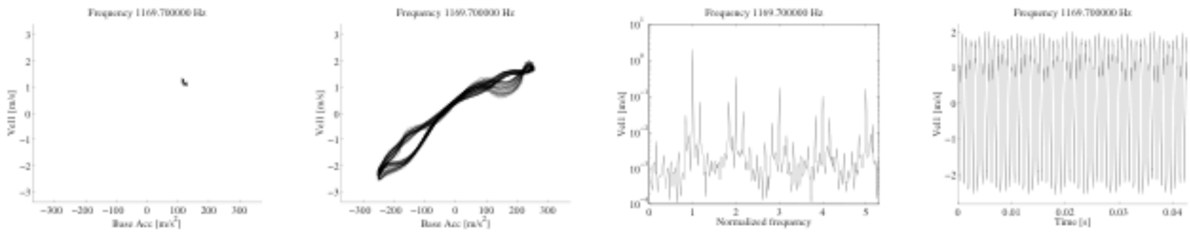
Figure 5 Bifurcation diagram and Waterfall diagram at 1.0V downwards

For the case of 250N preload, the bifurcation diagrams and the waterfall spectrum diagrams are shown in figs.4,5. Looking to these diagrams, it is possible to notice that the response is periodic until 0.9V. For 0.9V amplitude the bifurcation presents a single vertical line close to 1200Hz; at the same frequency the waterfall shows a broad band response with a clear presence of sidebands, thus suggesting the arising of a quasiperiodic motion.

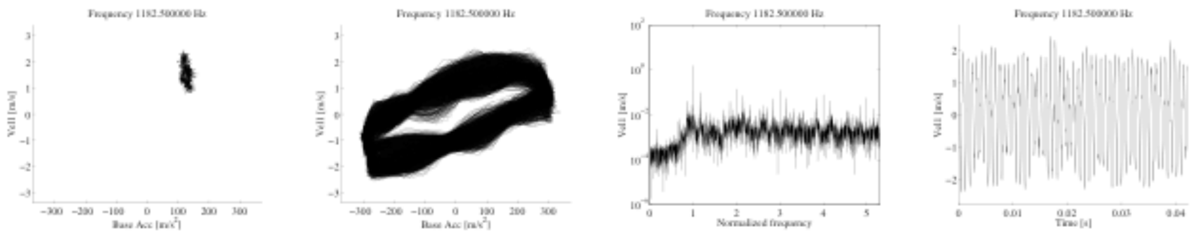
The additional test (run 2), performed at an amplitude of 1V and with a refined 0.1Hz step, is extremely interesting; the response is different with respect to the run performed with 1Hz step. In particular, in the bifurcation diagram (fig. 4) there are at least two regions with non stationary response, while for the fast run there was only a narrow non stationary band at 1200Hz with 1Hz width. In the present case, the non-stationary response bands are 1170-1182.5Hz and 1198-1210Hz. Looking at fig. 4 it can be clearly noticed that the lowest non-stationary region (1170-1183Hz) is split into two sub- regions, since at 1175Hz in the downwards run the complexity of the system increases moving to a full spectrum.



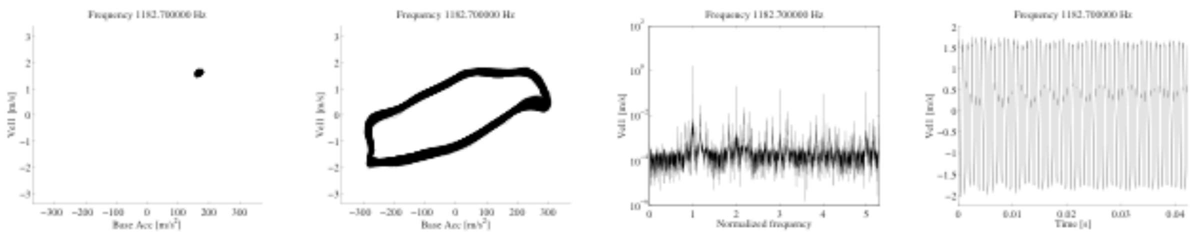
i) $250N$ $1.0V$ DW run 2 - $1169.50Hz$



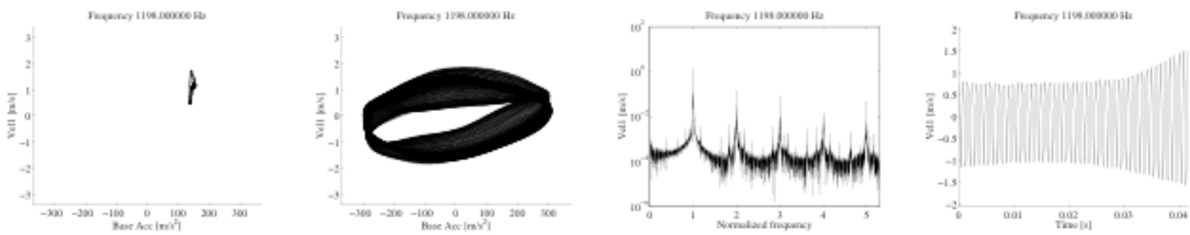
ii) $250N$ $1.0V$ DW run 2 - $1169.70Hz$



iii) $250N$ $1.0V$ DW run 2 - $1182.50Hz$



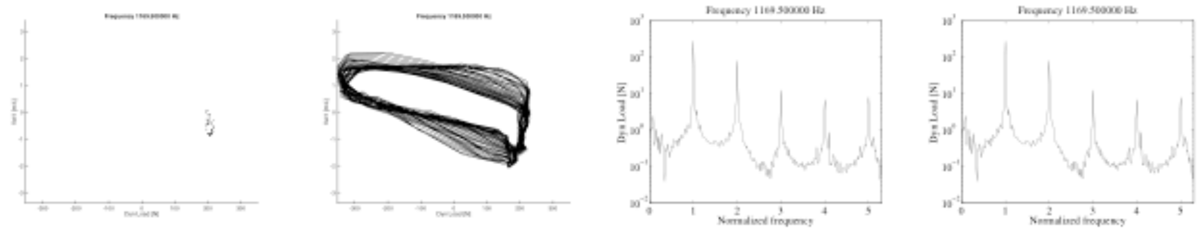
iv) $250N$ $1.0V$ DW run 2 - $1182.70Hz$



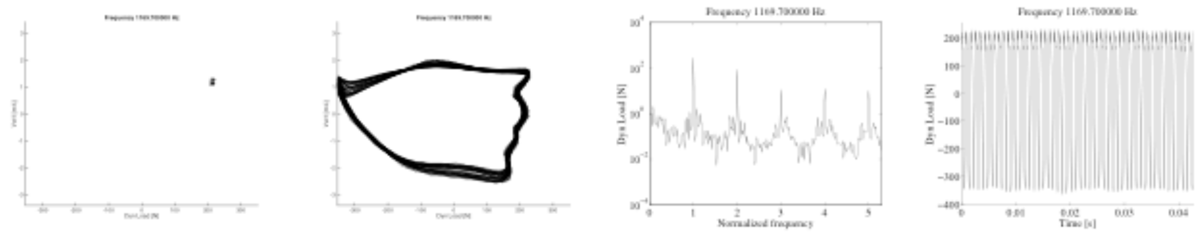
v) $250N$ $1.0V$ DW run 2 - $1198.00Hz$

Figure 6. Base acceleration - Lateral velocity: Poincaré maps, phase diagrams, spectrum and time history at 250N 1.0V run 2 downwards

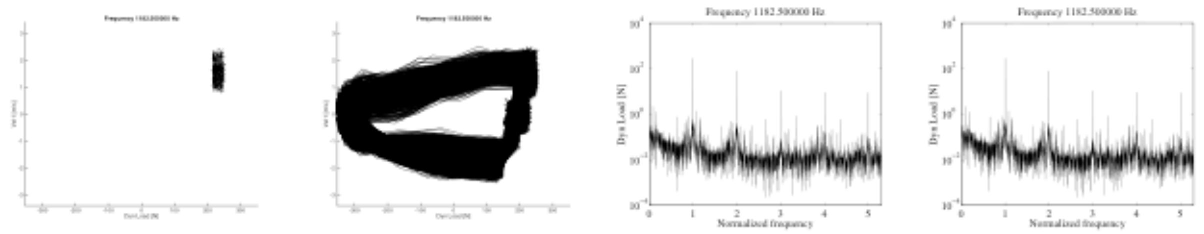
Now some details of the dynamics are analyzed more deeply. At 1169.50Hz the response is quasiperiodic, as confirmed by the Poincaré map both in the (base acceleration, lateral vibration) and in (dynamic axial load, lateral vibration) plane, see fig. 6 and 7. At 1182.50Hz the system presents a chaotic response, with a fractal Poincaré map. At higher frequency, 1182.70 the response is stationary again, as confirmed by the Poincaré map and the phase portrait.



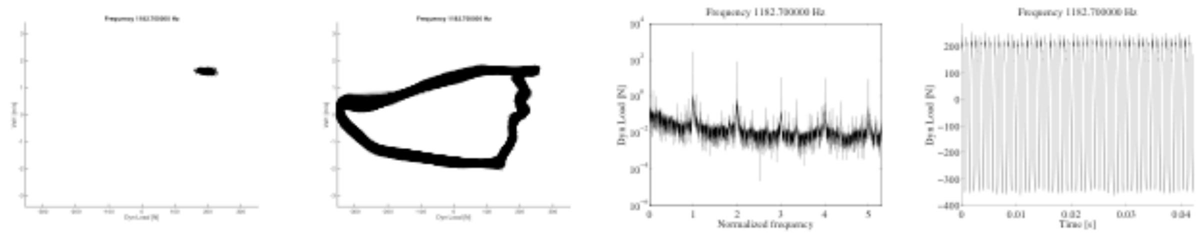
i) $250N$ $1.0V$ DW run 2 - $1169.50Hz$



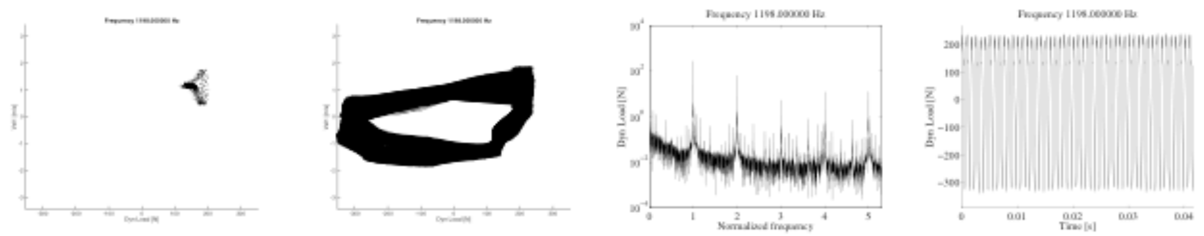
ii) $250N$ $1.0V$ DW run 2 - $1169.70Hz$



iii) $250N$ $1.0V$ DW run 2 - $1182.50Hz$



iv) $250N$ $1.0V$ DW run 2 - $1182.70Hz$



v) $250N$ $1.0V$ DW run 2 - $1198.00Hz$

Figure 7: Dynamic load - Lateral velocity: Poincar´e maps, phase diagrams, spectrum and time history at 250N 1.0V run 2 downwards

The second non stationary region starts at 1198Hz with a quasiperiodic response, clearly visible in the Poincaré maps in figg. 6 and 7. Note that on the (dynamic axial load, lateral vibration) plane the response can look similar to a chaotic attractor, but it is more likely to be a closed curve in the Poincaré map and the high thickness of the curve is due to a larger uncertainty in the dynamic load with respect to the base acceleration measure.

Conclusions

The nonlinear dynamics of pre-compressed circular cylindrical shells has been investigated by experiments. A new setup has been developed in order to apply both static a dynamic axial load to the shell. The tests performed with a sinusoidal excitation clarify the role of the preload in enlarging instability regions. These tests have been performed using an open loop strategy with a novel technique for providing the excitation signal and for ensuring a proper timing between output and input signal. The signal is a sine step joined with sine sweep connections. This procedure, which is able to provide a continuous signal during the whole duration of a test performed at varying frequency, allows for following solutions with a limited stability, and to observe highly subharmonic responses by experiments.Another important feature pointed out by the experiments is the coexistence of more than one stable solution in the pre-loaded shell excited with a large sinusoidal signal. The tests show that the dynamic scenario is completely different if the frequency step is reduced from 1Hz to 0.1Hz, with larger instability regions and a completely different amplitude-frequency diagram.

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