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A PARSIMONIOUS DEFAULT PREDICTION MODEL FOR ITALIAN SMEs

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Abstract

In the light of the fundamental role played by small and medium enterprises (SMEs) in the economy of many countries including Italy and of the specific treatment of this issue within the Basel II regulation, the aim of this work is to build a default prediction model for the Italian SMEs. Specifically, we develop a logit model based on financial ratios: using the AIDA database, we focus the attention on a specific region in Italy, Emilia Romagna, where SMEs represent the firms' majority . We find that a parsimonious model based on only four explanatory variables fits well the default data.

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1. Introduction

Small and medium enterprises (SMEs) play a very important role in the economic system of many countries and particularly in Italy. One of the main problems of Italian SMEs is to recover money to finance their investments: the role of banks in Italy is very important since they are the only subject issuing loans directly to SMEs. The aim of this work is to develop a default prediction model for the Italian SMEs, focusing the attention on a specific geographic area, namely the Emilia Romagna region, where SMEs represent the firms' majority.

The model we propose is a logit model based on balance-sheet data. A wide range of models for the estimation of the corporates' default probability have been developed: these models can be classified according to the type of data required. The models for pricing risky debt, having their milestone in the Merton model, are based on market data and therefore they are not suitable for small (not quoted) enterprises; on the contrary statistical models, such as those based on discriminant analysis and binary choice models, mainly use accounting data which are available for all enterprises regardless of their size. This paper focuses on balance-sheet data which are public so that the model proposed lends itself to be used not only by banks but by any economic agent who may be interested in the firm's credit quality.

An additional reason to develop specific models for the estimation of the probability of default (PD) for SMEs lies in the Basel II regulation. The estimation of the obligors' PD is a fundamental issue for banks adopting the Internal Ratings Based (IRB) approach. Moreover Basel II requires these banks to build a rating systems and provides a formula for the calculation of minimum capital requirements where the PD is the main input. Basel II recognizes a different treatment for the exposures towards SMEs, which benefit from a reduction of the capital requirement proportional to their size.

The paper is organized as follows. The literature related to default prediction, in particular for SMEs, is briefly presented in Section 2. Section 3 illustrates relevant issues related with the dataset used and the approach adopted, while Section 4 presents the results obtained. The last Section concludes.

2. Literature overview

There is a wide range of default prediction models, i.e. models that assign a probability of failure or a credit score to firms over a given time horizon. The literature on this topic has developed especially in connection with Basel II, which allows banks to set up an internal rating system, that is, a system to assign ratings to the obligors and to quantify the associate PDs. As stressed in the introduction, some sophisticated models available in the literature can be used only if market data on stocks (structural models) or corporate bonds and asset swaps (reduced-form models) are available. As for SMEs, for which market data are generally not available, either heuristic (e.g. neural network) or statistical models can be applied.

Beaver (1966) and Altman (1968) first used discriminant analysis (DA) to predict default. In order to overcome the limits inherent in DA (e.g. strong hypotheses on explanatory variables, equal variance-covariance matrix for failed and not failed firms), logit and probit models have been widely adopted[1]. An important advantage of the latter models is the immediate interpretation of the output as a default probability. A seminal paper in this respect is the one by Ohlson (1980), who analysed a dataset of US firms over the years 1970-1976 and estimated a logit model with nine financial ratios as regressors. Despite the diffusion of the pricing models based on market data, the logit/probit models based on accounting data are nowadays widely used. Recently Beaver (2005), by analysing a dataset of US firms over the period 1962-2002, has shown that balance-sheet financial ratios still preserve their predictive ability, even if market-based variables partly encompass accounting data.

A relatively new approach, based on machine learning, is the Maximum Expected Utility (MEU). This model, developed at the Standard & Poor's Risk Solutions Group (see Friedman and Sandow, 2003), is based on the maximization of the expected utility of an investor who chooses her investment strategy based on her beliefs and on the data. Marassi and Pediroda (2008) applies this approach to a dataset of Italian firms.

Focusing on SMEs, a few recent works use logit/probit models, or some evolution of the same, for PD estimation: Altman & Sabato (2007) use a dataset of US SMEs, Altman and Sabato (2005) analyse separately US, Australian and Italian SMEs, Behr and Güttler (2007) and Fantazzini and Figini (2009) analyse German data, Fidrmuc and Heinz (2009) use data from Slovakia. Despite some differences among these analyses, a convergence emerges on some types of financial indicators, which can be grouped into five categories: leverage, liquidity, profitability, coverage, activity (see Altman and Sabato 2007).

[1] A number of papers, among which Lennox (1999) and Altman and Sabato (2007), show that probit/logit models outperform DA model in default prediction.

3. The construction of the Data Set

The sample for the empirical analysis is entirely drawn from AIDA, a financial database powered by Bureau Van Dijk which contains the balance sheet data of all the Italian firms. Indeed we use public data only, while banks usually build their models on private data (e.g. default on single bank loans) taken from Credit Registers.

Given the aim of our research, we restrict our attention to SMEs. In order to construct an appropriate data set, there are a number of issues we have to tackle. The first one is the very same definition of SME, for which we stick to the Basel II rule. The definition given by the European Union[2] refers both to the number of employees and to the sales: firms are considered small if they have less than euro 50 million in sales or less than 250 employees. The Basel Committee on Banking Supervision (BCBS), for the purpose of capital requirements, imposes a criterion based on sales only to discriminate between SMEs and corporates: firms with annual sales less than 50 million euros are considered SMEs and this imply for the intermediary a reduction in capital requirement[3] proportional to the firm's size. In our sample we have included only firms with annual sales lower than 50 million euros[4], consistently with the Basel II definition. This choice is motivated by the ultimate aim of this work: the estimated PDs are in fact to be used as input in the Basel II capital requirement formula.

As for the geographic focus, we concentrate on a particular area, the Emilia Romagna region, in order to develop a model able to capture the specific features of the firms in this region, since it is highly representative of SMEs.

In our sample we consider balance sheet data for 2004 to estimate the one-year PD. Another relevant issue is the definition of default to be used in the classification. In order to classify defaulted firms in our sample, we need first of all to adopt a definition of default, since literature does not provide a univocal one. We refer to Altman and Hotchkiss (2006) for the various definition: the authors highlight four terms, *failure, insolvency, default* and *bankruptcy*, which are used interchangeably in the literature but have different meaning and refer to different situations in different countries' bankruptcy law.

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[2] Commission Recommendation 96/280/EC of April 3, 1996, updated in 2003/361/EC of May 6, 2003. See http://europa.eu/scadplus/leg/en/lvb/n26026.htm.

[3] The reduction applies to the capital function through the correlation, which is reduced by a maximum of 0.04 for the smallest firms. This correction is justified by the assumption that defaults of small firms are less correlated and therefore less risky on the whole for the portfolio.

[4] From the SMEs original data set we deleted firms with sales less than 100 000 euros since we believe that such small firms may be very different from typical firms working in industrial sectors in terms of operational, financial and economic features.

The BCBS (2006) adopts a wide default definition in that "a default is considered to have occurred with regard to a particular obligor when either or both of the two following events have taken place.

- The bank considers that the obligor is unlikely to pay its credit obligations to the banking group in full, without recourse by the bank to actions such as realising security (if held).
- The obligor is past due more than 90 days on any material credit obligation to the banking group Overdrafts will be considered as being past due once the customer has breached an advised limit or been advised of a limit smaller than current outstandings."

Often default definitions for credit risk models concern single loan defaults of a company versus a bank, as also emerges from the above Basel II instructions. This is the case for banks building models based on their portfolio data, that is relying on single loans data which are reserved (for example Altman and Sabato 2005 develop a logit model for Italian SMEs based on the portfolio of a large Italian bank). However, traditional structural models (i.e. Merton-type models) refer to a firm-based definition of default: a firm defaults when the value of the assets is lower than the value of the liabilities, that is when equity is negative.

In this work default is intended as the end of the firm's activity, i.e. the status where the firm needs to liquidate its assets for the benefit of its creditors. In practice, we consider a default occurred when a specific firm enters a bankruptcy procedure as defined by the Italian law. The reason for this choice lies in the data availability but it is also motivated by the objective of the paper: our aim is to define a model, based on public and accessible data, that measures the health state of the firms and enables any economic subject interested in a specific firm's health (i.e. suppliers, customers, lenders, etc.) to estimate the probability of a particular firm to get bankrupted.

In practice, in order to create our sample from the AIDA database, we associate the event of default to the absence of deposited balance sheet[5]: for the Italian Law, firms must not deposit their balance sheet at the firms registry (*Registro delle Imprese*[6]) if, in a particular year, a bankruptcy proceeding starts. In general, a bankruptcy proceeding occurs when a firm is configured as an insolvent debtor and it can start after a specific request of the insolvent debtor, one or more creditors, the Public Prosecutor or the Law Court. According to these observations, we build our sample for the year 2004 by focusing on two groups of firms:

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[5] Even if AIDA provides a flag to distinguish currently failed firms, it is not possible to automatically select firms failed in a particular year.

[6] The "Registro delle Imprese" is the Italian registry office which collects the balance sheet information of all the Italian firms.

1. active firms: firms that are currently operative (i.e. not bankrupted)[7];
2. bankrupted firms: firms that are currently failed and whose last balance sheet was registered in 2005.

We assume that failed firms which deposited their last balance sheet in 2005 entered the bankruptcy proceeding in 2006: therefore we analyse the balance sheet data from one to two years before bankruptcy to estimate the probability of default.

The total default rate in the sample is about 0.6%[8].

4. The empirical analysis

In line with most of the literature based on accounting data, we use a binary logistic regression model. The default probability in a logit model is estimated by equation (1):

$$PD_i = P(Y_{i,t+1} = 1) = \frac{\exp(\alpha + \sum_{k=1}^{R} \beta_k X_{ik,t})}{1 + \exp(\alpha + \sum_{k=1}^{R} \beta_k X_{ik,t})} \tag{1}$$

where:

$$Y_{i,t+1 \quad i=1,\dots,n} = \begin{cases} 1\; if\;\; obligor\; i\; defaults\; in\; t+1 \\ 0\; if\;\; obligor\; i\; does\; not\; defaults\; in\; t+1 \end{cases}$$

$$X_{ik,t \quad i=1,\dots,n} = k^{th}\;\; regressor\;\; for\; obligor\; i\; in\; t$$

We quantify the dependent variable according to the definition of default given in Section 3, while we consider balance sheet variables as regressors. The main issue is precisely the selection of appropriate and informative balance sheet variables, as explained in the sub-section.

4.1. Selection of the predictors

In order to select the appropriate regressors, we start by considering a number of variables which have been largely used in the default prediction literature, namely we choose 16 financial ratios, presented in

[7] The current status refers to the time of the data collection, i.e. January 2008.

[8] It has to be noted that the default rate is very low if compared with some other works : this difference is due to the definition of default adopted, which is a consequence of the type of data available. For example, in Altman and Sabato (2005) any delay (more than 90 days) in the payments is counted as default, while in the present paper only the firms actual defaults are considered.

Table 1, related to the main aspects of a company's financial profile (leverage, liquidity, profitability, coverage, activity).

Table 1 List of candidate predictors

Financial ratio	**Categoria**
Inventory/Sales (**IS**)	ACTIVITY
Sales/Asset (**SALESA**)	ACTIVITY
Short Term Debt/Equity (**STDE**)	LEVERAGE
Long Term Liabilities/Asset (**LTLA**)	LEVERAGE
Equiy/Asset (**EQUITYA**)	LEVERAGE
Ebit/Asset (**EBITA**)	PROFITABILITY
Ebit/Sales (**ES**)	PROFITABILITY
Economic Value Addded/Asset (**EVAA**)	PROFITABILITY
Net Income/Asset (**NIA**)	PROFITABILITY
Working Capital/Asset (**WCA**)	LIQUIDITY
Cash/Asset (**CA**)	LIQUIDITY
Working Capital/Sales (**WCA**)	LIQUIDITY
Working Capital/Current Liabilities (WCC)	LIQUIDITY
Cash/Current/Liabilities (**CCL**)	LIQUIDITY
Current Liabilities/Asset (**CLA**)	LIQUIDITY
Ebit/Interest Expenses (**EIE**)	COVERAGE

We select among these candidate predictors by means of a backward elimination procedure based on the Schwartz Information Criterion (SIC). The resulting model is illustrated in Table 2: the estimation results show that all the coefficients display the expected sign and are significant.

The equity ratio (EQUITYA) indicates the relative proportion of equity to all used to finance a company's assets. In general, we expect that a higher equity ratio implies a decrease in an SME's default risk and the model confirms this presumption. The current ratio measures whether a firm has enough resources to pay its debts over the next 12 months. The Ebit/Asset ratio measures the ability of generating income without tax distorsion: the higher this ratio, the more healty should be the firm and hence the lower is the PD. The Long term liabilities / Asset ratio quantifies the long term debt compared to the short term one: higher long term liabilities means (by construction) lower short term ones, and, for this reason, the higher is this ratio the lower is the PD. A high value for the Sales/Asset indicator means good performances on the market and therefore a low PD.

Table 2 Estimation output

Estimated equation: $PD = 1/(1 + \exp(2.86 + 3.46 LTLA + 3.52 EBITA + 11.18 EQUITYA + 0.43 SALESA))$				
Variable	Estimated Coefficient	Std.Error (Huber /White)	z-Stat.	Prob.
CONSTANT	**-2.8654**	0.3467	-8.2679	**0.000**
EQUITYA	**-11.1832**	2.9199	-3.8299	**0.000**
EBITA	**-3.5190**	1.3478	-2.6110	**0.009**
LTLA	**-3.4596**	0.7688	-4.4999	**0.000**
SALESA	**-0.4315**	0.2393	-1.8034	**0.071**
Mean dep. var.	0.00573	S.D. dep. Var.	0.07547	
S.E. regession	0.07201	Akaike I. C.	0.05913	
Sum sq. res.	85.9835	Schwarz I.C.	0.06146	
Log Likelihood	-485.410	Hannan Quinn I.C.	0.05990	
Restr. Log lik.	-585.159	Avg. log lik.	-0.02927	
LR stat. (5 d.f.)	199.498	Mc Fadden R-sq	**0.1705**	
Prob. (LR stat.)	0.000			

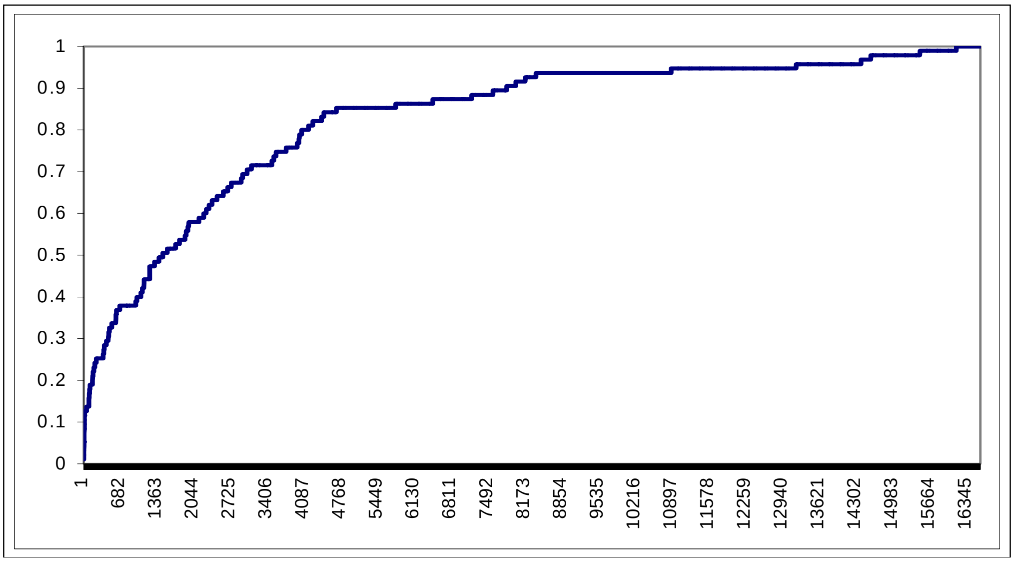
4.2. Model performance

The performances of default prediction model can be measured in different ways: an exhaustive presentation of the available validation techniques can be found in BCBS (2005).

Consistently with most of the literature, we evaluate the performance of our model by means of the Cumulative Accuracy Profile (CAP) and the associate Accuracy Ratio (AR), which measures the ability of the model to maximize the distance between the defaulted and non-defaulted firms[9]. Figure 1 shows the in sample CAP curve for our model; the associate AR is 66.84%.

[9] See Sobehart *et al.*, 2001 and Engelman et al. (2003) for a discussion of the CAP curve and the Accuracy Ratio.

Figure 1 Cumulative accuracy profile of the model



While common goodness of fit measures for binary choice models rely on the choice of a particular cut-off value to discriminate between the two states, the AR indicator is free of arbitrary choices. Table 3 shows the error rates for some values of the discriminating cut-off: obviously type I error increases with increasing cut-off values while type II error decreases; the average error rate is low when the cut-off value is fixed at the level of the sample default rate.

Table 3 Error Rates

Cut-off	Type I error rate	Type II error rate	Avg error rate
0.00576	14.74%	30.82%	22.78%
0.01	31.58%	17.37%	24.47%
0.05	87.37%	0.1%	43.73%
0.1	87.37%	0.03%	43.70%

Note: Type I error refers to failed firms classified as not failed; Type II error refers to not failed classified as failed.

5. Conclusions and further research

In this work we have estimated a logit default prediction model for Italian SMEs in the Emilia Romagna region. The model behaves fairly well in sample: this result confirm the validity of limited dependent variable models with financial ratios as predictors to represent default events. We find that a parsimonious model with four predictors, namely the equity ratio, the long term liabilities over asset ratio, the ebit over asset ratio and the sales over asset ratio, is sufficient to fit default events in our sample. In particular, the equity ratio on its own explain very well defaults:

this means that the idea underlying the Merton approach, based on the relation between assets, liabilities and equity, also works for SMEs, even if the application of the Merton model requires market data.

Based on the results obtained so far, we plan to extend the analysis by enlarging the sample in two directions. First, since our estimation sample covers one year of data, it does not allow to consider business cycle effects, while the analysis would benefit from the observations of defaults over different economic phases. Second, it would be useful to separately consider firms belonging to different industrial sectors.

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