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are call options more informative than put options?
evidence from the DAX index options market*

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The relation between implied and realised volatility:
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Abstract

The aim of this paper is to investigate the relation between implied volatility, historical volatility and realised volatility in the Dax index options market. Since implied volatility varies across option type (call versus put) we run a horse race of different implied volatility estimates: implied call, implied put and average implied that is a weighted average of call and put implied volatility with weights proportional to traded volume. Two hypotheses are tested in the Dax index options market: unbiasedness and efficiency of the different volatility forecasts. Our results suggest that all the three implied volatility forecasts are unbiased (after a constant adjustment) and efficient forecasts of future realised volatility in that they subsume all the information contained in historical volatility.

Keywords: Implied Volatility, Volatility Forecasting, Option type, trading volume.
JEL classification: G13, G14.

1. Introduction.

Volatility is a key variable in option pricing models and risk management techniques and has drawn the attention of many theoretical and empirical studies aimed at assessing the best way in order to forecast it. Among the various models proposed in the literature in order to forecast volatility, we distinguish between option based volatility forecasts that use prices of traded options in order to unlock volatility expectations and time series volatility models that use historical information in order to predict future volatility (following Poon and Granger (2003), in

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this set we group predictions based on past standard deviation, ARCH conditional volatility models and stochastic volatility models). Many empirical studies have tested the forecasting power of implied volatility versus a time series volatility model.

Some early contributions find evidence that implied volatility is a biased and\or inefficient forecast of future realised volatility. Canina and Figlewski (1993) use a data set of daily closing prices of options on the S&P 100 from March 1983 to March 1987 and find a poor relationship between implied and realised volatility. In the same market, Day and Lewis (1992) examine the predictive power of implied volatility over a longer time period, from 1983 to 1989, and find that it is not better than standard time series models such as GARCH and EGARCH. Jorion (1995) uses data on currency options and finds that implied volatility is a biased but efficient predictor of future realised volatility. Lamourex and Lastrapes (1993) examine the information content of implied volatility extracted from options on ten stocks from 1982 to 1984 and find that implied volatility is biased and inefficient. Although the results of some of these studies (e.g. Day and Lewis (1992), Lamourex and Lastrapes (1993)) are affected by overlapping samples, as recalled by Christensen, Hansen and Prabhala (2001), or mismatching maturities between the option and the volatility forecast horizon, they constitute early evidence against the unbiasedness and information efficiency of implied volatility.

More recently, numerous papers analyse the empirical performance of implied volatility in various option markets, ranging from indexes, futures or individual stocks and find that implied volatility is an unbiased and\or efficient forecast of future realised volatility. In the index options market, Christensen and Prabhala (1998) examine the relation between implied and realized volatility using S&P100 options, over the time period 1983-1995. They look for a possible regime shift around October 1987 and use non-overlapping samples and instrumental variables in order to account for possible errors in variables. They found that implied volatility is a good predictor of future realized volatility. Ederington and Guan (2002) analyse the S&P 500 futures options market and find that implied volatility is an efficient forecast of future realised volatility. Christensen, Hansen and Prabhala (2001) use options on the S&P 100 and non overlapping samples and find evidence for the efficiency of implied volatility as a predictor of future realised volatility. In the futures options market Szakmary et al. (2003) consider options on 35 different Futures contracts on a variety of asset class. They find that implied volatility, while not a completely unbiased estimate of future realised volatility, has more informative power than past realised volatility. In the stock options market, Godbey and Mahar (2005) analyse the information content of implied call and put volatility extracted from options on 460 stocks that compose the S&P500 index. They find that implied volatility contains some

information on future realised volatility that is superior both to past realised volatility and to a GARCH(1,1) estimate. Moreover they highlight that the information content of implied volatility decreases as option volumes decrease.

Other papers analyse the performance of the VIX volatility index, that measures the implied volatility of a basket of options on the S&P500 (prior to 2003 the S&P100 was used). Among them, Blair, Poon and Taylor (2001b) find that the VIX index is an unbiased estimator of future realized volatility. Bandi and Perron (2006) investigate the long-run relation between implied and realised volatility in the VIX index over the period 1988-2003. They obtain that implied volatility is an unbiased estimate of future realised volatility.

As option implied volatility differs depending on strike price of the option (the so-called smile effect), time to maturity of the option (term structure of volatility) and option type (call versus put) which option class yields implied volatilities that are most representative of the markets' volatility expectations is still an open debate. As for the moneyness dimension, most of the studies use at the money options (or close to the money options) since they are the most heavily traded and thus the most liquid. As for the time to maturity dimension, the majority of the studies use options with time to maturity one month in order to make it coincide with the sampling frequency and the estimation horizon of realised volatility. As for the option type, call options are more used than put options.

As far as we know, little is the evidence about the different information content of call or put prices. Fleming (1998) investigates the implied-realised volatility relation in the S&P100 options market and finds that call implied volatility has slightly more predictive power than put implied volatility. In the same market, Christensen and Hansen (2002) find that both call and put implied volatilities are informative of future realized volatility, even if call implied volatility performs slightly better than put implied volatility. All these studies use American type options and do not explicitly take into account the dividend payments. These two variables influence in a different manner call and put option prices, and may have altered the comparison if not properly addressed.

Moreover, given that option prices are observed with measurement errors (stemming from finite quote precision, bid-ask spreads, non-synchronous observations and other measurement errors) small errors in any of the input may produce large errors in the implied volatility (see e.g. Hentshle (2003)). As noted in Moriggia, Muzzioli and Torricelli (2007) the use of both call and put options in the volatility estimation, highly improves the pricing performance of option pricing models based on implied binomial trees.

The aim of the paper is twofold. In the first place we explore the relation between call and put implied volatilities in the Dax index option market. The market is chosen for two main reasons: First the options are European, therefore the estimation of the early exercise premium is not needed and can not influence the results. Second, the Dax index is a capital weighted performance index composed of 30 major German stocks and is adjusted for dividends, stocks splits and changes in capital. Since dividends are assumed to be reinvested into the shares, they do not affect the index value. In the second place, we look for a combination of call and put prices in a single estimate, in order to convey the information from both call and put prices and cancel possible errors across option type.

The plan of the paper is the following: in section 2 we illustrate the data set used, the sampling procedure and the variables definitions. In section 3 we describe the methodology used in order to address the unbiasedness and efficiency of the different volatility forecasts. In section 4 we report the results of the univariate and encompassing regressions and we test for robustness our methodology in order to see if some errors in variables problem may have affected our results. As in Godbey and Mahar (2006) it is highlighted that traded volume has a positive relation with the information content of option implied volatility, in Section 5 we analyse the different information content of implied volatility by grouping options into four quartiles according to increasing trading volume. Based on the results of Section 5, in section 6 we combine call and put implied volatilities in a single estimate: a trade weighted average implied volatility and we compare the performance of this estimator on the same data set. The last section concludes.

2. The Data set, the sampling procedure and the variables definitions.

Our data set[2] consists of closing prices of at the money call and put options on the DAX-index, with maturity one month recorded from 19 July 1999 to 6 December 2006. Each record reports the strike price, expiration month, transaction price and total trading volume of the day separately for call and put prices. We have a total of 1928 observations. As for the underlying we use the DAX-index closing prices recorded in the same time period. As a proxy for the risk-free rate we use the one month Euribor rate.

DAX-options started trading on the German Options and Futures Exchange (EUREX) in August 1991. They are European options on the DAX-index, which is a capital weighted performance index composed of 30 major German stocks and is adjusted for dividends, stocks

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[2] The data source for Dax-index options, Dax index and the risk-free rate is Data-Stream.

splits and changes in capital. Since dividends are assumed to be reinvested into the shares, they do not affect the index value, therefore we do not have to estimate the dividend payments. Moreover the fact that the options are European avoids the estimation of the early exercise premium. This latter feature is very important since our data set is by construction less prone to estimation errors if compared to the majority of previous studies that use American style options. DAX-index options are quoted in index points, carried out one decimal place. The contract value is EUR 5 per DAX index point. The tick size is 0.1 of a point representing a value of EUR 0.50. They are cash settled, payable on the first exchange trading day immediately following the last trading day. The last trading day is the third Friday of the expiration month, if that is an exchange day, otherwise the exchange trading day immediately prior to that Friday. The final settlement price is the value of the DAX determined on the basis of the collective prices of the shares contained on the DAX index as reflected in the intra-day trading auction on the electronic system of the Frankfurt Stock Exchange. Expiration months are the three near calendar months within the cycle March, June, September and December as well as the two following months of the cycle June and December.

In order to avoid measurement errors, the data set has been filtered according to the following filtering constraints. First, in order not to use stale quotes, we eliminate dates with trading volume less than ten contracts. Second, we eliminate dates with option prices violating the standard no arbitrage bounds. After the application of the filters, we are left out with 1860 observations out of 1928.

As for the sampling procedure, in order to avoid the telescoping problem described in Christensen, Hansen and Prabhala (2001), we use monthly non-ovelapping samples. In particular, we collect the prices recorded on the Wednesday that immediately follows the expiry of the option (third Saturday of the expiry month) since the week immediately following the expiration date is one of the most active. These options have a fixed maturity of almost one month (from 17 to 22 days to expiration). If the Wednesday is not a trading day we move to the trading day immediately following.

The implied volatility, provided by Datastream, is obtained by inverting the Black and Scholes formula as a weighted average of the two options closest to being at the money i.e. with strikes one below and one above the underlying price, with weights equal to the distance to the moneyness (for example if the DAX-index is 5355 and the closest strikes are 5400 and 5350 the implied volatility of the 5400 strike will be weighted 5/50 against the implied volatility 5350 strike which is weighted 45/50). The implied volatility is computed for call options (σ_c) and for put options (σ_p).

Implied volatility is an ex-ante forecast of future realised volatility on the time period until the option expiration. Therefore we compute the realised volatility (σ_r) in month t, as the sample standard deviation of the daily index returns over the option's remaining life:

$$s_r = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (R_i - \overline{R})^2}$$

where R_i is the return of the DAX-index on day i and $\overline{R}$ is the mean return of the Dax-INDEX in month t. We annualize the standard deviation by multiplying it by $\sqrt{252}$.

In order to examine the predictive power of implied volatility versus a time series volatility model, following prior research (see e.g. Christensen and Prabhala (1998), Jiang and Tian (2005)), we choose to use the lagged (one month before) realized volatility as a proxy for historical volatility (σ_h).

Descriptive statistics for volatility and log volatility series are reported in Table 1. We can see that on average realized volatility is lower than implied volatility estimates, with call implied volatility being slightly higher than put implied volatility. As for the standard deviation, realised volatility is slightly more volatile than both implied volatility estimates. The volatility series are highly skewed (long right tail) and leptokurtic.

In line with the literature (see e.g. Jiang and Tian (2005)) we decided to use the natural logarithm of the volatility series instead of the volatility itself in the empirical analysis for the following reasons. First log-volatility series conform more closely to normality than pure volatility series, this is documented in various papers and it is the case in our sample (see Table 1). Second, natural logarithms are less likely to be affected by outliers in the regression analysis.

Table 1. Descriptive statistics.

Statistic	s_c	s_p	s_r	$\ln s_c$	$\ln s_p$	$\ln s_r$
mean	0,2404	0,2395	0,2279	-1,51	-1,52	-1,6
std dev	0,11	0,11	0,12	0,41	0,41	0,49
skewness	1,43	1,31	1,36	0,49	0,4	0,41
kurtosis	4,77	4,21	4,37	2,73	2,71	2,46
Jarque Bera	41,11	30,28	33,68	3,69	2,68	3,54
p-value	0,00	0,00	0,00	0,16	0,26	0,17

3. The methodology.

The information content of implied volatility is examined both in univariate and in encompassing regressions. In univariate regressions, realized volatility is regressed against one

of the three volatility forecasts (implied call (σ_c), implied put (σ_p), historical volatility (σ_h)) in order to examine the predictive power of each volatility estimator. The univariate regressions are the following:

$$\ln(\sigma_r) = a + b \ln(\sigma_i) \tag{1}$$

where σ_r = realized volatility and σ_i= volatility forecast, $i=h,c,p$.

In encompassing regressions, realized volatility is regressed against two or more volatility forecasts in order to distinguish which one has the highest explanatory power. We choose to compare pairwise one implied volatility forecast (call, put) with historical volatility in order to see if implied volatility subsumes all the information contained in historical volatility. The encompassing regressions used are the following:

$$\ln(\sigma_r) = a + b \ln(\sigma_i) + g \ln(\sigma_h) \tag{2}$$

where σ_r = realized volatility, σ_i= implied volatility, $i=c,p$ and σ_h = historical volatility.

Moreover, we compare call and put implied volatilities in order to understand if the information carried by call (put) prices is more valuable than the information carried by put (call) prices:

$$\ln(\sigma_r) = a + b \ln(\sigma_p) + g \ln(\sigma_c) \tag{3}$$

where σ_r = realized volatility, σ_c= implied call volatility and σ_p= implied put volatility.

Following Christensen and Prabhala (1998) three are the hypotheses tested in univariate regressions (1). The first hypothesis concerns the amount of information about future realized volatility contained in the volatility forecast. If the volatility forecast contains some information, then the slope coefficient should be different from zero. Therefore we test if $b = 0$ and we see whether it can be rejected. The second hypothesis is about the unbiasdness of the volatility forecast. If the volatility forecast is an unbiased estimator of future realised volatility, then the intercept should be zero and the slope coefficient should be one (H$_0$: $a = 0$ and $b = 1$). In case this latter hypothesis is rejected, we see if at least the slope coefficient is equal to one (H$_0$: $b = 1$) and, if confirmed, we interpret the volatility forecast as unbiased after a constant adjustment. Finally if implied volatility is efficient then the error term should be white noise and uncorrelated with the information set.

In encompassing regressions (2) three are the hypotheses to be tested. The first is about the efficiency of the volatility forecast: in encompassing regressions (2) we test whether the volatility forecast (implied call, implied put) subsumes all the information contained in historical volatility. In affirmative case the slope coefficient of historical volatility should be equal to zero, (H$_0$: $g = 0$). Moreover, as a joint test of information content and efficiency we test in equations (2) if the slope coefficients of historical volatility and implied volatility (call, put) are equal to

zero and one respectively (H[0]: $g = 0$ and $b = 1$). Following Jiang and Tian (2005), we ignore the intercept in the latter null hypothesis, and if our null hypothesis is verified, we interpret the volatility forecast as unbiased after a constant adjustment.

Finally we investigate the different information content of call and put implied volatility. To this end we test, in augmented regression (3), if $g = 0$ and $b = 1$, in order to see if put implied volatility subsumes all the information contained in call implied volatility.

Differently from other papers (see e.g. Christensen and Prabhala 1998, Christensen and Hansen (2002)) that use American options on dividend paying indexes, our data set of European style options on a non-dividend paying index avoids measurement errors that may arise in the estimation of the dividend yield and the early exercise premium. Nonetheless, as we are using closing prices for the index and the option that are non- synchronous (15 minutes difference) and we are ignoring bid ask spreads, some measurement errors may still affect our estimates. Therefore we adopt an instrumental variable procedure (IV), we regress call (put) implied volatility on an instrument (in univariate regressions) and on an instrument and any other exogenous variable (in encompassing and augmented regressions) and replace fitted values in the original univariate and encompassing regressions. As the instrument for call (put) implied volatility we use both historical volatility and past call (put) implied volatility as they are possibly correlated to the true call (put) implied volatility, but unrelated to the measurement error associated with call (put) implied volatility one month later. As an indicator of the presence of errors in variables we use the Hausman (1978) specification test statistic[3].

4. The results.

The results of the OLS univariate (equation (1)), encompassing (equation (2)), and augmented (equation (3)) regressions are reported in Table 2 (p-values in parentheses). In all the regressions the residuals are normal, omoschedastic and not autocorrelated (the Durbin Watson statistic is not significantly different from two and the Breusch-Godfrey LM test confirms non autocorrelation up to lag 12[4]). The columns χ^{2a} and χ^{2b} report the statistic (p-values in

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[3] The Hausman specification test is defined as: $m = \dfrac{\left(\hat{b}_{IV} - \hat{b}_{OLS}\right)^2}{Var(\hat{b}_{IV}) - Var(\hat{b}_{OLS})}$ where: $\hat{b}_{IV}$ is the beta obtained through the TSLS procedure, $\hat{b}_{OLS}$ is the beta obtained through the OLS procedure and $Var(x)$ is the variance of the coefficient x. The Hausman specification test is distributed as a $\chi^2(1)$.

[4] In the regressions that include as explanatory variable the lagged realised volatility, the Durbin's alternative has been computed and only in equation (1) it was possible to obtain a result that has confirmed the non autocorrelation of the residuals. The results of the Durbin's alternative and of the Breusch-Godfrey LM test are available upon request.

parentheses) of a χ^2 test for the null hypothesis $a = 0$ and $b = 1$ in equation (1), and $g = 0$ and $b = 1$ in equations (2) and (3) respectively. The superscripts ***, **, * indicate that β is insignificantly different from one at the 10%, 5%, and 1% critical level respectively. The superscripts +++, ++, + indicate that β is insignificantly different from zero at the 10%, 5%, and 1% critical level respectively.

First of all, in the three univariate regressions all the beta coefficients are significantly different from zero: this means that all the three volatility forecasts (implied call, implied put and historical) contain some information about future realised volatility. However, the null hypothesis that any of the three volatility forecasts is an unbiased estimate of future realized volatility is strongly rejected in all cases. In particular, in our sample, realized volatility is on average a little lower than the two implied volatility forecasts, suggesting that implied overpredicts realised volatility, in line with the results found in Lynch and Panigirtzoglou (2003). The adjusted R^2 is the highest for the put implied volatility, followed by the call implied volatility. The historical volatility has the lowest adjusted R^2. Therefore among the two implied volatility forecasts, the put implied is ranked first in explaining future realized volatility, while historical volatility is the last. The null hypothesis that β is insignificantly different from one can not be rejected at the 10% critical level for the two implied volatility estimates, while it is strongly rejected for historical volatility. Therefore we can consider both implied volatility estimates as unbiased after a constant adjustment given by the intercept of the regression.

In encompassing regressions (2) we compare pairwise one implied volatility forecast (call, put) with historical volatility in order to understand if implied volatility subsumes all the information contained in historical volatility. The results are striking and provide strong evidence for both the unbiasdness and efficiency of both implied volatility forecasts. First of all, from the comparison of univariate and encompassing regressions, the inclusion of historical volatility does not improve the goodness of fit according to the adjusted R^2. In fact, the slope coefficient of historical volatility is not significantly different from zero at the 10% level in all the two encompassing regressions (2), indicating that both call and put implied volatilities subsume all the information contained in historical volatility. The slope coefficient of both call and put implied volatilities is not significantly different from one at the 10% level and the joint test of information content and efficiency $g = 0$ and $b = 1$ does not reject the null hypothesis, indicating that both implied volatility estimates are efficient and unbiased after a constant adjustment.

In order to see if put implied volatility has more predictive power than call implied volatility, we test in augmented regression (3) if $g = 0$ and $b = 1$. We see that only the slope

coefficient of put implied volatility is significantly different from zero, while the slope coefficient of call implied volatility is not significantly different from zero. The joint test $g = 0$ and $b = 1$ does not reject the null hypothesis, providing evidence for the superiority of put implied volatility with respect to call implied volatility.

Finally, in order to test for robustness our results, and see if implied volatility has been measured with errors, we adopt an instrumental variable procedure (IV) and run a two stage least squares. The Hausman (1978) specification test reported in the last column of Table 2 indicates that the errors in variables problem is not significant neither in univariate regressions (1), nor in encompassing regressions (2), nor in augmented regression (3)[5]. Therefore we can trust the OLS regressions results.

Table 2. OLS regressions.

Dependent variable: log realized volatility								
Independent variables								
Intercept	**ln(s_c)**	**ln(s_p)**	**ln(s_h)**	**Adj. R^2**	**DW**	**$\chi^{2\,a}$**	**$\chi^{2\,b}$**	**Hausman test**
-0,01		1,05***		0,77	1,73	13,139		0,10021
(0,915)		(0,000)				(0,00)		
-0,018	1,047***			0,76	1,77	13,139		0,25128
(0,853)	(0,000)					(0,00)		
-0,29			0,82	0,65	2,12	7,517		
(0,008)			(0,000)			(0,02)		
-0,02	0,938***		0,103+++	0,76	1,87		1,288	0,47115
(0,850)	(0,000)		(0,400)				(0,53)	
-0,01		0,9631***	0,082+++	0,77	1,80		1,158	0,95521
(0,915)		(0,000)	(0,489)				(0,56)	
0,0006	0,372	0,6861***		0,77	1,74		2,04	0,14977
(0,994)	(0,244)	(0,033)					(0,35)	

Note: The number in brackets are the p-values. The χ^{2a} report the statistic of a χ^2 test for the joint null hypothesis $a = 0$ and $b = 1$ (p-values in parentheses) in the following univariate regressions $\ln(s_r) = a + b \ln(s_i)$, where σ_r = realized volatility and σ_i = volatility forecast, i=h,c,p. The χ^{2b} report the statistic of a χ^2 test for the joint null hypothesis $g = 0$ and $b = 1$ (p-values in parentheses) in the following regressions: $\ln(s_r) = a + b \ln(s_i) + g \ln(s_h)$, $\ln(s_r) = a + b \ln(s_p) + g \ln(s_c)$, where σ_r = realized volatility, σ_h = historical volatility and σ_i = volatility forecast, i=c,p. The superscripts ***, **, * indicate that the slope coefficient is insignificantly different from one at the 10%, 5%, and 1% critical level respectively. The superscripts +++, ++, + indicate that the slope coefficient is insignificantly different from zero at the 10%, 5%, and 1% critical level respectively The last column reports the Hausman (1978) specification test statistic (one degree of freedom) 5% critical level = 3,841.

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[5] In augmented regression (3) the instrumental variables procedure is used for the variable ln(σ_p).

Our results, that point to a better performance of put implied volatility w.r.t call implied, are very different from the ones obtained both in Fleming (1998) and in Christensen and Hansen (2002). The difference can possibly be attributed to the option exercise feature, that in our case is European and not American, and to the underlying index features, that in our case does not require the dividend payment estimation. An other possible explanation stems from the very same characteristics of the data set used. In particular in our case put implied volatility was on average lower than the call implied one, while in Christensen and Hansen (2002) the opposite is true. As usually implied volatility overpredicts realised volatility, if a choice has to be made between call and put implied volatility, a rule of thumb can be to choose the lowest one between the two.

5. The role of trading volume in forecasting volatility.

Implied volatility is a forward looking estimate of future realised volatility. As such, we expect actively traded options to be more informative of future realised volatility than less traded options. Various papers have investigated the role of trading volume in influencing the predictive power of implied volatility. By analysing the predictive power of implied volatility for individual stocks, Mayhew and Stivers (2003) and Godbey and Mahar (2006) find that the predictive power of implied volatility increases with option trading volume. Donaldson and Kamstra (2005) investigate the role of trading volume in the information content of ARCH versus implied volatility forecasts, given by the VIX index. They find that trading volume is important in increasing the informativeness of the volatility forecast.

In line with these contributions, in this section we investigate if the liquidity of the option type, call versus put, proxied by the total trading volume of the day in each option class, is a key determinant of the information content of implied volatility. To this end we group implied volatility of call and put separately into four quartiles, according to increasing trading volume. Table 3 reports the minimum, maximum and average volume of contracts traded in the four quartiles for both call and put options. We can see that put options are more actively traded than call options in each quartile. For each quartile and each option type we run univariate regressions (1) and we collect the R^2, as a measure of goodness of fit. In Appendix 1 we report the results of the regressions of log realized volatility on call or put log implied volatility in each quartile. Figure 1 shows how the R^2 varies in the four quartiles for each option type.

Differently from previous papers, the evidence is mixed and it is not simple to extrapolate a one to one relation. For both call and put options the highest quartile has the highest

forecasting power. For call options, trading volume has an u-shaped relation with forecasting power, with the highest and lowest quartiles being the best. For put options, trading volume has a swinging relation with forecasting power and is less volatile than the one for call options. The R^2 are on average higher for put options than for call options: this may be explained by the average higher trading volume of put options. Therefore it seems that trading volume has some positive relation with forecasting power, in particular for the highest quartiles.

Table 3. The volume of contract traded for call and put options.

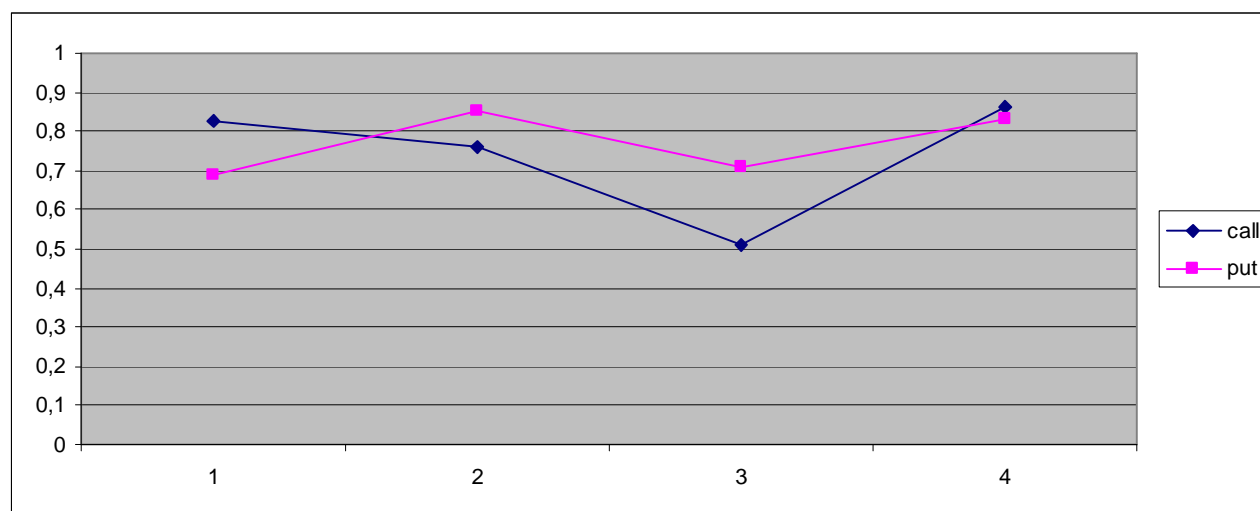|	Call	Put
Min	13601	15329
Max	229546	235190
Average	75337	89873
1st quartile	13601 - 67587	15329 – 70294
2nd quartile	67587 - 121574	70294 – 125260
3rd quartile	121574 - 175560	125260 – 180225
4th quartile	175560 - 229546	180225 – 235190

Figure 1. The R^2 in the four quartiles for call and put options.

6. A combination of call and put options.

Many papers in the literature have addressed the issue of combining implied volatilities extracted from options with different strike price or type in a single estimate (see e.g. Ederington and Guan (2002)). The most used weighting schemes are based on the vega of the option or the trading volume, whereas less used schemes are equally weighted proportions or weights based on the elasticity of option price to volatility. Weighting schemes based on the vega or the trading

volume lead to the same result of favouring at the money options, since at the money options have the highest vega and the highest trading volume. The forecasting performance of these schemes has been empirically tested against the performance of individual implied volatilities and in general weighting schemes that favour at the money options have performed better than others.

Given that prices are observed with measurement errors (stemming from finite quote precision, bid-ask spreads, non-synchronous observations and other measurement errors) small errors in any of the input may produce large errors in the implied volatility. Quoting Hentshle (2003): "Unfortunately many authors preclude the cancellation of errors across puts and calls by using only the more liquid out of the money options. Unless underlying asset prices and dividend rates are observed with high precision, this practice can result in a substantial loss of efficiency". Moreover, as noted in Moriggia, Muzzioli and Torricelli (2007) the use of both call and put options in the volatility estimation, highly improves the pricing performance of option pricing models based on implied binomial trees.

Therefore, in this section we investigate how to combine call and put implied volatilities in a single estimate, in order to convey the information from both call and put prices and cancel possible errors across option type[6]. In the logarithmic specification, natural candidates for the weights that we may assign to call and put implied volatilities, would be the estimated coefficients of regression (3). However, as the beta coefficient of call implied volatility is not significantly different from zero, it is not possible to find an optimal combination of the two with constant weights through time.

Based on the results of Section 5 and in line with the approach by Christensen and Hansen (2002), that proposes to favour the most actively traded options, we construct a weighted average of call and put implied volatilities (σ_m), where the weights are the relative trading volume of each option class on the total trading volume:

$$s_m = \frac{s_c V_c + s_p V_p}{V_c + V_p}$$

where V_i is the trading volume of option i, $i=c,p$, c for call and p for put. The weighting rule favours the most actively traded options.

Descriptive statistics of average implied volatility and log average implied volatility are reported in Table 4. Average implied volatility is slightly higher than realised volatility. Similarly to the results in Table 1, we can see that the natural logarithm of average implied

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[6] Even if the Hausman specification test pursued in Section 2 highlights that the errors in variables problem does not matter in our case, the use of both option classes can still be deemed useful in eliminating noise.

volatility conforms more to normality than the plain average implied volatility series. Therefore it will be used as explanatory variable in univariate and encompassing regressions (1) and (2).

In order to analyse the performance of the obtained average implied volatility estimate, we run both univariate and encompassing regressions (1) and (2) with $s_i = s_m$. Furthermore, in order to test for robustness our results, we look for possible errors in variables. The results are reported in Table 5. In all the regressions the residuals are normal, omoschedastic and not autocorrelated (the Durbin Watson statistic is not significantly different from two and the Breusch-Godfrey LM test confirms non autocorrelation up to lag 12[7]).

Table 4. Descriptive statistics for average implied volatility.

Statistic	s_m	$\ln s_m$
mean	0,2398	-1,51
std dev	0,11	0,41
skewness	1,38	0,46
kurtosis	4,5	2,73
Jarque Bera	35,85	3,32
p-value	0,00	0,19

In univariate regression (1), the beta coefficient of average implied is significantly different from zero, but the null hypothesis that average implied is an unbiased estimate of future realized volatility is strongly rejected. The null hypothesis that β is insignificantly different from one can not be rejected at the 10% critical level: therefore we can consider average implied volatility as unbiased after a constant adjustment given by the intercept of the regression. With respect to the performance of the other volatility forecasts reported in Table 2, the adjusted R^2 is the highest for average implied volatility. Therefore we conclude that average implied has the highest forecasting power if compared to call or put implied volatility.

In encompassing regression (2) we compare average implied volatility with historical volatility in order to understand if average implied volatility subsumes all the information contained in historical volatility. The results provide strong evidence for both the unbiasedness and efficiency of the average implied volatility forecast. First of all, from the comparison of univariate and encompassing regression, the inclusion of historical volatility does not improve the goodness of fit according to the adjusted R^2. In fact, the slope coefficient of historical volatility is not significantly different from zero at the 10% level, indicating that average implied

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[7] In the regression that include as explanatory variable the lagged realised volatility, the Durbin's alternative has been computed, but it was not possible to obtain a result. The results of the Durbin's alternative and of the Breusch-Godfrey LM test are available upon request.

subsume all the information contained in historical volatility. The slope coefficient of average implied volatility is not significantly different from one at the 10% level and the joint test of information content and efficiency $g = 0$ and $b = 1$ does not reject the null hypothesis, indicating that average implied volatility is efficient and unbiased after a constant adjustment. With respect to the performance of the other volatility forecasts reported in Table 2, average implied has the highest forecasting power. The better performance of average implied can be attributed to the fact that it contains more information, being an average of both call and put implied volatilities weighted by trading volume.

Table 5. OLS regressions of realised volatility on average implied volatility.

Dependent variable: log realized volatility							
Independent variables							
Intercept	$\ln(s_m)$	$\ln(s_h)$	Adj. R^2	DW	$\chi^{2\,a}$	$\chi^{2\,b}$	Hausman test
0,0022	1,059***		0,78	1,73	13,73		0,00019
(0,981)	(0,000)				(0,00)		
0,0013	1***	0,055+++	0,78	1,78		1,157	1,136729
(0,989)	(0,000)	(0,648)				(0,56)	

Note: The number in brackets are the p-values. The χ^{2a} report the statistic of a χ^2 test for the joint null hypothesis $a = 0$ and $b = 1$ (p-values in parentheses) in the following univariate regression $\ln(s_r) = a + b \ln(s_m)$, where σ_r = realized volatility and σ_m = average implied volatility. The χ^{2b} report the statistic of a χ^2 test for the joint null hypothesis $g = 0$ and $b = 1$ (p-values in parentheses) in the following encompassing regression: $\ln(s_r) = a + b \ln(s_m) + g \ln(s_h)$, where σ_r = realized volatility, σ_h = historical volatility and σ_m = average implied volatility. The superscripts ***, **, * indicate that the slope coefficient is insignificantly different from one at the 10%, 5%, and 1% critical level respectively. The superscripts +++, ++, + indicate that the slope coefficient is insignificantly different from zero at the 10%, 5%, and 1% critical level respectively. The last column reports the Hausman (1978) specification test statistic (one degree of freedom): 5% critical level = 3,841.

Finally, in order to test for robustness, we adopt an instrumental variable procedure (IV), we regress average implied volatility on an instrument (in univariate regression (1)) and on an instrument and any other exogenous variable (in encompassing regression (2)) and replace fitted values in the original univariate and encompassing regressions. As the instrument for implied volatility we use both historical volatility and past average implied volatility as they are possibly correlated to the true implied volatility, but unrelated to the measurement error associated with implied volatility one month later. The Hausman (1978) specification test reported in the last

column of Table 5 indicates that the errors in variables problem is not significant both in univariate and in encompassing regressions.

7. Conclusions.

In this paper we have investigated the relation between implied volatility, historical volatility and realised volatility in the Dax index options market. Since implied volatility varies across option type (call versus put) we have run a horse race of different implied volatility estimates: implied call, implied put and average implied, that is a weighted average of call and put implied volatility, with weights proportional to traded volume.

Two hypotheses have been tested: unbiasedness and efficiency of the different volatility forecasts. Our results suggest that all the three implied volatility forecasts (implied call, implied put, average implied) contain more information about future realised volatility than historical volatility. In particular, all the three implied volatility estimates are unbiased (after a constant adjustment) and efficient forecasts of realised volatility in that they subsume all the information contained in historical volatility. Differently from previous research (Christensen and Hansen, 2002) in our sample put implied volatility has more predictive power than call implied volatility. This is an interesting result and is a warning against the a-priori choice of using call implied volatility.

Among the three implied volatility forecasts, the average implied is ranked first in explaining future realized volatility, followed by put implied volatility, and call implied volatility. The better performance of average implied can be attributed to the fact that it contains more information, being a trade weighted average of both call and put implied volatilities and permits error cancellation across option type.

Appendix 1.

We report in Table A1 the results of the OLS regressions of log realized volatility on log implied call or put volatility in each quartile (standard errors in brackets). $\ln(\sigma_c)^i$, $i=1,...4$ is the log implied call volatility in quartile i, $\ln(\sigma_p)^i$, $i=1,...4$ is the log implied put volatility in quartile i.

Table A1. OLS regressions of log realized volatility on log implied call or put volatility in each quartile.

Dependent variable: log realized volatility in each quartile										
Independent variables										
intercept	$\ln(s_c)^1$	$\ln(s_c)^2$	$\ln(s_c)^3$	$\ln(s_c)^4$	$\ln(s_p)^1$	$\ln(s_p)^2$	$\ln(s_p)^3$	$\ln(s_p)^4$	R^2	DW
0,308	1,299								0,825	1,587
(0,153)	(0,000)									
-0,378		0,811							0,761	2,123
(0,460)		(0,000)								
-0,210			0,926						0,513	2,015
(0,504)			(0,000)							
0,104				1,118					0,861	2,800
(0,468)				(0,000)						
0,349					1,299				0,693	2,282
(0,257)					(0,000)					
-0,042						1,040			0,846	1,952
(0,790)						(0,000)				
-0,159							0,952		0,713	1,704
(0,469)							(0,000)			
-0,021								1,021	0,830	2,277
(0,895)								(0,000)		

Note: p-values in brackets.

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