



## A measurement of the CP-conserving component of the decay

$$K_S^0 \rightarrow \pi^+ \pi^- \pi^0$$

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## Abstract

The NA48 Collaboration has measured the amplitude of the CP-conserving component of the decay  $K_S^0 \rightarrow \pi^+\pi^-\pi^0$  relative to  $K_L^0 \rightarrow \pi^+\pi^-\pi^0$ . For the characteristic parameter  $\lambda$ , the values  $\text{Re } \lambda = 0.038 \pm 0.010$  and  $\text{Im } \lambda = -0.013 \pm 0.007$  have been extracted. These values agree with earlier measurements and with theoretical predictions from chiral perturbation theory.

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## 1. Introduction

The  $K_S^0 \rightarrow \pi^+\pi^-\pi^0$  decay amplitude is dominated by two angular momentum components,  $l = 0$  and  $l = 1$  ( $l$  is the angular momentum of the neutral pion with respect to the system of the two charged pions); higher angular momentum states are suppressed because the kaon mass is close to the three pion mass. In this analysis we have measured the CP conserv-

ing transition to the  $l = 1$  state through its interference with the dominant  $K_L^0 \rightarrow \pi^+\pi^-\pi^0$  decay. We neglect any effects from CP violation in mixing and decay, which are totally negligible at our level of sensitivity.

The amplitudes for the decays of neutral kaons into three pions ( $A_L^{3\pi}$  for  $K_L^0$  and  $A_S^{3\pi}$  for  $K_S^0$ ) can be parameterized in terms of the Dalitz variables  $X$  and  $Y$ , which are defined as

$$X = \frac{s_{\pi^-} - s_{\pi^+}}{m_{\pi^\pm}^2}, \quad Y = \frac{s_{\pi^0} - s_0}{m_{\pi^\pm}^2} \quad (1)$$

with  $s_\pi = (p_K - p_\pi)^2$ ,  $s_0 = \frac{1}{3}(s_{\pi^+} + s_{\pi^-} + s_{\pi^0})$  and  $p_K$  and  $p_\pi$  being the 4-momenta of the kaon and the pion, respectively. The Dalitz variable  $X$  is a measure of the difference of the energies of the two charged pions in the kaon's rest system while  $Y$  is a measure of the energy of the neutral pion in the kaon's rest system.

The  $l = 1$  and the  $l = 0$  components of the decay  $K_S^0 \rightarrow \pi^+\pi^-\pi^0$  can be separated by the fact that the amplitude of the  $l = 1$  process is antisymmetric in  $X$  while the  $l = 0$  process is symmetric in  $X$ . Therefore the  $l = 1$  contribution can be extracted by separately integrating over the regions  $X > 0$  and  $X < 0$  and subtracting the results from each other. We consider the distribution

$$V(t) = \frac{N_{3\pi}^{X>0}(t) - N_{3\pi}^{X<0}(t)}{N_{3\pi}^{X>0}(t) + N_{3\pi}^{X<0}(t)}, \quad (2)$$

where  $N_{3\pi}^{X>0}(t)$  [ $N_{3\pi}^{X<0}(t)$ ] is the number of decays of neutral kaons into  $\pi^+\pi^-\pi^0$  at time  $t$  with a value of the Dalitz variable  $X$  larger [smaller] than zero [1,2].

In the NA48 set-up, where  $K_S^0$  and  $K_L^0$  are produced in equal amounts at a fixed target, the interference of the  $l = 1$  component of  $K_S^0 \rightarrow \pi^+\pi^-\pi^0$  with the  $l = 0$  component of the dominant  $K_L^0 \rightarrow \pi^+\pi^-\pi^0$  decay can be observed. It can be described by a complex pa-

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parameter [3]

$$\lambda = \frac{\int_{-\infty}^{\infty} dY \int_0^{\infty} dX A_L^{*3\pi(l=0)}(X, Y) A_S^{3\pi(l=1)}(X, Y)}{\int_{-\infty}^{\infty} dY \int_0^{\infty} dX |A_L^{3\pi(l=0)}(X, Y)|^2}, \quad (3)$$

which has been extracted by fitting the distribution defined above:

$$V(t) \approx 2D(E) [\operatorname{Re}(\lambda) \cos(\Delta mt) - \operatorname{Im}(\lambda) \sin(\Delta mt)] \times \frac{e^{-\frac{t}{2}(\frac{1}{\tau_S} + \frac{1}{\tau_L})}}{e^{-\frac{t}{\tau_L}}}. \quad (4)$$

$\Delta m$  is the mass difference between  $K_L^0$  and  $K_S^0$ ,  $\tau_L$  and  $\tau_S$  are the respective lifetimes, and the energy-dependent ‘‘dilution’’  $D(E)$  is the difference in the relative abundances of  $K^0$  and  $\bar{K}^0$  at production for a kaon energy of  $E$ :

$$D(E) = \frac{K^0 - \bar{K}^0}{K^0 + \bar{K}^0}. \quad (5)$$

## 2. Experimental setup

The data presented here were taken in a high-intensity neutral beam at the CERN Super Proton Synchrotron during 89 days in 2002. Protons of 400 GeV with an average intensity of  $5 \times 10^{10}$  per 4.8 s spill (within a cycle of 16.2 s) impinged on a beryllium target, where secondary particles were produced. Charged particles were removed by a sweeping magnet and a 5.1 m long collimator, which selected a beam of neutral particles at an angle of 4.2 mrad with respect to the proton beam.

After the collimator, the neutral beam and the decay products propagated in an 89 m long vacuum tank, where most of the short-lived kaons ( $K_S^0$ ) and neutral hyperons ( $\Lambda^0$ ,  $\Xi^0$ ), as well as a small fraction of the long-lived neutral kaons ( $K_L^0$ ) decayed. Undecayed particles ( $K_L^0$ , neutrons) continued to a beam dump via a vacuum pipe that passed through all of the downstream detectors.

Downstream of the vacuum tank there was a magnetic spectrometer consisting of four drift chambers inside a helium-filled tank and a magnet with a horizontal transverse momentum kick of 265 MeV/c. The spatial resolution was 120  $\mu\text{m}$  per view, and the mo-

mentum resolution was  $\delta p/p = (0.48 \oplus 0.015p)\%$ , with momentum  $p$  in GeV/c.

After the spectrometer there was a scintillator hodoscope, for the accurate timing of charged particles (yielding a time resolution on single tracks of 250 ps), and a liquid-krypton electromagnetic calorimeter, for the measurement of photons and electrons, with an energy resolution of  $\sigma(E)/E = (3.2/\sqrt{E} \oplus 9/E \oplus 0.42)\%$  ( $E$  in GeV). These were followed by a hadron calorimeter and a muon detector. A more detailed description of the detector has been given elsewhere [4].

## 3. The trigger

Decays into  $\pi^+\pi^-\pi^0$  made up only a small part of the event rate in the detector. This was due to the small expected branching ratio for the  $K_S^0$  decay and to the long lifetime of the  $K_L^0$ . To obtain a statistically significant sample of  $\pi^+\pi^-\pi^0$  decays, a very selective trigger was used to suppress the high two-particle background, which came mostly from  $K_S^0 \rightarrow \pi\pi$  and from  $\Lambda^0 \rightarrow p\pi^-$ .

When the level-1 trigger indicated an event with at least two charged tracks and a minimum energy deposition of 30 GeV in the calorimeters, a fast on-line processor calculated the momenta of the charged tracks, and hence the invariant mass of the decaying neutral particle under the assumptions of  $K_S^0 \rightarrow \pi^+\pi^-$ ,  $\Lambda^0 \rightarrow p\pi^-$ , and  $\bar{\Lambda}^0 \rightarrow \bar{p}\pi^+$ . Events were accepted by the trigger if none of the calculated masses matched the physical mass of the particle in question (difference of squares of the calculated mass and the nominal mass [5] larger than two percent). Additional cuts were applied to the relative momenta of the two charged particles ( $p_{\text{larger}}/p_{\text{smaller}} < 3.5$ , to further suppress  $\Lambda^0$  decays) and to the separation of the two tracks at the entry of the spectrometer ( $> 5$  cm in drift chamber 1, to remove photon conversions in the upstream Kevlar window). To allow a measurement of the trigger efficiency, a downscaled sample of minimum-bias events (1/35) was recorded in parallel, where only the level-1 trigger condition was fulfilled. In the course of data-taking, the high voltage in the drift chambers was occasionally adjusted to maintain stable operating conditions. As a result, the efficiency of the level-2 trigger varied between  $(75 \pm 4)\%$  and

$(86 \pm 4)\%$ . By comparing the analysis of different time periods, it was checked that this variation did not introduce any spurious asymmetries.

#### 4. Data analysis

For the offline analysis, events were selected with two tracks of opposite charge from the same vertex (distance of closest approach less than 3 cm), and at least two electromagnetic clusters (within  $\pm 20$  ns). For each pair of clusters, the  $\pi^0$ -mass was calculated assuming that the pion decayed at the position of the charged vertex. If there were more than two clusters, the cluster pair yielding the best  $\pi^0$ -mass was retained.

In order to reduce acceptance effects near the edges of the detectors, each charged track was required to cross the drift chambers at a minimum distance of 12 cm and a maximum distance of 110 cm from the beam axis. Likewise, each photon cluster was required to be more than 15 cm from the beam axis and more than 10 cm from the outer edge of the calorimeter.

In addition, the photon clusters had to be well separated from tracks ( $> 15$  cm) to avoid mismeasurements due to energy sharing. Tracks were identified as charged pions rather than electrons by demanding that their electromagnetic energy deposition was less than 90 percent of the track momentum. Cuts were applied on the charged vertex (less than 3 cm lateral distance

from the  $K^0$  beam line, and between 800 and 5400 cm from the target), and on the masses reconstructed for  $K^0 \rightarrow \pi^+\pi^-\pi^0$  and  $\pi^0 \rightarrow \gamma\gamma$  ( $m_{K^0}^{\text{PDG}} \pm 10.5$  MeV and  $m_{\pi^0}^{\text{PDG}} \pm 7.8$  MeV [5]). These cuts also excluded  $K_S^0 \rightarrow \pi^+\pi^-$  decays. The reconstructed kaon energy was required to be between 30 and 166 GeV.  $\Lambda^0$  hyperons were further suppressed by cutting harder than in the trigger on the masses for the hypotheses  $\Lambda^0 \rightarrow p\pi^-$  and  $\bar{\Lambda}^0 \rightarrow \bar{p}\pi^+$  ( $m_{\Lambda^0}^{\text{PDG}} \pm 21$  MeV [5]) and on the momentum ratio of the two charged particles ( $p_{\text{larger}}/p_{\text{smaller}} < 2.9$ ). No  $\Lambda^0$  hyperons survive after these two cuts. The mass cuts were optimized to maintain a high efficiency without introducing substantial background. A total of 19 million events passed all these cuts.

Fig. 1 shows the distributions of the position of decay vertices along the beam line ( $z$  vertex) and the reconstructed  $K^0$  energy after applying the analysis cuts.

Fig. 2 shows the distribution of the  $K^0$  invariant mass after cuts.

Applying these cuts and correcting for acceptance and trigger efficiency as described below, the experimental distribution  $V(t)$  defined in (2) was obtained. The values of  $\text{Re}\lambda$  and  $\text{Im}\lambda$  were extracted by fitting equation (4) to the experimental data. Due to the energy dependence of the dilution  $D(E)$ , the fit was made in bins of kaon energy (see Fig. 3), with 8 equal-sized energy bins ranging from 30 to 166 GeV. The

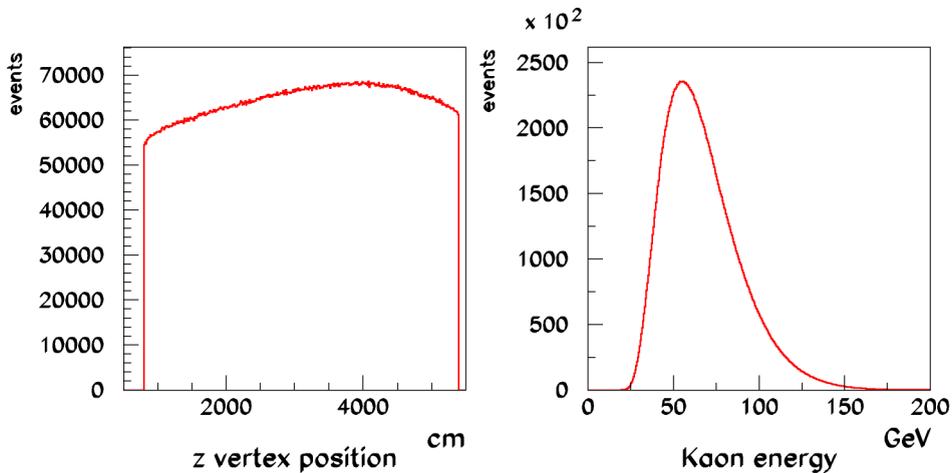


Fig. 1. Position of the decay vertex along the beam line (left, measured from target position) and reconstructed  $K^0$  energy (right) for the data sample of 19 million events.

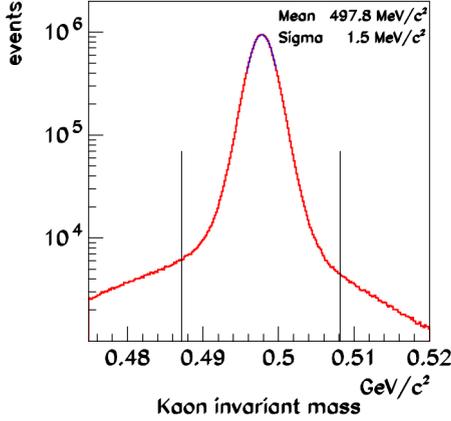


Fig. 2. The  $K^0$  invariant mass. The vertical lines indicate the  $K^0$  mass cut used in the analysis.

values for the external parameters  $\Delta m$ ,  $\tau_L$  and  $\tau_S$  were taken from [5]. For the dilution  $D(E)$ , we used a quadratic fit to the values measured in [6] (energy  $E$  given in GeV)

$$D_{\text{measured}}(E) = -0.167 + \frac{6.10}{10^3}E - \frac{1.72}{10^5}E^2. \quad (6)$$

The errors for the individual points measured in [6] vary between 5 percent at the lowest kaon energies (75 GeV) and 25 percent at the highest energies (165 GeV). We assume the uncertainties at the different energies to be uncorrelated. To take into account the difference in proton energy and production angle between [6] and our own measurement, we used measurements with charged kaons [7], from which we derived an empirical correction factor (energy  $E$  given in GeV)

$$D_{\text{corr}}(E) = 1.28 - \frac{3.82}{10^3}E + \frac{5.38}{10^5}E^2 - \frac{3.32}{10^7}E^3 + \frac{7.15}{10^{10}}E^4. \quad (7)$$

To fit Eq. (4), we then use

$$D(E) = D_{\text{measured}}(E) \times D_{\text{corr}}(E). \quad (8)$$

Based on the uncertainties in [6,7], the overall relative uncertainty in  $D(E)$  is estimated at about 15 percent.

The aim of the experiment was to measure a small effect that is antisymmetric in the Dalitz variable  $X$  (the interference of  $K_S^0$  and  $K_L^0$  in the decay into  $\pi^+\pi^-\pi^0$ ) over a large background that is symmetric in  $X$  (from the CP-conserving decay  $K_L^0 \rightarrow$

$\pi^+\pi^-\pi^0$ ). Therefore, care had to be taken to exclude any detector effects that could artificially introduce such an asymmetry in the  $K_L^0$  decay.

The sign of  $X$  equals the sign of the momentum difference of the  $\pi^+$  and the  $\pi^-$  in the kaon rest system. The pion momenta in the kaon rest system are correlated to their momenta in the laboratory system. Any momentum-dependent difference in the acceptance or the trigger efficiency for positive and negative pions may therefore introduce a possible bias. Due to the field of the spectrometer magnet, positive and negative particles populated the detector in a different way. In order to compensate such effects, the field of the spectrometer magnet was reversed weekly during the data taking.

An artificial asymmetry can be caused by the detector acceptance only if this acceptance is asymmetric in  $X$ . However, if there is a real, physical asymmetry, its value may be distorted even by a symmetric  $X$ -dependence of the acceptance. The NA48 detector simulation (based on the GEANT3 package [8]) showed a symmetric decrease in the acceptance for high  $|X|$ . Using the simulation, the acceptance was calculated as a function of  $|X|$ , kaon decay time and kaon energy. The effect was then corrected by weighting each event in the data in inverse proportion to the calculated acceptance for the corresponding value of  $|X|$ , kaon decay time and kaon energy. This procedure resulted an increase of  $10 \pm 5$  percent in the extracted values of  $\text{Re } \lambda$  and  $\text{Im } \lambda$ .

In the drift chambers, signal attenuation near the ends of some wires created a left–right acceptance asymmetry on the detector periphery, especially for strongly deviated low-momentum particles. This was compensated in the offline reconstruction by the redundancy of chamber planes, and there was no visible effect on the minimum-bias data sample. However the data collected with the fast level-2 trigger algorithm, which did not use the full redundancy, showed an asymmetry in  $X$  at all kaon lifetimes, which was strongest (2 percent) at the lowest kaon energies (30 GeV). This asymmetry was reversed when the magnetic field of the spectrometer magnet was inverted. Using the  $X$ -symmetric amplitude of  $K_L \rightarrow \pi^+\pi^-\pi^0$ , the detector and trigger simulation accurately reproduced this behavior.

The Monte Carlo distributions from the simulation were used to correct the data. The correction resulted

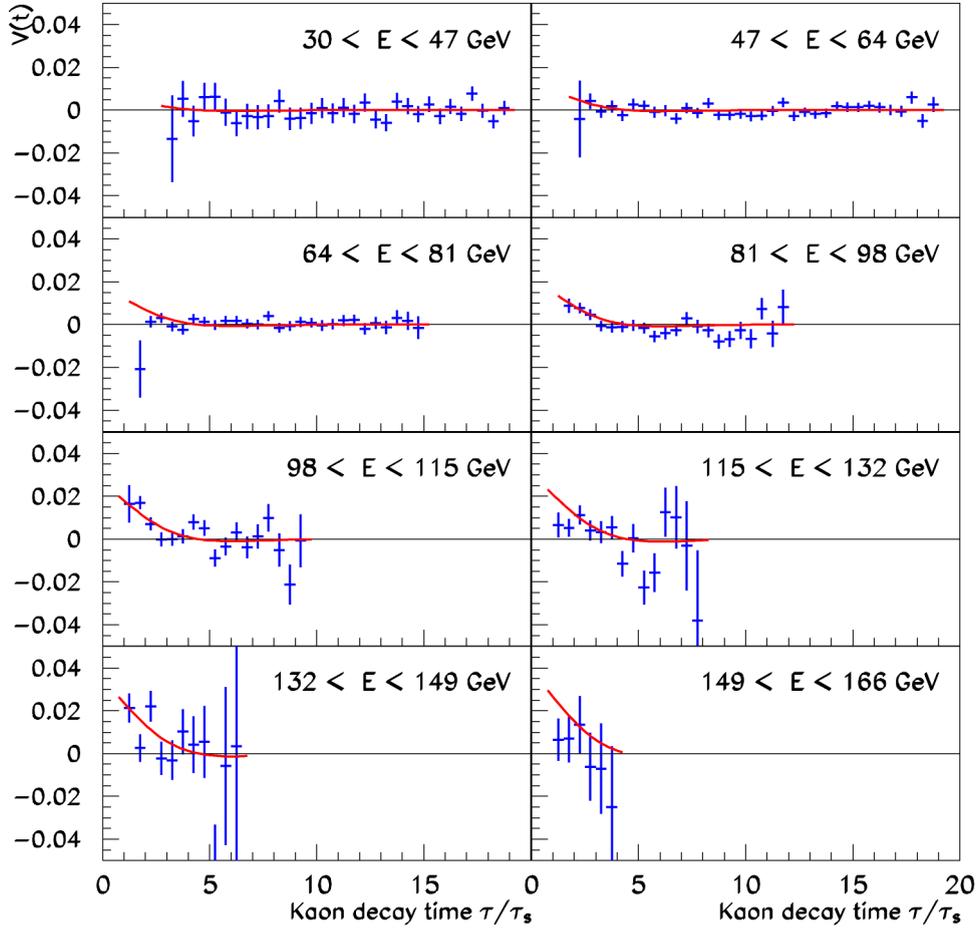


Fig. 3. The experimental distribution  $V(t)$  (points with error bars) and the function obtained by a  $\chi^2$  fit (smooth curve) in bins of time and energy. The  $\chi^2$  per degree of freedom is  $169.35/162 = 1.045$ . The fit was made simultaneously over all energy bins, but is mainly constrained by the intermediate energy bins. The data were compensated for acceptance and trigger efficiency by applying Monte Carlo corrections and by averaging over samples with opposite magnetic field in the spectrometer.

in a shift of about  $3 \times 10^{-2}$  in both  $\text{Re } \lambda$  and  $\text{Im } \lambda$ , with opposite sign for the two field orientations of the spectrometer magnet. The simulation program had been developed, extensively used and validated during previous studies by the collaboration [4,9]. After acceptance and trigger corrections, the individual results for the two field orientations differ by about  $1.4\sigma$ . To exclude possible additional effects the Monte Carlo corrected data samples with positive and negative fields in the spectrometer were subsequently averaged (taking the arithmetic mean of the observed distributions  $V(t)$  in formula (2), which is equivalent to normalizing the data taken in the two field orientations to the same number of events and summing them before apply-

ing (2)). Slightly different results were obtained when using either only the Monte Carlo correction (without averaging over the two magnetic field orientations), or only averaging over the magnetic fields (without the Monte Carlo correction). We use half the difference between these two last approaches as a contribution to our systematic error ( $\pm 0.004$  in  $\text{Re } \lambda$  and  $\pm 0.003$  in  $\text{Im } \lambda$ ).

Another cross-check was to analyse the data with a tight outer radius cut of 50 cm in the drift chambers and otherwise follow the above procedure. This was prompted by the fact that the observed drift chamber inefficiencies were highest at large radii. The results in  $\text{Re } \lambda$  and  $\text{Im } \lambda$  differed by about 0.001 while

the statistical errors increased by about the same amount.

The other major contribution to the systematic uncertainty comes from the  $K^0/\bar{K}^0$  dilution  $D(E)$  defined in formula (5) and leads to an additional uncertainty of  $\pm 0.005$  in  $\text{Re } \lambda$  and of  $\pm 0.003$  in  $\text{Im } \lambda$ . By varying the experimental cuts quoted above within reasonable bounds and the external parameters from [5] within their errors it was seen that all other possible sources of systematic uncertainties were negligible.

## 5. Results

The following values have been obtained for the real and imaginary parts of  $\lambda$ :

$$\text{Re } \lambda = +0.038 \pm 0.008_{\text{stat}} \pm 0.006_{\text{syst}}, \quad (9)$$

$$\text{Im } \lambda = -0.013 \pm 0.005_{\text{stat}} \pm 0.004_{\text{syst}}. \quad (10)$$

Adding both errors in quadrature,

$$\text{Re } \lambda = +0.038 \pm 0.010, \quad (11)$$

$$\text{Im } \lambda = -0.013 \pm 0.007. \quad (12)$$

These values can be compared with the theoretical numbers of  $\text{Re } \lambda = +0.031$  and  $\text{Im } \lambda = -0.006$  obtained in [3] by using results from chiral perturbation theory [2].

Fig. 4 shows the distribution  $V(t)$  defined in (2) derived from the data, and the curve obtained from (4) with the fitted values of  $\text{Re } \lambda$  and  $\text{Im } \lambda$ . These values

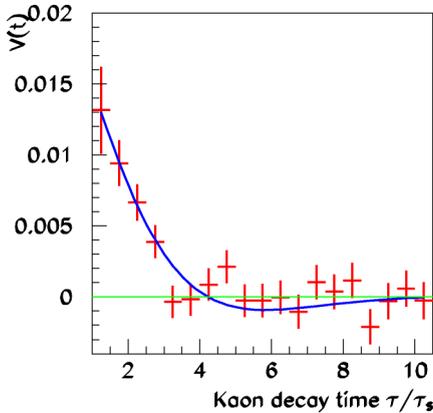


Fig. 4. The experimental distribution  $V(t)$  (points with error bars, data summed over all kaon energies) and the function obtained from the fitted values for  $\text{Re } \lambda$  and  $\text{Im } \lambda$  for a mean kaon energy of 87 GeV.

were not obtained by fitting this histogram directly, but by fitting all data in bins of energy. This is important because of the dependence of the dilution  $D(E)$  on the kaon energy.

The real and imaginary parts of  $\lambda$  are strongly correlated, as can be seen from Fig. 5, which shows the measured values and contours of equal  $\chi^2$  in the complex plane. Taking into account only the statistical errors, the correlation coefficient between  $\text{Re } \lambda$  and  $\text{Im } \lambda$  is 0.66.

From the real part of  $\lambda$ , the branching ratio for the CP-conserving component of the decay  $K_S^0 \rightarrow \pi^+\pi^-\pi^0$  can be obtained. Using the same approximation and the external parameters as in [3], we get

$$\begin{aligned} \text{BR}(K_S^0 \rightarrow \pi^+\pi^-\pi^0) \\ = (4.7^{+2.2}_{-1.7}(\text{stat})^{+1.7}_{-1.5}(\text{syst})) \times 10^{-7}. \end{aligned}$$

The results from this experiment agree with chiral perturbation theory and with two other measurements with comparable errors [3,10]. It is worth noting that experiment [3] used tagged kaons, so that its result is independent of any measurements or calculations of the dilution  $D(E)$  while [10] was carried out at higher kaon energies and therefore higher dilution than our measurement. It should also be noted that all three experiments obtain a nonzero negative value for the imaginary part of  $\lambda$  (which is predicted to be small).

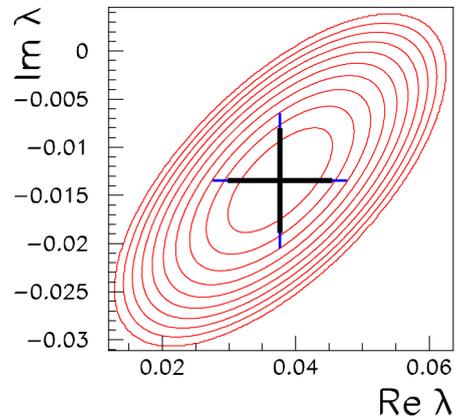


Fig. 5. The fitted values of  $\text{Re } \lambda$  and  $\text{Im } \lambda$  in the complex plane with the contours for a  $\chi^2$  of 1 to 10 above the minimum  $\chi^2$ . The shorter error bars and the contours refer only to the statistical uncertainty while the longer error bars correspond to the total uncertainty.

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