Introduction to the papers of TWG04: Geometry Teaching and Learning

Keith Jones, Michela Maschietto, Joris Mithalal-Le Doze, Chrysi Papadaki

Introduction to the papers of TWG04: Geometry Teaching and Learning

Keith Jones[1], Michela Maschietto[2], Joris Mithalal-Le Doze[3] and Chrysi Papadaki[4]

[1]University of Southampton, School of Education, United Kingdom; d.k.jones@soton.ac.uk

[2]University of Modena e Reggio Emilia, Dipartimento di Educazione e Scienze Umane, Italy; michela.maschietto@unimore.it

[3]Université de Lyon, S2HEP, ESPE de Lyon, France; joris.mithalal@ens-lyon.org

[4] Universität Bremen, AG Didaktik der Mathematik, Germany; chrysi@uni-bremen.de

Keywords: Geometry education, instruments, proof and argumentation, spatial skills, visualization.

Introduction

Around 25 participants attended the Thematic Working Group 4 sessions during CERME 11. 75% of them were Europeans, and the other 25% came from the Middle East or from Australia. Five discussion sessions were dedicated to specific topics (manipulation, artifacts, visualization, teacher education), and each contribution (in total 18 papers and 3 posters) was presented and discussed during 20 minutes. The last two sessions were dedicated to small-group debates that supported the writing of the final report that was presented on the final day of the conference.

The call for papers, the contributions, and the discussions, addressed classical issues in geometry education such as the role of manipulation, instruments, investigation and modeling, the ways of describing and training visualization processes or spatial skills, and the role of language in geometry, including problem solving, argumentation or proof. It appeared that the multiple frameworks or methods that helped in addressing these issues were sometimes very close, or sometimes shed different lights upon the phenomena we observed. In order to benefit from these multiple viewpoints, we decided that, rather than summarizing our work following each initial issue, we should identify general questions that reflected the heart of the discussions; these were:

- How is it possible to describe how space intervenes in "doing geometry"?
- What is at stake in learning geometry, from cognitive and didactical points of view?
- Which transversal competencies have to be taken into account in the teaching of geometry, and how are they interrelated?

The way we addressed these questions shows continuity in the group's work across the CERME conferences. Schematically speaking, we could say that CERME 8 was more about what geometry is, CERME 9 about what is at stake when doing geometry, and CERME 10 about the various theoretical approaches of these questions. We built our discussion on this basis. We managed to address more efficiently these questions, and to understand better the similarity or complementarity of the participants' points of view.

Space in "doing geometry"

One of the toughest theoretical issues was about space, and the mutual understanding between psychology and mathematics education. We identified during CERME 11 that, on the one hand,

psychology considered that visualization was a part of "spatial skills" and that, on the other hand, to mathematics education spatial skills were a part of the visualization process. It seemed that this was not only a matter of word meaning, and we tried to investigate these opposite points of view.

We used the identification by Perrin & Godin (2018)[1] of three spaces involved in doing geometry: the physical one, the graphical one, and the geometrical[2] one (see Fig. 1). In a way, geometry consists in establishing relations between these spaces, and solving geometry problems needs to "grasp space" and to make the information usable in another kind of space.

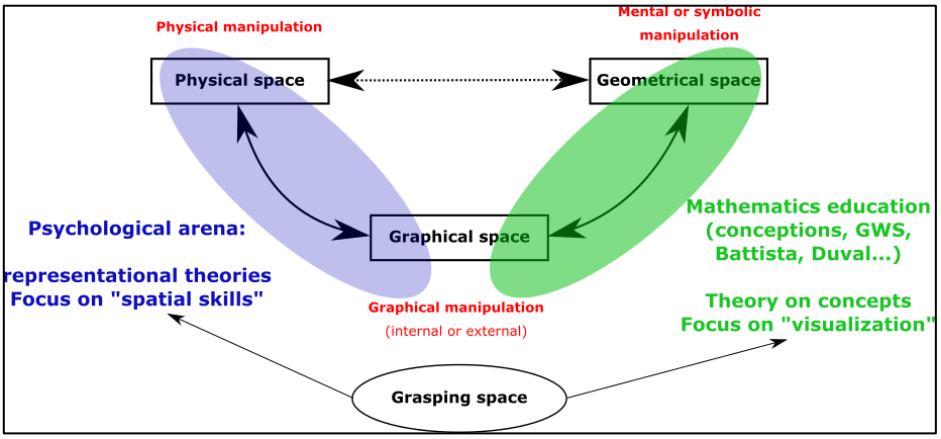

Figure 1: three spaces involved in doing geometry, two points of view on it.

On the one hand, psychological points of view in the group (see Heil; Conceicão) were mainly used in order to explore the articulation between physical and graphical space: How do children manage to represent physical space? What are the difficulties? How does it intervene in their solving physical space problems? This point of view put forth representational issue. Visualization is one of the ways to grasp and interpret information; then it is no more than one of the components of the spatial skills.

On the other hand, mathematics education traditionally focuses on the links between graphical and geometrical space. The main issue is the correct use of graphical information to elaborate concepts or to work on ideal objects, or the graphical representation of idealities (see previous TWG4 reports and Downton; Gridos; Jones; Palatnik). In this way, visualization also embraces geometrical knowledge or specific treatments of graphical space. Spatial skills are one of the treatments performed on the graphical space, so in this case they are just a part of visualization.

This clarification aims at improving mutual understanding, and then collaboration, not only by explaining the discrepancies but also by identifying complementary issues and showing that these points of view are in fact two sides of general matters about space.

---

[1] Perrin-Glorian, M.-J., & Godin, M. (to be published). Géométrie plane : pour une approche cohérente du début de l'école à la fin du collège. In proceedings of the CORFEM, *Ressources pour la formation des professeurs. Savoirs mathématiques à enseigner au collège et au lycée*. A preliminary version is available on https://hal.archives-ouvertes.fr/hal-01660837v2/document

[2] By geometrical space, we refer to ideal objects and their relations, which are mainly elaborated by/in discourse.

We can add that we do not pretend that the relations between physical and geometrical space are not relevant, and some of the works presented in the group examined it (Favilla, Luppi & Maschietto). They highlighted the potential of such direct geometrical interpretation of physical experience, including an instrumental point of view. In this case, though, neither visualization nor spatial skills are at the core of the studies.

Learning geometry

The second main focus of our work was about "learning geometry". It has to be noted that, even if specific learning topics were examined, the group discussions investigated how general competencies (such as creativity, flexibility, fluency, language, beliefs, etc.) may affect the learning of these specific topics (see Brunheira; Favilla; Gridos; Mendes; Palatnik). In this perspective, the role of everyday life, physical experience, manipulation or spatial skills in creating mental images or developing abstract concepts was more strongly highlighted than in the previous topic (Brunheira; Heil; Palatnik). The role of tools and artefacts was also discussed, highlighting, on the one hand, the limitation of their use (because of difficult instrumental genesis, but also for intrinsic reasons), and on the other hand, their potential, including higher education where it appears that tools, games, and manipulatives are very helpful but generally seen as something that is not needed (Bjørkås; Katter).

Language, and more specifically the emergence of geometrical lexicon, appeared as one great issue in the learning of geometry (Bulf, Favilla, Haj Yahya). Many contributions pointed out that constructing the meanings of the words used in geometry is a long and complex process that cannot be reduced to "vocabulary" issue. These meanings are the result of more general practices (including manipulation), negotiation, social interaction in problem solving contexts, combined to the cultural background – including everyday meaning that influences the understanding of the words. This dynamic and progressive learning of specific lexicon and meanings was coherent with the works on mathematical discourse, but it has to be noted that discourse itself was not studied in this case. This remains an open discussion field in this group.

It was connected to many contributions (Albano; Bernabeu; Brunheira; Gridos; Jones; Palatnik, Vieira da Silva) about argumentation, justification, reasoning or proving. This topic combined various levels of considerations, embracing the multiple facets of proof: required operation for proving in geometry (such as the analysis of figures as components and relations), relations between arguing, reasoning and justifying, or about the specific writing process that is required by formal proof. One contribution proposed a general overview, showing variations of the type of language used during the proving process.

Teaching geometry and teacher education

These two issues have been unified into one only discussion group, as many contributions addressed general topics, relevant for both of them. The participants raised four great topics involved in the teaching of geometry: problem solving, manipulation with tools (including drawings), visualization, and proof (see Fig. 2). Language has been added considering some contributions showing how it is linked to manipulation with tools and to visualization.

Some contributions used these topics to analyze precisely the pre-service and in-service teachers' geometrical knowledge, focusing on specific parts of the diagram we propose (Brunheira; Bulf; Nechache; Haj Yahya; Mendes). It has to be noted that the contributions mixed analysis about specific, local, geometrical knowledge, and more general concerns as described in Fig.2, in a very convincing way.

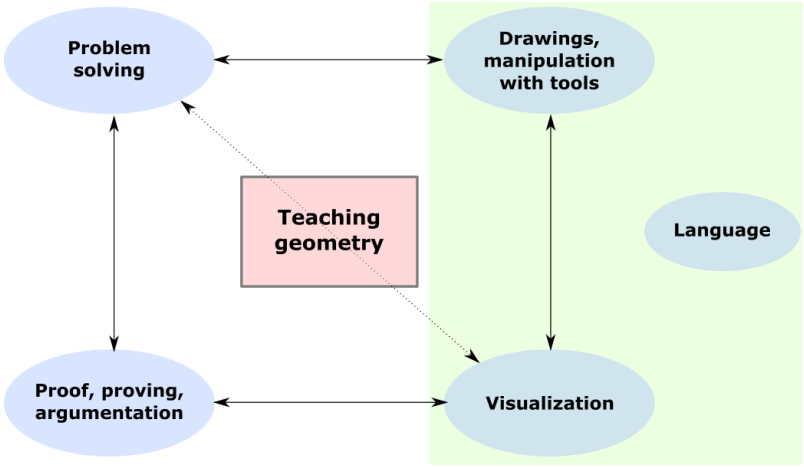

Figure 2. Topics involved in teaching geometry and teacher education

Other contributions proposed results about the relations between these topics (Bjorkas; Boavida; Bulf; Delgado; Mendes; Palatnik). The proving process needs the pupils to identify relations in the drawings, so both manipulation and visualization should be considered in the teaching of proof, and in the teacher training curricula. Moreover, by using specific artifacts (e.g. geoboard), by promoting specific strategies or by giving access to multiple solutions of a single problem, teachers promote efficient manipulation and visualization and, consequently, support solving problems skills development and the understanding of geometrical concepts. Then, teaching sequence design may take into account the relations we indicated in the diagram, and the difficulty to coordinate multiple poles of geometrical activity that was raised by some contributions, and by previous works in the group (e.g. about Geometrical Working Space or language). In a general way, this indicates that neither teaching nor teacher training should be only focused on mathematical contents or on a specific pole, but it should embrace the coordination between many of these poles.

Ultimately, we would like to mention that these general components involved in the teaching of geometry were less intertwined with specific topics, and then were helpful when examining how interactions with other fields (such as arts education) may be productive.

Perspectives and conclusion

As is clear in the papers that follow this introduction, the participants contributed to enrich the understanding of some classical issues in geometry education, and to develop more topical ones. We hope that a careful reading of these papers may also reflect that the work and the discussions promoted mutual understanding about both the frameworks and the issues they address. This seemed to be more productive than seeking a unified and unique framework, and we believe this to be a major contribution of CERME in general, to be continued over the next sessions of the group.