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FRICTION FACTOR EVALUATION OF COMPRESSIBLE MICROFLOWS USING 1D FANNO FLOW-BASED NUMERICAL MODEL

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Abstract

A numerical model based on the conventional Fanno flow theory for the friction factor evaluation of the gas flows inside microchannels has recently been developed by Cavazzuti et al. [1]. The current work aims to compare this numerical model with experimental results of microchannels. Pressure drop experiments are performed for a circular cross sectional microtube and a rectangular microchannel with Nitrogen gas as working fluid. The hydraulic diameters of microtube and microchannel are 100 μm and 69.4 μm respectively. Rectangular microchannel with an aspect ratio (height to width) of 0.036 is chosen for the comparison. This allows to treat the rectangular microchannel as parallel plate in the numerical Fanno model. During experiments stagnation pressure at the inlet is increased such that maximum Reynolds number is 3000 in the case of microchannel and 7600 for microtube. Results show that for the evaluation of average friction factor in both considered geometries, there exists a good match between Fanno-based 1D numerical model and experimental results in the laminar regime whereas comparison worsens as the flow approaches choking. Limitations as well as the potential reasons for the discrepancies between the developed model and experiments will be discussed.

KEYWORDS: pressure drop, internal flow, flow choking, microtube

MATHEMATICAL MODEL FOR FANNO FLOW

Fanno theory is based on conservation equations (mass, momentum and energy), ideal gas equation and the definitions of Mach number. The differential form of these equations for a channel of hydraulic diameter D_h along the streamwise direction x can be written as:

$$\begin{aligned}
\frac{\mathrm{d}u}{u} &= -\frac{\mathrm{d}\rho}{\rho} = \frac{\gamma \mathrm{Ma}^2}{2(1-\mathrm{Ma}^2)}\frac{f\mathrm{d}x}{D_\mathrm{h}} \\
\frac{\mathrm{d}T}{T} &= \frac{-\gamma(\gamma-1)\mathrm{Ma}^4}{2(1-\mathrm{Ma}^2)}\frac{f\mathrm{d}x}{D_\mathrm{h}} \\
\frac{\mathrm{d}p}{p} &= \frac{-\gamma \mathrm{Ma}^2[1+(\gamma-1)\mathrm{Ma}^2]}{2(1-\mathrm{Ma}^2)}\frac{f\mathrm{d}x}{D_\mathrm{h}} \\
\frac{\mathrm{dMa}}{\mathrm{Ma}} &= \frac{\gamma \mathrm{Ma}^2[1+(\gamma-1)\mathrm{Ma}^2/2]}{2(1-\mathrm{Ma}^2)}\frac{f\mathrm{d}x}{D_\mathrm{h}}
\end{aligned} \tag{1}$$

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where p, ρ, T and u are cross sectional average pressure, density, temperature and axial velocity respectively. Ma and f denote local Mach number and friction factor respectively whereas γ represents the ratio of specific heats ($c_\mathrm{p}/c_\mathrm{v}$) of the gas. A detailed description of the numerical solution procedure has been given in [2] and for the completeness a short description will be repeated here. The length of the channel is discretized in $'n'$ number of segments. Given the stagnation temperature and pressure boundary conditions at both inlet and outlet, solution of the system is carried out as follows:

- As an initial condition, Mach number at the inlet is guessed using the last expression of Eq. (1) between node $(i-1)$ and node (i) assuming γ and f from the previous node
- All the other quantities are computed and corrected in an iterative loop marching in the streamwise direction up to convergence. In particular, the dynamic pressure and temperature correction factors as well as the friction factor are computed as recommended in [1]. Using these correction factors and friction factor, temperature (static, dynamic, total) and pressure (static, dynamic, total) are computed by solving the Eq. (1). Thermophysical properties are then updated from computed static temperature, velocity is evaluated by imposing mass conservation, and finally the Mach number is corrected from the velocity just computed.
- Once the flow quantities are evaluated along the whole length of the channel using the above methodology, the resulting downstream stagnation pressure is compared to the given boundary condition and this is used to guess another inlet Mach number and repeat the procedure if needed.

EXPERIMENTAL SETUP

A schematic of experimental test bench used in this work is shown in Fig. 1. Pressurized Nitrogen gas is allowed to enter the test section perpendicular to the axial direction of microchannel or microtube (MC/MT) and leaves perpendicularly as well through outlet manifold. For a detailed description of sensors used and associated uncertainties, reader is referred to [3]. A commercial fused silica MT is used for cicrcular cross

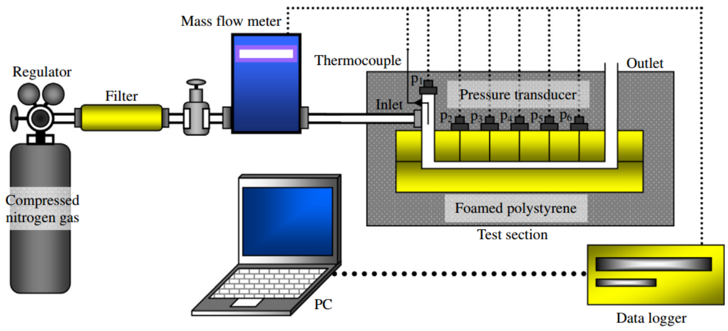

Figure 1: Schematic of experimental setup

section whereas MC is fabricated by etching silicon wafers in a $30\,\%$ KOH solution by weight at 80°C. The surface roughness (ϵ) of the MC was measured using a surface profilometer (Sloan DEKTAK IIA) and it was 0.002 which deems the considered channel as smooth. The length (L) of the MC is $26.9\,\mathrm{mm}$ whereas it is $13.3\mathrm{mm}$ for MT. Inlet stagnation pressure is varied to achieve the desired mass flow rate through the MC/MT and pressure is measured at the inlet and outet of MC/MT. In case of MC, five axial pressure taps also allowed local pressure reading using absolute pressure sensors. Stagnation temperature of Nitrogen gas is also measured at the inlet of MC/MT. Electrical signals from all the sensors are fed to the data logger and finally transmitted to PC for further post processing. Dimensions of the channels that is width (w), height (h) and aspect ratio α can be found in Table 1.

Table 1: Channels geometry used for experiments.

Channel	h (µm)	w (µm)	L (mm)	α	D_h (µm)
MC	36	992	26.9	0.036	69
MT	100	100	13.3	1	100

Considering one dimensional flow of ideal gas, average Fanning friction factor between inlet 'in' and outlet 'out' of a MC with hydraulic diameter D_h and length L can be defined by the following expression for a compressible flow [4]:

$$f_{\mathrm{f,av}} = \frac{D_\mathrm{h}}{L}\left[\left(-2\ln\frac{p_\mathrm{in}}{p_\mathrm{out}} + 2\ln\frac{T_\mathrm{in}}{T_\mathrm{out}}\right) - \left(\frac{1}{\dot{G}^2 R\left(T_\mathrm{in} + \frac{u_\mathrm{in}^2}{2c_\mathrm{p}}\right)}\right)\right.$$
$$\left.\times\left(\frac{p_\mathrm{out}^2 - p_\mathrm{in}^2}{2} + \frac{B^2}{2}\ln\frac{p_\mathrm{out} + \sqrt{p_\mathrm{out}^2 + B^2}}{p_\mathrm{in} + \sqrt{p_\mathrm{in}^2 + B^2}} + \frac{1}{2}\left(p_\mathrm{out}\sqrt{p_\mathrm{out}^2 + B^2} - p_\mathrm{in}\sqrt{p_\mathrm{in}^2 + B^2}\right)\right)\right] \quad (2)$$

where, $B^2 = 4\beta \times \frac{\dot{G}^2 R^2}{2c_\mathrm{p}} \times \left(T_\mathrm{in} + \frac{u_\mathrm{in}^2}{2c_\mathrm{p}}\right)$. $\dot{G}$ is mass flow per unit area ($\dot{G} = \frac{\dot{m}}{A}$). β, kinetic energy recovery coefficient, is taken as 2 for laminar and 1 for turbulent flow.

RESULTS

Numerical solution for both geometries were carried out using the described methodology with the experimental boundary conditions of stagnation temperature and pressure at the inlet as well as atmospheric pressure at the outlet of the MT/MC outlet manifold. A comparison between numerical and experimental results of MC is made in Fig. 2. There exists a good agreement between the experimental f_f and current 1D numerical model in the laminar flow regime as shown in Fig. 2a. Since pressure was measured only at the five axial locations, therefore in the case of experimental results, a semi-local constant value of f_f calculated between two consecutive pressure taps is presented. In general for high values of stagnation pressure (Re), numerical results underestimate the f_f. This can also be seen by observing the comparison of experimentally measured pressures with numerically estimated ones in Fig. 2c. Pressure drop is underestimated by Fanno model at higher inlet stagnation pressures. There has been a very good match in local pressure estimations in the laminar and early transitional regime, however. Similar trend is also exhibited by both local gas temperature and Mach number comparisons. For MT, since local measurements of pressure and temperature were not available therefore a comparison of average f_f only, calculated between inlet and outlet is used for comparison. There exists a very good agreement between numerical model and experimental results in the laminar regime where both follow the Poiseuille's law as shown in Fig. 3a. Experimental results of MT show transitional regime between Re of 2400-2600 and then f_f stays below the Blasius law whereas 1D model follows it. A potential reason can be the flow choking in transitional regime for the MT. It is worth mentioning here that f_f correlations for laminar and turbulent flow regimes used in numerical model are itself developed from an exhaustive set of CFD simulations and are not sensitive to the flow choking during transitional flow regime. These aspects will be addressed further.

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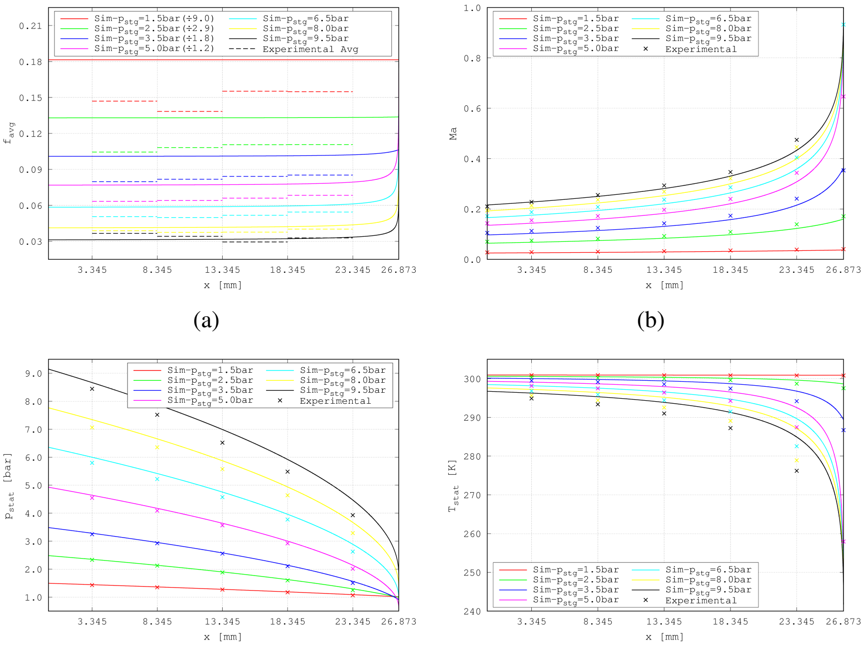
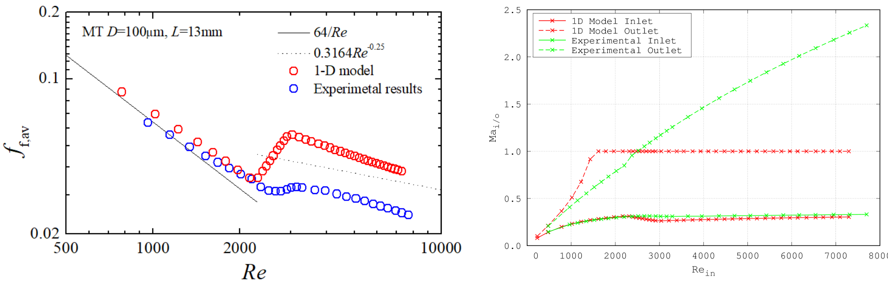


(a) (b) (c) (d)

Figure 2: Comparison between 1D numerical model and experiments for MC: local/semi-local friction factor (**a**), local Mach number (**b**), local pressure (**c**), and local temperature (**d**).



(a) (b)

Figure 3: Comparison between 1D numerical model and experiments for MT: average friction factor (**a**), Mach number (**b**).

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