Bending of beams in finite elasticity and some applications

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The 2D Rivlin solution concerning the finite bending of a prismatic solid has been recently extended by accounting for the complete 3D displacement field [1]. In particular, the relationship between the principal and transverse (anticlastic) deformation of a bent solid has been investigated, founding the coupling relationships among three kinematic parameters which govern the problem. Later, based on the formulation reported in [1], and making reference to a (hyper)elastic material, the formulation has been extended to slender beams by introducing some simplifying assumptions [2]. This leads to a challenging relation between the external bending moment m and the curvature R_0^{-1} of the longitudinal axis, which involves both the constitutive and geometric parameters of the beam. This relation can be viewed as a generalization of the *Elastica* [3].

However, such a relationship can be simplified through a series expansion, thus obtaining a reliable moment-curvature relation as follows [4]:

$$m(s) = \frac{4(a+b)(a+4b+3c)}{a+3b+2c}\frac{1}{R_0(s)} + O(R_0^{-3}(s)), \qquad r(s) = \frac{a+3b+2c}{b+c}R_0(s), \tag{1}$$

being a, b, c the constitutive parameters involved in the stored energy function according to a compressible Mooney-Rivlin material, whereas r denotes the anticlastic radius of the cross section [1]. In eqn $(1)_1$ the radius of curvature R_0 depends on the curvilinear abscissa s describing the beam axis in its deformed configuration. The rotation θ of the beam cross section follows from the derivative of the curvature with respect abscissa s, i.e. $\theta'(s) = R_0^{-1}(s)$. Thus, the axial and vertical components of the displacement field and the rotation of the beam cross section are found to be coupled in a set of three equations in integral form, which is handled in an iterative procedure in order to analyse elastic structures exhibiting deformations and displacements both large.

Some basic structural schemes under both dead and live loads are here investigated, thus assessing the deformed configuration and the arising internal forces into the beam. It is found that the magnitude of the external loads strongly affects the qualitative distribution of the axial and shear forces and the bending moment in the inflexed beam, giving rise to a solution which completely differs to that corresponding to infinitesimal strains and small displacements.

References

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