Looking at the SDEWES Index from a Multi-Criterion Decision Analysis perspective

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ABSTRACT

The SDEWES Index is obtained by aggregating several numerical indicators related to sustainable development. In the context of Multi-Criterion Decision Analysis (MCDA) this index can be seen as the solution to the "ranking problematic" for an underlying decisional problem. Accordingly, in this work we look at the SDEWES Index from an MCDA point of view. First, we consider some theoretical aspects, in particular the one usually referred to as "rank reversal". Then we consider some (classic as well as original) visual tools for decision aid, showing how they can be adapted and exploited.

KEYWORDS

SDEWES Index, Multi-Criterion Decision Analysis, rank reversal, compensatory methods, visual tools, principal components analysis, GAIA, TOPSIS.

INTRODUCTION

The SDEWES (Sustainable Development of Energy, Water, and Environment Systems) Index is a recently proposed benchmarking tool related to sustainable development. This tool has two main features:

- it addresses local systems, in particular, it applies to the evaluation of *cities*;
- it adopts an integrated approach, i.e., it is a *composite* index spanning across different (and often complementary) aspects.

An in-depth analysis of the SDEWES Index goes far beyond the scope of this work. We refer the reader to [1-5] for a technical description of the approach, and for an appraisal of its historical development and its future growth. Nevertheless, we point out a couple of relevant features here.

First, the computation of the SDEWES Index involves a substantial amount of work for data collection. Looking at things the other way round: a city to be evaluated via the SDEWES Index must have a SEAP (Sustainable Energy Action Plan) and maintain reliable statistics on its local energy system. Furthermore, collected data usually require pre-processing, e.g. for computing main indicators based on sub-indicators; in later versions, preprocessing also includes *winsorization* techniques to get rid of outliers. We somehow skip all these issues here: actually, our work starts *after* data collection and pre-processing.

Second, the SDEWES Index is descriptive in nature but it also has a relevant *prescriptive* value. Indeed, it may allow city planners to learn successful policies and/or to find promising directions for enhancing the sustainability of local energy systems. This can be obtained in several ways, the most obvious ones being enhancing awareness and identifying best practices. A more involved effect is related to "city pairing" [3,5]. Roughly speaking, cities showing similar performances across the whole set of indicators may be "paired" to each other, thus defining a network of "similar" cities. Cities in such a network may cooperate to develop common policies. In a similar way, one may design a network of "opposite" cities, that may balance each other's weaknesses by sharing their successful practices. Last but not least, the SDEWES Index may be adopted as the "objective function" in the process of selecting new environmental policies or projects. In other words, a city (or a network of cities) should consider solutions that will improve the value of multiple indicators, that is, take integrated actions addressing several aspects of sustainability [5].

Being integrated and quantitative, the SDEWES Index is a *numerical* aggregation of measures arising from different indicators. These measures are expressed in many different *scales*, ranging from purely qualitative or *ordinal* (that essentially sort elements into *categories*) to strongly *cardinal* ones, that have a sound physical meaning and specific units (such as "EURO" or "tons of CO_2"). The aggregation of multiple evaluations on different scales is the subject of *Multi-Criterion Decision Analysis* (MCDA) [6]. Essentially, the goal of MCDA is to develop reliable tools for *decision support*. This includes, among many other aspects, the analysis of the properties of aggregation methods, and the use of advanced visual tools for enhancing the comprehension of the decisional problems.

In fact, the SDEWES Index can be seen as the "solution" of an underlying MCDA problem. In particular, it addresses the "ranking problematic" [6, ch. 5], i.e., establishing a complete order among a set of alternatives. Actually, the index is the result of applying a very well-known MCDA methodology, namely the *weighted sum method*. In this work we look at the SDEWES Index from an MCDA point of view. More precisely, we consider the ranking problem underlying the index, and we try to get some insight on this problem applying some concepts and tools developed in the context of MCDA. We pursue two main lines of thought.

On one side, we point out a paradoxical effect that may arise when the SDEWES Index is applied to an increasing number of cities. This effect is often referred to as "rank reversal" in the MCDA literature, and is known to affect many existing methodologies, see for example [7,8]. Rank reversal may be rather disappointing, in particular when the index is adopted as a prescriptive tool. We suggest some possible ways to circumvent these problems.

On the other side, we apply visual tools to the SDEWES Index, actually to the underlying MCDA problem. In particular, we adapt the GAIA visual methodology [6, ch. 6], [9] and we apply it to the city samples addressed in [1-3]. This allows us to reveal interesting relations between different dimensions, indicators, and set of cities. In addition, we suggest a simple visual tool that may help, in particular, to reveal some information that is disregarded by the SDEWES Index.

The layout of this paper is as follows. In Section 2 we discuss the rank reversal effect. In Section 3 we consider the adaptation and application of visual tools. In Section 4 we draw some conclusions.

THE SDEWES INDEX AS AN MCDA METHODOLOGY

The indicators considered by the SDEWES Index define a two-level hierarchy. At the top level we find seven *macro-criteria,* referred to as *dimensions* and denoted by D_1, D_2,…, D_7. At the bottom level we find the actual criteria, i.e., the *main indicators*. There are exactly five indicators (criteria) for each dimension (macro-criterion), and this gives an MCDA problem with m=35 criteria. The *alternatives* (or *actions*) correspond to cities, that may vary depending on the sample. Here we consider the samples from [1-3], containing n=58 cities overall.

For each $x \in [1,7]$ and $y \in [1,5]$ denote by $E_{x,y}(C_j)$ the evaluation of city C_j according to the y^{th} criterion of dimension D_x. The computation of the SDEWES Index involves three steps:

1. Normalization of evaluations within each indicator;
2. Computation of a sub-index for each dimension;
3. Aggregation of sub-indexes.

Normalization maps evaluations into the interval [0,1]; for maximization criteria, the normalized value is defined as

$$I_{x,y}(C_j) = \frac{E_{x,y}(C_j) - m_{x,y}}{M_{x,y} - m_{x,y}}$$

while for minimization criteria the equation is

$$I_{x,y}(C_j) = \frac{E_{x,y}(C_j) - M_{x,y}}{m_{x,y} - M_{x,y}}$$

where $m_{x,y}$ and $M_{x,y}$ denote the minimum and the maximum values $E_{x,y}(C_j)$ across all cities. Note that for each criterion the best (resp. worst) value is mapped into the normalized value 1 (resp. 0). Then, for each dimension we define an *aggregated sub-index*

$$A_x(C_j) = \sum_{y=1}^{5} I_{x,y}(C_j)$$

Note that each sub-index is normalized in [0,5].
Finally, the SDEWES Index is obtained as

$$SI(C_j) = \sum_{x=1}^{7} \alpha_x A_x(C_j)$$

where $\alpha_x = 0.225$ for *x*=1 and *x*=5, and $\alpha_x = 0.11$ for the other dimensions. Note that the weights α sum to one, thus the index is normalized in [0,5]. In fact, the SDEWES Index can be seen as the result of the well-known *weighted sum* MCDA method, where each alternative is assigned a *score* defined as the weighted sum of the normalized evaluations w.r.t. the criterion weights; in this case, the y^{th} indicator of dimension D_x has weight α_x, therefore

$$SI(C_j) = \sum_{x=1}^{7}\sum_{y=1}^{5} \alpha_x I_{x,y}(C_j)$$

Note that the weighted sum is a *totally compensatory* method, where the weaknesses of an alternative can be compensated by its strengths. This means that the SDEWES Index of a city can be good even if some of the indicators have a quite poor evaluation. More precisely, a score $SI(C_j)$ does not depend on the *dispersion* of the values $I_{x,y}(C_j)$ or $A_x(C_j)$, i.e., on these values being rather similar across the whole set of indicators or spread in a large interval. On the contrary, other MCDA methods are sensitive to dispersion: this is the case e.g. of TOPSIS, see [10]. Later on, we propose a visual tool that captures the information related to dispersion.

Rank Reversal

The "rank reversal" effect occurs when the order of preference between two alternatives changes if an alternative is added to or removed from the decision problem. This means that the relative ranking of two alternatives may depend on the other alternatives. Rank reversal is rather ubiquitous in MCDA ranking methodologies [7,8]. We illustrate this effect with the following example, where we have four alternatives evaluated on two maximization criteria as follows:

C_1	1	200
C_2	2	100
C_3	1	75
C_4	0	100

Both criteria have weight one. Here we apply the same weighted sum technique used for the SDEWES Index, and we show the normalized evaluations, and the corresponding scores, for the sets of alternatives S_1={C_1, C_2}, S_2={C_1, C_2, C_3}, and S_3={C_1, C_2, C_3, C_4}:

	S_1			S_2			S_3		
C_1	0	1	*1*	0	1	1	*0.5*	1	*1.5*
C_2	1	0	*1*	1	0.2	1.2	*1*	0.2	*1.2*
C_3				0	0	0	*0.5*	0	*0.5*
C_4							*0*	0.2	*0.2*

If considered alone, the two first alternatives get equal score; if we add C_3, C_2 is the winner, while by (further) adding C_4 we make C_1 win. Note that rank reversal between C_1 and C_2 is related to normalization, and more precisely, due to the fact that the minimum evaluations *decrease* by adding new alternatives. Moreover, rank reversal is due to the addition of poor (actually, *dominated*) alternatives. More precisely, poor alternatives can *increase*, up to different extents, the score of "good" ones. In addition, it turns out that the addition of "good" alternatives can *decrease* some scores: in the above example, the scores of C_1 and C_2 would decrease if the evaluation of C_4 on the second criterion was greater than 200.

It should be clear from the above example that the SDEWES Index is prone to rank reversal. This implies that non only the index, but also the relative preference between two cities may (repeatedly) change, as long the sample of evaluated cities is extended.

Towards a stable index?

As a matter of fact, rank reversal is almost ubiquitous in existing MCDA methodologies [7,8]. Being prone to this phenomenon does not seem to seriously affect the descriptive power of the SDEWES Index. However, it may have some impact on its prescriptive value, due to the fact that the index is not *stable along time*. Indeed, as long as new cities are added to the sample, the score of a city may be artificially increased or decreased, regardless of the actual evolution of its local energy system. This seems to imply that the SDEWES Index cannot be considered as a reliable tool for monitoring the evolution of energy systems along time, and more important, for evaluating the impact of sustainable development policies. Therefore, the question arises of whether it is possible, and relevant, to address this "stability issue".

A possible objection is that energy systems are *inherently dynamic*: technological development, as well as social pressure, lead to setting higher and higher standards, to which a city should continuously struggle for complying. Accordingly, a local system should be evaluated in relation to other evolving systems, rather than based solely on its own features. From this point of view, a scoring method such as the SDEWES Index should be considered an inherently dynamic tool, and stability should not be an issue. We believe that this objection is only partially convincing: It is acceptable that scores decrease as higher standards are set (i.e., "good" cities are added) but the addition of "poor" cities should not lead, in our opinion, to improving a score. More important, a reliable tool for evaluating the impact of development policies would be definitely useful. On the other hand, solving the stability issue is not a trivial task, since it requires to deal with such an elusive phenomenon as rank reversal. We do not have any simple solution to propose, yet we can sketch three different attitudes towards this issue.

<u>*Wait and see*</u>. Stick to the current methodology, and keep expanding the city sample: possibly, the index (more precisely, the bounds $m_{x,y}$ and $M_{x,y}$ used for normalization) will become more

and more stable, also due to winsorization. A sufficiently reliable tool might be obtained in a short time.

Multiple indexes. Maintain several successive versions, e.g. the *58-city Index*, the *120-city Index*, and so on. Within each version, the normalization bounds are kept unchanged, so that the evolution of each city can be monitored. Moreover, to allow for consistent comparisons, "new" cities may be evaluated according to different versions; eventually, this may yield some normalized value $I_{x,y}(C_j)$ lying outside of the interval [0,1], however, $A_x(C_j)$ and $SI(C_j)$ are not likely to lie outside the interval [0,5], due to the compensatory nature of the weighted sum.

Stabilize Index. Define a version of the index once and for all. In practice, this requires to devise a normalization technique that does not depend on the city sample, i.e., to define a set of *utility functions* that map any (reasonably possible) value of $E_{x,y}(C_j)$ into a normalized value $I_{x,y}(C_j)$. This task mat be relatively easy from a mathematical point of view, but requires a quite deep understanding of the meaning and properties of all the indicators.

VISUAL TOOLS FOR THE SDEWES INDEX

Since long time, visual tools have been part of MCDA methodologies and software. Here we are interested in those tools representing the overall structure of the decision problem. In particular, we concentrate on the GAIA methodology, and we restrict ourselves to 2D representation, even if 3D options are already available in the *Visual Promethee* package [9].

A GAIA-like tool

The GAIA methodology addresses the "description problematic" [6, ch. 5], and has been developed as a visual companion of the PROMETHEE method [6, ch. 6]. Alternatives, as well as criteria and weights, are represented by points on the *GAIA plane*, identified by the axes *U* (horizontal) and *V* (vertical). A third axis *W* allows to define the secondary planes (*U*,*W*) and (*V*,*W*). This representation is obviously approximated, since it only shows the projections on the plane of points in a space of dimension *p*, i.e., the number of criteria. Nevertheless, it may reveal several aspects of the decision problem, such as conflicting criteria or sensitivity to changes in the weights, and many more. The method also provides a measure of the "quality" of the representation, which can be seen as the percentage of information retained after projection.

In order to find the axes *U*, *V* and *W*, GAIA applies Principal Component Analysis (CPA) to the matrix of *profiles* computed by PROMETHEE. The profile of an alternative is a (row) vector of scores, one for each criterion. Profiles are particularly suitable for CPA since they are normalized in [-1,1] and *centered*, i.e., the sum of scores over all alternatives is zero for each criterion.

Obviously, in the context of the SDEWES Index we do not have profiles, but we can still apply PCA to the available evaluations. We have two possibilities here, namely, apply PCA at the top level or at the bottom level of the criteria hierarchy. In the former case, a city C_j is represented by the sub-indexes $A_x(C_j)$, for *x*=1,2,…,7. In the latter case, we apply PCA separately for each dimension D_x, and a city C_j is represented by its evaluations $E_{x,y}(C_j)$, *y*=1,2,…,5. In both cases, let *M* be the $n \times p$ *evaluation matrix* where *n* is the number of cities and *p* is either 7 or 5; each city C_j is represented by row *j* in *M*. The CPA method for finding the axes *U*, *V*, *W*, and the qualities of the projection planes, can be summarized as follows.

Algorithm CPA
Input: an evaluation matrix M
Output: axes U, V, W; quality of planes (U,V), (U,W), (V,W)

1) Compute the matrix of *centered evaluations* C: $C_{jk} = M_{jk} - \bar{M}^k$, where $\bar{M}^k$ is the average of the values in column k of M;
2) Compute the *normalized matrix* N: $N_{jk} = C_{jk}/\|C_{\cdot k}\|_2$, where $C_{\cdot k}$ is column k of C;
3) Compute the $p\times p$ *correlation matrix* $A = N^T N$;
4) Compute the three largest eigenvalues $\lambda_1 \geq \lambda_2 \geq \lambda_3$ of A, and the corresponding eigenvectors x_1, x_2 and x_3; let $U= x_1$, $V= x_2$, and $W= x_3$;
5) For the planes (U,V), (U,W) and (V,W), the quality is given by $100(\lambda_1 + \lambda_2)/\mathrm{tr}(A)$, $100(\lambda_1 + \lambda_3)/\mathrm{tr}(A)$ and $100(\lambda_2 + \lambda_3)/\mathrm{tr}(A)$, respectively.

We can now compute the coordinates of relevant points on the plane (U,V):

- city C_j has coordinates $(N_{j\cdot}^T U, N_{j\cdot}^T V)$, where $N_{j\cdot} \in \mathbb{R}^p$ is row j of N;
- criterion (or dimension) k has coordinates (U_k, V_k);
- the vector of weights α has coordinates $(w^T U, w^T V)$, where $w = \alpha/\|\alpha\|_2$;

the computation for the secondary planes (U,W) and (V,W) is similar.

In the following we show some figures obtained by our GAIA-like tool. We consider the city samples used in [1], [2] and [3], referred to as MED, SEE and AUT, respectively. The city of Instanbul, used both in [1] and in [2], is showed separately. Figure 1 shows the top-level representation, namely, the dimensions $D_1,\ldots,D_7$ and the weights α, represented by the red "stick". Recall [6, ch. 6] that the stick represents the direction along which better alternatives are found, in our case, the direction along which the SDEWES Index increases.

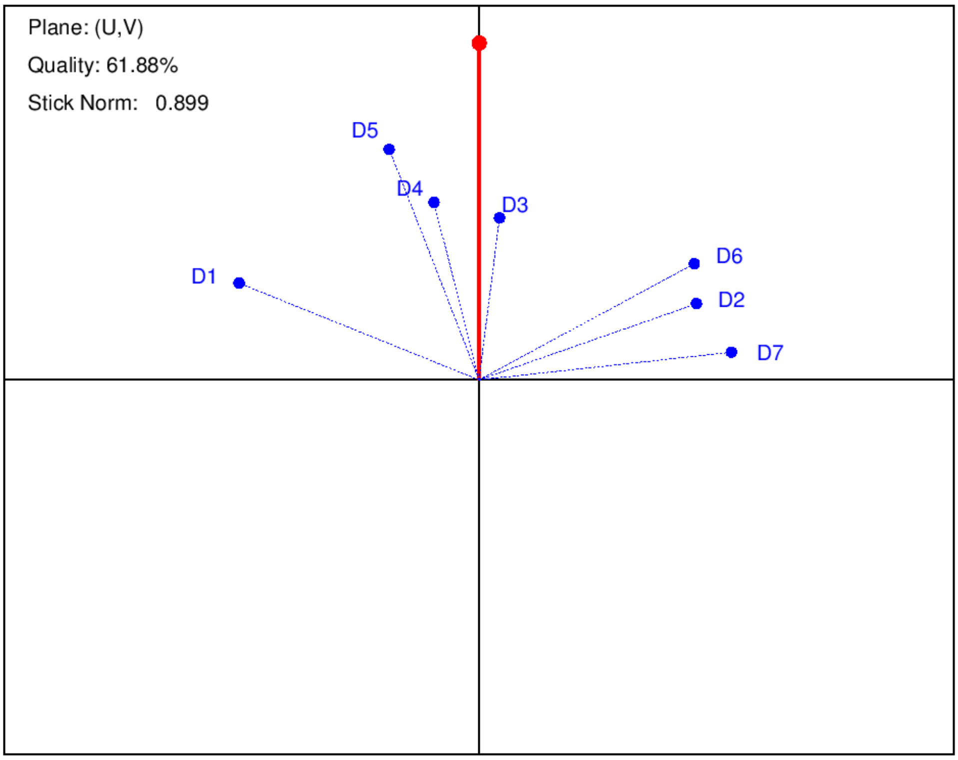

Figure 1. Top level, dimensions and weights, plane (U,V)

Figure 1 reveals some interesting facts. First of all, the stick coincides almost perfectly with the vertical axis *V*, in particular, its length is almost one: recall that the stick is the projection of a unit length vector. This means that the direction of maximal dispersion of the aggregated sub-indexes $A_x(C_j)$ (the principal axis *U*) is *orthogonal* to the direction along which the SDEWES Index increases. Here it turns out that $\lambda_1 = 39.2$ and $\lambda_2 = 22.7$ which means, roughly speaking, that the dispersion of the sub-indexes along *U* is about twice the dispersion of the index values, which accounts for about 1/5 of the total dispersion.

Moreover, we can see that some dimensions are "more relevant" than others to determine the index. Dimension D_5 is the most relevant, since it has the greatest vertical coordinate, i.e., longest projection onto the stick; dimension D_7 is the least relevant, being almost orthogonal to the stick. Dimensions also have quite different projections on the axis *U*. Two criteria pointing in opposite directions on the GAIA plane are likely to be conflicting, or more precisely, such that good alternatives for on criterion are (statistically) bad for the other. Here we may observe that dimension D_1 is somehow conflicting with dimensions D_2, D_6 and D_7. This is confirmed by Figure 2, where we consider the auxiliary plane (*U*,*W*): note that the stick is almost orthogonal to this plane. Informally speaking, these conflicts mean that: innovation and social wellness come at the expenses of energy consumption, while saving measures are more developed where consumption is higher. Figure 2 also shows that D_3 and D_4 have opposite projections on axis *W*. Actually, both D_3 and D_4 are positively correlated to the index (see Figure 1) but show a "residual" conflicting behaviour along axis *U* and *W*.

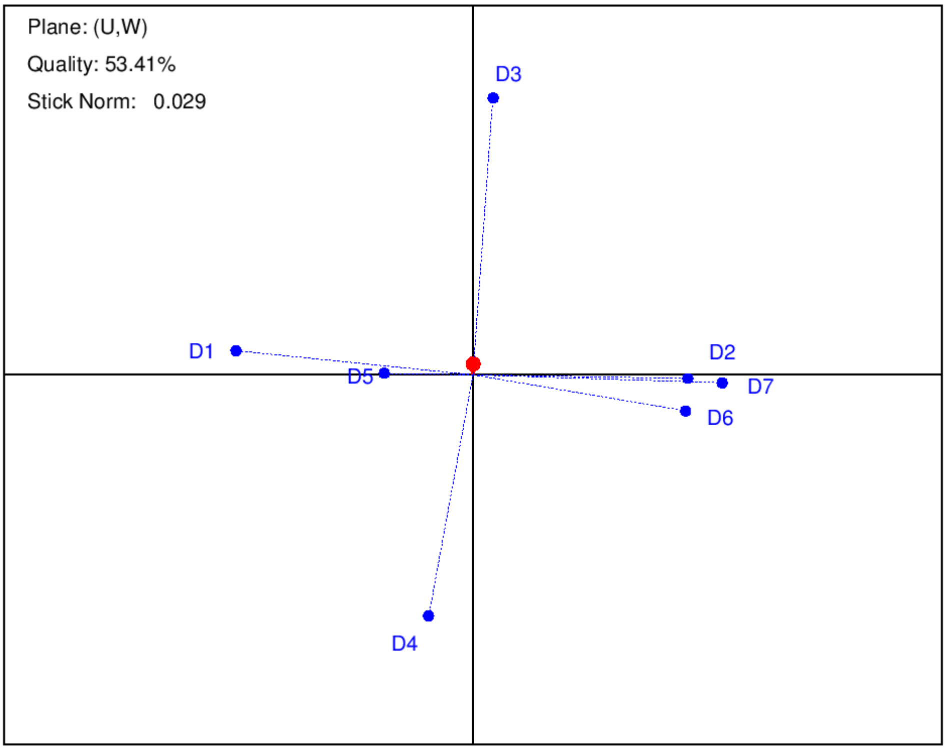

Figure 2. Top level, dimensions and weights, plane (*U*,*W*)

Figure 3 shows the projection of the cities on the (*U*,*V*) plane. The convex hull of each sample is shown too. The following observations can be drawn.

- MED and SEE cities are concentrated into relatively small areas, while AUT cities are spread in a larger portion of the plane;

- overall, SEE cities have a slightly better index w.r.t. MED cities (recall that the index increases along axis V) while the indexes of AUT cities have a larger dispersion;
- SEE and MED cities appear towards the left, i.e., they perform better in dimensions D_1 and (up to a minor extent) D_4 and D_5; AUT cities appear on the right, i.e., they are better for dimensions D_2, D_6 and D_7.

The auxiliary planes (U,W) and (V,W) show similar patterns, and are not reported here.

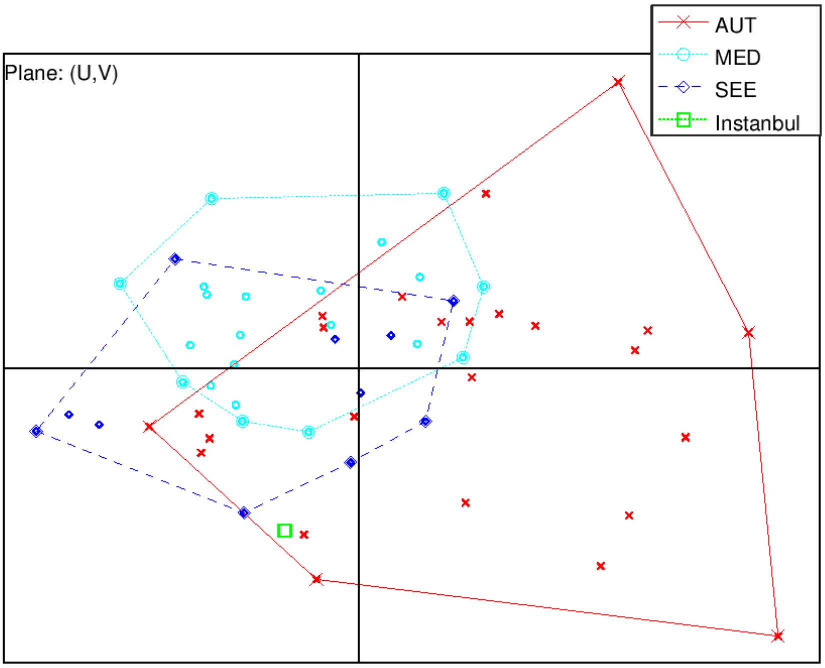

Figure 3. Top level, cities, plane (U,V)

In the following we consider the bottom level representation for dimension D_1. In Figure 4 we show the five indicators of D_1, denoted as $I_1, I_2,\ldots,I_5$, as well as the stick, on the plane (U,V). We can observe that the most relevant indicators are I_1, I_2 and I_3, while I_4 and I_5 are less relevant (almost orthogonal to the stick) and conflicting. This conflict is not surprising, since I_4 and I_5 measure yearly use of heating and cooling systems, respectively. Figure 5 shows the city projections on the same plane. Also in this case, the AUT sample is more spread around w.r.t. MED and SEE. However, here MED cities perform slightly better than SEE cities, while AUT cities have the best results. Not surprisingly, the pattern obtained considering dimension D_1 differs from the one obtained considering all the dimensions.

A visual representation of dispersion

As discussed earlier, the Weighted Sum method is totally compensatory, i.e., not sensitive to dispersion of indicator values. Accordingly, the SDEWES Index does not take dispersion into consideration. On the other hand, distribution patterns of values are relevant for city pairing. Actually, the GAIA methodology allows to reveal some information about dispersion, and this feature was quite evident in the previous figures. However, we are not aware of any MCDA methodology (either a ranking method or a visual support system) that explicitly addresses dispersion as a component of the decision process. Devising such a methodology goes definitely out of the scope of this work. Here we consider a rather straightforward approach to visualizing dispersion, and devise a quite simple "visual companion" for the Weighted Sum method, in particular for the SDEWES Index. In this tool, a city is represented by a point in the plane, where the horizontal coordinate is the SDEWES Index, and the vertical one is a measure of dispersion.

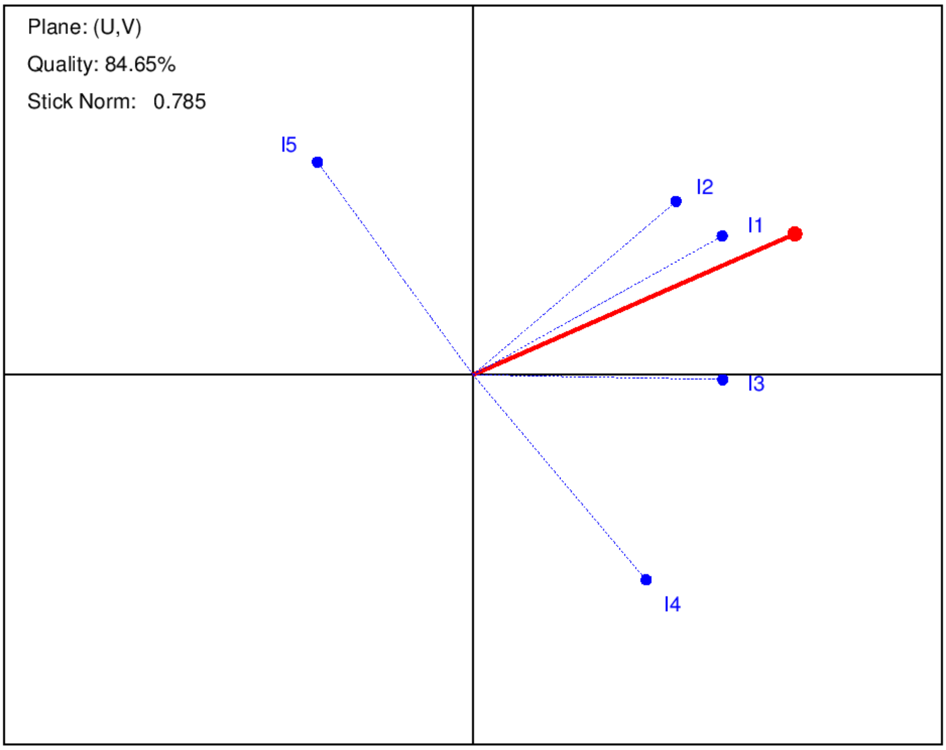

Figure 4. Bottom level, dimension D_1, indicators and weights, plane (U,V)

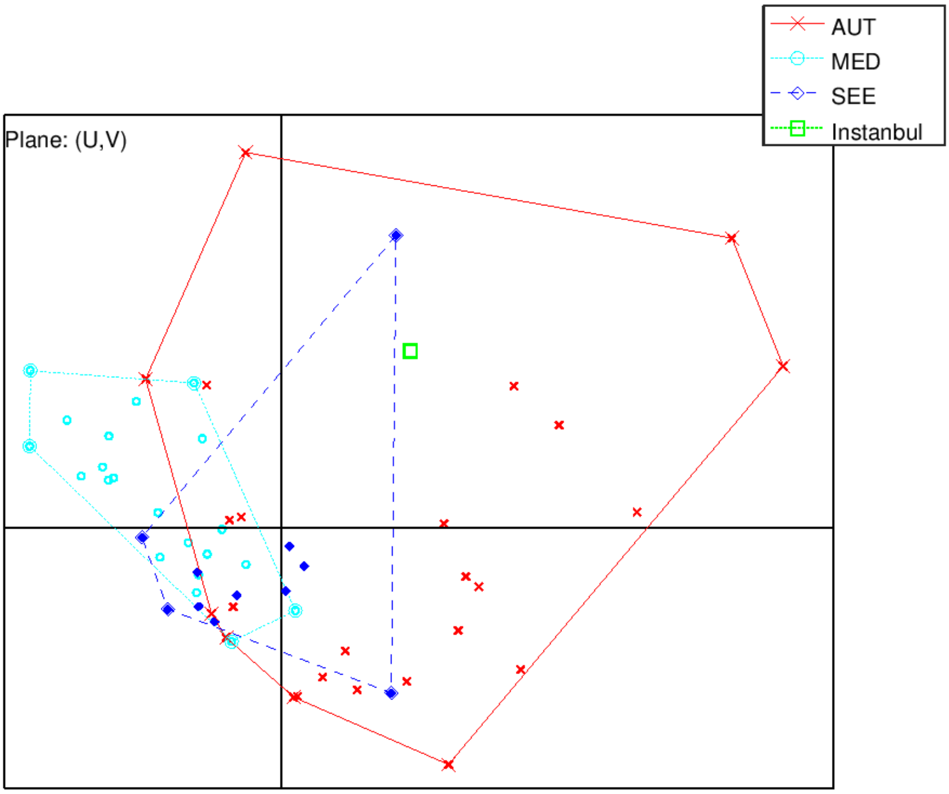

Figure 5. Bottom level, dimension D_1, cities, plane (U,V)

As a matter of facts, there are many different measures of dispersion that can be adopted: here we follow a straightforward geometrical approach. We only consider the top level of the hierarchy, i.e., the aggregated indexes $A_x(C_j)$; each city C_j is represented by row j in the normalized matrix N, see Algorithm CPA for details. Thus city C_j is a point $N_{j.}$ in the space of dimension p=7, and weights are represented by the unit vector $w = \alpha/\|\alpha\|_2$. For each city C_j we let $N_{j.} = \pi_j w + d_j$, where $\pi_j = N_{j.}^T w$. Note that π_j is the length of the projection of $N_{j.}$ onto the axis defined by w, while d_j is orthogonal to this axis, and $\|d_j\|_2$ is the distance of $N_{j.}$ from the axis. It can be easily checked that

$$SI(C_j) = \frac{\|\alpha\|_1}{\|\alpha\|_2}\pi_j$$

so we take

$$D_j = \frac{\|\alpha\|_1}{\|\alpha\|_2}d_j$$

as a measure of the dispersion of the aggregated sub-indexes representing city C_j. Figure 6 represents the city samples in [1-3] as before; each city C_j is a point of coordinates $(SI(C_j), D_j)$.

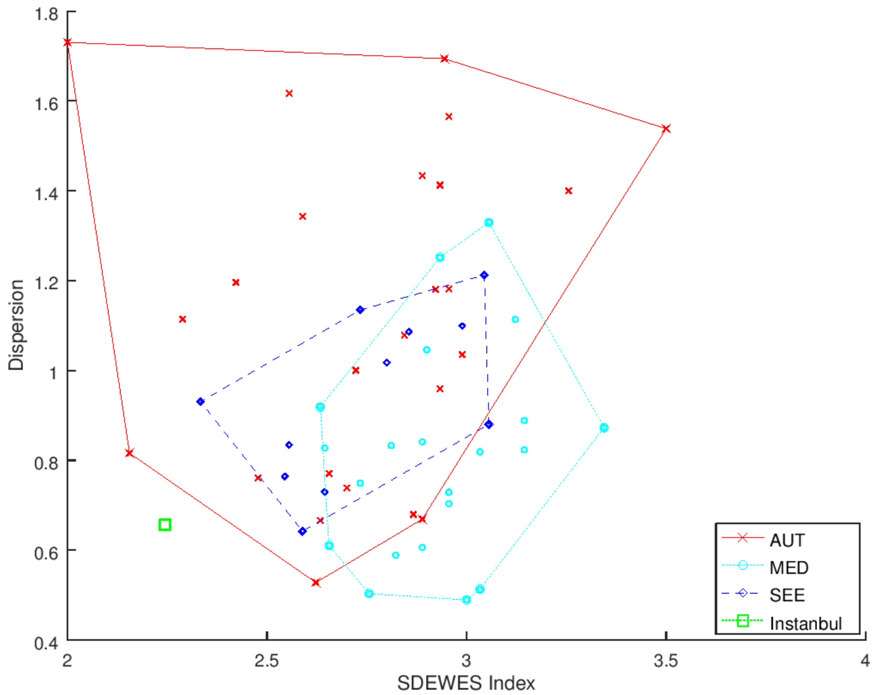

Figure 6. Cities on the Index/Dispersion plane

Observe that some of the observations made on Figure 3 apply to Figure 6 as well. This is not surprising, since we know that:

- the weight vector w, that defines the horizontal axis in Figure 6, almost coincides with the vertical axis V in the GAIA plane;
- the horizontal axis U captures almost half of the total dispersion measured by D_j.

In addition, Figure 6 seems to reveal that the best scores on the SDEWES Index come at the expenses of a greater dispersion.

CONCLUSIONS

In this work we have analysed the SDEWES Index from an MCDA point of view. This led us to draw a few observations.

First, the SDEWES Index as currently conceived raises a stability issue. This issue is not overwhelming, does not really affect the relevance of the Index, and is likely to fade away with time. Nevertheless, we believe that fixing the stability issue would enhance the relevance of the Index. Unfortunately, finding a stabilized version does not seem to be a trivial task.

Second, visual decision support tools can be adapted (or devised) to act as a visual companion of the SDEWES Index. Based on the examples reported here, we believe that these tools can be useful, not only to reveal information somehow hidden in the collected data, but also to enhance the comprehension of the scoring process and of its results. Clearly, further work on this issue is needed.

Finally, we pointed out that the SDEWES Index disregards data dispersion, despite the fact that data dispersion patterns have a relevant prescriptive value. This raises the question of how to consider data dispersion within an MCDA methodology. We believe that this is an interesting direction for research in the MCDA area.

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