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Quantum walks of two interacting particles on percolation graphs

Ilaria Siloi[1], Claudia Benedetti[2], Enrico Piccinini[3], Matteo G. A. Paris[2], Paolo Bordone[4]

[1] Department of Physics, University of North Texas, Denton, TX 76203, USA.
[2] Quantum Technology Lab, Dipartimento di Fisica, Università degli Studi di Milano, I-20133, Milano, IT
[3] ARCES Research Center, Via Toffano 2/2, I-40125, Bologna, IT
[4] Dipartimento di Scienze Fisiche, Informatiche e Matematiche, Università degli Studi di Modena e Reggio Emilia, I-41125, Modena, IT

E-mail: `paolo.bordone@unimore.it`

Abstract. We address the dynamics of two indistinguishable interacting particles moving on a dynamical percolation graph, i.e., a graph where the edges are independent random telegraph processes whose values jump between 0 and 1, thus mimicking percolation. The interplay between the particle interaction strength, initial state and the percolation rate determine different dynamical regimes for the walkers. We show that, whenever the walkers are initially localised within the interaction range, fast noise enhances the particle spread compared to the noiseless case.

1. Introduction

Quantum walks (QWs) are the quantum analogue of classical random walks and describe the propagation of quantum particles over a discrete lattice with non zero tunneling amplitudes between adjacent sites [1]. Two-particles quantum walks are paradigmatic systems to address the interplay between particle indistinguishability and particle interaction. Besides the fundamental interest, two-particle quantum walks are implemented on different platforms also with the aim of studying multiple quantum interference and to simulate physical, chemical, and biological complex systems [2]. Experimental realisations of QWs are subject to different sources of noise e.g. imperfections, defects or external perturbations - that may dramatically affect the dynamical behaviour of the walkers [3, 4]. As a consequence of the external noise, the regular structures underlying the QWs may be altered, and the ordered lattices may turn into irregular graphs. Among dynamically varying graphs, a central role is played by dynamical percolation graphs, which are random graphs where edges are created and destroyed in time, according to some stochastic process. While, to analyse more realistic scenarios, in a previous work [4] we focused on the case of a non-Gaussian random telegraph noise which randomises the tunneling amplitudes between adjacent sites still retaining for them a finite value, in this paper we address the decoherent dynamics of two indistinguishable and interacting particles over one-dimensional percolation graphs, with links that appear and disappear randomly in time. The percolation approach is indeed quite intriguing in quantum mechanics, and can even be thought as related to the cellular automaton interpretation of quantum mechanics itself [5].

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2. The physical model

The continuous-time quantum walk of two indistinguishable particles on percolation graphs is described by the Hamiltonian:

$$H_2(t) = H_1(t) \otimes \mathbb{I} + \mathbb{I} \otimes H_1(t) + H_{\text{int}} \tag{1}$$

where the single-particle Hamiltonian H_1 and the interaction Hamiltonian H_{int} read:

$$H_1 = \epsilon\mathbb{I} + \sum_x J g_x(t) \Big(|x\rangle\langle x+1| + |x+1\rangle\langle x| \Big), \quad H_{\text{int}} = U_p(|x-y|) \sum_{x,y=1}^{N} |x,y\rangle\langle x,y|. \tag{2}$$

$\mathbb{I}$ is the single-particle identity matrix and the term $\epsilon\mathbb{I}$ only determines a rescaling of the energies and can be omitted without loss of generality, J is the coupling constant, $\{|x\rangle\}$ represents the orthonormal basis of the site of the graphs, i.e. the walkers positions, while the set $\{|x,y\rangle\}$ describes the orthonormal basis for the two-particle QW, where one occupies site x and the other site y. $g_x(t)$ is a stochastic process describing the percolating links, i.e. it is described as the non-Gaussian random telegraph noise with a switching rate γ and autocorrelation function $C(t) = \langle g_x(t) g_y(0)\rangle = \delta_{x,y}\frac{1}{4}e^{-2\gamma t}$, whose values jump between 0 and 1, indicating the absence and presence of edges respectively [3]. U_p is the strength of the interaction between the two walkers, as a function of their distance:

$$U_p(|x-y|) = \begin{cases} U & \text{if} \quad x = y \text{ and } p = \text{bosons} \\ U/3 & \text{if } |x-y| = 1 \text{ and } p = \text{fermions} \end{cases}, \tag{3}$$

i.e. bosons experience on-site interaction energy while fermions only nearest-neighbours interaction. The initial state of the two walkers is taken in the form: $|\psi_0\rangle = \frac{1}{\sqrt{2}}(|x,y\rangle \pm |y,x\rangle)$ to take into account their indistinguishability. Since Eq. (1) conserves the symmetry of the wavefunction, the initial state of the walkers fixes the statistics of the particles. The time evolution of the initial state is determined by ensemble-averaging the dynamics over all possible realisations of the noise $\left\langle \Lambda_r \rho_0 \Lambda_r^\dagger(t) \right\rangle_{\{g_x(t)\}}$ where $\Lambda_r(t) = \mathcal{T} e^{-i\int_0^t H_2(s)ds}$ is the evolution operator for the single realisation , $\mathcal{T}$ is the time-ordering operator and $\rho_0 = |\psi_0\rangle\langle\psi_0|$. Hereafter, we use natural unit $\hbar = 1$ and we set $J = 1$ such that all quantities will be given in unit of the coupling constant.

3. Results and discussion

In order to characterise the dynamics of the two-particle QW on percolation graphs, we consider a 1D lattice with N=80 nodes, and we fix the switching rate of the percolating links $\gamma = 10$ (in unit of J). We simulate the dynamics, by numerically calculating the ensemble average over 1000 different percolation realisations, using a specific GPU accelerated code [7]. In order to characterize the dynamics, we evaluate the temporal behaviour of the single particle variance $\sigma^2(t) = \sum_x \langle x^2(t)\rangle - \langle x(t)\rangle^2$, with $\langle x^k\rangle = \sum_i i^k \rho^1_{ii}(t)$ and ρ^1 the single particle density matrix. To complete our analysis, we also evaluate the occupation number of the lattice sites during the evolution $\langle n_k(t)\rangle = 2\sum_j \rho_{kj,kj}(t)$, where $\rho_{kj,kj}(t)$ are the populations of the two-particle density matrix in the $\{|k,j\rangle\}$ basis. In the following we present our results for both bosons and fermions.

In Fig. 1 we show the behaviour of the single-particle variance as a function of time (in unit of J), for both bosons and fermions. In each panel we compare the dynamics for the regular lattice without noise (black lines) and the case of percolation graphs (dashed red lines). In the left part of the figure, we consider nearest-neighbours sites as initial state for fermions, while bosons initially occupy the same node. An important result emerges here: for suitable interaction strength regime ($U < 6$ for fermions and $U < 2$ for bosons), the single-particle variance for

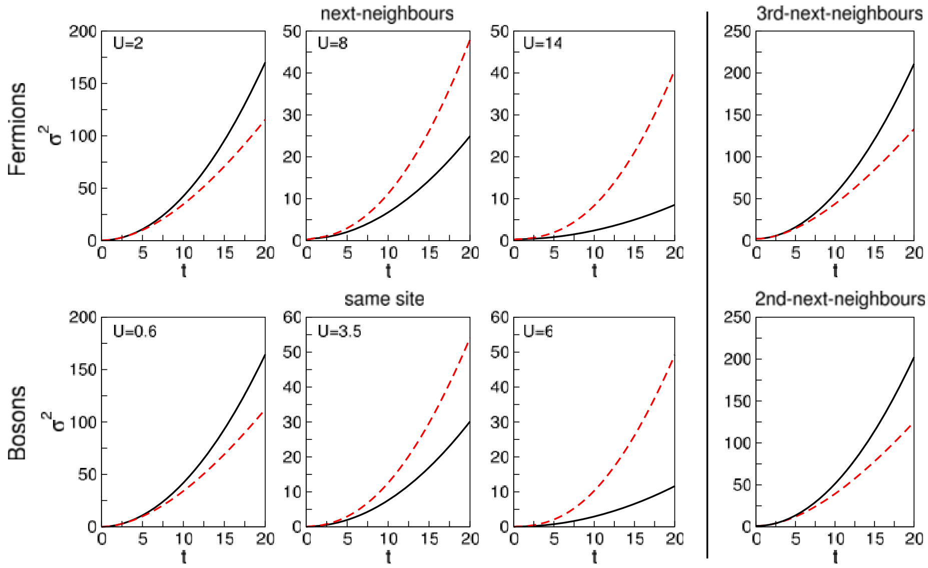

Figure 1. Single-particle variance as a function of dimensionless time t over a graph of $N = 80$ sites, for different values of the interaction strength U. Solid black lines are for regular noiseless lattices, while the dashed red lines represent the case of percolation graphs with $\gamma = 10$. The two panels in the right part of the figure are for fermions and bosons starting in third-next and second-next-neighbours respectively. In the latter case the results are essentially independent on the value of U.

percolation graphs is smaller with respect to the noiseless case, independently on the particle statistics. But as the interaction strength grows, we observe a crossover in its behaviour, and σ^2 for percolation graphs exceeds the noiseless case. This means that the particles can spread faster over sites in the lattice, due to fast ($\gamma = 10$) percolation. This happens when the two particles are initially located on nearest-neighbours sites (fermions) or same-site (bosons), that is, if they are positioned within the range of interaction (3). One may notice that particles undergo a different dynamics if the initial positions are outside the range of interaction (U=0), as shown in the rightmost panels of Fig. 1 where the two particles start from third-next (fermions) and second-next (bosons) -neighbours. In this case, indeed, the crossover between percolation and noiseless variance never happen and the free walkers always spread faster on the noiseless graph with respect to the percolation graph.

The effect of dynamical percolation is also further analysed by studying the occupation number $\langle n(t)\rangle$ of the lattice sites. This is shown in Fig. 2. The top panels show the distribution of the occupation number in the noiseless case, for strong interactions. As known [4], the particles tend to localise, with minor components of the wavefunction travelling away from the initial sites. The situation changes if we consider the dynamics of the QW over a percolation graph. Indeed, the particles spread more over the lattice, with wavefunction components that occupy a significantly larger number of the nodes, in agreement with the behaviour of the single-particle variance. This dynamical behaviour can be explained in terms of the band structure of the Hubbard model. In the absence of interaction (U=0), the band structure consist of a unique band identical for bosons and fermions. For finite interaction strength, a small band is formed, called mini-band, whose energy at the edge of the first Brillouin zone is given approximately by $U/3$ for fermions and U for bosons. The remaining states are contained in the main band, which ranges approximately from -4J to 4J [6]. Even if it is not possible to define proper band structures since the translational invariance is broken by the noise, we can consider fast percolating noise

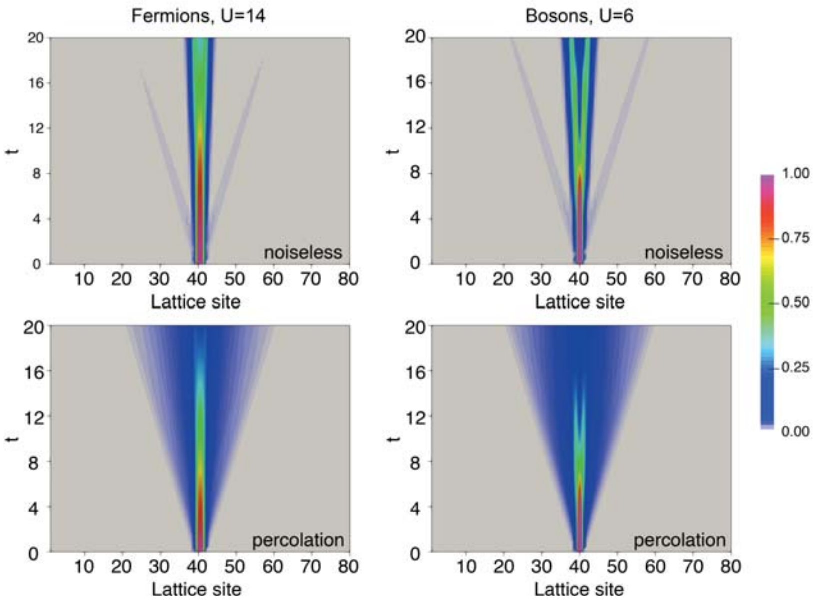

Figure 2. Occupation number $\langle n_k(t)\rangle$ as a function of sites k and dimensionless time t, for fermions and bosons in the noiseless and percolation lattice of $N = 80$ sites. Particle interaction is set $U = 14$ for fermions and $U = 6$ for bosons. Fermions start in nearest-neighbours sites, while bosons are initially localised on the same site.

as a perturbation to the noiseless dynamics and, to first-order approximation, we can explain the QW features in terms of bands [4]. In this framework, we can state that noise provides a redistribution of the wavefunction components among the subbands and it allows to access a new regime where the particles acquire faster propagation contributions, thus breaking the localisation induced by the strong interaction. This redistribution and the consequent crossover of the single-particle variance happens whenever the particles initially belong to the mini-bands and, because of noise, they are allowed to access other sub-bands. On the contrary, whenever particles are located outside the range of interaction, i.e. in the main sub-band, noise cannot add any faster contribution, thus the noiseless dynamics is always faster than the one obtained on a percolation graph.

For the sake of completeness, we comment also on the dynamics of two-particle QW in the presence of slow percolation, i.e. when $\gamma \ll 1$. The percolation rate in this case is very small and in the limit we reach a static percolation regime. This situation does not allow for any propagation of the walker and the combined effect of noise and interaction strongly localise the particle on their initial sites.

We conclude by remarking that localisation-breaking noise is an important feature that can be exploited in many contexts especially for protocols for quantum transport and communication, where noise can be engineered in order to obtain an enhancement of quantum properties.

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