Effects of toroidal inhomogeneities on the effective properties of a composite

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The present work focuses on the problem of rigid inhomogeneity of toroidal shape embedded in an elastic matrix. Inhomogeneities of this kind occur both in natural and man-made materials. Analytical modeling of materials with such microstructure has not been well developed. In the homogenization schemes, the inhomogeneities are usually assumed to be of ellipsoidal shape. This unrealistic assumption is largely responsible for insufficient linkage between methods of micromechanics and materials science applications. While for 2-D non-elliptical inhomogeneities many analytical and numerical results have been obtained, only a limited number of numerical results and approximate estimates are available for non-ellipsoidal 3-D shapes [1]. Most of them are related to pores and cracks. For the toroidal shapes, Argatov and Sevostianov [2] used asymptotic methods to evaluate the contribution of a thin rigid toroidal inhomogeneity into overall stiffness; Onaka et al.[3] derived analytical expressions for components of the Eshelby tensor for a toroidal inclusion. The problem of the effective conductivity (thermal or electric) of a material containing toroidal insulating inhomogeneities has been addressed in [4], where an analytic solution is presented for the steady-state temperature distribution in an infinite conductive medium containing an insulated toroidal inclusion, under uniform heat flux in an arbitrary direction. The temperature flux on the torus surface is then determined as a function of torus parameters. This result is then used to determine resistivity contribution tensor for the toroidal inhomogeneity and for calculation of effective conductive properties of a material containing multiple inhomogeneities of this shape.
A general analytical solution is developed here for the problem of an infinite elastic medium containing a rigid toroidal inhomogeneity, under remotely applied uniform strain. The traction vector on the torus surface is determined as a function of torus parameters and strain components applied at infinity by using the approach developed in [5]. The results are utilized to calculate the components of the fourth-rank stiffness contribution tensor of the rigid toroidal inhomogeneity that are required for calculation of the overall elastic properties of a material containing multiple toroidal inhomogeneities. The analytical results are verified by comparison with FEM calculations.

References

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