A Repetitive Control Scheme Based on B-Spline Trajectories Modification ⋆

Luigi Biagiotti * **Lorenzo Moriello** ** **Claudio Melchiorri** **

* *Department of Engineering "Enzo Ferrari", University of Modena and Reggio Emilia, Italy (e-mail:luigi.biagiotti@unimore.it).*
** *Department of Electrical, Electronic and Information Engineering "Guglielmo Marconi", University of Bologna, Italy (e-mail: {claudio.melchiorri},{lorenzo.moriello2}@unibo.it)*

Abstract: In many applications of interest in industrial robotics, tasks are cyclic and must be repeated over and over. In this context, it seems natural to exploit the intrinsic properties of repetitive control schemes, where the cyclic nature of "disturbances" and/or unmodeled dynamic effects can be exploited to reduce the tracking errors. In this paper, we propose a new repetitive control scheme, where the main idea consists in the modification of the reference trajectory in order to compensate for the periodic undesired effects. By exploiting the dynamic filters for the B-spline generation, it is possible to integrate the trajectory planning within a repetitive control scheme able to modify in real-time the reference signal with the aims of nullify interpolation errors. By means of an extensive experimental activity on a servo mechanism pros and cons of the proposed approach are analyzed.

Keywords: Repetitive Control, Learning Control, Splines, Robot control.

1. INTRODUCTION

Repetitive Control (RC) schemes, firstly proposed by Inoue et al. (1981a,b), may represent a quite natural choice in cases in which the task to be executed is periodic in time. As a matter of fact, in these cases from a control point of view it is required to track and/or reject periodic exogenous signals, that can be considered known since they refer to planned trajectories whose cycle time is usually known in advance. In this context, RC represents a relatively simple and effective approach, since it aims at cancelling tracking errors over repetitions by learning from previous iterations. Many surveys, see e.g. Cuiyan et al. (2004), Wang et al. (2009), report the successful use of RC in a number of applications, such as high accuracy trajectory tracking of servomechanism, torque vibration suppression in motors, noise cancellation in power supply, industrial robotics, and so on.

In this paper, a novel repetitive control scheme is presented and its performance are analyzed through an extensive experimental activity. The scheme is based on a proper modification of the reference trajectory for the plant, which is supposed to be already controlled. A similar idea has been already proposed in the continuous-time domain by Hara et al. (1988), where a two-degree-of-freedom local control, and a plug-in type RC is used to update the reference trajectory. The novelty of this paper consists in assuming that the reference trajectories are defined by spline functions, which are de-facto the standard tool used in the industrial field for planning complex motions interpolating a set of given via-points, see Biagiotti and Melchiorri (2008). Thanks to the possibility of generating B-spline trajectories by means of dynamic filters as shown Biagiotti and Melchiorri (2010), the trajectory planner has been inserted in an external feedback control loop that modifies in real-time the control points of the B-spline curve so that the tracking error at the desired via-points converges to zero. The proposed control scheme has been directly developed in the discrete time-domain, and is characterized by a very low computational complexity. Moreover, the application of this control scheme is independent by the particular control law of the plant, which may be an important feature when dealing with not accessible control systems e.g. factory controllers of industrial robots. The paper is organized as follows. In Sec. 2 a general overview of the filters for B-spline generation is given both in the continuous and in the discrete-time domain. Then in Sec. 3 the repetitive control scheme based on B-spline filters is illustrated. The experimental results reported in Sec. 4 allow to highlight pro and cons of the proposed approach, and the final remarks are discussed in Sec. 5.

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2. SET-POINT GENERATION VIA B-SPLINE FILTERS

As shown in Biagiotti and Melchiorri (2010) a B-spline trajectory of degree p can be generated by means of a chain of p dynamic filters defined as

$$M(s) = \frac{1 - e^{-sT}}{Ts}$$

fed by the staircase signal $p(t)$ obtained by maintaining the value of each control point p_i defining the curve for the entire period $iT \leq t < (i+1)T$, by means of a zero-order hold $H_0(s)$. For computer controlled systems equipped with digital controllers with sampling period T_s, the B-spline reference trajectory can be computed at time-instants kT_s by Z-transforming the chain of p filters $M(s)$ with zero-order hold. In this way the system of Fig. 1 is

$F_1(z^{-1}) =$	z^{-1}
$F_2(z^{-1}) =$	$\frac{1}{2}z^{-1} + \frac{1}{2}z^{-2}$
$F_3(z^{-1}) =$	$\frac{1}{6}z^{-1} + \frac{4}{6}z^{-2} + \frac{1}{6}z^{-3}$
$F_4(z^{-1}) =$	$\frac{1}{24}z^{-1} + \frac{11}{24}z^{-2} + \frac{11}{24}z^{-3} + \frac{1}{24}z^{-4}$
$F_5(z^{-1}) =$	$\frac{1}{120}z^{-1} + \frac{26}{120}z^{-2} + \frac{66}{120}z^{-3} + \frac{26}{120}z^{-4} + \frac{1}{120}z^{-5}$

Table 1. Expression of the filter $F_p(z^{-1})$ for different values of p.

obtained, where $F_p(z^{-1})$ is a FIR filter whose expression is reported in Tab. 1 for the most common values of the B-spline degree p In this case, the sequence p_i of the control points is transformed in the staircase sequence p_k, with sampling time T_s, by means of an up-sampling operation with replication

$$p_k = p_i, \qquad k = iN, iN+1, \ldots, (i+1)N-1 \tag{1}$$

where N denotes the ratio, supposed to be an integer, between T and T_s. The samples of the B-spline sequence are then generated by the filter denoted by $M_p(z)$ and coincide with the value of the continuous-time trajectory at time instants kT, i.e $q_k = q(kT)$, see Fig. 2.

2.1 Control points computation

We assume here that the tasks to be performed are cyclic, and therefore that the trajectories to be tracked are repetitive. Accordingly, in order to define the ideal spline trajectory passing through the via-points $q_i^\star$ at time instants $t_i = iT$, the so-called periodic B-splines are adopted, that is B-spline functions defined by control points periodic with period n:

$$p_j = p_{j+n}, \qquad j = 0, \ldots, n-1 \tag{2}$$

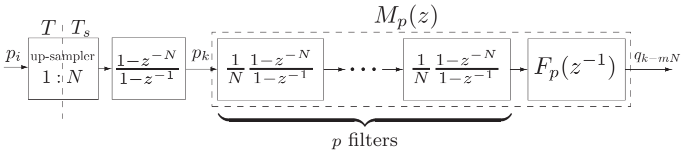


Fig. 1. Filter for the generation of discrete-time B-spline trajectories of degree p.

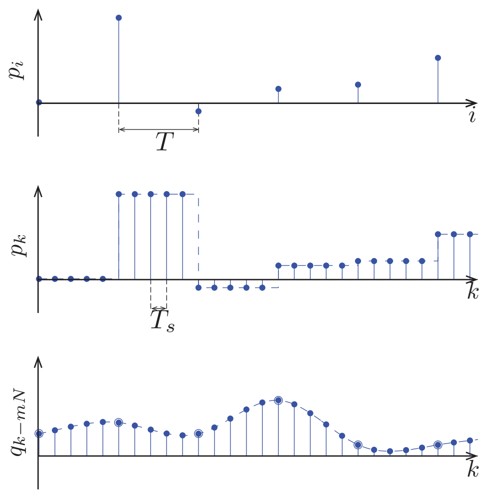
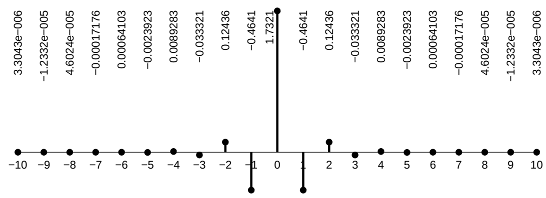
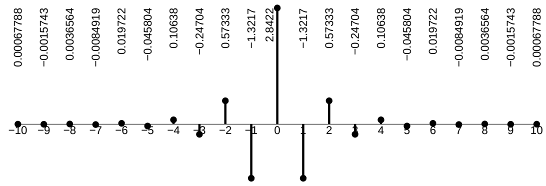


Fig. 2. Control points sequence p_i defining a cubic B-spline and related reference trajectory q_{k-mN} with $m = 2$ obtained with the dynamic filter of Fig. 1.



(a)



(b)

Fig. 3. Impulse response $h(n)$ of the filter (4) (a) and of the filter (5) (b).

By imposing the interpolation conditions $q(iT) = q_i^\star$, $i = 0, \ldots, n-1$, a system composed by n algebraic equations in the unknowns $p_i^\star$ is obtained, i.e.

$$\boldsymbol{A}_p\,\boldsymbol{p}^\star = \mathbf{q}^\star \tag{3}$$

where

$$\boldsymbol{p}^\star = [p_0^\star,\ p_1^\star,\ p_2^\star,\ \ldots,\ p_{n-3}^\star,\ p_{n-2}^\star,\ p_{n-1}^\star]^T$$
$$\mathbf{q}^\star = [q_0^\star,\ q_1^\star,\ q_2^\star,\ \ldots,\ q_{n-3}^\star,\ q_{n-2}^\star,\ q_{n-1}^\star]^T$$

and the expression of $\boldsymbol{A}_p$ is reported in Appendix A as a function of p. System (3) can be efficiently solved being $\boldsymbol{A}_p$ a band matrix, see Yamamoto (1979). However, it is worth noticing that the control points $p_i^\star$ can be calculated only when the overall set of via-points is given and therefore this procedure is suitable only for off-line computation. If it is necessary to calculate online the control points as soon as new via-points are provided, it is possible to observe that equation (3) can be seen as a dynamic relationship between via-points and control points (for more details see Biagiotti and Melchiorri (2013)), that in the domain of the Z-transform can be expressed as

$$\frac{P(z)}{Q(z)} = \frac{6}{z + 4 + z^{-1}} \tag{4}$$

for cubic B-splines, and

$$\frac{P(z)}{Q(z)} = \frac{120}{z^2 + 26z + 66 + 26z^{-1} + z^{-2}} \tag{5}$$

for quintic B-splines. Unfortunately, both filters (4) and (5) are unstable system and consequently they cannot be used for computing the sequence $p_i^\star$ from $q_i^\star$. This is a direct consequence of the fact that the interpolation procedure is a global problem that involves all the points $q_i^\star$. However, it is possible to approximate the interpolation process by taking into account only a small set of points $q_i^\star$. This approach leads to a FIR filter defined by

$$H(z) = \sum_{n=-r}^{r} h(n)\, z^{-n} \tag{6}$$

that approximates the impulse response of (4) and (5) within a prescribed tolerance according to the value of r. The sequences $h(n)$ for $p = 3$ and $p = 5$ are reported in Fig. 3, while their analytical expression is given in

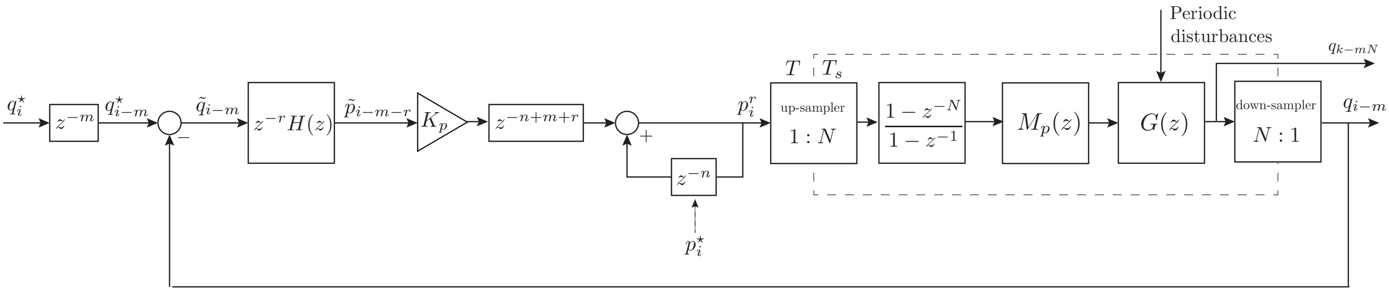


Fig. 5. Discrete-time repetitive control scheme based on discrete-time B-spline filter.

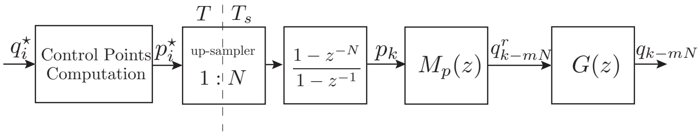


Fig. 4. Set-point generation by means of a B-spline filter for a (controlled) discrete-time system $G(z)$.

Appendix B. Note that in both cases the value of $h(n)$ becomes extremely small as $|n|$ grows. The quality of the approximation depends on the order $2r+1$ of the filter, but it is worth noticing that the choice $r = 4$ guarantees an approximation error with respect to the exact solution of (3) smaller than 0.5%. Moreover, since $H(z)$ is not a causal filter, in order to practically implement the transformation between via points and control points it is necessary to introduce a delay equal to r which makes the filter feasible, that is

$$H'(z) = z^{-r}\, H(z) = \sum_{n=0}^{2r} h(n-r)\; z^{-n}. \qquad (7)$$

3. IMPROVING B-SPLINES TRACKING VIA CONTROL POINTS MODIFICATION

In standard digital control applications, the reference trajectory generated by the discrete B-spline filter of previous section is provided to a dynamic system $G(z)$, as illustrated in Fig. 4. The control points $p_i^\star$ are usually computed off-line by solving (3).

Since the scheme of Fig. 4 has a standard cascade structure without control actions but with the only purpose of generating arbitrarily complex trajectories for the plant $G(z)$, the capabilities of $G(z)$ to track such a kind of signals are implicitly assumed. Therefore, the system $G(z)$ is assumed to be a controlled plant, with a standard closed-loop structure, whose frequency response is characterized by a typical low-pass behavior with a static gain as close as possible to the unity. As explained in control textbooks (see for instance Ogata (1997) among many others), in order to follow the input signal accurately the bandwidth of system $G(z)$ must be large enough, and in particular larger than the maximum frequency of the input signal, that in the case of the B-splines introduced in previous section can be assumed to be ω_0. Therefore, a cutoff frequency of the plant $\omega_c \gg \omega_0$ guarantee a good tracking performance. However perfect tracking (with zero tracking error $e = q - q^r$) is generally not achieved because $G(e^{j\omega T})$ is equal to one only approximatively and may be affected by external disturbances.

If the tasks to be performed are cyclic, and also "external" disturbances share the same property, it is possible to implement a procedure for modifying the reference signal in order to guarantee that the interpolation error at the given via-points $q_i^\star$ asymptotically vanishes. Since a B-spline curve is completely determined by the position of its control points, the modification of the trajectory can be obtained by directly acting on them, by means of the scheme of Fig. 5 based on the RC approach. In this scheme, both the trajectory generator and the plant $G(z)$ are inserted in a discrete-time control loop that, on the basis of the interpolation error $\tilde{q}_i = q_i^\star - q_i$, modifies in real-time the control points sequence (denoted by p_i^r) from the initial value $p_i^\star$. In the scheme of Fig. 5, the filter $H(z)$ is used to transform in run time the interpolation error $\tilde{q}_i$ in an error in the control points position $\tilde{p}_i$. The sequence $\tilde{p}_i$ multiplied by the constant $K_p \le 1$ (usually $K_p = 1$) and properly delayed in time is provided to the filter

$$\frac{1}{1 - z^{-n}} \qquad (8)$$

used to compute the reference sequence of points p_i^r for the discrete-time interpolator based on B-splines and the controlled plant. Note that the initial value of the output of filter in (8) has been set to $p_i^\star$, that is the sequence of the control points defining the ideal trajectory.

According to the theory of discrete-time repetitive control, see Tsai et al. (1988), that exploits the internal model principle of Francis and Wonham (1975), the presence in the control loop of the transfer function (8) assures asymptotic perfect tracking of a periodic signal with period n (in this case the number of the desired via points $q_i^\star$) provided that the whole system is stable. In Biagiotti et al. (2015) it has been found that the asymptotic stability of the RC scheme is guaranteed by the condition

$$||K_p\, G_{\rm wc} - 1|| < 1. \qquad (9)$$

where the constant (complex) number

$$G_{\rm wc} = \max_{\omega \le \omega_0} |G(e^{j\omega T_s})|\; \mathrm{e}^{\,j \min\limits_{\omega \le \omega_0} \{\arg G(e^{j\omega T_s})\}}$$

takes into account the maximum gain variation and the maximum (negative) phase displacement caused by $G(z)$. The use of the B-spline filter allows to restrict the range of variation of ω to the interval $[0, \omega_0]$ because, as already noted, the spectrum of the reference signal can be neglected outside this interval. If the condition mentioned at the beginning of this section about the tracking capabilities of $G(z)$ is met, i.e. $G(e^{j\omega T_s}) \approx 1$ for $0 \le \omega \le \omega_c$, and being $\omega_0 \ll \omega_c$ the condition (9) always holds true.

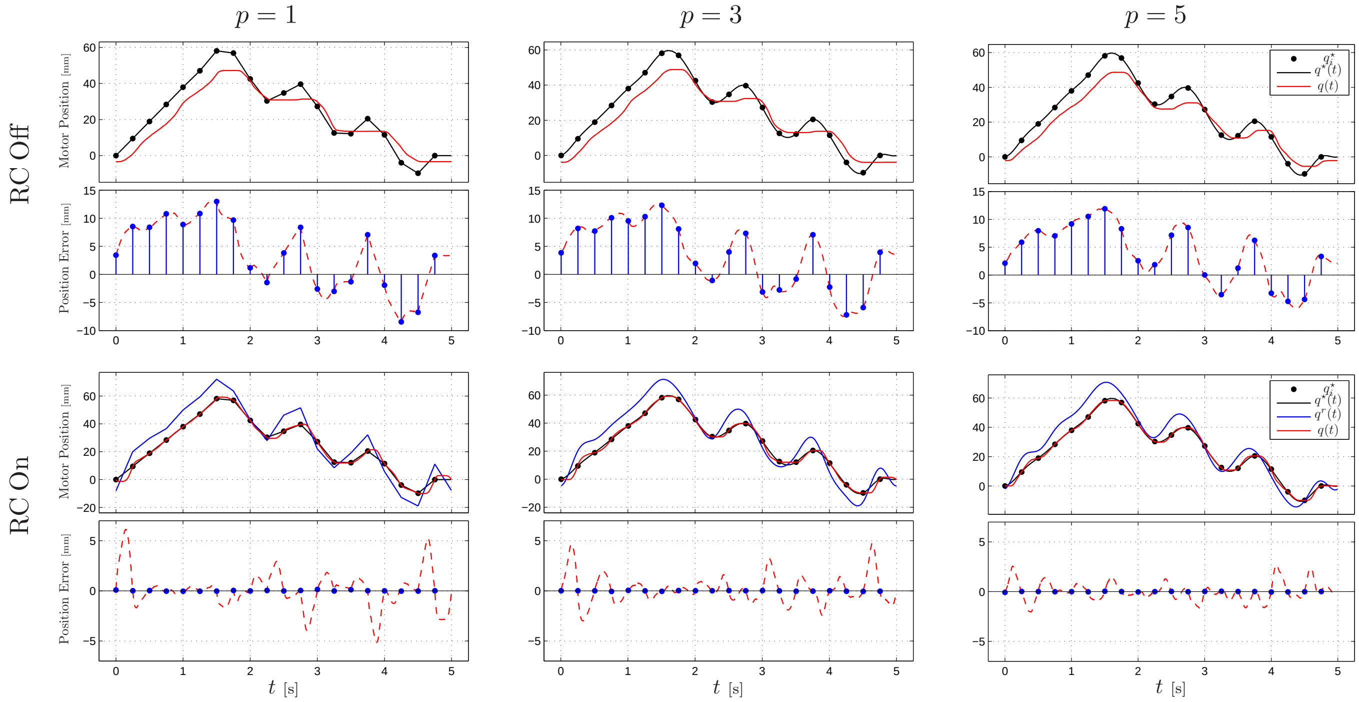


Fig. 6. Reference trajectory and actual position of the motor A and related interpolation error $\tilde{q}_i$ without and with RC mechanism as a function of p ($T = 0.25$ s).

4. EXPERIMENTAL VALIDATION

In order to experimentally test the proposed method the setup of Fig. 7 has been arranged. This system is characterized by two linear motors, LinMot PS01-37x120, rigidly connected along the axis of motion. Linear motor A is controlled by means of a position controller properly set up [1] to track a desired periodic motion defined by a uniform B-spline trajectory. On the other side, the linear motor B, equipped with a force/current controller, is used to generate an external periodic disturbance that emulates a mechanical load connected to the actuator A or the inertial coupling that exists among different axes of a robot manipulator. In particular, in the experiments the simple relation

$$F_{dist} = -k\, q_m(t) - c\, \dot{q}_m(t)$$

that reproduces a spring-damper system has been assumed, with the parameters $k = 500$ [N m] and $c = 100$ [N m s^{-1}]. The control system is based on the servo controller LinMot E2010-VF that performs the basic current control, while the position control (based on a standard velocity/position cascade control scheme) and the force control have been implemented on a standard PC with a Pentium IV 3 GHz processor and 1 GB of RAM equipped with a Sensoray 626 data acquisition board, used to communicate with the servo controller. The position of the motor is measured by an incremental encoder with a resolution of $1\mu m$ integrated in the stator. The real-time operating system RTAI-Linux on a Debian SID distribution with Linux kernel 2.6.17.11 and RTAI 3.4 allows the position controller to run with a sampling period $T_s = 500\mu s$. For the design of the control scheme and of trajectory generator, the MatLab/Simulink/RealTime Workshop environment has been used.

In order to test the performances of the system with the RC scheme, a trajectory passing through $n = 20$ via-points is considered. Once that the shape of the B-spline trajectory and its control-points, which depend only on the given via-points, have been fixed, the only parameters of the trajectory generator that can be changed are the knot span T (and accordingly the total duration of the trajectory) and the order p of the spline. In Fig. 6, the behavior of the system with and without RC modification of the trajectory is shown, along with the interpolation errors $\tilde{q}_i$, for different values of the degree p. When the RC is not activated the tracking error, intentionally quite

[1] In order to better highlight the behavior of the RC mechanism, the integral control term which is present in the position control loop of the actuator has been disabled.

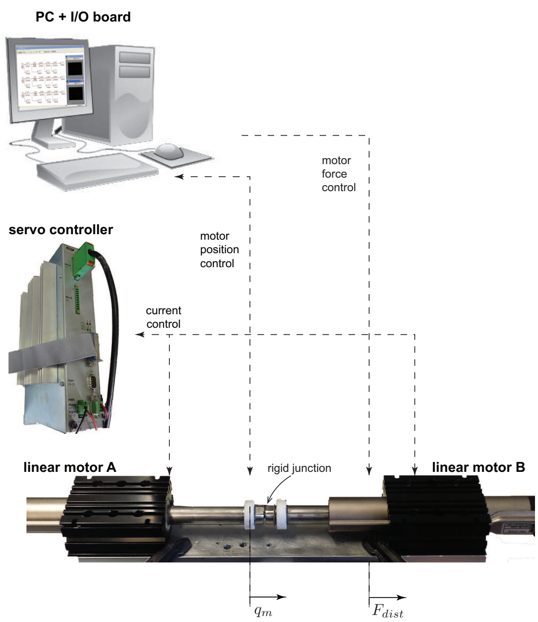


Fig. 7. Experimental setup.

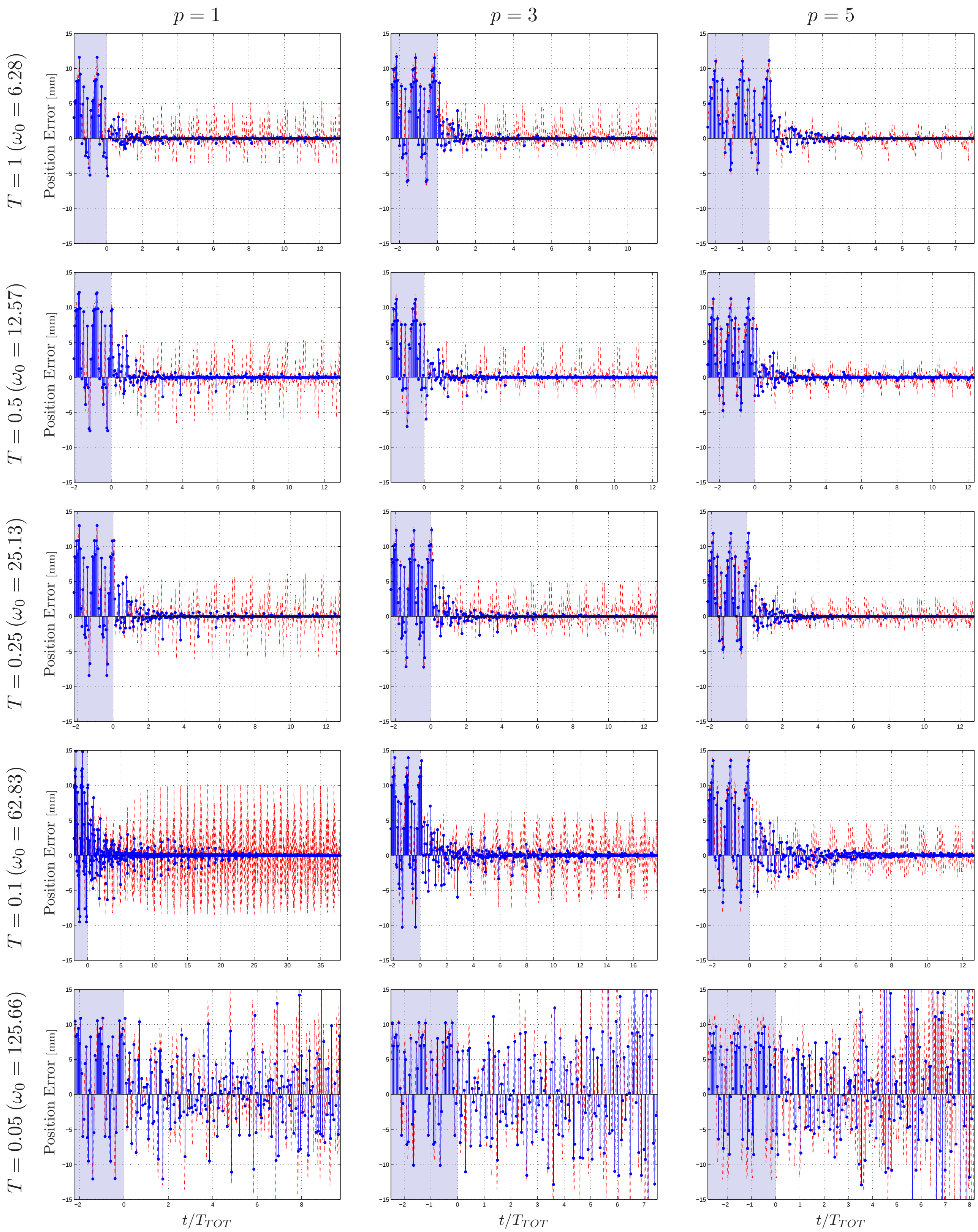


Fig. 8. Interpolation error $\tilde{q}_i$ at sampling instants iT as a function of p and T. On the x-axis, t/T_{TOT}, being $T_{TOT} = nT$ the total duration of the desired spline trajectory, represents the number of iterations.

large due to the noticeable external disturbance, seems to be not influenced by p. On the contrary, when the RC is activated (after 15 cycles), even if the interpolation error $\tilde{q}_i$ at sampling time iT is negligible, during the inter-samples the tracking error is strongly affected by p.

The same conclusions can be deduced from the results illustrated in Fig. 8, where the tracking errors obtained with the RC for different values of T and p are shown. The stability of the overall control system only depends on T, as stated in Sec. 3. As a matter of fact the system is stable until ω_0 is smaller than the cutoff frequency of the plant ($\omega_c \approx 63$ rad/s). But when $T = 0.05$ s and accordingly $\omega_0 = 125.6637$ rad/s overcomes ω_c the control system becomes unstable, independently of p. By analyzing Fig. 8, it is clear that the amplitude of the inter-sample oscillation depends on p, and in particular it decreases as p grows. This appear reasonable, since practical experience suggests that smoother reference signals, represented by B-spline of higher degree p, are usually better tracked by physical plants.

Finally, the role played by the gain K_p usually assumed equal to one has been investigated (in this case the plots are not reported because of space limits). Values of K_p smaller than one reduce the rate of convergency of the error q_i but they do not influence the stability of the system or the magnitude of the inter-sample error.

5. CONCLUSIONS

In this paper, motion planning and reactive control have been integrated in order to obtain a perfect tracking of a desired set of via-points. By considering tasks performed cyclically, which are quite common in the industrial and robotics field, a trajectory generation based on B-spline has been enhanced with a RC-type mechanism that modifies in real-time the control points defining the spline in order to nullify the tracking error at the desired points. The effectiveness and the robustness of the proposed approach has been demonstrated by implementing the control scheme on a test bed composed by two actuators: a position servo system and a source of load/disturbances. The role played by the free parameters of the controller (degree p of the trajectory and knot span T) have been investigated. If the controlled plant is able to track a given B-spline trajectory it is possible to implement the proposed mechanism without care about the stability of the overall system, which is always guaranteed. The use of of higher values of p allows to reduce inter-sample oscillations, which are a well-known side effect of RC.

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Appendix A. MATRICES OF THE SYSTEM FOR CONTROL-POINTS COMPUTATION

If $p = 1$, $\boldsymbol{A}_1 = \boldsymbol{I}_n$, being $\boldsymbol{I}_n$ the $n \times n$ identity matrix.

If $p = 3$,

$$\boldsymbol{A}_3 = \frac{1}{6}\begin{bmatrix} 4 & 1 & 0 & & & \cdots & 0 & 1 \\ 1 & 4 & 1 & 0 & & \cdots & & 0 \\ \vdots & & & & \ddots & & & \vdots \\ 0 & & & \cdots & 0 & 1 & 4 & 1 \\ 1 & 0 & & \cdots & & 0 & 1 & 4 \end{bmatrix}.$$

If $p = 5$,

$$\boldsymbol{A}_5 = \frac{1}{120}\begin{bmatrix} 66 & 26 & 1 & 0 & & & \cdots & 0 & 1 & 26 \\ 26 & 66 & 26 & 1 & 0 & & \cdots & & 0 & 1 \\ 1 & 26 & 66 & 26 & 1 & & \cdots & & & 0 \\ \vdots & & & & \ddots & \ddots & \ddots & & & \vdots \\ 0 & & & \cdots & 0 & 1 & 26 & 66 & 26 & 1 \\ 1 & 0 & & \cdots & & 0 & 1 & 26 & 66 & 26 \\ 26 & 1 & 0 & \cdots & & & 0 & 1 & 26 & 66 \end{bmatrix}.$$

Appendix B. COEFFICIENTS OF THE FIR FILTER FOR CONTROL-POINTS COMPUTATION

The coefficients $h(n)$ of the FIR filter $H(z)$ in (6) for $p = 3$ can be computed as

$$h(n) = \frac{1-\alpha}{1+\alpha}\alpha^{|n|}$$

where $\alpha = -2 + \sqrt{3}$ is the stable pole of (4), see Biagiotti and Melchiorri (2013).

For $p = 5$ the coefficients of the approximating FIR filter are

$$h(n) = c_1\,\alpha_1^{|n|} + c_2\,\alpha_2^{|n|}$$

where α_1 and α_2 are the unstable poles of (5) defined by

$$\alpha_i = \frac{1}{2}(2 + u_i + \sqrt{4\,u_i + u_i^2}), \qquad i = 1, 2$$

with $u_i = -15 \pm \sqrt{105}$, and the coefficients c_i are

$$c_1 = \frac{\alpha_1(-1+\alpha_1)(-1+\alpha_2)^2}{(\alpha_1-\alpha_2)(-1+\alpha_1\alpha_2)(1+\alpha_1)}$$

$$c_2 = \frac{\alpha_2(-1+\alpha_2)(-1+\alpha_1)^2}{(\alpha_2-\alpha_1)(-1+\alpha_1\alpha_2)(1+\alpha_2)}.$$