This is a pre print version of the following article:

CAPM-based capital budgeting and nonadditivity / Magni, Carlo Alberto. - In: JOURNAL OF PROPERTY INVESTMENT AND FINANCE. - ISSN 1463-578X. - STAMPA. - 26:5(2008), pp. 388-398. [10.1108/14635780810900251]

Terms of use:

The terms and conditions for the reuse of this version of the manuscript are specified in the publishing policy. For all terms of use and more information see the publisher's website.

12/10/2024 10:35

(Article begins on next page)

CAPM-based capital budgeting and nonadditivity

Carlo Alberto Magni
Università di Modena e Reggio Emilia, Dipartimento di Economia Politica
viale Berengario 51, 41100 Modena, Italy
Email: magni@unimo.it, tel. +39-059-2056777, fax +39-059-2056937

Original version: March 2006
This version: May 2008

Abstract. This paper deals with the CAPM-derived capital budgeting criterion, and in particular with Rubinstein's (1973) criterion, according to which a project is profitable if the project rate of return is greater than the risk-adjusted cost of capital, where the latter depends on the project's disequilibrium systematic risk. It is shown that the disequilibrium net present value implied by this criterion, widely used in corporate finance, is nonadditive. Four proofs are provided: (i) a counterexample taken from Copeland and Weston (1988), (ii) a *modus-tollens* argument showing that this notion of NPV is incompatible with additivity, (iii) a formalization showing that this NPV does not fulfil the principle of description invariance (iv) an example showing that CAPM-minded evaluators may incur arbitrage losses. The disequilibrium NPV should therefore be dismissed in investment decisions and valuations.

Running Title. CAPM-based capital budgeting and nonadditivity

Keywords and phrases. Investment, decision, valuation, NPV, disequilibrium, CAPM, capital budgeting, nonadditivity, framing effects.

JEL codes. G11, G12, G30, G31, M21

Electronic copy available at: http://ssrn.com/abstract=1081193

Introduction

This paper deals with project valuation and capital-budgeting decision-making in accept-reject situations. In applied corporate finance, it is usual to use the net-present-value concept in order to appraise investments and make decisions. The net present value (NPV) requires cash flow discounting by making use of the so-called risk-adjusted cost of capital, which serves the purpose of accounting for risk. A widespread model employed for computing the cost of capital is the Capital Asset Pricing Model (CAPM). Rubinstein (1973) and other scholars in the late Sixties and Seventies have provided the link between CAPM and capital budgeting decisions: in their classical contributions they show that, if the CAPM assumptions are met, then the cost of capital is a function of the systematic risk, which is given by the beta of the project. The latter depends in turn on the *cost* of the project, which implies that it is a *disequilibrium* beta. The corresponding NPV is therefore a *disequilibrium* NPV.

In the corporate finance literature this procedure is widespread not only as a *decision* rule but also as a *valuation* tool. For example, Bøssaerts and Odegaard (2001) endorse the use of cost-based betas and disequilibrium NPVs "to value a risky cash flow" (p. 60); Copeland and Weston mostly take for granted that an investor should use the disequilibrium NPV for both decision and valuation; in Weston and Copeland (1988) they give the formula of the disequilibrium NPV referring to it as ''the risk-adjusted method for evaluating projects'' (p. 381). These authors provide several numerical examples to illustrate the implementation of this NPV (e.g., Copeland and Weston, 1983, p. 135; Weston and Copeland, 1988, pp. 372–375 and 379–381; Copeland and Weston, 1988, pp. 415–418). Jones and Dudley (1978, p. 378) compute the required rate of return of a mispriced asset by discounting cash flows with a cost-based discount rate, that is by using the disequilibrium NPV (see their Tables 18.2 and 18.3). Lewellen (1977) uses the disequilibrium NPV approach for valuation. While stating his rule in a return-rate-exceeds-cost-of-capital fashion, Rubinstein (1973) himself refers to the disequilibrium NPV upholding it not only for decision purposes but for valuation purposes as well: in the last sentence of the second paragraph at p. 174 of his paper the author writes that the (risk-adjusted) cost of capital is the "appropriate discount rate for the project". Some pages earlier he makes the same claim, when he writes of "risk-adjusted discount rate for the project" and repeats the expression "discount rate" just after the sentence (p. 172). If Rubinstein thinks the (disequilibrium) cost of capital is the appropriate discount rate for the project, then he thinks that the disequilibrium NPV is actually the correct NPV not only for making decisions but for *valuing* projects as well. Also, in footnote 14 at p. 174, he writes, referring to mutually exclusive

Electronic copy available at: http://ssrn.com/abstract=1081193

investments: "This result follows immediately from equation (c) of footnote 10 and is equivalent to accepting the project with the highest net present value". He is then claiming that the disequilibrium NPV is a correct tool not only for accept-reject decisions, but for ranking projects as well. Also, in footnote 8 he uses the expression "present value risk-adjusted discount rate . . . form". That is, he refers to the (disequilibrium) NPV and uses the term "value" to mean the result of the (cost-based) discounting process.

Barring some isolated warnings against the use of this methodology (Grinblatt and Titman, 1998; Ekern, 2006) there is no debate in applied finance about this taken-for-granted methodology. This paper aims at showing that the disequilibrium NPV, firmly entrenched in the literature and in the practice and validly deducted from the CAPM as a decision rule for accept-reject decisions, it is nonetheless unsafe both as a valuation tool and as a decision rule: it does not comply with the additivity principle or, to say it in different terms, it does not comply with the principle of description invariance nor with the no-arbitrage principle.

The paper is structured as follows. Section 1 briefly formalizes Rubinstein's criterion and shows that it logically implies the use of disequilibrium NPV for decision-making. Section 2 is devoted to showing the NPV's pitfalls, which should induce to avoid its use for valuation and decision-making. In particular, (i) a numerical example is illustrated, taken from Copeland and Weston (1988); (ii) a logical argument (modus tollens) is provided to highlight inconsistencies; (iii) it is shown that framing effects arise if this NPV is used; (iv) it is underlined that the evaluator abiding by the disequilibrium NPV method is open to arbitrage losses. Some remarks conclude the paper.

1. Disequilibrium (cost-based) beta and disequilibrium NPV

Assume that there exists a security market satisfying the assumptions of the CAPM. In such a case, any asset traded in the security market lies on the Security Market Line (SML) and its rate of return is given by the relation

$$r_j = r_f + \lambda \operatorname{cov}(r_j, r_m) \tag{1a}$$

with $\lambda := \dfrac{\bar{r}_m - r_f}{\sigma_m^2}$ (see other notational conventions at the end of the paper) or, equivalently,

$$\frac{\bar{\mathrm{F}}_j - V_j}{V_j} = r_f + \lambda \frac{\operatorname{cov}(\mathrm{F}_j, r_m)}{V_j}. \tag{1b}$$

Eqs. (1a)-(1b) may be restated in net-present-value terms:

$$\mathrm{NPV}_j = -V_j + \frac{\overline{\mathrm{F}}_j}{1 + r_f + \lambda \dfrac{\mathrm{cov}(\mathrm{F}_j, r_m)}{V_j}} = 0 . \tag{1c}$$

Solving (1c) for V_j we have the well-known certainty-equivalent form $V_j = \dfrac{\overline{\mathrm{F}}_j - \lambda \,\mathrm{cov}(\mathrm{F}_j, r_m)}{1 + r_f}$.

Let us now suppose a project j is available to a firm and decision must be taken about undertaking it or not. In a classical pellucid paper, Rubinstein (1973, pp. 171-172 and footnote 10) proves that, if the above relation holds, and if the objective is shareholder maximization, then the project is worth undertaking if

$$\frac{\overline{\mathrm{F}}_j - I_j}{I_j} > r_f + \lambda \frac{\mathrm{cov}(\mathrm{F}_j, r_m)}{I_j} \tag{2a}$$

where the right-hand side is often called risk-adjusted cost of capital. Criterion (2a) is mathematically equivalent to

$$\mathrm{NPV}_j = -I_j + \frac{\overline{\mathrm{F}}_j}{1 + r_f + \lambda \dfrac{\mathrm{cov}(\mathrm{F}_j, r_m)}{I_j}} > 0 . \tag{2b}$$

It is worth noting that the covariance term in (2a)-(2b) depends on project cost I_j, not on the equilibrium value of the project. Therefore, Rubinstein's covariance term is a *disequilibrium* (cost-based) covariance, not an *equilibrium* covariance.

Senbet and Thompson (1978) show that the capital budgeting criteria proposed by Hamada (1969), Bierman and Hass (1973), Rubinstein (1973), Stapleton (1971), Bogue and Roll (1974) are equivalent. The same criterion is also found in Litzenberger and Budd (1970), where they explicitly acknowledge the equivalence of Mossin's criterion, Hamada's criterion, and Tuttle and Litzenberger's (1968) criterion. (In the Appendix of this paper the equivalence of Rubinstein's criterion and Mossin's criterion is shown).

As it is clear from eq. (2b), for practical implementation this criterion requires to fulfil the following steps:

- Forecast end-of-period payoff from the project in the possible states of nature
- Forecast market rate of return in the possible states of nature
- Compute covariance between project's payoff and market rate of return
- Divide covariance by the project's cost
- Determine the market price of risk λ and, therefore the risk-adjusted cost of capital
- Discount expected end-of-period payoff at the risk-adjusted cost of capital
- Subtract cost

(for multi-period projects generalization is straightforward). Hence, according to the (disequilibrium) NPV supporters, the sign of eq. (2b) tells decision makers whether the project is profitable, and the discounted end-of-period payoff gives the project's value (see Copeland and Weston, 1983, 1988 and Weston and Copeland, 1988, for several numerical examples).

2. Nonadditivity

Additivity in valuation is a major tenet in finance. The net present value is acceptable as a meaningful notion only if it is additive. Formally, additivity means that

$$\mathrm{NPV}_j + \mathrm{NPV}_k = \mathrm{NPV}_{j+k} \qquad \text{for all } j, k. \tag{3}$$

In words, picking *any* pair of projects *j* and *k*, the sum of their NPVs must equal the NPV of that project (*j*+*k*) obtained by summing the cash flows of the two projects. We now show that condition eq. (3) is not fulfilled if the disequilibrium NPV in eq. (2b) is used for valuation.

2.1 Counterexample

Let us consider a very simple numerical example taken from the classical textbook by Copeland and Weston (1988). At pages 414-418 the authors present two projects and calculate their risk-adjusted cost of capital. They employ $\mathrm{cov}(r_j, r_m) = \dfrac{\mathrm{cov}(\mathrm{F}_j, r_m)}{I_j}$ to compute the risk-adjusted cost of capital (see their eq. (12.30)). Tables 1 and 2 collect all the relevant data and calculations made by the authors (rates are given in percentage). As the reader may see, the costs of capital are −9.33% for project 1 and 14% for project 2. Using these costs of capital it is very simple for an investor to calculate the NPVs of the projects. We find

$$\mathrm{NPV}_1 = -100 + \frac{\frac{1}{3}(105+115+95)}{1+(-0.0933)} = 15.808 \tag{4}$$

$$\mathrm{NPV}_2 = -100 + \frac{\frac{1}{3}(107.5+100+102.5)}{1+0.14} = -9.356\,. \tag{5}$$

Let us now consider the project obtained by summing the cash flows of project 1 and project 2 and let us calculate the cost of capital and the NPV by using again the very same formulas (see Tables 3 and 4). We find a cost of capital of 2.33%. The NPV of project (1+2) is therefore

$$\mathrm{NPV}_{1+2} = -200 + \frac{\frac{1}{3}(212.5+215+197.5)}{1+0.0233} = 3.583\,. \tag{6}$$

This means that

$$\mathrm{NPV}_1 + \mathrm{NPV}_2 = 15.808 - 9.356 = 6.452 \neq 3.583 = \mathrm{NPV}_{1+2}\,.$$

In other terms, condition (3) does not hold.

It is worth noting that if we suppose that project 1's cost is 104.6 (other things equal), then we find

$$\mathrm{NPV}_1 = 10.464 \qquad \mathrm{NPV}_2 = -9.356 \qquad \mathrm{NPV}_{1+2} = -1.091 \tag{7}$$

so that

$$\mathrm{NPV}_1 + \mathrm{NPV}_2 = 1.108 \neq -1.091 = \mathrm{NPV}_{1+2}\,.$$

Additivity is not satisfied and, in addition, we have two NPVs of opposite sign, leading to different decisions about undertaking the same course of action: if the course of action is seen as the sum of two separate projects (to be both undertaken or both rejected), then the course of action is accepted; if cash flows are seen as aggregate amounts so that the gross alternative (1+2) is evaluated as a unique alternative, then the course of action is rejected.

2.2 Modus tollens

Let us consider project *j* whose initial outflow is I_j and the final payoff is the random sum F_j. Let us assume that

(i) additivity holds

(ii) the disequilibrium NPV is used for valuation.

It is easy to show that these assumptions imply that the project's NPV can be any real number. In fact, let α be the desired NPV and choose a pair $(h^*, k^*) \in R^2$ such that

$$k^* = \left[(\alpha + I_j)\left(R_f + \frac{\lambda}{(I_j - h^*)} \right) \mathrm{cov}(\mathrm{F}_j, r_m) - \overline{\mathrm{F}}_j \right] \frac{R_f (I_j - h^*)}{\lambda \, \mathrm{cov}(\mathrm{F}_j, r_m)} .$$

where we let $R_f := 1 + r_f$. Manipulating algebraically we get to

$$\alpha = \left[-(I_j - h^*) + \frac{\overline{\mathrm{F}}_j - k^*}{R_f + \frac{\lambda}{(I_j - h^*)} \mathrm{cov}(\mathrm{F}_j, r_m)} \right] + \left[-h^* + \frac{k^*}{R_f} \right]$$

By assumption (ii), a net present value is calculated as in eq. (2b), and α may be interpreted as the sum of two projects' NPVs: the first project costs $I_j - h^*$ and pays off the random sum $\mathrm{F}_j - k^*$, the second project costs h^* and pays off the certain amount k^*. Let us call j_1 the first risky project and j_2 the second riskless project. Given that $\mathrm{F}_j = (\mathrm{F}_j - k^*) + k^*$ and $I_j = (I_j - h^*) + h^*$ we evidently have $j = j_1 + j_2$ (the two project are constituents of project j). By assumption (i) we have

$$\alpha = \mathrm{NPV}_{j_1} + \mathrm{NPV}_{j_2} = \mathrm{NPV}_j .$$

As α is any real number, then the NPV of project j is whatever number one wants it to be. To avoid this nonsense, one is bound to conclude, by modus tollens, that the two assumptions (i) and (ii) cannot simultaneously hold. In other terms, the disequilibrium NPV as expressed in eq. (2) and deducted by Rubinstein (1973) on the basis of the CAPM is incompatible with the notion of additivity.

2.3 Description invariance

Let us consider again project j above and let

$$f(h,k) := \left(-(I_j - h) + \frac{\overline{\mathrm{F}}_j - k}{R_f + \dfrac{\lambda \operatorname{cov}(\mathrm{F}_j, r_m)}{(I_j - h)}} \right) + \left(-h + \frac{k}{R_f} \right)$$

with $(h,k) \in \mathrm{R}^2$. Additivity implies that $f(h,k)$ is constant under changes in *h* and *k*:

$$f(h_1,k_1) = f(h_2,k_2) \qquad \text{for any} \quad h_1,k_1,h_2,k_2 \in \mathrm{R}. \tag{8}$$

To see that condition (8) does not hold, we just need to calculate the first partial derivatives of the function. After simple algebraic manipulations we find

$$\frac{\partial f(h,k)}{\partial h} = -\frac{\lambda \operatorname{cov}(\mathrm{F}_j, r_m)(\overline{\mathrm{F}}_j - k)}{[R_f(I_j - h) + \lambda \operatorname{cov}(\mathrm{F}_j, r_m)]^2}, \tag{9a}$$

and

$$\frac{\partial f(h,k)}{\partial k} = \frac{1}{R_f} - \frac{1}{R_f + \dfrac{\lambda \operatorname{cov}(\mathrm{F}_j, r_m)}{(I_j - h)}}. \tag{9b}$$

which are not identically zero. This means that the function $f(h,k)$ is not invariant with respect to *h* and *k* or, equivalently, additivity is not fulfilled. Therefore, valuation changes depending on the way a course of action is depicted (see Magni, 2002, section 4) and evaluators do not abide by the principle of description invariance, whose violations are known as "framing effects" (Tversky and Kahneman, 1981; Kahneman and Tversky, 1984; Soman, 2004).

2.4 Arbitrage loss

As seen, an NPV-minded decision-maker (DM) adopting eq. (2) may frame courses of action in different but logically equivalent ways obtaining different valuations. This implies that he is open to possible arbitrage losses, as in the following case. Suppose an arbitrageur offers an NPV-minded DM an agreement according to which they exchange the same cash flows generated by project 1, with the arbitrageur taking a short position (he will be the borrower) and the DM a long position (he will be the lender); but the arbitrageur warns the DM that if he accepts this agreement he will have to pay a 15-euros fee. The NPV of project 1 is 15.808 euros (see eq. (4)), which represents the maximum fee the DM is willing to pay in order to accept the agreement. Being 15<15.808 he accepts. Now suppose the arbitrageur offers the DM the

opportunity of exchanging project (1+2)'s cash flows where the arbitrageur will be now the lender and the DM will be the borrower; but if the DM accepts he will receive a 4-euros prize. The NPV for the DM is –3.583 euros (consider eq. (6) changed in sing), but 4>3.583 so he accepts again. Finally, the arbitrageur offers the DM the opportunity of exchanging the same cash flows generated by project 2 with the arbitrageur being the borrower and the DM acting as the lender; if he accepts, he will be rewarded by the arbitrageur with a 10-euros prize. Project 2's NPV is –9.356 (see eq. (5)), but as 10>9.356 he accepts again. As a result, the NPV-minded DM is trapped in an arbitrage loss of 1 euro (net cash flows for the DM are summarized in Table 5. The arbitrageur's cash flows are the same reversed in sign).[1]

Conclusions

This paper deals with the well-established CAPM-derived capital budgeting criterion and the notion of disequilibrium cost of capital, which Rubinstein (1973) fosters in his classical paper and which mathematically implies the use of the disequilibrium NPV for capital budgeting. The use of disequilibrium NPVs is widespread in applied corporate finance, both in the literature and in the practice, both as a valuation tool and a decision rule. There is no debate in the literature about possible flaws of this capital budgeting rule. This paper aims at highlighting that the disequilibrium NPV is nonadditive and may not be used for *valuation*, given that it is nonadditive. As for decision-making, while it is indeed logically deducted by from the CAPM (eq. (2b) is mathematically equivalent to eq. (2a), and eq. (2a) is deductively implied by the CAPM), its very use for decision-making is unsafe, because it leaves decision makers open to arbitrage losses.

[1] If one changes the framing and aggregates the fee/prize and the initial cash flow for each alternative, and calculates NPVs, then the arbitrage loss does not occur. This is actually a violation of description invariance (existence of arbitrage losses should not depend on the way the evaluator frames the problem).

Appendix

Mossin (1969, p. 755, left column) shows that, assuming the market is in equilibrium, an investment Z will be undertaken by a firm *l* if and only if

$$\frac{1}{1+r_f}(\overline{\mathrm{F}}_Z - R\,\mathrm{cov}(\mathrm{F}_Z, V_m)) > I_Z \tag{A.1}$$

where F_Z is the cash flow generated by the project, V_m is the end-of-period value of the security market, I_Z is the investment cost, and

$$R = \frac{\overline{\mathrm{F}}_l - (1+r_f)V_l}{\mathrm{cov}(\mathrm{F}_l, V_m)} \tag{A.2}$$

with $\overline{\mathrm{F}}_l$=free cash flow of firm *l*, V_l=market value of firm *l*. Dividing both sides of (A.1) by I_Z we have

$$\frac{\overline{\mathrm{F}}_Z}{I_Z} - R\,\mathrm{cov}(r_Z, V_m) > 1 + r_f\,. \tag{A.3}$$

As $\frac{\overline{\mathrm{F}}_Z}{I_Z} - 1 = \bar{r}_Z$, eq. (A.3) becomes

$$\bar{r}_z > r_f + R\,\mathrm{cov}(r_Z, V_m)\,. \tag{A.4}$$

Letting V_0 be the current value of the market, we have $\mathrm{cov}(r_Z, V_m) = V_0\,\mathrm{cov}(r_Z, r_m)$. Therefore, we have, using (A.2) ,

$$\bar{r}_z > r_f + \frac{\overline{\mathrm{F}}_l - (1+r_f)V_l}{\mathrm{cov}(\mathrm{F}_l, V_m)} V_0\,\mathrm{cov}(r_Z, r_m) \tag{A.5}$$

which boils down to

$$\bar{r}_z > r_f + \frac{\overline{\mathrm{F}}_l - (1+r_f)V_l}{\mathrm{cov}(\mathrm{F}_l, r_m)} \mathrm{cov}(r_Z, r_m) \tag{A.6}$$

whence

$$\bar{r}_z > r_f + \frac{\bar{r}_l - r_f}{\mathrm{cov}(r_l, r_m)} \mathrm{cov}(r_Z, r_m) \tag{A.7}$$

where r_l is the rate of return on firm l.[2] The term $\frac{\bar{r}_l - r_f}{\text{cov}(r_l, r_m)}$ "is the same for all companies" (Mossin, 1969, p. 755, right column), so that

$$\frac{\bar{r}_l - r_f}{\text{cov}(r_l, r_m)} = \frac{\bar{r}_m - r_f}{\sigma_m^2}.$$

As a result, eq. (A.7) becomes

$$\bar{r}_Z > r_f + \lambda \,\text{cov}(r_Z, r_m)$$

or

$$\text{NPV}_Z = -I_Z + \frac{\bar{\text{F}}_Z}{1 + r_f + \lambda \,\text{cov}(r_Z, r_m)(\bar{r}_m - r_f)} > 0$$

which coincides with eqs. (2a) and (2b) respectively, with j=Z.

Q.E.D.

[2] V_l and r_l refer to the value and rate of return of firm l prior to investment Z.

References

Bierman, H and Hass, J.E. (1973), Capital budgeting under uncertainty: a reformulation. *Journal of Finance*, 28(1), 119–129, March.

Bogue, M.C. and Roll, R. (1974), Capital budgeting of risky projects with 'imperfect markets' for physical capital, *Journal of Finance*, 601–613, May.

Bøssaerts, P.L. and Odegaard, B.A. (2001). Lectures on Corporate Finance. World Scientific, Singapore.

Copeland, T.E. and Weston, J.F. (1983). *Solutions Manual for Financial Theory and Corporate Policy*, second ed. Addison-Wesley Publishing Company.

Copeland, T. and Weston, J.F. (1988), *Financial Theory and Corporate Finance*. Addison-Wesley Publishing Company, Inc.

Ekern, S. (2006). A dozen consistent CAPM-related valuation models – so why use the incorrect one?. *Department of Finance and Management Science, Norwegian School of Economics and Business Administration (NHH)*. Bergen, Norway. Available online at <http://www.nhh.no/for/dp/2006/0606.pdf>

Grinblatt, M. and Titman, S. (1998). *Financial Markets and Corporate Strategy*. Irwin/McGraw-Hill.

Hamada, R. S. (1969), Portfolio analysis, market equilibrium and corporation finance, *Journal of Finance*, 24(1), 13–31, March.

Jones Jr., R.G. and Dudley, D. (1978), *Essential of Finance*. Englewood Cliffs. NJ: Prentice-Hall.

Kahneman, D. and Tversky, A. (1984), Choices, values and frames. *American Psychologist*, 39, 341–350.

Lewellen, W.G. (1977), Some observations on risk-adjusted discount rates. *Journal of Finance* 32 (4), 1331–1337.

Magni, C.A. (2002) , Investment decisions in the theory of finance: Some antinomies and inconsistencies, *European Journal of Operational Research*, 137, 206-217

Mossin, J. (1969), Security pricing and investment criteria in competitive markets, *American Economic Review*, 59(5), 749–756, December.

Litzenberger, R.H. and Budd, A.P. (1970), Corporate investment criteria and the validation of risk assets, *Journal of Financial and Quantitative Analysis*, 5(4), 395–418, December.

Rendleman, R. (1978), Ranking errors in CAPM capital budgeting applications, *Financial Management*, 7(4), 40–44, Winter.

Rubinstein, M. (1973), A Mean-variance synthesis of corporate financial theory, *Journal of Finance*, 28, 167–182, March.

Senbet, L.W. and Thompson, H.E. (1978), The equivalence of mean-variance capital budgeting models, *Journal of Finance*, 23(29), 395–401, May.

Soman (2004), Framing, loss aversion, and mental accounting. In D. J. Koehler and N. Harvey, Eds.,*Blackwell Handbook of Judgment and Decision Making*, pp. 379–398, Oxford, UK: Blackwell.

Stapleton, R.C. (1971), Portfolio analysis, stock valuation and capital budgeting decision rule for risky projects, *Journal of Finance*, 26, 95–117, March.

Tuttle, D.L. and Litzenberger, R.H. (1968), Leverage, diversification and capital market effects on a risk-adjusted capital budgeting framework, *Journal of Finance,* 23(3), 427–443.

Tversky, A. and Kahneman, D. (1981), The framing of decisions and the psychology of choice. *Science*, 211, 453–458.

Weston, J.F. and Copeland, T.E. (1988), *Managerial Finance*, second British ed. London, UK: Cassell Educational Limited.

Notational conventions used in the paper

F_j = End-of-period cash flow of asset/project j

$\overline{\mathrm{F}}_j$ = Expected end-of-period cash flow of asset/project j

$\bar{r}_j$ = Expected rate of return of asset/project j

r_m = Market rate of return

σ_m^2 = Variance of market rate of return

NPV_j = NPV of asset/project j

I_j = Cost of project j

r_j = Rate of return of asset or project j

r_f = Risk-free rate of return

$\bar{r}_m$ = Expected market rate of return

V_j = Value of asset/project j

$\mathrm{cov}(\cdot,\cdot)$ = Covariance operator

Table 1. Copeland and Weston's example –cash flows and rates of return							
		Market		**Project 1** $(I_1 = 100)$		**Project 2** $(I_2 = 100)$	
	Probability	r_m	r_f	F_1	r_1	F_2	r_2
State 1	0.333	26	4	105	5	107.5	7.5
State 2	0.333	14	4	115	15	100	0
State 3	0.333	20	4	95	–5	102.5	2.5

Table 2. Copeland and Weston's example –relevant statistics and values				
	$\bar{r}_j$	$\mathrm{cov}(r_j, r_m)$	β_j	i_j
Project 1	5.00	−0.002	−0.833	−9.33
Project 2	3.33	0.0015	0.625	14.00
Market	20.00	0.0024	1.000	

Table 3. Project (1+2) –cash flow and rates of return			
		Project (1+2) $(I_{1+2} = 200)$	
	Probability	F_{1+2}	r_{1+2}
State 1	0.333	212.5	6.25
State 2	0.333	215	7.50
State 3	0.333	197.5	1.25

Table 4. Project (1+2) –relevant statistics and values				
	$\bar{r}_{1+2}$	$\mathrm{cov}(r_{1+2}, r_m)$	β_{1+2}	i_{1+2}
Project C	4.166	−0.00025	−0.104	2.33

Table 5. Arbitrage loss		
	Time 0	Time 1
First contract (cash flows)	–100	F_1
First contract (fee)	–15	0
Second contract (cash flows)	200	$-(F_1+F_2)$
Second contract (prize)	4	0
Third contract (cash flows)	–100	F_2
Third contract (prize)	10	0
Net Cash Flows	–1	0