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# Modeling waste production into two-machine one-buffer transfer lines

Elisa Gebennini \*

Dipartimento di Scienze e Metodi dell'Ingegneria,

Università degli Studi di Modena e Reggio Emilia,

Via Amendola 2, 42122 Reggio Emilia, Italy

Stanley B. Gershwin

Laboratory for Manufacturing and Productivity – MIT Department of Mechanical Engineering,

Massachusetts Institute of Technology,

77 Massachusetts Avenue, Cambridge, Massachusetts 02139, USA

## Abstract

The paper focuses on analytical models of two-machine one-buffer Markov lines including waste production. The aim is to compute the probability of producing good parts, referred as *effective* efficiency, when waste production is related to stoppages of the first machine. This problem is common in industrial fields where parts are generated by a continuous process, e.g. in high speed beverage packaging lines.

Two innovative models including waste production are presented: the *WP*-Basic Model extends the model of a basic two-machine one-buffer transfer line; the *WP*-RP Model extends the model of a two-machine one-buffer transfer line with a restart policy operating on the first machine (i.e., when the first machine is blocked because the buffer is filled, it is not allowed to resume production until the buffer becomes empty). The two previous models are improved by distinguishing, at any time step the first machine is operational, whether it is producing a good or a bad part.

The probabilities of the system being in any feasible state are analytically derived for both the *WP*-Basic Model and the *WP*-RP Model. Then, the obtained probabilities are used to determine the performance measures of interest, i.e., waste probability and *effective* efficiency. Finally,

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\*Corresponding author. E-mail: elisa.gebennini@unimore.it (E. Gebennini). Tel: +39 0522 522672.

some numerical results are provided to illustrate the effectiveness of the *WP-Basic Model* and the *WP-RP Model*.

# 1 Introduction

This paper presents an analytical method for calculating the *effective* efficiency of two-machine one-buffer lines with Markovian machines having equal and constant processing times. The *effective* efficiency is defined as the probability of producing a good part in any time step.

Parts produced in the manufacturing system of interest may be either “good” or “bad”. It is assumed that bad parts, i.e., parts that do not meet the required quality standards, are produced by the first machine of the line under specific conditions. Then, both bad and good parts pass through the buffer and enter the second machine that is supposed to ensure a defect-free process.

## 1.1 Problem formulation and motivation

In this work, the condition under which bad parts are produced is the occurrence of stoppages of the first machine. Such stoppages can be caused by two disjoint sets of events: (i) operational failures and (ii) blocking events. While the former refers to the internal behaviour of the machine (depending on the machine time-to-failure distribution); the latter concerns a more complex phenomenon related to the interactions with the rest of the line (i.e., the buffer and the second machine). Specifically, a blocking event occurs when the downstream buffer is full so as there is not enough storage space to keep incoming parts. This situation may occur if the second machine is down while the first one is still operational, depending on the buffer level and the maximum buffer size. This paper assumes that when the first machine is repaired (according to the repair probability of the very machine) or leaves the blocking condition (according to the repair probability of the second machine and the buffer level), it produces a certain and constant number of bad parts. Hence, waste production not only depends on internal machine failures but also on events occurring in other parts of the line.

This scenario is common in practice in the food and beverage industry. Let us consider the first machine of an automatic packaging line, called “filling machine”, that fills packages with liquid dairy products or soft drinks. Such a machine takes in packaging material and liquid food. The production process is a continuous and aseptic process where the packaging material passes through a heated hydrogen peroxide bath. If the process is interrupted for any reason (e.g. a failure or a blocking event) a portion of the packaging material remains in contact with the hydrogen peroxide for too long. As a consequence, the first packages produced when the stoppage is removed must be

rejected as waste. It can be assumed that the number of waste packages per stoppage is constant. The filled packages released by the filling machine (“good” or “bad”) pass to an accumulation conveyor and then to the rest of the line. Since the most important feature of this accumulation conveyor, from the point of view of the issue addressed in this paper, is its storing capacity, we refer to it simply as a finite FIFO buffer (see Li and Meerkov, 2009).

The machines in the buffer downstream are inspection stations and packaging machines (e.g. straw applicators, film wrappers, etc.) that do not compromise the integrity of the filled packages. Thus, it is acceptable to assume that the machines following the filler machine in the line ensure a waste-free process. This is an interesting industrial application of a manufacturing system where a number of bad parts are produced in the event of any stoppages of the first machine.

Note that in many automatic manufacturing lines parts of different sizes and shapes are produced at a very high speed. For example, the filling machine of an automatic packaging line produces thousands of packages per hour. Thus, an important production issue is the risk of jamming, sliding or tumbling. When a “jamming” occurs a manual intervention of an operator is needed to restore the operational state of the machine. In this paper jamming is treated as failures. This is correct if it is assumed that a machine may only fail when it is operating. In general, we can distinguish between *time dependent* and *operation dependent* failures (Buzacott and Hanifin, 1978). The former depend only on the amount of time since the last repair, and the latter depend only on the number of operations that have been performed since the last repair. In semiconductor industry, for example, the majority of interruptions are time-based (SEMI, 1992). Nevertheless, in order to address automatic manufacturing lines where jamming may interrupt the production flow, the assumption of operation dependent failures is adopted in this paper.

Even if the occurrence of a specific failure is unpredictable, we can represent the time to the next failure (time to failure, TTF) by means of mass probability functions. To facilitate mathematical tractability, TTF is commonly assumed to be distributed according an exponential law, that is, to consider the failure occurrence process as it were completely memory-less. The same is assumed to model the time needed to restore the operational state of a machine (time to repair, TTR) once the failure occurred. Although those assumptions could be felt invasive, practical evidence showed that TTF and TTR assume distributions very close to the exponential, especially when automated machines are considered (Perrica et al., 2008). Hence, both failure and repair processes can be conveniently modelled as memory-less processes.

The above considerations justify the modelling approach adopted in this paper. Specifically, the system under study can be modelled as a *transfer line* with Markovian machines decoupled by finite

buffers.

## 1.2 Literature review

Extensive reviews on transfer/production line modelling are provided by Dallery and Gershwin (1992), Papadopoulos et al. (1993) and Gershwin (2002).

For reason of mathematical tractability, exact analytical models of this kind of manufacturing systems are available only for short lines, i.e., lines made up of two machines decoupled by an intermediate buffer. Thus, also the analytical models presented in this paper address the simple configuration of two-machine one-buffer lines. Although this may seem restrictive, the contribution of the present study is of practical relevance even when longer lines are under analysis. Note that if the line under analysis is a long line with only one buffer, the first machine and/or the second machine of the model could be equivalent machines representing series of machines in the real system. In case of more complex lines (with several buffers decoupling adjacent machines), approximated techniques can be adopted. The interesting point is that the most widely used methods for analysing long lines, e.g. decomposition techniques (see Gershwin, 1987; Dallery et al., 1989; Levantesi et al., 2003), are based on the evaluation of a series of two-machine one-buffer sub-systems. Thus, the development of accurate analytical models for short lines is of great practical importance to advance the understanding of more complex and longer lines.

In recent years, transfer/production line models including quality control and quality requirements have been developed. The reader may refer to works such as Colledani and Tolio (2000), Kim and Gershwin (2005), Aksoy and Gupta (2005), Li and Huang (2007), Colledani and Tolio (2009). These models treat a system where any part has a certain probability of having a quality defect.

On the other hand, this paper addresses a different kind of problem where waste production is related to the occurrence of specific events, i.e., stoppages of the first machine. Note that stoppages of the first machine are due to not only internal failures but also events occurring in other parts of the line causing the blocking of the first machine itself.

A similar problem is addressed in Liberopoulos et al. (2007) by developing analytical expressions for important performance measures of an automatic transfer line where the quality of the material trapped in the stopped workstations deteriorates with time. Nevertheless, the study does not consider storage buffers along the line. A first attempt to address waste production into a two-machine one-buffer transfer line is presented in Gebennini et al. (2009) and Gebennini et al. (in press). Nonetheless, the *effective* efficiency is evaluated by means of an approximate formula, valid only under certain conditions (e.g. low values for both the failure probability and the amount of

waste per stoppage).

Hence, this study contributes to the current literature by analytically expressing the probability of waste production and, as a consequence, by providing the *effective* efficiency of a two-machine one-buffer line. The new models presented in this paper extend the results from the well-known basic two-machine one-buffer model (refer to Gershwin, 2002) and from the two-machine one-buffer model with restart policy in Gebennini et al. (2009). The improvement consists in evaluating whether a good or a bad part is produced at any time step by considering both internal machine failures and machine-machine interactions, partially mitigated by the intermediate finite buffer. The rational is to take the aforementioned previous models without waste production, easier to be solved, as a starting point, and to “disaggregate” the states as needed.

The remainder of the paper is organized as follows. Section 3 introduces some notation and discusses the new modelling approach with waste production. The results from Section 3, i.e., the probabilities of being in any state with the first machine producing a good or a bad part, are used in Section 4 to derive the expressions of the performance measures of interest, i.e., waste probability and *effective* efficiency. Section 5 presents some numerical results in order to demonstrate the robustness of the new models. Section 6 draws some conclusions and outlines future work.

## 2 Modeling approach

The manufacturing system under study is a two-machine one-buffer transfer line with Markovian machines.

If waste production is not taken into consideration and the behaviour of each machine only depends on its own failure/repair probability and on the level of the intermediate buffer, the transfer line can be studied by means of a discrete-time discrete-state Markov model. Specifically, we refer to two previous models:

- the model of the simplest line acting as follows (for major details see Gershwin, 2002): when the buffer is neither full nor empty both machines may fail and be repaired according to their own failure/repair probabilities and, accordingly, the buffer level rises or falls; when the buffer is empty the second machine is “starved”, i.e., there are no parts to work on; when the buffer is full the first machine is “blocked”, i.e., there is no storage space to hold parts in the downstream. Starvation and blocking are negative phenomena resulting in a loss of production capacity (the machine is operational but prevented from processing parts). This model is called *Basic Model* in the following;

- the model with “restart policy” acting as follows (for major details see Gebennini et al., 2009):  
the machines may become blocked and starved as in the *Basic Model*, nevertheless the first machine is not allowed to leave the blocking state as soon as the buffer level starts to decrease. Briefly, once the first machine gets blocked it is put into a “controlled idle state” in order to allow the buffer level to diminish. The aim is to reduce the probability of subsequent blocking events that can occur if the first machine resumes production when the buffer level is just below the maximum buffer size. Such a control policy is implemented in several industrial applications, especially in case of costly machine outages. This is the case, for example, of automatic packaging lines where it is important to keep the filling machine working with the minimum of disruption. This model is called *RP Model* in the following.

The Basic Model and the RP Model allow to solve the Markov chains representing the system with or without the restart policy if no waste production is taken into consideration. The states constituting this kind of Markov chain, referred here as the “aggregate” Markov chains, depends only on the buffer level and the condition of the two machines (up or down), without distinguishing the production of good or bad parts.

In the present work, taking the “aggregate” Markov chains of the Basic Model and the RP Model as a starting point, additional information about waste production is provided. The new models are called *WP-Basic Model* and *WP-RP Model*.

In both cases (with or without restart policy), waste production is assumed to be related to specific events that may occur in the system, i.e., the interruption of the process performed by the first machine. Such an interruption can be caused either by an operational failure of the first machine (situation denoted by the subscript “f” in the following) or by a blocking event (situation denoted by the subscript “b” in the following), i.e., the blockage of the outgoing flow due to the buffer being full. When the first machine resumes operation, since either it is repaired or the buffer level decreases, a constant number  $\mathcal{W}$  of bad parts are produced before the good ones. If a further stoppage occurs while the first machine is still producing bad parts, waste production is interrupted.

Given such a more complex system behaviour, the procedure applied in this paper and discussed in details in Section 3.1 and Section 3.2 can be summarised as follows:

- Verify the adoption of a restart policy in the manufacturing line under analysis and refer to the RP Model if the restart policy is applied, to the Basic Model otherwise;
- Evaluate the “aggregate” Markov chain related to the RP Model or the Basic Model without distinguishing between bad and good parts;

- Develop a “disaggregate” Markov chain for considering waste production (with or without restart policy), i.e., a new Markov chain where two more components are added: a first component indicating whether the latest event causing waste production is an operational failure (denoted by the subscript “f”) or a blocking event (denoted by the subscript “b”), and a second component specifying the number of bad parts  $w$  produced since recovery from that latest event, with  $0 \leq w \leq \mathcal{W}$ ;
- Relate the “disaggregate” Markov chain to the “aggregate” Markov chain by identifying overlapping states (e.g. in the “disaggregate” Markov chain, since bad parts can be produced by the first machine only, states with the first machine down have a non-zero probability only if waste production is zero; these states coincide with the corresponding states in the “aggregate” Markov chain).

Briefly, this study first evaluates the “aggregate” Markov chain, easier to be solved by applying the Basic Model or the RP Model, and switches to the “disaggregate” Markov chain only when it is necessary, i.e., when the system is in states where waste can be produced. This procedure is convenient because some of the states of the “disaggregate” Markov chain coincide with those of the “aggregate” Markov chain whose solution is easily provided by the Basic Model and the RP Model. For example, when the first machine is down there is no need to distinguish between bad or good parts.

In the following, after a description of the model assumptions, the basic case without restart policy is addressed in Section 3.1 which describes the *WP*-Basic Model. Then, Section 3.2 discusses how some probabilities change for the *WP*-RP Model when a restart policy is adopted.

## 2.1 Notation and assumptions

The system state is defined as  $(n, \alpha_1, \alpha_2, w)$ , where:

- $n = 0, \dots, N$  is the buffer level, being  $N$  the maximum buffer size;
- $\alpha_i = 0, 1$  is the condition of machine  $i$ , with  $i = 1, 2$ : if  $\alpha_i = 0$  machine  $i$  is down, if  $\alpha_i = 1$  machine  $i$  is operational;
- $w$  indicates the bad part the first machine is processing given that a total of  $\mathcal{W}$  bad parts are expected to be produced as a consequence of the last stoppage:  $w = 0$  if the first machine is processing a good part;  $w = 1, \dots, \mathcal{W}$  if the first machine is processing a bad part (specifically, the bad part  $w$  of the  $\mathcal{W}$  bad parts related to the last stoppage).



The probabilistic model of the system is studied in steady state.

Let  $\mathbf{p}^w(n, \alpha_1, \alpha_2, w)$  be the probability of state  $(n, \alpha_1, \alpha_2, w)$ .

Let  $\bar{\mathbf{p}}(n, \alpha_1, \alpha_2)$  be the probability of a state defined only by the machines' conditions and the buffer level if waste production is not taken into account. We assume that  $\bar{\mathbf{p}}(n, \alpha_1, \alpha_2)$  is given by solving the “aggregate” Markov chain of the aforesaid two-machine systems without waste production, i.e.,  $\bar{\mathbf{p}}(n, \alpha_1, \alpha_2)$  is obtained by the Basic Model in Gershwin (2002) if no restart policy is adopted or by the RP Model in Gebennini et al. (2009) if there is a restart policy.

The main assumptions of the Basic Model and the RP Model without waste production are retained here. For the sake of clarity, the most important ones are briefly recalled in the following:

- If the buffer is full the first machine is said to be *blocked*, if the buffer is empty the second machine is said to be *starved*;
- The two machines have equal and constant processing times and start their operations at the same instant;
- Time is scaled so that processing one part takes one time step;
- Both machines have geometrically distributed times between failures and times to repair: the constant parameters  $p_i$  and  $r_i$  represent the failure and the repair probability of machine  $i$ , with  $i = 1, 2$ ;
- Operational failures are assumed to be operation dependent according to the discussion in Section 1;
- Repairs and operational failures occur at the beginnings of the time steps, and changes in the buffer level take place at the end of the time steps;
- Parts are not destroyed or rejected at any stage in the system.

Some new assumptions regarding waste production are introduced as follows:

- Each stoppage of the first machine, due to either an operational failure or a blocking event, causes waste production at the restart;
- No defects are produced at the second machine;
- The total amount of waste produced per stoppage of the first machine is a constant value  $\mathscr{W}$ . The first bad part is produced as soon as the first machine is repaired or leaves the block condition. Next parts are bad parts until either it completes the batch of  $\mathscr{W}$  bad parts or another

failure/blocking event occurs. In the latter case, the first machine stops producing waste and, when it resumes production, a new batch of  $\mathcal{W}$  bad parts starts to be processed;

- Waste produced by the first machine at each restart moves forward through the buffer and it is processed by the second machine. Thus, waste can be detected only at the end of the line.

## 2.2 Aggregate and disaggregate Markov chains

In the so-called “aggregate” Markov chains (with or without restart policy) the system state depends only on the buffer level and the condition of the two machines (up or down), without distinguishing the production of good or bad parts. The solution of the “aggregate” Markov chains, denoted as  $\bar{\mathbf{p}}(n, \alpha_1, \alpha_2)$ , is supposed to derive from the Basic Model in Gershwin (2002) if no restart policy is adopted or the RP Model in Gebennini et al. (2009) if there is a restart policy.

In the “disaggregate” Markov chains (with or without restart policy) information about the quality of the part under production (“good” or “bad”) is added. The objective of this work is to find the solution of “disaggregate” Markov chains, denoted as  $\mathbf{p}^w(n, \alpha_1, \alpha_2, w)$ , in order to be able to express the performance measures of interest, i.e., waste probability and *effective* efficiency (see Section 4).

Given a certain buffer level  $n$  and machines’ conditions  $\alpha_1, \alpha_2$ , either the system is producing a good part ( $w = 0$ ) or it is producing bad part  $w = 1, \dots, \mathcal{W}$ . Hence, there exists a relationship between the probability  $\mathbf{p}^w(n, \alpha_1, \alpha_2, w)$  and the probability  $\bar{\mathbf{p}}(n, \alpha_1, \alpha_2)$  computed by considering the same line with no distinction between good and bad parts. The relationship is as follows, for  $n = 0, \dots, N$ ,  $\alpha_1, \alpha_2 = 0, 1$  and  $w = 0, \dots, \mathcal{W}$ :

$$\sum_{w=0}^{\mathcal{W}} \mathbf{p}^w(n, \alpha_1, \alpha_2, w) = \bar{\mathbf{p}}(n, \alpha_1, \alpha_2). \quad (1)$$

Moreover, since waste can be produced only by the first machine, if  $\alpha_1 = 0$ ,  $n = 0, \dots, N$  and  $\alpha_2 = 0, 1$  we have:

$$\mathbf{p}^w(n, 0, \alpha_2, 0) = \bar{\mathbf{p}}(n, 0, \alpha_2), \quad (2)$$

$$\mathbf{p}^w(n, 0, \alpha_2, w) = 0, \quad w = 1, \dots, \mathcal{W}. \quad (3)$$

Since waste production is related to two types of events, i.e., operational failures of the first machine and blocking events, the two cases can be treated separately when  $w \neq 0$ . This is convenient when  $\alpha_1 = 1$  (and the first machine is not blocked) since the probabilities of states with  $w \neq 0$  are zero when  $\alpha_1 = 0$ , as stated by equation (3). Thus, for  $n = 0, 1, \dots, N - 1$  (the first machine is not

blocked),  $\alpha_2 = 0, 1$  and  $w \neq 0$ , the probability  $\mathbf{p}^w(n, 1, \alpha_2, w)$  can be expressed as follows:

$$\mathbf{p}^w(n, 1, \alpha_2, w) = \mathbf{p}_f^w(n, 1, \alpha_2, w) + \mathbf{p}_b^w(n, 1, \alpha_2, w), \quad w = 1, \dots, \mathcal{W}, \quad (4)$$

where

- $\mathbf{p}_f^w(n, 1, \alpha_2, w)$  with  $w = 1, \dots, \mathcal{W}$ , is the probability that the first machine is producing bad part  $w$  related to the last operational failure;
- $\mathbf{p}_b^w(n, 1, \alpha_2, w)$  is the probability that the first machine is producing bad part  $w$  related to the last blocking event.

Note that  $\mathbf{p}_f^w(n, 1, \alpha_2, w)$  and  $\mathbf{p}_b^w(n, 1, \alpha_2, w)$  are independent since failures are assumed to be operation-dependent and, consequently, when the first machine is blocked it cannot fail.

If the first machine is blocked, i.e., when  $n = N$  and  $\alpha_2 = 0$ , no part can be processed. We adopt the following convention:

$$\mathbf{p}^w(N, 1, 0, 0) = \bar{\mathbf{p}}(N, 1, 0), \quad (5)$$

$$\mathbf{p}^w(N, 1, 0, w) = 0, \quad w = 1, \dots, W. \quad (6)$$

The probability of producing a good part  $\mathbf{p}^w(n, 1, \alpha_2, 0)$ , for  $n = 0, 1, \dots, N-1$  and  $\alpha_2 = 0, 1$ , can be expressed, according to equation (1), as:

$$\mathbf{p}^w(n, 1, \alpha_2, 0) = \bar{\mathbf{p}}(n, 1, \alpha_2) - \sum_{w=1}^{\mathcal{W}} \mathbf{p}^w(n, 1, \alpha_2, w). \quad (7)$$

In the following section, the WP-Models for addressing the “disaggregate” Markov processes are discussed in detail by distinguishing between the scenarios with and without restart policy.

### 3 Models with waste production (WP- Models)

#### 3.1 The WP-Basic Model

In this case the probabilities  $\bar{\mathbf{p}}(n, \alpha_1, \alpha_2)$ , for  $n = 0, \dots, N$  and  $\alpha_i = 0, 1$  with  $i = 1, 2$ , are given by the Basic Model without waste production described in Gershwin (2002). The “aggregate” Markov chain related to the Basic Model is depicted in Figure 1.

[PUT FIGURE 1 HERE]

In the following the *WP*-Basic Model taking waste production into consideration is discussed. Since waste can be produced only by the first machine, the focus is here on states with  $\alpha_1 = 1$ . All the steady-state probabilities of the *WP*-Basic Model are reported in Appendix A.

### 3.1.1 Operational failures

In this section we address the probability of waste production due to operational failures of the first machine. Thus, we focus on the term  $\mathbf{p}_f^w(n, 1, \alpha_2, w)$ , for  $n = 0, \dots, N-1$ ,  $\alpha_2 = 1, 0$  and  $w = 1, \dots, \mathcal{W}$ . If  $n = N$  the only non-transient state is that with  $\alpha_2 = 0$  and equations (5) and (6) hold.

Let us consider first the case where the first bad part is produced, i.e.,  $w = 1$ . Then, the probabilities for bad part  $w$ , with  $w = 2, \dots, \mathcal{W}$ , are derived.

- *States with  $w = 1$ .*

The first machine can be processing the first bad part of the  $\mathcal{W}$  bad parts related to a previous failure if it has just been repaired, or, in other words, if the first machine was down during the previous time step and a repair has occurred at the beginning of the current time step. Thus, we are interested in transitions from states with  $\alpha_1 = 0$  (and, as a consequence, with  $w = 0$ ) to states with  $\alpha_1 = 1$ .

If  $n = 2, \dots, N-1$ , the possible states where the first machine could be processing bad part  $w = 1$  are states  $(n, 1, 0, 1)$  and  $(n, 1, 1, 1)$ . As shown in Figure 2, the internal state  $(n, 1, 0, 1)$  can be reached only from two states (among all the possible states with  $\alpha_1 = 0$ ), i.e., from state  $(n-1, 0, 0, 0)$ , if the first machine is repaired and the second machine stays down, and state  $(n-1, 0, 1, 0)$ , if the first machine is repaired and the second fails. The internal state  $(n, 1, 1, 1)$  can be reached from state  $(n, 0, 0, 0)$ , if both machines are repaired, and from state  $(n, 0, 1, 0)$ , if the first machine is repaired and the second machine does not fail. This leads to the following equations:

$$\mathbf{p}_f^w(n, 1, 0, 1) = r_1(1 - r_2)\mathbf{p}^w(n-1, 0, 0, 0) + r_1 p_2 \mathbf{p}^w(n-1, 0, 1, 0) \quad n = 2, \dots, N-1, \quad (8)$$

$$\mathbf{p}_f^w(n, 1, 1, 1) = r_1 r_2 \mathbf{p}^w(n, 0, 0, 0) + r_1(1 - p_2)\mathbf{p}^w(n, 0, 1, 0) \quad n = 2, \dots, N-1, \quad (9)$$

where  $\mathbf{p}^w(n, 0, 0, 0)$  and  $\mathbf{p}^w(n, 0, 1, 0)$  are given by equation (2) and supposed to be known, according to the assumption that the probabilities  $\bar{\mathbf{p}}$  are provided by the Basic Model in Gershwin (2002).

[PUT FIGURE 2 HERE]

If  $n = 0, 1$  some states are transient and consequently have zero steady-state probability as explained in Gershwin (2002). The only non-transient state where the first machine could be processing bad part  $w = 1$  is state  $(1, 1, 1, 1)$  that can be reached from states  $(1, 0, 0, 0)$ ,  $(1, 0, 1, 0)$  and  $(0, 0, 1, 0)$ . Thus,

$$\mathbf{p}_f^w(1, 1, 1, 1) = r_1 r_2 \mathbf{p}^w(1, 0, 0, 0) + r_1 (1 - p_2) \mathbf{p}^w(1, 0, 1, 0) + r_1 \mathbf{p}^w(0, 0, 1, 0). \quad (10)$$

- *States with  $w = 2, \dots, \mathscr{W}$ .*

Let us consider the generic bad part  $w$  with  $w = 2, \dots, \mathscr{W}$  produced as a consequence of a previous failure of the first machine.

The possible internal states where the first machine could be processing bad part  $w$  are states  $(n, 1, 0, w)$  and  $(n, 1, 1, w)$ . The system can be in one of these two states in the current time step only if it was in a feasible state producing bad part  $w - 1$  during the previous time step, and no failure of the first machine has occurred. Specifically, state  $(n, 1, 0, w)$  can get from state  $(n - 1, 1, 0, w - 1)$  if the first machine does not fail and the second machine is not repaired; it can get from state  $(n - 1, 1, 1, w - 1)$  if the second machine fails and the first machine does not. The system reaches state  $(n, 1, 1, w)$  from state  $(n, 1, 1, w - 1)$ , if neither of the machines fail, from state  $(n, 1, 0, w - 1)$ , if the first machine does not fail and the second machine is repaired. Thus, we obtain the following equations:

$$\mathbf{p}_f^w(n, 1, 0, w) = (1 - p_1)(1 - r_2) \mathbf{p}_f^w(n - 1, 1, 0, w - 1) + (1 - p_1)p_2 \mathbf{p}_f^w(n - 1, 1, 1, w - 1), \quad (11)$$

$$w = 2, \dots, \mathscr{W}, \quad n = 3, \dots, N - 1,$$

$$\mathbf{p}_f^w(n, 1, 1, w) = (1 - p_1)r_2 \mathbf{p}_f^w(n, 1, 1, w - 1) + (1 - p_1)(1 - p_2) \mathbf{p}_f^w(n, 0, 1, w - 1), \quad (12)$$

$$w = 2, \dots, \mathscr{W}, \quad n = 2, \dots, N - 1.$$

As regards the lower boundary, the non-transient states where the first machine could be processing bad part  $w$  with  $w > 1$  are  $(2, 1, 0, w)$  and  $(1, 1, 1, w)$ . By considering transitions not including failures of the first machine, we have

$$\mathbf{p}_f^w(1, 1, 1, w) = (1 - p_1)(1 - p_2) \mathbf{p}_f^w(1, 1, 1, w - 1), \quad w = 2, \dots, \mathscr{W}, \quad (13)$$

$$\mathbf{p}_f^w(2, 1, 0, w) = (1 - p_1)p_2 \mathbf{p}_f^w(1, 1, 1, w - 1), \quad w = 2, \dots, \mathscr{W}. \quad (14)$$

The remaining states are transient with zero steady-state probability.

### 3.1.2 Blocking events

In this section we address the probability of waste production due to blocking events. Thus, we consider now the term  $\mathbf{p}_b^w(n, 1, \alpha_2, w)$  for  $n = 0, \dots, N-1$ ,  $\alpha_2 = 1, 0$ , and  $w = 1, \dots, \mathcal{W}$ .

When the system leaves state  $(N, 1, 0, 0)$ , defined according to (5), the buffer level decreases to  $N-1$  and the first machine resumes processing a part. If waste production is taken into account, this part is a bad part and, specifically, it is the first bad item of the  $\mathcal{W}$  bad parts related to that restart. Thus, the system gets state  $(N-1, 1, 1, 1)$ .

If the first machine does not fail or get blocked again, it keeps on processing bad parts until it completes the whole batch of  $\mathcal{W}$  bad parts. Otherwise, waste production is interrupted. In other words, the first machine processes bad parts related to a previous blocking event until the system remains in states with  $n = N-1$ . This happens if neither of the machines fails. Thus,

$$\mathbf{p}_b^w(N-1, 1, 1, w) = [(1-p_1)(1-p_2)]^{w-1} r_2 \mathbf{p}^w(N, 1, 0, 0), \quad w = 1, \dots, \mathcal{W}. \quad (15)$$

Since no waste due to blocking events can be produced in states with  $n \neq N-1$ , we have:

$$\mathbf{p}_b^w(n, 1, \alpha_2, w) = 0, \quad n \neq N-1, \quad \alpha_2 = 1, 0, \quad w = 1, \dots, \mathcal{W}. \quad (16)$$

## 3.2 The WP-RP Model

The restart policy described in Gebennini et al. (2009) applies to the first machine each time it gets blocked, i.e., the buffer fills up. Specifically, the first machine is put into the so-called “controlled idle state”, i.e., it is forced to remain idle even when the buffer level starts to drop. The “controlled idle state” persists until the buffer empties again.

In order to model such a transfer line with restart policy, two complementary Markovian behaviors are considered and, consequently, the state space is divided into two partitions that are briefly recalled in the following:

1. the “Standard Operation Partition” includes states where both machines can fail and be repaired according to their own parameters and, as a consequence, the buffer level can fluctuate within the range  $[0; N]$ ; once the buffer level reaches its maximum capacity  $N$ , the first machine gets blocked and the system leaves states belonging to this partition, i.e., it enters the *buffer drainage partition*.

2. the “Buffer Drainage Partition” is entered by the system when, being the buffer at level  $N$ , a repair on the second machine occurs and the buffer level begins to decrease. The first machine is put into the “controlled idle” state: it is prevented from processing parts and it cannot fail. The system leaves states of this partition, i.e., it returns in the *standard operation partition*, when the buffer level drops below the value  $n = 2$  and the second machine is operational. The choice of leaving this partition at  $n = 2$  allows the system to come back into the *standard operation partition* with  $n = 1$ , i.e., with the second machine not starved.

The “aggregate” Markov chain related to the RP Model is depicted in Figure 3.

[PUT FIGURE 3 HERE]

Note that if waste production is taken into consideration according to the assumptions listed in Section 2.2 bad parts can be produced only in states belonging to the Standard Operation Partition.

Similarly as for the Basic Model, waste due to operational failures and to blocking events are discussed separately in the following. The probability of producing a good part is still expressed according to equation (1).

All the steady-state probabilities of the *WP*-Basic Model are summarized in Appendix B.

### 3.2.1 Operational failures

As regards waste due to operational failures, the expressions developed for the case without restart policy are still valid (see Section 3.1.1). Thus,  $\mathbf{p}_f^w(n, 1, \alpha_2, w)$ , can be expressed by equations (2), (3), (5), (6), (8), (9) and (10), for  $n = 0, \dots, N - 1$ ,  $\alpha_2 = 0, 1$  and  $w = 1, \dots, \mathcal{W}$ . In this case the probabilities  $\bar{\mathbf{p}}$  are given by the RP Model by Gebennini et al. (2009) considering the “Standard Operation Partition”.

### 3.2.2 Blocking events

As regards the occurrence of blocking events, a new set of state probabilities has to be formalized.

The first machine produces the first bad part of the  $\mathcal{W}$  bad parts related to the previous blocking event when the system leaves the Buffer Drainage Partition and comes back to the Standard Operation Partition in a state with  $n = 1$  and both the machines operational. As a consequence, if  $w = 1$  the only non-zero probability state is  $(1, 1, 1, 1)$ . Consider now the second bad part, i.e.,  $w = 2$ . The second bad part can be produced in a state with  $\alpha_1 = 1$  that can be reached from  $(1, 1, 1, 1)$  in a time step. The possible states are  $(2, 1, 0, 2)$  and  $(1, 1, 1, 2)$ . If  $w = 3$ , the states with non-zero probabili-

ties are states  $(1, 1, 1, 3)$ ,  $(2, 1, 0, 3)$ ,  $(2, 1, 1, 3)$  and  $(3, 1, 0, 3)$ . A similar reasoning is valid also for  $w > 3$ .

Thus, in case of waste due to blocking events, we obtain that:

- States  $(1, 1, 1, w)$  have a non-zero probability for  $w = 1, \dots, \mathcal{W}$ . Bad part  $w$  is produced when the system leaves state  $(N, 1, 0, 0)$  and no failures occur. Thus,

$$\mathbf{p}_b^w(1, 1, 1, w) = r_2 \mathbf{p}^w(N, 1, 0, 0) [(1 - p_1)(1 - p_2)]^{w-1} \quad w = 1, \dots, \mathcal{W}. \quad (17)$$

- The generic state  $(n, 1, 0, w)$  with  $n = 2, \dots, N - 1$  has a non-zero probability only if  $w \geq n$ . Moreover, if  $\mathcal{W} < N$  bad parts related to a blocking event can be produced only in states with a buffer level  $n \leq \mathcal{W}$ . Otherwise, if  $\mathcal{W} \geq N$  the first machine could get blocked again while it is still producing bad parts. In this case, the current waste production is interrupted and, at the restart, it will start to produce other  $\mathcal{W}$  bad parts. Thus, by setting  $W_1 = \min\{\mathcal{W}; N - 1\}$ , we have that the only non-zero stationary probabilities are as follows:

$$\mathbf{p}_b^w(n, 1, 0, w) = (1 - p_1)(1 - r_2) \mathbf{p}_b^w(n - 1, 1, 0, w - 1) + (1 - p_1)p_2 \mathbf{p}_b^w(n - 1, 1, 1, w - 1), \quad (18)$$

$$n = 2, \dots, W_1, \quad w = n, \dots, W.$$

- The generic state  $(n, 1, 1, w)$  has a non-zero probability only if  $w \geq n + 1$ . Moreover, bad parts related to a blocking event can be produced only in states with either  $n \leq \mathcal{W} - 1$  (if  $\mathcal{W} < N$ ) or  $n \leq N - 1$  (if  $\mathcal{W} \geq N$ ). By setting  $W_2 = \min\{\mathcal{W} - 1; N - 1\}$ , the following expression for the non-zero stationary probabilities can be derived:

$$\mathbf{p}_b^w(n, 1, 0, w) = (1 - p_1)r_2 \mathbf{p}_b^w(n, 1, 0, w - 1) + (1 - p_1)(1 - p_2) \mathbf{p}_b^w(n, 1, 1, w - 1), \quad (19)$$

$$n = 2, \dots, W_2, \quad w = n + 1, \dots, W.$$

- The remaining states not discussed above have zero steady-state probability for  $w \neq 0$ .

## 4 Waste probability and effective efficiency

The approach discussed in Section 3 provides the probability that the first machine is producing either a good part or bad part  $w$  with  $w = 1, \dots, W$ . This is given for both the case without restart



policy (*WP*-Basic Model) and the case with restart policy (*WP*-RP Model).

In both cases, the expression of *waste probability* in a time step, i.e., the probability that the first machine is producing any bad part during a time step, is as follows:

$$P_w = \sum_{n=0}^{N-1} \sum_{w=1}^{\infty} \sum_{\alpha_2=0}^1 \mathbf{p}^w(n, 1, \alpha_2, w). \quad (20)$$

Similarly, the *effective efficiency*  $E_w$ , i.e., the probability that the system produces a good part in a time step, can be expressed as follows:

$$E_w = \sum_{n=0}^{N-1} \sum_{\alpha_2=0}^1 \mathbf{p}^w(n, 1, \alpha_2, 0). \quad (21)$$

Such information is missing in previous models that do not distinguish between bad and good parts. Gershwin (2002) and Gebennini et al. (2009) only present the exact expression of the *total efficiency*, that is denoted as  $E$  for the whole line and as  $E_i$  for machine  $i$  with  $i = 1, 2$ . The *total efficiency*  $E_i$  is the probability that machine  $i$  processes a part (bad or good) during a time step. Since the system is studied in steady state, we have:

$$E_1 = \sum_{\substack{n < N \\ \alpha_1 = 1}} \bar{\mathbf{p}}(n, \alpha_1, \alpha_2), \quad E_2 = \sum_{\substack{n > 0 \\ \alpha_2 = 1}} \bar{\mathbf{p}}(n, \alpha_1, \alpha_2), \quad E = E_1 = E_2, \quad (22)$$

where  $\bar{\mathbf{p}}(n, \alpha_1, \alpha_2)$  are provided by the two-machine models without waste production for  $n = 0, \dots, N$  and  $\alpha_i = 0, 1$  with  $i = 1, 2$ .

Note that the *effective efficiency*  $E_w$  can also be expressed in terms of *total efficiency* and *waste probability*. So,

$$E_w = E - P_w. \quad (23)$$

## 5 Numerical results

The main objective of this paper is to present an analytical method for computing the *effective efficiency*  $E_w$  of a two-machine one-buffer transfer line with good accuracy. This performance measure can be computed whether a restart policy is adopted or not.

An approximate formula for the *effective efficiency* was given in Gebennini et al. (2009) and Gebennini et al. (in press). It is recalled as follows:

$$E_w = E - W(f_b + p_1 E), \quad (24)$$

where  $E$  is the *total* efficiency of the line and  $f_b$  is the blocking frequency of the first machine. Note that this approximate formula is valid only if we assume that the first machine cannot fail or get blocked during waste production. This is because the approximate formula (24) does not take into consideration that waste production, as well as the production of good parts, can be interrupted by a further stoppage of the first machine.

In the sequel, the accuracy estimates are obtained by comparing the *effective* efficiency  $E_w$  resulting from a simulation model programmed in PERL language,  $E_w^{\text{sim}}$ , with the corresponding values deriving from the new analytical models,  $E_w^{\text{ana}}$ , and the approximate formula,  $E_w^{\text{app}}$ . Specifically, for each scenario, 4 simulation runs (each of them 1,000,000 time units long) were generated for both the basic line and the line with restart policy. The objective is to show that the new analytical models with waste production presented in this paper accurately predict the performance of the two-machine one-buffer systems under study, with a better fit to simulation results than the approximate formula.

In this section, we used the same examples as in Gebennini et al. (in press) to show significant results. Table 1 lists the values of the input parameters for the case in which the second machine is more reliable than the first machine (denoted as “case\_1”), and for the opposite case in which the first machine is the most reliable machine (denoted as “case\_2”).

Table 2 and Table 3 summarize the comparison results for “case\_1” and “case\_2”, respectively. In both cases,  $E_w^{\text{ana}}$  is evaluated according to the *WP-Basic Model* for the basic line and according to the *WP-RP Model* for the line with restart policy. It can be noted that the accuracy of the *WP-Basic Model* and the *WP-RP Model* is always acceptable. On the contrary, the approximate formula can give significant errors. This observation is more evident in the basic line, where blocking events have a greater impact (as discussed in Gebennini et al., in press), and when the first machine is less reliable (“case\_1”). In such situations a method that does not take into consideration interruptions of the waste production may lead to significant errors.

Another interesting example is given in Table 4. Table 5 shows that the approximate formula (24) is acceptable only when the failure probability  $p_1$  and the amount of waste per stoppage  $\mathcal{W}$  are small enough. As the amount of waste per stoppage  $\mathcal{W}$  increases, the error with the simulation results also increases. On the other hand, the results from *WP-Basic Model* and the *WP-RP Model* are always consistent with the results of the simulation runs.

Finally, note that the proposed models allow the line designer to decide whether to adopt the restart policy or not. In “case\_1” (see Table 2) the basic line performs better than the line with restart policy when the amount of waste per stoppage is  $\mathcal{W} \leq 4$  or when  $\mathcal{W} \leq 6$  and the buffer size is  $N \geq 60$  parts. On the contrary, as the amount of waste per stoppage increases or the phenomenon of

blocking of the first machine becomes more significant (e.g. when the buffer is small or in “case\_2” where the first machine is the more reliable than the second machine) the adoption of a restart policy becomes convenient. This is not surprising since the main positive effect of the restart policy is to reduce the blocking frequency of the first machine and, as a consequence, its waste production. Finally, note that the line with restart policy performs better than the basic line in most scenarios. This is more evident as  $\mathcal{W}$  increases. This is not surprising since the main positive effect of the restart policy is to reduce the blocking frequency of the first machine and, as a consequence, its waste production. In order to better understand this phenomenon, let us consider the following input parameters: failure/repair probabilities as in Table 4,  $N = 200$  and  $\mathcal{W}$  varying in the range of 0 to 20 bad parts per stop. Figure 4 shows the trend of  $E_w$  for both the basic case (dashed line) and the case with restart policy (solid line). If no waste is produced (i.e.,  $\mathcal{W} = 0$ ) the basic line performs better than the line with restart policy. This is because the total production time of the first machine is longer in the basic line where the first machine is never forced to remain idle (i.e., it never enters the “controlled idle” state). Nevertheless, when  $\mathcal{W} > 0$  the application of a restart policy makes it possible to reduce the blocking frequency of the first machine and, as a consequence, to gain in terms of *effective* efficiency.

[PUT FIGURE 4 HERE]

[PUT TABLE 1 HERE]

[PUT TABLE 2 HERE]

[PUT TABLE 3 HERE]

[PUT TABLE 4 HERE]

[PUT TABLE 5 HERE]

## 6 Conclusions and further research

In this paper, an analytical method for evaluating the *effective* efficiency of a two-machine one-buffer line with waste production is presented. We assume that waste production is related to specific events that may occur in the system. In particular, the first machine produces a certain amount of bad parts each time it resumes processing after any stoppage (due to either an operational failure or a blocking event).

In particular, the work extends the Basic Model presented in Gershwin (2002) and the RP Model in Gebennini et al. (2009) and Gebennini et al. (in press) by distinguishing whether the first machine is producing a good or a bad part. The expressions of the probabilities of producing any bad part are derived in order to compute the waste probability and the *effective* efficiency of the line for both the basic case and the case with restart policy.

Comparisons with simulation results show that the new models offer very good accuracy by overcoming the precision errors of the approximate formula presented in Gebennini et al. (2009).

Future work will be directed towards applying the method to real long lines by including it in decomposition techniques (see Gershwin, 1987; Dallery et al., 1989; Levantesi et al., 2003) based on the evaluation of a series of two-machine one-buffer sub-systems. Moreover, further studies may improve the model, e.g., by avoiding that bad parts enter the buffer, by including an intermediate restart level, by distinguish different sizes of bad part batches. Finally, it would be interesting to investigate extensions of the proposed approach to different behaviours related to other types of events that may occur in the system, e.g. production lines with order-selection and switch-over.

## A Steady-State Probabilities - The WP-Basic Model

$$\mathbf{p}^{\mathbf{w}}(n, 0, 0, w) = \begin{cases} 0 & \text{if } w > 0 \\ \bar{\mathbf{p}}(n, 0, 0) & \text{if } w = 0 \end{cases}, \quad n = 0, \dots, N, \quad (25)$$

$$\mathbf{p}^{\mathbf{w}}(n, 0, 1, w) = \begin{cases} 0 & \text{if } w > 0 \\ \bar{\mathbf{p}}(n, 0, 1) & \text{if } w = 0 \end{cases}, \quad n = 0, \dots, N, \quad (26)$$

$$\mathbf{p}^{\mathbf{w}}(n, 1, 0, w) = \begin{cases} \mathbf{p}_{\mathbf{f}}^{\mathbf{w}}(n, 1, 0, w) + \mathbf{p}_{\mathbf{b}}^{\mathbf{w}}(n, 1, 0, w) & \text{if } w > 0 \\ \bar{\mathbf{p}}(n, 1, 0) - \sum_{w=1}^W (\mathbf{p}_{\mathbf{f}}^{\mathbf{w}}(n, 1, 0, w) + \mathbf{p}_{\mathbf{b}}^{\mathbf{w}}(n, 1, 0, w)) & \text{if } w = 0 \end{cases}, \quad n = 0, \dots, N, \quad (27)$$

$$\mathbf{p}^{\mathbf{w}}(n, 1, 1, w) = \begin{cases} \mathbf{p}_{\mathbf{f}}^{\mathbf{w}}(n, 1, 1, w) + \mathbf{p}_{\mathbf{b}}^{\mathbf{w}}(n, 1, 1, w) & \text{if } w > 0 \\ \bar{\mathbf{p}}(n, 1, 1) - \sum_{w=1}^W (\mathbf{p}_{\mathbf{f}}^{\mathbf{w}}(n, 1, 1, w) + \mathbf{p}_{\mathbf{b}}^{\mathbf{w}}(n, 1, 1, w)) & \text{if } w = 0 \end{cases}, \quad n = 0, \dots, N, \quad (28)$$

where the probabilities  $\bar{\mathbf{p}}(n, \alpha_1, \alpha_2)$  are given by the model without waste production (refer to Gershwin, 2002), and where

- waste probabilities related to failures are

$$\mathbf{p}_{\mathbf{f}}^{\mathbf{w}}(0, 1, 0, w) = 0, \quad w = 1, \dots, \mathcal{W}, \quad (29)$$

$$\mathbf{p}_{\mathbf{f}}^{\mathbf{w}}(0, 1, 1, w) = 0, \quad w = 1, \dots, \mathcal{W}, \quad (30)$$

$$\mathbf{p}_{\mathbf{f}}^{\mathbf{w}}(1, 1, 0, w) = 0, \quad w = 1, \dots, \mathcal{W}, \quad (31)$$

$$\mathbf{p}_{\mathbf{f}}^{\mathbf{w}}(1, 1, 1, w) = \begin{cases} r_1 \mathbf{p}^{\mathbf{w}}(0, 0, 1, 0) + r_1 r_2 \mathbf{p}^{\mathbf{w}}(1, 0, 0, 0) + r_1 (1 - p_2) \mathbf{p}^{\mathbf{w}}(1, 0, 1, 0) & \text{if } w = 1 \\ (1 - p_1)(1 - p_2) \mathbf{p}_{\mathbf{f}}^{\mathbf{w}}(1, 1, 1, w - 1) & \text{if } w > 1 \end{cases}, \quad (32)$$

$$\mathbf{p}_{\mathbf{f}}^{\mathbf{w}}(2, 1, 0, w) = \begin{cases} r_1 (1 - r_2) \mathbf{p}^{\mathbf{w}}(1, 0, 0, 0) + r_1 p_2 \mathbf{p}^{\mathbf{w}}(1, 0, 1, 0) & \text{if } w = 1 \\ (1 - p_1) p_2 \mathbf{p}_{\mathbf{f}}^{\mathbf{w}}(1, 1, 1, w - 1) & \text{if } w > 1 \end{cases}, \quad (33)$$

$$\mathbf{p}_{\mathbf{f}}^{\mathbf{w}}(n, 1, 0, w) = \begin{cases} r_1 (1 - r_2) \mathbf{p}^{\mathbf{w}}(n - 1, 0, 0, 0) + r_1 p_2 \mathbf{p}^{\mathbf{w}}(n - 1, 0, 1, 0) & \text{if } w = 1 \\ (1 - p_1)(1 - r_2) \mathbf{p}_{\mathbf{f}}^{\mathbf{w}}(n - 1, 1, 0, w - 1) + (1 - p_1) p_2 \mathbf{p}_{\mathbf{f}}^{\mathbf{w}}(n - 1, 1, 1, w - 1) & \text{if } w > 1 \end{cases}, \quad (34)$$

$$n = 3, \dots, N-1,$$

$$\mathbf{p}_f^w(n, 1, 1, w) = \begin{cases} r_1 r_2 \mathbf{p}^w(n, 0, 0, 0) + r_1 (1 - p_2) \mathbf{p}^w(n, 0, 1, 0) & \text{if } w = 1 \\ (1 - p_1) r_2 \mathbf{p}_f^w(n, 1, 0, w-1) + (1 - p_1)(1 - p_2) \mathbf{p}_f^w(n, 1, 1, w-1) & \text{if } w > 1 \end{cases}, \quad (35)$$

$$n = 2, \dots, N-1,$$

$$\mathbf{p}_f^w(N, 1, 0, w) = 0, \quad w = 1, \dots, \mathcal{W}, \quad (36)$$

$$\mathbf{p}_f^w(N, 1, 1, w) = 0, \quad w = 1, \dots, \mathcal{W}, \quad (37)$$

- waste probabilities related to blocking events are

$$\mathbf{p}_b^w(n, 1, 0, w) = 0 \quad w = 1, \dots, \mathcal{W}, \quad n = 0, \dots, N, \quad (38)$$

$$\mathbf{p}_b^w(n, 1, 1, w) = 0, \quad w = 1, \dots, \mathcal{W}, \quad n = 0, \dots, N-2, \quad (39)$$

$$\mathbf{p}_b^w(N-1, 1, 1, w) = \begin{cases} r_2 \mathbf{p}^w(N, 1, 0, 0) & \text{if } w = 1 \\ (1 - p_1)(1 - p_2) \mathbf{p}_b^w(N-1, 1, 1, w-1) & \text{if } w > 1 \end{cases}, \quad (40)$$

$$\mathbf{p}_b^w(N, 1, 1, w) = 0, \quad w = 1, \dots, \mathcal{W}. \quad (41)$$

## B Steady-State Probabilities - The WP-RP Model

$$\mathbf{p}^w(n, 0, 0, w) = \begin{cases} 0 & \text{if } w > 0 \\ \bar{\mathbf{p}}(n, 0, 0) & \text{if } w = 0 \end{cases}, \quad n = 0, \dots, N, \quad (42)$$

$$\mathbf{p}^w(n, 0, 1, w) = \begin{cases} 0 & \text{if } w > 0 \\ \bar{\mathbf{p}}(n, 0, 1) & \text{if } w = 0 \end{cases}, \quad n = 0, \dots, N, \quad (43)$$

$$\mathbf{p}^w(n, 1, 0, w) = \begin{cases} \mathbf{p}_f^w(n, 1, 0, w) + \mathbf{p}_b^w(n, 1, 0, w) & \text{if } w > 0 \\ \bar{\mathbf{p}}(n, 1, 0) - \sum_{w=1}^W (\mathbf{p}_f^w(n, 1, 0, w) + \mathbf{p}_b^w(n, 1, 0, w)) & \text{if } w = 0 \end{cases}, \quad n = 0, \dots, N, \quad (44)$$

$$\mathbf{p}^w(n, 1, 1, w) = \begin{cases} \mathbf{p}_f^w(n, 1, 1, w) + \mathbf{p}_b^w(n, 1, 1, w) & \text{if } w > 0 \\ \bar{\mathbf{p}}(n, 1, 1) - \sum_{w=1}^W (\mathbf{p}_f^w(n, 1, 1, w) + \mathbf{p}_b^w(n, 1, 1, w)) & \text{if } w = 0 \end{cases}, \quad n = 0, \dots, N, \quad (45)$$

where the probabilities  $\bar{\mathbf{p}}(n, \alpha_1, \alpha_2)$  are given by the RP Model without waste production (refer to Gebennini et al., 2009) considering the Standard Operation Partition (where the first machine is not in the “controlled idle state”), and where

- waste probabilities related to failures are

$$\mathbf{p}_f^w(0, 1, 0, w) = 0, \quad w = 1, \dots, \mathcal{W}, \quad (46)$$

$$\mathbf{p}_f^w(0, 1, 1, w) = 0, \quad w = 1, \dots, \mathcal{W}, \quad (47)$$

$$\mathbf{p}_f^w(1, 1, 0, w) = 0, \quad w = 1, \dots, \mathcal{W}, \quad (48)$$

$$\mathbf{p}_f^w(1, 1, 1, w) = \begin{cases} r_1 \mathbf{p}^w(0, 0, 1, 0) + r_1 r_2 \mathbf{p}^w(1, 0, 0, 0) + r_1 (1 - p_2) \mathbf{p}^w(1, 0, 1, 0) & \text{if } w = 1 \\ (1 - p_1)(1 - p_2) \mathbf{p}_f^w(1, 1, 1, w - 1) & \text{if } w > 1 \end{cases}, \quad (49)$$

$$\mathbf{p}_f^w(2, 1, 0, w) = \begin{cases} r_1 (1 - r_2) \mathbf{p}^w(1, 0, 0, 0) + r_1 p_2 \mathbf{p}^w(1, 0, 1, 0) & \text{if } w = 1 \\ (1 - p_1) p_2 \mathbf{p}_f^w(1, 1, 1, w - 1) & \text{if } w > 1 \end{cases}, \quad (50)$$

$$\mathbf{p}_f^w(n, 1, 0, w) = \begin{cases} r_1 (1 - r_2) \mathbf{p}^w(n - 1, 0, 0, 0) + r_1 p_2 \mathbf{p}^w(n - 1, 0, 1, 0) & \text{if } w = 1 \\ (1 - p_1)(1 - r_2) \mathbf{p}_f^w(n - 1, 1, 0, w - 1) + (1 - p_1) p_2 \mathbf{p}_f^w(n - 1, 1, 1, w - 1) & \text{if } w > 1 \end{cases}, \quad (51)$$

$$n = 3, \dots, N - 1,$$

$$\mathbf{p}_f^w(n, 1, 1, w) = \begin{cases} r_1 r_2 \mathbf{p}^w(n, 0, 0, 0) + r_1 (1 - p_2) \mathbf{p}^w(n, 0, 1, 0) & \text{if } w = 1 \\ (1 - p_1) r_2 \mathbf{p}_f^w(n, 1, 0, w - 1) + (1 - p_1)(1 - p_2) \mathbf{p}_f^w(n, 1, 1, w - 1) & \text{if } w > 1 \end{cases}, \quad (52)$$

$$n = 2, \dots, N - 1,$$

$$\mathbf{p}_f^w(N, 1, 0, w) = 0, \quad w = 1, \dots, \mathcal{W}, \quad (53)$$

$$\mathbf{p}_f^w(N, 1, 1, w) = 0, \quad w = 1, \dots, \mathcal{W}, \quad (54)$$

- waste probabilities related to blocking events are

$$\mathbf{p}_b^w(0, 1, 0, w) = \mathbf{p}_b^w(1, 1, 0, w) = 0, \quad w = 1, \dots, \mathcal{W}, \quad (55)$$

$$\mathbf{p}_b^w(1, 1, 1, w) = r_2 \mathbf{p}^w(N, 1, 0, 0) [(1 - p_1)(1 - p_2)]^{w-1}, \quad w = 1, \dots, \mathcal{W}, \quad (56)$$

$$\mathbf{p}_{\mathbf{b}}^{\mathbf{w}}(n, 1, 0, w) = \begin{cases} (1 - p_1)(1 - r_2)\mathbf{p}_{\mathbf{b}}^{\mathbf{w}}(n, 1, 0, w - 1) + (1 - p_1)p_2\mathbf{p}_{\mathbf{b}}^{\mathbf{w}}(n - 1, 1, 1, w - 1), & \text{if } w \geq n \\ 0 & \text{if } w < n \end{cases},$$

(57)

$$n = 1, \dots, W_1,$$

$$\mathbf{p}_{\mathbf{b}}^{\mathbf{w}}(n, 1, 0, w) = 0, \quad n = W_1 + 1, \dots, N,$$

(58)

$$\mathbf{p}_{\mathbf{b}}^{\mathbf{w}}(n, 1, 1, w) = \begin{cases} (1 - p_1)r_2\mathbf{p}_{\mathbf{b}}^{\mathbf{w}}(n, 1, 0, w - 1) + (1 - p_1)(1 - p_2)\mathbf{p}_{\mathbf{b}}^{\mathbf{w}}(n - 1, 1, 1, w - 1), & \text{if } w > n \\ 0 & \text{if } w \leq n \end{cases},$$

(59)

$$n = 1, \dots, W_2,$$

$$\mathbf{p}_{\mathbf{b}}^{\mathbf{w}}(n, 1, 1, w) = 0, \quad n = W_2 + 1, \dots, N,$$

(60)



## References

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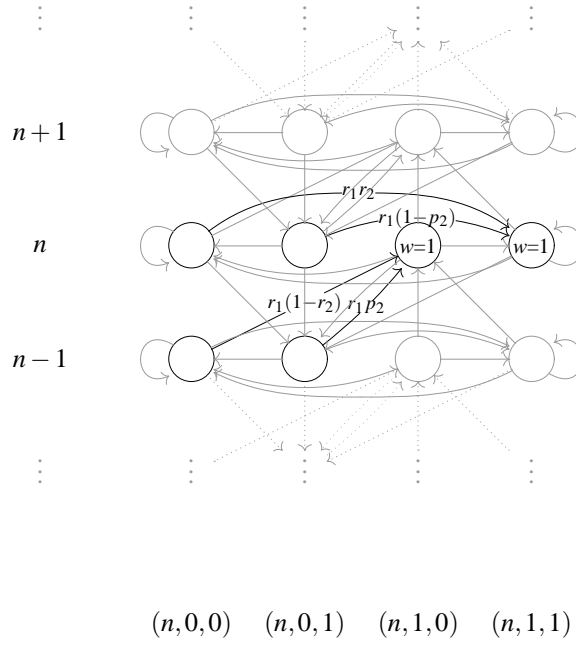


Figure 2: *WP*-Basic Model: how to get state  $(n, 1, 0, 1)$ .

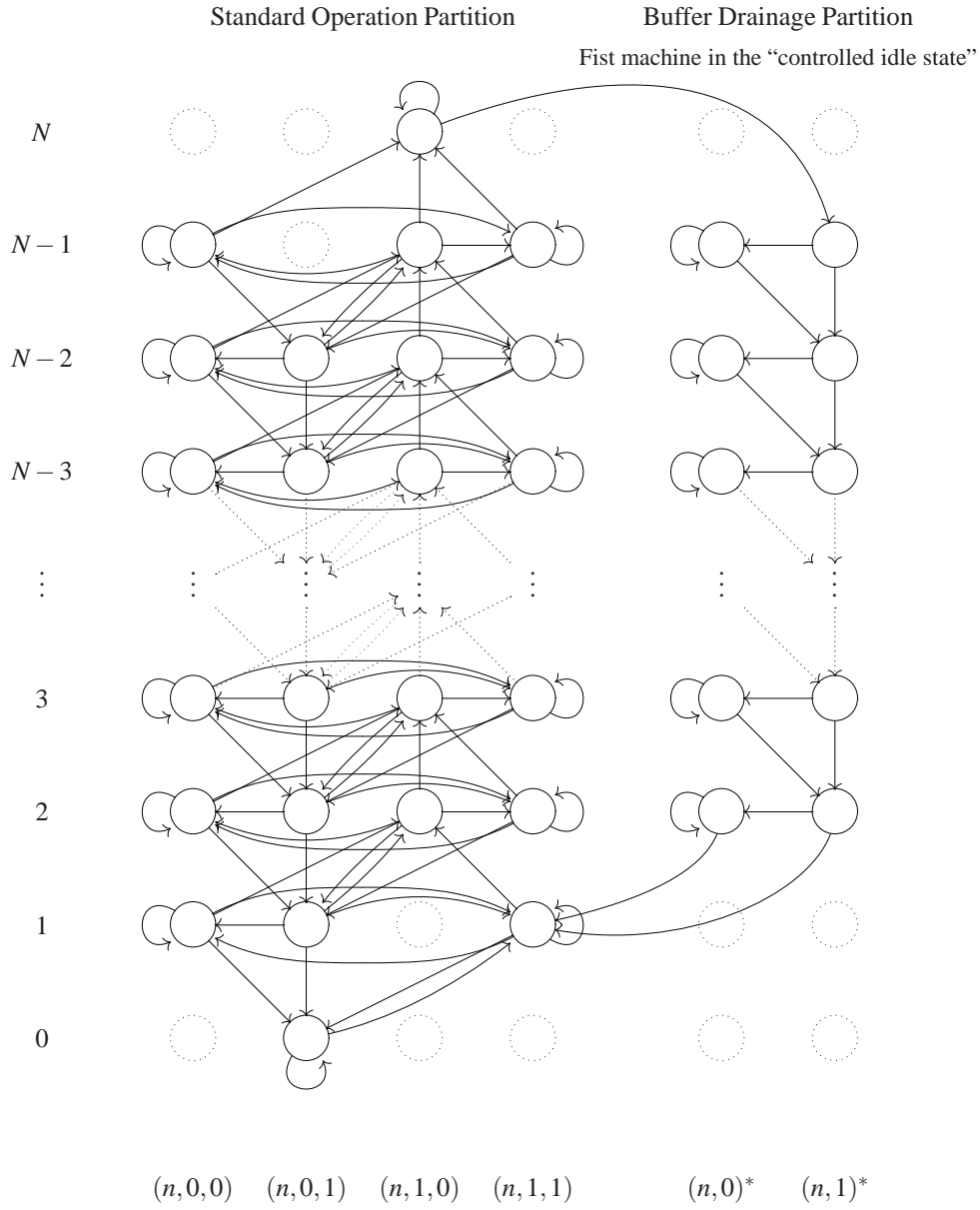


Figure 3: RP Model: Markov chain.

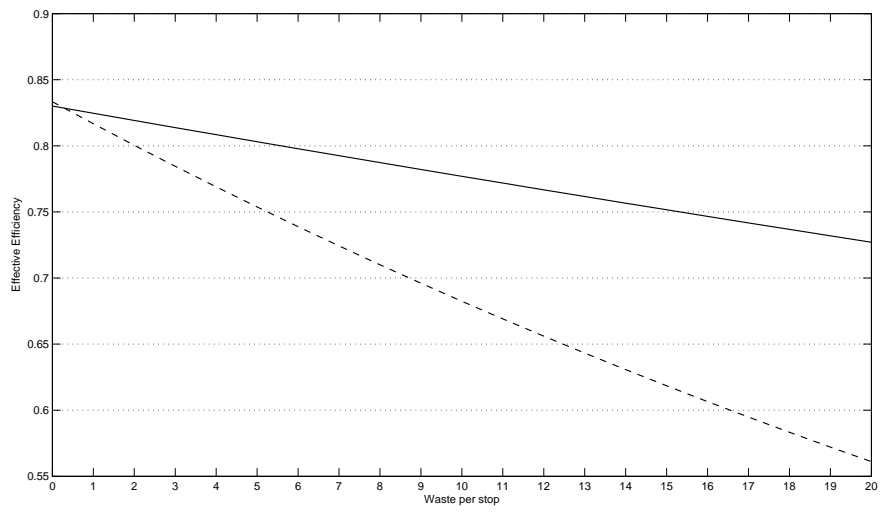


Figure 4: Basic line (dashed) vs. line with restart policy (solid).

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Parameter	case_1	case_2
$p_1$	0.06	0.03
$p_2$	0.05	0.05
$r_1$	0.2	0.2
$r_2$	0.2	0.2
$N$	[20;40;60;80]	[20;40;60;80]
$\mathcal{W}$	[2;4;6;8;10]	[2;4;6;8;10]

Table 1: Input parameters for “case\_1” and “case\_2”.

Scenario		Basic Line			Line with Restart Policy		
$N$	$\mathcal{W}$	$E_w^{\text{sim}}$	$E_w^{\text{ana}}$	$E_w^{\text{app}}$	$E_w^{\text{sim}}$	$E_w^{\text{ana}}$	$E_w^{\text{app}}$
100	2	0.677	0.677	0.674	0.674	0.674	0.671
	4	0.598	0.598	0.581	0.595	0.596	0.58
	6	0.527	0.527	0.488	0.526	0.526	0.488
	8	0.465	0.465	0.395	0.465	0.465	0.396
	10	0.411	0.411	0.302	0.410	0.411	0.305
80	2	0.675	0.675	0.672	0.671	0.671	0.668
	4	0.595	0.595	0.578	0.592	0.593	0.577
	6	0.525	0.525	0.485	0.523	0.523	0.485
	8	0.463	0.463	0.391	0.462	0.462	0.394
	10	0.408	0.408	0.297	0.408	0.408	0.303
60	2	0.671	0.671	0.668	0.665	0.665	0.663
	4	0.591	0.590	0.573	0.587	0.587	0.572
	6	0.520	0.520	0.478	0.519	0.519	0.481
	8	0.457	0.457	0.384	0.458	0.458	0.39
	10	0.402	0.402	0.289	0.404	0.404	0.299
40	2	0.663	0.663	0.659	0.655	0.655	0.652
	4	0.581	0.580	0.562	0.577	0.577	0.561
	6	0.509	0.509	0.465	0.509	0.509	0.471
	8	0.446	0.446	0.367	0.449	0.449	0.381
	10	0.400	0.391	0.270	0.395	0.396	0.29
20	2	0.639	0.639	0.635	0.629	0.629	0.626
	4	0.553	0.553	0.531	0.551	0.551	0.535
	6	0.479	0.478	0.426	0.483	0.483	0.444
	8	0.414	0.413	0.322	0.423	0.423	0.353
	10	0.358	0.357	0.218	0.370	0.370	0.262

Table 2: Comparison results - “case\_1”:  $p_1 = 0.06$ ,  $p_2 = 0.05$ ,  $r_1 = r_2 = 0.2$ .

Scenario		Basic Line			Line with Restart Policy		
$N$	$\mathcal{W}$	$E_w^{\text{sim}}$	$E_w^{\text{ana}}$	$E_w^{\text{app}}$	$E_w^{\text{sim}}$	$E_w^{\text{ana}}$	$E_w^{\text{app}}$
100	2	0.723	0.723	0.720	0.744	0.745	0.744
	4	0.654	0.653	0.640	0.699	0.699	0.695
	6	0.593	0.591	0.560	0.656	0.657	0.646
	8	0.537	0.536	0.480	0.616	0.617	0.597
	10	0.488	0.486	0.400	0.579	0.579	0.548
80	2	0.723	0.722	0.720	0.742	0.742	0.742
	4	0.654	0.653	0.640	0.697	0.697	0.692
	6	0.592	0.591	0.560	0.654	0.654	0.643
	8	0.537	0.536	0.480	0.614	0.614	0.594
	10	0.487	0.486	0.399	0.576	0.576	0.545
60	2	0.722	0.722	0.719	0.738	0.739	0.738
	4	0.653	0.652	0.639	0.693	0.693	0.688
	6	0.591	0.590	0.558	0.649	0.649	0.638
	8	0.536	0.534	0.478	0.609	0.609	0.589
	10	0.486	0.484	0.398	0.571	0.571	0.539
40	2	0.719	0.719	0.716	0.731	0.731	0.73
	4	0.649	0.649	0.635	0.684	0.684	0.679
	6	0.587	0.586	0.554	0.640	0.640	0.629
	8	0.531	0.530	0.472	0.599	0.599	0.578
	10	0.481	0.479	0.391	0.560	0.560	0.527
20	2	0.705	0.704	0.701	0.711	0.711	0.71
	4	0.632	0.631	0.616	0.660	0.660	0.655
	6	0.568	0.566	0.531	0.613	0.613	0.6
	8	0.510	0.508	0.446	0.570	0.569	0.546
	10	0.458	0.457	0.360	0.528	0.528	0.491

Table 3: Comparison results - “case\_2”:  $p_1 = 0.03$ ,  $p_2 = 0.05$ ,  $r_1 = r_2 = 0.2$ .

Parameter	Value
$p_1$	0.006
$p_2$	0.02
$r_1$	0.1
$r_2$	0.1
$N$	[20; 40; 60; 80]
$\mathcal{W}$	[2; 4; 6; 8; 10]

Table 4: Input parameters (case with low failure probability).

Scenario		Basic Line			Line with Restart Policy		
$N$	$\mathcal{W}$	$E_w^{\text{sim}}$	$E_w^{\text{ana}}$	$E_w^{\text{app}}$	$E_w^{\text{sim}}$	$E_w^{\text{ana}}$	$E_w^{\text{app}}$
100	2	0.800	0.800	0.800	0.815	0.815	0.815
	4	0.769	0.769	0.767	0.803	0.803	0.803
	6	0.739	0.739	0.733	0.792	0.792	0.791
	8	0.710	0.710	0.700	0.780	0.780	0.779
	10	0.682	0.682	0.667	0.769	0.769	0.768
80	2	0.800	0.800	0.800	0.813	0.813	0.813
	4	0.769	0.769	0.766	0.801	0.801	0.801
	6	0.739	0.739	0.733	0.789	0.789	0.788
	8	0.710	0.710	0.700	0.777	0.777	0.776
	10	0.682	0.682	0.666	0.765	0.765	0.764
60	2	0.800	0.800	0.799	0.810	0.810	0.81
	4	0.768	0.768	0.766	0.797	0.797	0.797
	6	0.738	0.738	0.732	0.784	0.784	0.783
	8	0.709	0.709	0.699	0.771	0.772	0.77
	10	0.681	0.681	0.665	0.759	0.759	0.757
40	2	0.797	0.797	0.797	0.804	0.804	0.804
	4	0.765	0.765	0.763	0.789	0.789	0.789
	6	0.735	0.735	0.729	0.775	0.775	0.774
	8	0.706	0.706	0.695	0.761	0.761	0.76
	10	0.678	0.678	0.662	0.747	0.748	0.745
20	2	0.789	0.789	0.788	0.790	0.790	0.79
	4	0.755	0.755	0.753	0.772	0.772	0.772
	6	0.724	0.723	0.717	0.754	0.754	0.753
	8	0.693	0.693	0.682	0.736	0.736	0.734
	10	0.664	0.664	0.647	0.719	0.719	0.715

Table 5: Comparison results -  $p_1 = 0.006$ ,  $p_2 = 0.02$ ,  $r_1 = r_2 = 0.1$ .