NONLINEAR VIBRATIONS OF FUNCTIONALLY GRADED CYLINDRICAL SHELLS: EFFECT OF COMPANION MODE PARTICIPATION

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In this paper, the effect of the companion mode participation on the nonlinear vibrations of functionally graded (FGM) cylindrical shells is analyzed. The Sanders-Koiter theory is applied to model the nonlinear dynamics of the system in the case of finite amplitude of vibration. The shell deformation is described in terms of longitudinal, circumferential and radial displacement fields. Simply supported boundary conditions are considered. The displacement fields are expanded by means of a double mixed series based on Chebyshev polynomials for the longitudinal variable and harmonic functions for the circumferential variable. Both driven and companion modes are considered. Numerical analyses are carried out in order to characterize the nonlinear response when the shell is subjected to an harmonic external load. A convergence analysis is carried out by considering a different number of axisymmetric and asymmetric modes. The present study is focused on modelling the nonlinear travelling-wave response of the shell in the circumferential direction with the companion mode participation.

1. Introduction

Functionally graded materials (FGMs) are composite materials obtained by combining and mixing two or more different constituent materials, which are distributed along the thickness in accordance with a volume fraction law.

The idea of FGMs was first introduced in 1984/87 by a group of Japanese material scientists [1]. They studied many different physical aspects such as temperature and thermal stress distributions, static and dynamic responses.

Loy et al. [2] analyzed the vibrations of the circular cylindrical shells made of FGM, considering simply supported boundary conditions. They found that the natural frequencies are affected by the constituent volume fractions and configurations of the materials.

Leissa [3] analyzed the linear dynamics of shells having different topologies, materials and boundary conditions, considering the most important shell theories, such as Donnell, Flugge and Sanders-Koiter.

A modern treatise on the shells dynamics and stability can be found in Ref. [4], where also FGMs are considered.

Pellicano et al. [5] considered the nonlinear vibrations of homogeneous isotropic shells with companion mode participation.

The method of solution used in the present work was presented in Ref. [6].

In this paper, the effect of the companion mode participation on the nonlinear vibrations of FGM cylindrical shells is analyzed. The Sanders-Koiter theory is applied to model the nonlinear dynamics of the system in the case of finite amplitude of vibration.

The shell deformation is described in terms of longitudinal, circumferential and radial displacement fields. Simply supported boundary conditions are considered.

The FGM is made of a uniform distribution of stainless steel and nickel, and the material properties are graded in the thickness direction, according to a volume fraction power-law distribution.

The solution method consists of two steps: 1) linear analysis and eigenfunctions evaluation; 2) nonlinear analysis, using an eigenfunction based expansion.

In the linear analysis, the displacement fields are expanded by means of a double series based on harmonic functions for the circumferential variable and Chebyshev polynomials for the longitudinal variable. A Ritz based method allows to obtain the approximate natural frequencies and mode shapes (eigenvalues and eigenvectors).

In the nonlinear analysis, the three displacement fields are re-expanded by using the approximate eigenfunctions. An energy approach based on the Lagrange equations is considered in order to reduce the nonlinear partial differential equations to a set of nonlinear ordinary differential equations.

Numerical analyses are carried out in order to characterize the nonlinear response when the shell is subjected to a harmonic external load.

A convergence analysis is carried out to obtain the correct number of axisymmetric and asymmetric modes able to describe the actual nonlinear behaviour of the shell.

Nonlinear amplitude-frequency curves with the companion mode participation are carried out; the time histories of the driven and companion modes are analyzed.

2. Equations of functionally graded materials

A generic material property P_{fgm} of an FGM depends on the material properties and the volume fractions of the constituent materials, and it is expressed in the form [2]

$$P_{fgm}(T,z) = \sum_{i=1}^{k} \widetilde{P_i}(T) V_{fi}(z) \tag{1}$$

where $\widetilde{P_i}$ and V_{fi} are the material property and volume fraction of the constituent material i.

The material property $\widetilde{P_i}$ of a constituent material can be described as a function of the temperature T(K) by Touloukian's relation (the index i is dropped for the sake of simplicity) [2]

$$\tilde{P}(T) = P_0(P_{-1}T^{-1} + 1 + P_1 T + P_2 T^2 + P_3 T^3) \tag{2}$$

where P_0, P_{-1}, P_1, P_2 and P_3 are the coefficients of temperature of the constituent material.

In the case of an FGM thin cylindrical shell with a uniform thickness h and a reference surface at its middle surface, the volume fraction V_f of a constituent material can be written as [2]

$$V_f(z) = \left(\frac{z + h/2}{h}\right)^p \tag{3}$$

where the power-law exponent p is a positive real number, $(0 \le p \le \infty)$, and z describes the radial distance measured from the middle surface of the shell, $(-h/2 \le z \le h/2)$, see Fig. 1.

Young's modulus E, Poisson's ratio ν and mass density ρ are expressed as [2]

$$E_{fgm}(T,z) = \big(E_2(T) - E_1(T)\big)\left(\frac{z + h/2}{h}\right)^p + E_1(T) \tag{4}$$

$$\nu_{fgm}(T,z) = \big(\nu_2(T) - \nu_1(T)\big)\left(\frac{z + h/2}{h}\right)^p + \nu_1(T) \tag{5}$$

$$\rho_{fgm}(T,z) = \big(\rho_2(T) - \rho_1(T)\big)\left(\frac{z + h/2}{h}\right)^p + \rho_1(T) \tag{6}$$

3. Sanders-Koiter theory of circular cylindrical shells

In Figure 1, an FGM circular cylindrical shell having radius R, length L and thickness h is represented; a cylindrical coordinate system $(O; x, \theta, z)$ is considered in order to take advantage from the axial symmetry of the structure, the origin O of the reference system is located at the centre of one end of the shell. Three displacement fields are considered: longitudinal $u(x, \theta, t)$, circumferential $v(x, \theta, t)$ and radial $w(x, \theta, t)$.

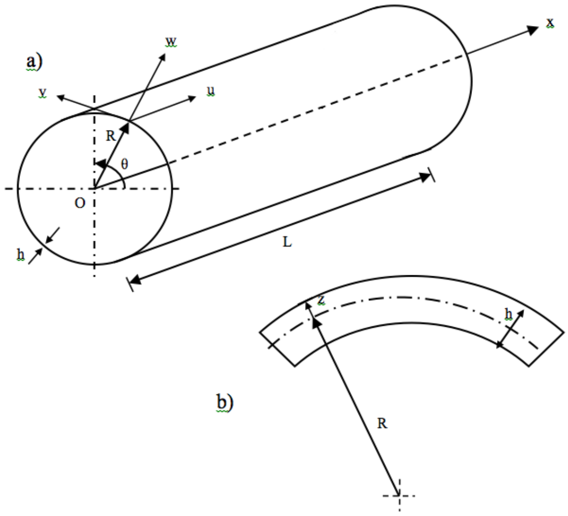

Figure 1. Geometry of the cylindrical shell. a) Complete shell; b) cross-section of the shell surface.

3.1 Elastic strain energy, kinetic energy, virtual work, damping forces

The Sanders-Koiter nonlinear theory of circular cylindrical shells, which is an eight-order shell theory, is based on the Love's "first approximation" [3]. The strain components $(\varepsilon_x, \varepsilon_\theta, \gamma_{x\theta})$ at an arbitrary point of the shell are related to the middle surface strains $(\varepsilon_{x,0}, \varepsilon_{\theta,0}, \gamma_{x\theta,0})$ and to the changes in the curvature and torsion $(k_x, k_\theta, k_{x\theta})$ of the middle surface of the shell by the following relationships [3]

$$\varepsilon_x = \varepsilon_{x,0} + zk_x \qquad \varepsilon_\theta = \varepsilon_{\theta,0} + zk_\theta \qquad \gamma_{x\theta} = \gamma_{x\theta,0} + zk_{x\theta} \tag{7}$$

where z is the distance of the arbitrary point of the cylindrical shell from the middle surface and (x, θ) are the longitudinal and angular coordinates of the shell, see Fig. 1.

The middle surface strains and changes in curvature and torsion are given by [3]

$$\varepsilon_{x,0} = \frac{\partial u}{L\partial\eta} + \frac{1}{2}\left(\frac{\partial w}{L\partial\eta}\right)^2 + \frac{1}{8}\left(\frac{\partial v}{L\partial\eta} - \frac{\partial u}{R\partial\theta}\right)^2 + \frac{\partial w}{L\partial\eta}\frac{\partial w_0}{L\partial\eta}$$

$$\varepsilon_{\theta,0} = \frac{\partial v}{R\partial\theta} + \frac{w}{R} + \frac{1}{2}\left(\frac{\partial w}{R\partial\theta} - \frac{v}{R}\right)^2 + \frac{1}{8}\left(\frac{\partial u}{R\partial\theta} - \frac{\partial v}{L\partial\eta}\right)^2 + \frac{\partial w_0}{R\partial\theta}\left(\frac{\partial w}{R\partial\theta} - \frac{v}{R}\right)$$

$$\gamma_{x\theta,0} = \frac{\partial u}{R\partial\theta} + \frac{\partial v}{L\partial\eta} + \frac{\partial w}{L\partial\eta}\left(\frac{\partial w}{R\partial\theta} - \frac{v}{R}\right) + \frac{\partial w_0}{L\partial\eta}\left(\frac{\partial w}{R\partial\theta} - \frac{v}{R}\right) + \frac{\partial w}{L\partial\eta}\frac{\partial w_0}{R\partial\theta}$$

$$k_x = -\frac{\partial^2 w}{L^2\partial\eta^2} \quad k_\theta = \frac{\partial v}{R^2\partial\theta} - \frac{\partial^2 w}{R^2\partial\theta^2} \quad k_{x\theta} = -2\frac{\partial^2 w}{LR\partial\eta\partial\theta} + \frac{1}{2R}\left(3\frac{\partial v}{L\partial\eta} - \frac{\partial u}{R\partial\theta}\right) \tag{8}$$

where $(\eta = x/L)$ is the nondimensional longitudinal coordinate.

In the case of FGMs, the stresses are related to the strains as follows [4]

$$\sigma_x = \frac{E(z)}{1 - \nu^2(z)}(\varepsilon_x + \nu(z)\varepsilon_\theta) \;\; \sigma_\theta = \frac{E(z)}{1 - \nu^2(z)}(\varepsilon_\theta + \nu(z)\varepsilon_x) \;\; \tau_{x\theta} = \frac{E(z)}{2(1 + \nu(z))}\gamma_{x\theta} \tag{9}$$

where $E(z)$ is the Young's modulus and $\nu(z)$ is the Poisson's ratio ($\sigma_z = 0$, plane stress).

The elastic strain energy U_s of a cylindrical shell is given by [4]

$$U_s = \frac{1}{2}LR\int_0^1\int_0^{2\pi}\int_{-h/2}^{h/2}(\sigma_x\varepsilon_x + \sigma_\theta\varepsilon_\theta + \tau_{x\theta}\gamma_{x\theta})\,d\eta d\theta dz \tag{10}$$

The kinetic energy T_s of a cylindrical shell (rotary inertia effect is neglected) is given by [4]

$$T_s = \frac{1}{2} LR \int_0^1 \int_0^{2\pi} \int_{-h/2}^{h/2} \rho(z)\,(\dot{u}^2 + \dot{v}^2 + \dot{w}^2)\,d\eta d\theta dz \tag{11}$$

where $\rho(z)$ is the mass density of the shell.

The virtual work W done by the external forces is written as [5]

$$W = LR \int_0^1 \int_0^{2\pi} (q_x u + q_\theta v + q_z w) d\eta d\theta \tag{12}$$

with (q_x, q_θ, q_z) as distributed forces in longitudinal, circumferential and radial direction.

The nonconservative damping forces are expressed by the Rayleigh's dissipation function [5]

$$F = \frac{1}{2} cLR \int_0^{2\pi} \int_0^1 (\dot{u}^2 + \dot{v}^2 + \dot{w}^2)\,d\eta d\theta \tag{13}$$

4. Vibration analysis

In the dynamic analysis of the shell, a two-steps procedure is considered [6]: i) the Rayleigh-Ritz method is applied to the linearized formulation of the problem to obtain an approximation of the eigenfunctions; ii) the displacement fields are re-expanded using the approximate eigenfunctions, the Lagrange equations are considered in conjunction with the fully nonlinear expression of the strain energy to obtain a set of nonlinear ordinary differential equations in modal coordinates.

4.1 Linear vibration analysis

In order to carry out a linear vibration analysis, only the quadratic terms are retained in Eqn. (10). A modal vibration, i.e. a synchronous motion, is obtained in the form [6]

$$u(\eta,\theta,t) = U(\eta,\theta) f(t) \quad v(\eta,\theta,t) = V(\eta,\theta) f(t) \quad w(\eta,\theta,t) = W(\eta,\theta) f(t) \tag{14}$$

where $u(\eta,\theta,t), v(\eta,\theta,t), w(\eta,\theta,t)$ are the displacement fields, $U(\eta,\theta)$, $V(\eta,\theta), W(\eta,\theta)$ represent the modal shape, $f(t)$ describes the time law, which is supposed to be the same for each displacement field (synchronous motion hypothesis).

The components of the modal shape are expanded by means of a double mixed series: the periodicity of deformation in the circumferential direction suggests the use of harmonic functions $(\cos n\theta\,, \sin n\theta)$, Chebyshev polynomials are considered in the longitudinal direction $T_m^*(\eta)$ [6]

$$U(\eta,\theta) = \sum_{m=0}^{M_u} \sum_{n=0}^{N} \tilde{U}_{m,n} T_m^*(\eta) \cos n\theta \qquad V(\eta,\theta) = \sum_{m=0}^{M_v} \sum_{n=0}^{N} \tilde{V}_{m,n} T_m^*(\eta) sin\, n\theta \qquad W(\eta,\theta) = \sum_{m=0}^{M_w} \sum_{n=0}^{N} \widetilde{W}_{m,n} T_m^*(\eta) \cos n\theta \tag{15}$$

where $T_m^*(\eta) = T_m(2\eta - 1)$, m is the number of longitudinal half-waves, n is the number of nodal diameters and $(\tilde{U}_{m,n}, \tilde{V}_{m,n}, \widetilde{W}_{m,n})$ are the generalized coordinates.

4.1.1 Boundary conditions

Simply supported – simply supported (S – S) boundary conditions are given by [3]

$$w = 0 \qquad v = 0 \qquad M_x = 0 \qquad N_x = 0 \qquad \text{for} \quad \eta = 0{,}1 \tag{16}$$

The previous conditions imply the following equations [6]

$$\sum_{m=0}^{M_w} \widetilde{W}_{m,n} T_m^*(\eta) = 0 \quad \sum_{m=0}^{M_v} \tilde{V}_{m,n} T_m^*(\eta) = 0 \quad \sum_{m=0}^{M_w} \widetilde{W}_{m,n} T_{m,\eta\eta}^*(\eta) = 0 \quad \sum_{m=0}^{M_u} \tilde{U}_{m,n} T_{m,\eta}^*(\eta) = 0 \quad \theta\epsilon[0,2\pi] \quad n\epsilon[0,N] \quad \text{for } \eta = 0{,}1 \tag{17}$$

The linear algebraic system given by Eqns. (16) can be solved analytically in terms of the coefficients $(\tilde{U}_{1,n}, \tilde{U}_{2,n}, \tilde{V}_{0,n}, \tilde{V}_{1,n}, \widetilde{W}_{0,n}, \widetilde{W}_{1,n}, \widetilde{W}_{2,n}, \widetilde{W}_{3,n})$, for $n\epsilon[0, N]$.

4.1.2 Rayleigh-Ritz procedure

The maximum number of variables needed for describing a generic vibration mode can be calculated by the relation $(N_p = M_u + M_v + M_w + 3 - r)$, with $(M_u = M_v = M_w)$ as maximum degree of the Chebyshev polynomials and r as number of equations for the boundary conditions. The number of degrees of freedom is computed by the relation $(N_{max} = N_p \times (N + 1))$, where N describes the maximum number of nodal diameters.

Equations (14) are inserted in the expressions of U_s and T_s (Eqns. $(10 - 11)$).

Consider now the Rayleigh quotient $R(\tilde{\mathbf{q}}) = \frac{V_{max}}{T^*}$, where V_{max} is the maximum of the potential energy, $T^* = \frac{T_{max}}{\omega^2}$, T_{max} is the maximum of the kinetic energy, ω is the circular frequency of the harmonic motion, $\tilde{\mathbf{q}} = \left[\ldots, \tilde{U}_{m,n}, \tilde{V}_{m,n}, \ \tilde{W}_{m,n}, \ldots\right]^T$ is a vector containing all the unknowns.

After imposing the stationarity to Rayleigh quotient, one obtains the eigenvalue problem [6]

$$(-\omega^2\mathbf{M} + \mathbf{K})\tilde{\mathbf{q}} = \mathbf{0} \tag{18}$$

which furnishes natural frequencies and modes of vibration (eigenvalues and eigenvectors).

The modal shape is given by the Eqns. (15), where coefficients $(\tilde{U}_{m,n}, \tilde{V}_{m,n}, \tilde{W}_{m,n})$ are substituted with $(\tilde{U}_{m,n}^{(j)}, \tilde{V}_{m,n}^{(j)}, \tilde{W}_{m,n}^{(j)})$, which are the components of the j-th eigenvector $\tilde{\mathbf{q}}_{\mathrm{j}}$ of the Eqn. (18).

The vector function $\boldsymbol{U}^{(j)}(\eta,\theta) = \left[U^{(j)}(\eta,\theta), V^{(j)}(\eta,\theta), W^{(j)}(\eta,\theta)\right]^T$ represents an approximation of the j-th mode of the original problem, see Ref. [6] for explanation.

4.2 Nonlinear vibration analysis

In order to carry In the nonlinear vibration analysis, the full expression of the elastic strain energy (10), containing terms up to the fourth order (cubic nonlinearity), is considered.

The displacement fields $u(\eta,\theta,t), v(\eta,\theta,t), w(\eta,\theta,t)$ are then expanded by using the linear mode shapes $U(\eta,\theta), V(\eta,\theta), W(\eta,\theta)$ obtained in the previous section [6]

$$u(\eta,\theta,t) = \sum_{j=1}^{N_u} U^{(j)}(\eta,\theta) f_{u,j}(t) \qquad v(\eta,\theta,t) = \sum_{j=1}^{N_v} V^{(j)}(\eta,\theta) f_{v,j}(t) \qquad w(\eta,\theta,t) = \sum_{j=1}^{N_w} W^{(j)}(\eta,\theta) f_{w,j}(t) \tag{19}$$

These expansions respect exactly the boundary conditions except for the free case; the synchronicity is relaxed as for each mode and component (u, v, w) different time laws are allowed.

Mode shapes $U^{(j)}(\eta,\theta), V^{(j)}(\eta,\theta), W^{(j)}(\eta,\theta)$ are known functions expressed in terms of polynomials and harmonic functions.

The Lagrange equations for forced vibrations are expressed in the following form [6]

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \qquad \text{for } i\epsilon[1, N_{max}] \qquad (L = T_s - U_s) \tag{20}$$

where the modal coordinates are now ordered in a vector $\mathbf{q}(t) = \left[\ldots f_{u,j}, f_{v,j}, f_{w,j}, \ldots\right]$, N_{max} depends on the number of modes considered in the expansions (19).

The generalized forces Q_i are obtained by differentiation of the Rayleigh's dissipation function F (13) and the virtual work done by external forces W (12), in the form [6]

$$Q_i = -\frac{\partial F}{\partial \dot{q}_i} + \frac{\partial W}{\partial q_i} \tag{21}$$

Expansions (19) are inserted into strain energy (10), kinetic energy (11), virtual work of the external forces (12) and damping forces (13). Using Lagrange Eqns. (20), a set of nonlinear ordinary differential equations (ODE) is then obtained.

5. Numerical results

In this section, the nonlinear vibrations of functionally graded circular cylindrical shells with different modal shape expansions and geometries are analyzed. Analyses are carried out on an FGM made of stainless steel and nickel. FGM properties are graded in the thickness direction according to a volume fraction distribution, where p is the power-law exponent. The material properties vs. coefficients of temperature at $T = 300$K are reported in Tab. 1 [2].

Table 1. Properties of stainless steel and nickel vs. coefficients of temperature.

	stainless steel			nickel		
	E	ν	ρ	E	ν	ρ
P_0	2.0×10^{11} Nm^{-2}	0.326	8166 kgm^{-3}	2.2×10^{11} Nm^{-2}	0.310	8900 kgm^{-3}
P_{-1}	0 K	0 K	0 K	0 K	0 K	0 K
P_1	3.1×10^{-4} K^{-1}	-2.0×10^{-4} K^{-1}	0 K^{-1}	-2.8×10^{-4} K^{-1}	0 K^{-1}	0 K^{-1}
P_2	-6.5×10^{-7} K^{-2}	3.8×10^{-7} K^{-2}	0 K^{-2}	-4.0×10^{-9} K^{-2}	0 K^{-2}	0 K^{-2}
P_3	0 K^{-3}	0 K^{-3}	0 K^{-3}	0 K^{-3}	0 K^{-3}	0 K^{-3}
P	2.1×10^{11} Nm^{-2}	0.318	8166 kgm^{-3}	2.0×10^{11} Nm^{-2}	0.310	8900 kgm^{-3}

5.1 Nonlinear response convergence analysis

The convergence analysis is carried out on a simply supported cylindrical shell excited with an harmonic force; a different number of asymmetric and axisymmetric modes is considered in the nonlinear expansions (19) of the displacement fields u, v, w, see Tab. 2.

Table 2. Asymmetric and axisymmetric modes inserted in the different nonlinear models.

6 dof model	9 dof model	12 dof model	15 dof model	18 dof model
mode (1,6) u, v, w	mode (1,6) u, v, w	mode (1,6) u, v, w	mode (1,6) u, v, w	mode (1,6) u, v, w
mode (1,12) v	mode (1,12) v	mode (3,6) u, v, w	mode (3,6) u, v, w	mode (3,6) u, v, w
mode (1,0) u, w	mode (3,12) v	mode (1,12) v	mode (1,12) v	mode (1,12) v
	mode (1,0) u, w	mode (3,12) v	mode (3,12) v	mode (3,12) v
	mode (3,0) u, w	mode (1,0) u, w	mode (1,18) v	mode (1,18) v
		mode (3,0) u, w	mode (1,0) u, w	mode (3,18) v
			mode (3,0) u, w	mode (1,0) u, w
			mode (5,0) u, w	mode (3,0) u, w
				mode (5,0) u, w
				mode (7,0) u, w

The FGM cylindrical shell is excited by means of an external modally distributed radial force $q_z = f_{1,6} \sin \eta \cos 6\theta \cos \Omega t$; the amplitude of excitation is $f_{1,6} = 0.0012 h^2 \varrho \omega_{1,6}^2$ and the frequency of excitation Ω is close to the mode (1,6), $\Omega \cong \omega_{1,6}$. The external forcing $f_{1,6}$ is normalized with respect to mass, acceleration and thickness; the damping ratio is equal to $\xi_{1,6} = 0.0005$. Nonlinear amplitudes $f_{u,1}, f_{v,1}, f_{w,1}$ of expansions (19) refer to the displacement fields u, v, w of mode (1,6).

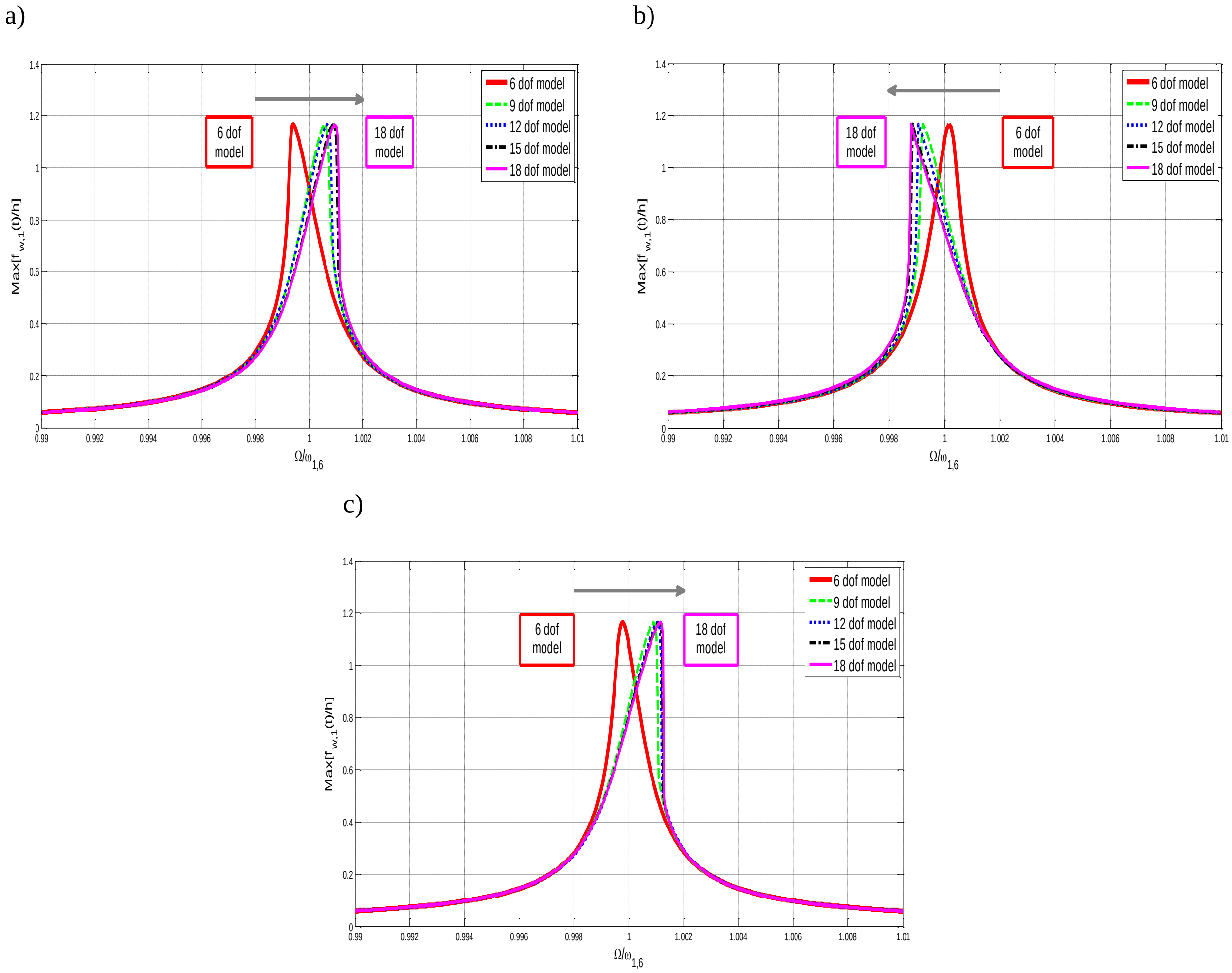

Figure 2. Comparison of nonlinear amplitude-frequency curves of the cylindrical shell.
a) $h/R = 0.002, L/R = 20, p = 1$; b) $h/R = 0.025, L/R = 20, p = 1$; c) $h/R = 0.050, L/R = 20, p = 1$.

In Figure 2(a), a comparison of nonlinear amplitude-frequency curves of the cylindrical shell $(h/R = 0.002, L/R = 20, p = 1)$ with different nonlinear expansions is shown; the shell is very thin and long. The nonlinear 6 dof model describes a wrong softening nonlinear behaviour, while the higher-order nonlinear expansions converge to a hardening nonlinear behaviour.

In Figure 2(b), a comparison of nonlinear amplitude-frequency curves of the cylindrical shell $(h/R = 0.025, L/R = 20, p = 1)$ with different nonlinear expansions is shown; the shell is moderately thick and long. The nonlinear 6 dof model describes a wrong slightly hardening nonlinear behaviour, the higher-order nonlinear expansions converge to a softening nonlinear behaviour.

In Figure 2(c), a comparison of nonlinear amplitude-frequency curves of the cylindrical shell $(h/R = 0.050, L/R = 20, p = 1)$ with different nonlinear expansions is shown; the shell is thick and long. The nonlinear 6 dof model describes a wrong slightly softening nonlinear behaviour, the higher-order nonlinear expansions converge to a hardening nonlinear behaviour.

From these convergence analyses, one can say that the 9 dof model gives satisfactory results with the minimal computational effort; therefore, in the following, the 9 dof model will be used.

5.2 Effect of the companion mode participation on the nonlinear response

In this section, the effect of the companion mode participation on the nonlinear response of the FGM shell is analyzed. In Figure 3(a), the amplitude-frequency curve with the companion mode participation is presented $(h/R = 0.025, L/R = 20, p = 1, mode\ (1,6))$. The response $f_{w,1}(t)$ with the companion mode participation, solid blue line, is very similar to the response without companion mode participation, dashed black line. Taking into account the companion mode, Figure 3(b), does not produce any variation except for a small region close to the resonance, where the companion mode is excited by means of a 1:1 internal resonance. The modal excitation does not excite directly the companion mode, which is excited in the frequency range $0.9996 < \Omega/\omega_{1,6} < 0.9999$.

In Figure 4(a), the time histories of the driven mode $(1,6)$, blue line, and companion mode, red line, for $\Omega/\omega_{1,6} = 0.9998$ are presented; the companion mode is initially not active, then an energy transfer takes place, the amplitude of the driven mode decreases and eventually the companion mode is excited. In Figure 4(b), enlarged view of Figure 4(a), a time phase shift between the modal coordinates (conjugate modes) close to $\pi/2$ is present, and a travelling wave takes place.

(a) (b)

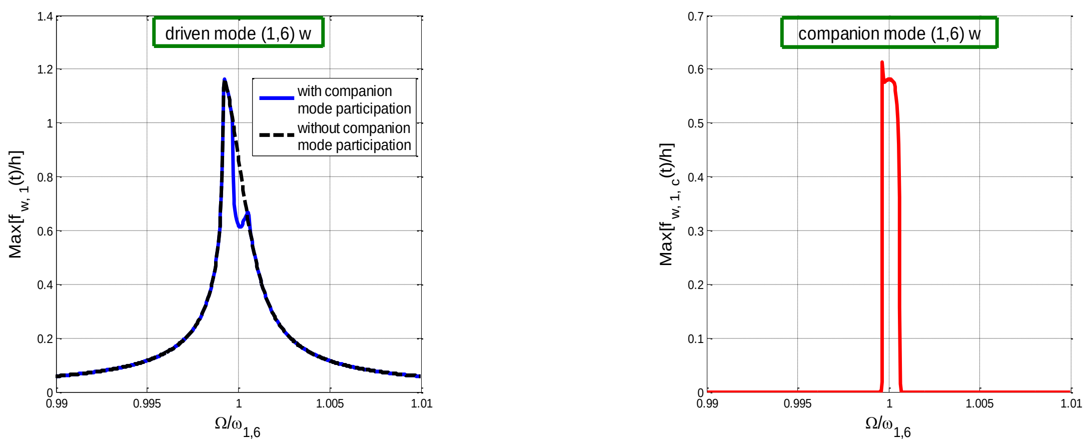

Figure 3. Amplitude-frequency curves of the cylindrical shell $(h/R = 0.025, L/R = 20, p = 1)$ with the companion mode participation. 14 dof model. (a) "– –", driven mode $(1,6)$ w without companion mode participation; "–", driven mode $(1,6)$ w with companion mode participation. (b) Companion mode $(1,6)$ w.

(a) (b)

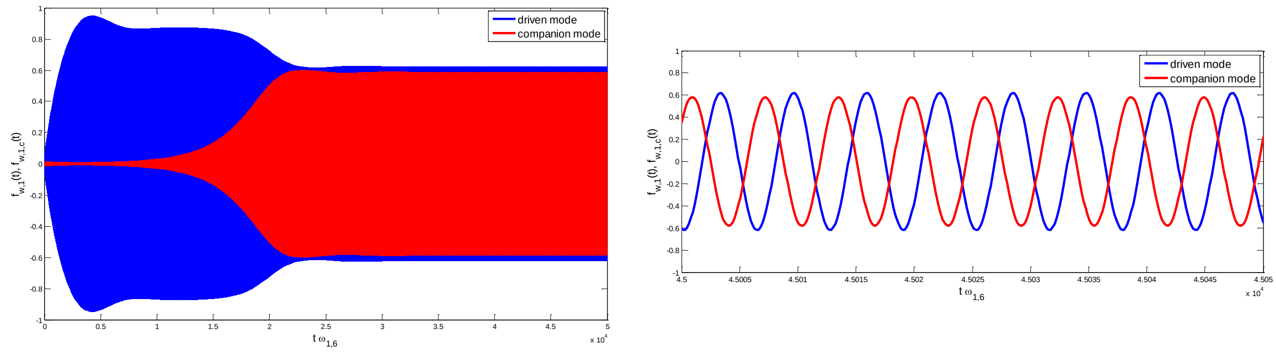

Figure 4. Time histories of the FGM shell $(h/R = 0.025, L/R = 20, p = 1)$.
"—", driven mode $(1,6)$ w with companion mode participation; "—", companion mode $(1,6)$ w.
(a) Transient included. (b) Steady state.

6. Conclusions

In this paper, the effect of the companion mode participation on the nonlinear vibrations of FGM circular cylindrical shells is analyzed. The Sanders-Koiter theory is applied to model the nonlinear dynamics of the system in the case of finite amplitude of vibration.

The functionally graded material is made of a uniform distribution of stainless steel and nickel, and the material properties are graded along the thickness direction, according to a volume fraction power-law distribution.

A convergence analysis is carried out by introducing in the series expansions of longitudinal, circumferential and radial displacement fields a different number of asymmetric and axisymmetric modes; the fundamental role of the axisymmetric modes is confirmed, and the role of the higher-order asymmetric modes is clarified in order to obtain the actual character of the shell nonlinearity.

The effect of the companion mode participation on the nonlinear response of the shell is analyzed. Both driven and companion modes are considered allowing for the travelling-wave response of the shell; amplitude-frequency curves with companion mode participation are obtained.

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