

THE BEGINNING OF THE ADVENTURE WITH PASCALINE AND E-PASCALINE

Michela Maschietto*, Sophie Soury-Lavergne^o

*Dipartimento di Educazione e Scienze Umane, Università di Modena e Reggio Emilia (IT)
^oS2HEP, Institut Français de l'Éducation, École Normale Supérieure de Lyon (FR)

The paper presents the idea of “duo of artefacts”, constituted by the pascaline (i.e., the arithmetical machine Zero+1) and its digital version e-pascaline. This “duo of artefacts” is proposed here to support student’s learning about the position notation in base ten at primary school. It also represents an example in which the development in technology (Cabri Elem environment) allows the relationships between material and virtual manipulatives to be discussed.

INTRODUCTION

The use of manipulatives in mathematics education is wide spread, especially at primary school level. There are two main differences among manipulatives that are available as pedagogical supports. A first distinction concerns the roots of these manipulatives: some of them are related, more or less, to the history of mathematics, others are expressly created for educational purposes. For instance, mathematical machines (Maschietto & Bartolini Bussi 2011, [1]) are reconstructions of historical devices that are present in mathematical treatises; they can be considered historical manipulative materials. A second distinction concerns their nature: beside the manipulative materials, virtual manipulative is present. The latter is defined as “an interactive, Web-based, visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge” (Moyer *et al.* 2002, p.373). In this paper, we consider two kinds of manipulative: a material one that is linked to the history of mathematics (the arithmetical machine Zero+1, called pascaline, see Figure 1 on the left) and its digital simulation (called e-pascaline, see Figure 2), constructed in the Cabri Elem environment.

Manipulatives have been especially considered from a student’s perspective. Researchers have been paid attention to the relationships between the use of manipulatives and student’s learning (Durmus & Karakirik 2006). In the last years, they are also considered in teacher education (Hunt *et al.* 2011), with a focus on the analysis and awareness of specific epistemological character of manipulatives (Nührenböcker & Steinbring 2008) for teachers. This position is coherent with the use of manipulatives (for instance, mathematical machines) in terms of tools of semiotic mediation (Bartolini Bussi & Mariotti 2008) and cultural analysis of content.

Research works usually focus on one kind of manipulatives, i.e. material or virtual ones, even if virtual manipulatives are proposed as simulations of (historical) materials. Several research works consider the design and analysis of use of digital tools in mathematics learning (Schnotz & Lowe 2003). But there are very few studies in mathematics education that analyse their articulation, although that topic is present in educational debate (Bartolini Bussi & Borba 2010; Maschietto & Bartolini Bussi 2011). In this paper, we intend to contribute to that kind of research.

In general, the aim of our work is to study why and how the use of technology is an adding value to the use of other kinds of teaching tools, in particular materials. Our idea is to provide the teachers of a *duo of artefacts*, that is a physical pedagogical material and its digital counterpart. Our hypothesis is that such a *duo of artefacts* will enlarge the learning experience of the students. For this, we aim to study the articulation between the two kinds of manipulative in teaching and learning mathematics at primary school level (Maschietto & Soury-Lavergne submitted).

This paper intends to contribute to the conference themes of using technology to support students' learning and of developments in technology for learning and teaching mathematics by the discussion of the idea of the *duo of artefacts* (pascaline, e-pascaline).

DUO OF ARTEFACTS

The starting point in the construction of the *duo of artefacts* was the research work (Maschietto 2011) on the use of the arithmetical machine Zero+1 (called pascaline by students and teachers that have used it; see Figure 1, on the left), a cultural material manipulative, within the mathematics laboratory methodology (Maschietto & Trouche 2010). The first step in the conception of the duo is the design of the virtual manipulative. The second step is the conception of tasks and scenario for the *duo of artefacts*. They are discussed in this section.

The arithmetical machine “pascaline”

The arithmetical machine Zero+1 [2] is inspired by the mechanical calculators, like the *Pascaline* (Figure 1, on the right) designed by Blaise Pascal (1623-1662) in 1642.



Figure 1. The arithmetical machine Zero+1 (on the left); Pascaline by B. Pascal (on the right)

Zero+1 is a small plastic tool (27 cm x 16 cm) and is composed of a base with a gear train of five wheels (each of them has ten teeth). This machine allows numbers to be written in decimal position notation. It works as a counter, because of the number to be carried in an automatic way. The wheels can be turned clockwise (with relation to make addition) and anticlockwise (with relation to make subtraction). For writing numbers and making operations two procedures are identified: iteration by the operator '+1' on units wheel, decomposition of numbers in hundreds, tens and units (Maschietto 2011).

Design of the e-pascaline

The e-pascaline (Figure 2) has been created within the authoring Cabri Elem environment, developed by the Cabrilog society. This authoring environment enables to design activity books, which consist of a succession of pages incorporating some representations of objects and sequence of tasks (for more details about the design of books with the Cabri Elem technology see (Mackrell *et al.* forthcoming) or (Laborde & Laborde 2011)).

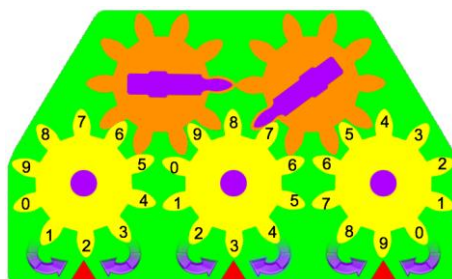


Figure 2. The e-pascaline

The e-pascaline has been created as a complex object very similar to the physical one. The shapes, the main constitutive elements and the colours are respected, but the e-pascaline is not a simple virtual reconstruction of the *Pascaline* (contrary to Bascoul's work [3]) or the Zero+1. The implementation of the e-pascaline has required additional design decisions, which are related to the authoring environment and to didactical choices (Maschietto & Soury-Lavergne submitted).

The design of the e-pascaline is mainly grounded on two theoretical frameworks, the instrumental approach (Rabardel & Bourmaud 2003) and the theory of semiotic mediation (Bartolini Bussi & Mariotti 2008). These frameworks are involved in our research methodology at two levels (Maschietto & Soury-Lavergne submitted): [1] they have been used to plan and analyse teaching experiments with the physical pascaline (teaching experiments with the pascaline have been carried out in Italian and French classes at primary school); [2] they support the choices for the design of the e-pascaline in terms of continuity and discontinuity between the material and digital artefacts. In particular, our choices in terms of continuity/discontinuity are based on the analysis of utilisation schemes interlinked with the analysis of the semiotic potentials of the artefacts. This kind of analysis allows characterising several components of the artefacts and their behaviours as a consequence of student's actions. For instance, the rotation of a tooth at a time (a discrete movement of wheels) is a fundamental feature of the pascaline that is kept in the e-pascaline. But each e-pascaline wheel turns by the use of two buttons that launch its animation (curved arrows at the bottom, see Figure 2 under each yellow wheel) in both directions. In such a way, wheel gestures of grasping a tooth, the upper arrow on the upper wheel or the whole wheel are not possible any more. They were eliminated because they contribute to the instrumental genesis by the students but they did not have a semiotic potential that contributes to didactical aims. This choice enacts a discontinuity in the instrumentation from the pascaline to the e-pascaline. On the other hand, the automatic turning of the wheels (for the number to be carried when making arithmetical operations) is preserved as it is a fundamental feature of the pascaline.

Interactive books with e-pascaline

Many different interactive e-books can include the e-pascaline as the principal component (Figure 3). Each e-book can be composed of several pages with the possibility to change several elements and behaviours for each of them and to add new elements. In such a way, e-books can organize and structure tasks for students and feedback, i.e. the reaction of the objects to the action of the user.

Concerning feedback, Mackrell *et al.* (forthcoming) have identified three levels of feedback that can be provided in Cabri Elem e-books: (i) the direct manipulation feedback, that can be linked to the perceptive aspects of the objects behaviour, (ii) the evaluation feedback, (iii) the strategy feedback that results from an analysis of student's strategy. The third level of feedback is strictly related to another feature of an e-book. With respect to the educational aim and tasks, important elements in the pages represent didactical variables, that are parameters whose values are chosen in order to foster or to prevent students' resolution strategies. This kind of control represents an important point in the discussion about the articulation of material and virtual manipulatives. Indeed, a shared argument is that virtual manipulatives allow eliminating some constraints that manipulative materials impose (Durmus & Karakirik 2006, p.121). Here, in the e-books it is possible to constraint the use of e-pascaline with respect to the educational aim. For instance, if we want to make operations with the minimum number of clicks, to promote the evolution of procedures from iteration to decomposition, we can hide the arrows moving the wheels if the students has turned too many times. This example of strategy feedback is also an example of added value for the use of the e-pascaline with respect to the pascaline. Moreover, in the pages of an e-book other components that are not the e-pascaline can be included: for instance, the "tools box" (see Figure 3 on the right, at the bottom), with tools to write texts and numbers on the page, to mark points and to reset the e-

pascaline. Also this choice is based on the analysis of teaching experiments carried out with the pascaline revealing that the pascaline is rarely used alone, but it belongs to a system of instruments for students.

Discussion

The e-pascaline is developed in the “Mallette” project, a French project supported by the Ministry of Education, directed by the IFÉ [4], in collaboration with the Laboratory of Mathematical Machines from the University of Modena e Reggio Emilia (collaboration between the two authors of the present contribution). It aims to improve the teaching and learning of mathematics at primary school by providing teachers with pedagogical devices that foster manipulation. Within the “Mallette” project, teaching experiments with the duo of artefacts is ongoing at grades 1 and 2 in France. In the first phase, the pascaline was introduced in the class with some tasks of writing numbers. The two main procedures - iteration and decomposition - have appeared, but in both cases, the students’ control has operated on the final display and not on the process to get it, which process relies on the number of “click” or the number of “tooth”. In the second phase, the e-pascaline has been introduced. For these teaching experiments, several e-books have been designed.

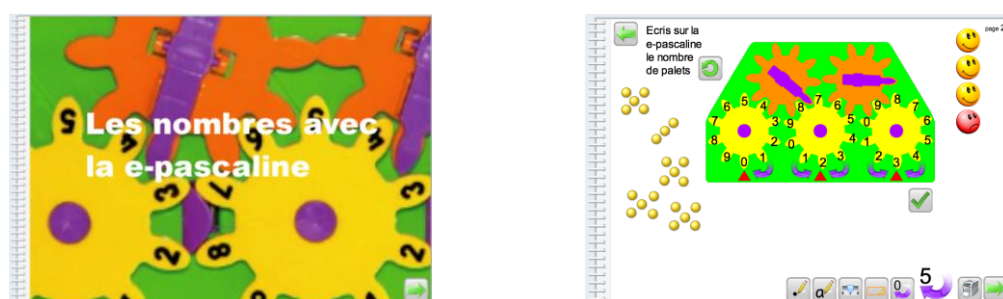


Figure 3. The cover of the e-book (on the left); the e-pascaline used by a student (on the right).

A first e-book is about the decimal system for writing number (Figure 3, on the left). The kind of task proposed to the students is the same from one page to another: to write a number with the e-pascaline. The difference consists in the way the target number is presented, by a collection of counters or by an oral message. The spatial organisation of the counters and the size of the numbers are didactical variables of the pages (Figure 4). Their choices will entail different strategies for students.

On each page, there are almost the same objects (see Figure 3, on the right): the e-pascaline, a reload button (to get a new number to be written with the e-pascaline), a set of tools (including a click counter), an evaluation button (to get a feedback from the system about the correctness of the number written on the e-pascaline) and arrows to move to the previous or next page.

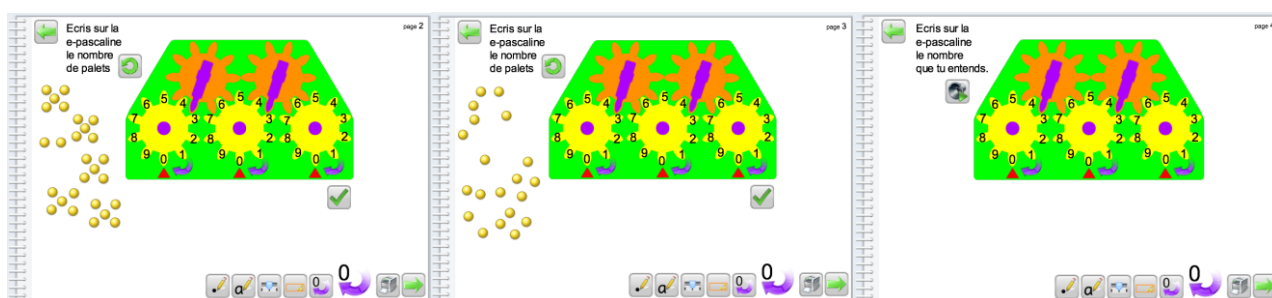


Figure 4. Page 2; Page 3; Page 5

In the first pages of the e-book (Figure 4, on the left and middle), counters up to 30 are randomly determined and displayed. Students have to determine the number of counters in the collection and write it in the e-pascaline. At least two strategies are possible: 1) first the student counts the counters, then it uses the iteration or decomposition procedure to write the number on the e-pascaline; 2) the student matches each counter with a click on the right arrow of the units wheel, and he stops when the collection has been enumerated. At the end, the student can check his solution: a smiley appears when the evaluation button is pushed (it is an example of evaluation feedback, Figure 3 on the right).

In the last pages of the e-book, the target number is given orally by a message that can be repeated as many times as needed (Figure 4, on the right). Since those pages contains the crucial task of numerical transcoding from oral to written registers, the e-book takes into account the irregularities of French words for number (the 100 first numbers are irregular from 0 to 16 and from 70 to 99).

The aim of the first page is also the appropriation of the elements of the e-book by the students, especially the reload button and the evaluation button. It allows initiating the instrumental genesis of the main elements of the pages. On the other hand, in our idea of the *duo of artefact*, the appropriation of e-pascaline can be supported by the use of the pascaline. This element represents an essential point of continuity. For instance, iteration and decomposition strategies are discussed in the phase of work with the pascaline. Moreover, during experimentations, we have observed some students using the pascaline while solving task on the e-book with the e-pascaline.

The evolution of the constraints from one page to another should lead the student to adapt and/or change his/her strategy. A didactical aim is that students are able first to understand and explain the mathematical meanings behind their utilisation schemes, then to formulate them in a mathematical text. This e-book provides an example to discuss the added value of the digital artefact with respect to the physical one. Added value results from the differences and common points between the two artefacts. These differences and common points enable to control the instrumental genesis, to impede some gesture and to reinforce some others. But the added value does not concern just the functioning of the e-pascaline. It also results from the way the task is proposed, the range of numbers, the possibility for the student to repeat the task and to have a personal feedback. Indeed, e-books are very engaging for the students, especially because of the evaluation feedback, as far as we have observed from the recent experiments. All those elements are important for students' learning.

With the ongoing teaching experiments, we will analyse the *duo of artefact* from the point of view of the students' learning. But it is not the only element. The introduction of the duo of artefacts also concerns the teacher that uses it for teaching. In this sense, it will be analysed from the point of view of the teachers' appropriation. So, we can have elements for studying why and how the use of technology is an adding value to the use of other kinds of teaching tools. And we can participate to the educational debate on virtual and material manipulatives.

NOTES

1. Laboratorio delle Macchine Matematiche <http://www.mmlab.unimore.it>; Associazione Macchine Matematiche www.macchinematematiche.org/
2. It is produced and sold by the Italian company "Quercetti" (<http://www.quercetti.it>).
3. A virtual reconstruction of the Pascal's *Pascaline*: <http://christophe.bascoul.free.fr/spip.php?article20>
4. <http://educmath.ens-lyon.fr/Educmath/recherche/equipements-associees/mallette12-13/>

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