

Smooth tensionful higher-codimensional brane worlds with bulk and brane form fields

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ABSTRACT: Completely regular tensionful codimension- n brane world solutions are discussed, where the core of the brane is chosen to be a thin codimension- $(n - 1)$ shell in an infinite volume flat bulk, and an Einstein-Hilbert term localized on the brane is included (Dvali-Gabadadze-Porrati models). In order to support such localized sources we enrich the vacuum structure of the brane by the inclusion of localized form fields. We find that phenomenological constraints on the size of the internal core seem to impose an upper bound to the brane tension. Finite transverse-volume smooth solutions are also discussed.

KEYWORDS: Large Extra Dimensions, Classical Theories of Gravity, Field Theories in Higher Dimensions, Space-Time Symmetries

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1 Introduction and summary

Brane world models have drawn a lot of attention in the last years since they provide an interesting scenario for the search of solutions to long standing particle physics puzzles as the cosmological constant problem and the hierarchy problem. In cosmology they might provide alternatives to dark matter and/or dark energy (see e.g. [1, 2]).

In the present manuscript we study brane world models with codimension larger than two, for a variety of situations. However, we are mostly interested in flat bulk models where the extra-dimensional volume is infinite and 4d gravity is brane-induced on the brane at short scales [3–5, 7] (see [6] for an orientifold derivation). Thin tensionful higher-codimensional solutions in flat space are known to give rise to singular backgrounds [8–10] and need to be regulated. One possible way to regulate such singularity is to "resolve" the brane, by giving it a non-trivial core, in the extra-dimensions, e.g. a thin spherical shell; in this latter case the brane results effectively codimension-one. This method has proven to be quite efficient in the codimension-two case, both for finite volume rugby-ball [11] solutions and for infinite-volume induced gravity ones.¹

For codimension larger than two it had been shown that a naive regularization of the higher-codimension brane by blowing the thin brane to a thin spherical shell lead to a no-go theorem [19], that we review later. A possible way-out to such no-go theorem was then found in [20, 21] by employing bulk higher curvature terms to regulate the bulk singularity. Another recent proposal for smoothing out higher-codimensional singularities is to consider a bulk Einstein-Skyrme model [22]. Here we present a different way-out: we keep bulk Einstein-Hilbert gravity but consider a richer brane vacuum structure by the inclusion of higher-rank (form) fields (this was suggested in [23] for a Z_2 -symmetric setup [24]) similarly to the codimension-two models that involve an axion field [25–29] whose vacuum expectation value cancels the tangential (to the brane profile) component

¹Tensionful codimension-two singularities are milder (conical) and do not, a priori, need to be regulated [12]. However, regularization via smoothing out the brane profile is often invoked in order to avoid subtleties associated to purely conical radially symmetric extra-dimensional space [13–18].

of the pressure. We explicitly show here that the inclusion of higher-rank fields works as well for our higher-codimensional solutions.

Another reason behind the present work is the study of new (higher-codimensional) brane cosmology models: as said above codimension-one regularization seems necessary for cosmological setups, at least in bulk Einstein gravity (for Lovelock gravity and/or broken spherical symmetry the situation might improve [30–33]). Such regularization allowed to study some cosmological properties of codimension-two setups using the moving brane approach [34, 35] or weak field limit [27, 36]. We can thus also see the present work as a possible framework where study cosmology on a generic-codimension brane world.

Finally we consider higher-codimensional induced gravity brane world models, in the light of more recent results [37, 38] where it was found that cascading higher-codimensional induced-gravity models are ghost-free, hence shedding new light on such induced gravity models, which have been sources of several controversies regarding their classical and quantum stability. In [39] it was also suggested that cancellation of ghost excitations might as well take place for resolved brane setups with codimension larger than two, provided tangential pressures are cancelled. We show later that, opposed to the codimension-two case, in our setup tangential pressures do not have to vanish and no strong fine-tuning between flux field and tension is a priori needed. However, phenomenological constraints on the size of the internal brane profile seem to impose, for this class of models, an upper bound (cfr. eq. (2.37)) to the brane tension, as opposed to the codimension-two case where the upper bound for the tension is due to a topological constraint (the conical deficit angle is bounded to be less than 2π).

2 Vanishing bulk cosmological constant

The brane world model we study in this section is described by the following action:

$$\begin{aligned}
 S &= \widehat{M}^{D-2} \int d^D x \sqrt{-\widehat{g}} \widehat{\mathcal{R}} \\
 &+ \int_{\Sigma} d^{D-1} x \sqrt{-g} \left[M^{D-3} \left(\mathcal{R} - \Lambda - \frac{1}{2 \cdot p!} F_{[p]}^2 \right) + 2\widehat{M}^{D-2} K_{\pm} \right] \\
 \Sigma &= \mathbf{R}^{D-n-1,1} \times \mathbf{S}_{\epsilon}^{n-1}
 \end{aligned} \tag{2.1}$$

Here Σ is a fat codimension- n source brane, whose geometry is given by the product $\mathbf{R}^{D-n-1,1} \times \mathbf{S}_{\epsilon}^{n-1}$, where $\mathbf{R}^{D-n-1,1}$ is the $(D-n)$ -dimensional Minkowski space, and $\mathbf{S}_{\epsilon}^{n-1}$ is a $(n-1)$ -sphere of radius ϵ (in the following we will assume that $n \geq 3$). The quantity $M^{D-3}\Lambda$ plays the role of the tension of the brane Σ and $F_{[p]}$ is the field strength of a $(p-1)$ -form potential $A_{[p-1]}$

$$F_{[p]} = dA_{[p-1]}, \quad F_{m_1 \dots m_p} = p \partial_{[m_1} A_{m_2 \dots m_p]} = \partial_{m_1} A_{m_2 \dots m_p} + \text{cyclic} \tag{2.2}$$

Also,

$$g_{mn} \equiv \delta_m^M \delta_n^N \widehat{g}_{MN} \Big|_{\Sigma}, \tag{2.3}$$

where x^m are the $(D - 1)$ coordinates along the brane (the D -dimensional coordinates are given by $x^M = (x^m, r)$, where $r \geq 0$ is a non-compact radial coordinate transverse to the brane, and the signature of the D -dimensional metric is $(-, +, \dots, +)$); finally, the $(D - 1)$ -dimensional Ricci scalar \mathcal{R} is constructed from the $(D - 1)$ -dimensional metric g_{mn} and K is the extrinsic curvature, with $K_{\pm} \equiv K_+ + K_-$. In the following we will use the notation $x^i = (x^\alpha, r)$, where x^α are the $(n - 1)$ angular coordinates on the sphere. Moreover, the metric for the coordinates x^i will be (conformally) flat:

$$\delta_{ij} dx^i dx^j = dr^2 + r^2 \gamma_{\alpha\beta} d\theta^\alpha d\theta^\beta, \tag{2.4}$$

where $\gamma_{\alpha\beta}$ is the metric on a unit $(n - 1)$ -sphere. Also, we will denote the $(D - n)$ Minkowski coordinates on $\mathbf{R}^{D-n-1,1}$ via x^μ (note that $x^m = (x^\mu, \theta^\alpha)$).

The bulk equations of motion are clearly given by

$$\widehat{G}_{MN} = 0 \tag{2.5}$$

and the boundary conditions for the fat brane can be obtained using Israel junction conditions

$$\left\langle K^m_n - \delta_n^m K \right\rangle_{\pm} = -\frac{1}{2\widehat{M}^{D-2}} T^m_n \tag{2.6}$$

where

$$T_{mn} = -M^{D-3} (2G_{mn} + g_{mn}\Lambda) + T_{mn}(F) \tag{2.7}$$

is the total energy-momentum tensor for the ‘‘matter’’ localized on the fat brane, with

$$T_{mn}(F) = \frac{M^{D-3}}{(p-1)!} \left(-\frac{1}{2p} F^2 g_{mn} + F_m{}^{l_2\dots l_p} F_n{}_{l_2\dots l_p} \right). \tag{2.8}$$

2.1 The no-go theorem

In order to better clarify our results let us first revise the no-go theorem associated to radially symmetric solutions in absence of the p -form term [19]. Let us consider the following ansatz for the background metric:

$$ds^2 = \exp(2A) \eta_{\mu\nu} dx^\mu dx^\nu + \exp(2B) \delta_{ij} dx^i dx^j, \tag{2.9}$$

where A and B are functions of r but are independent of x^μ and θ^α (that is, we are looking for solutions that are radially symmetric in the extra dimensions). The bulk equations of motion then read (here prime denotes derivative w.r.t. r):

$$(D - n) \left[\frac{D - n - 1}{2} (A')^2 + \frac{n - 1}{r} A' + (n - 1) A' B' \right] + (n - 1)(n - 2) \left[\frac{1}{2} (B')^2 + \frac{1}{r} B' \right] = 0 \tag{2.10}$$

$$(D - n) \left[A'' + \frac{D - n + 1}{2} (A')^2 + \frac{n - 2}{r} A' + (n - 3) A' B' \right] + (n - 2) \left[B'' + \frac{n - 3}{2} (B')^2 + \frac{n - 2}{r} B' \right] = 0 \tag{2.11}$$

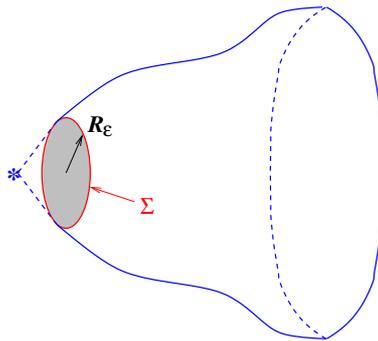


Figure 1. Pictorial representation of the infinite-volume smooth brane world.

Above, equation (2.11) is the $(\alpha\beta)$ equation, while equation (2.10) is the (rr) equation. Note that the latter equation does not contain second derivatives of A and B . The solution for B' is given by (we have chosen the plus root, which corresponds to solutions with infinite-volume extra space):

$$B' = -\frac{1}{r} - \frac{D-n}{n-2}A' + \sqrt{\frac{1}{r^2} + \frac{1}{\kappa^2}(A')^2}, \tag{2.12}$$

where we have introduced the notation

$$\frac{1}{\kappa^2} \equiv \frac{(D-n)(D-2)}{(n-1)(n-2)^2} \tag{2.13}$$

to simplify the subsequent equations.

Here we are interested in non-singular solutions such that A and B are constant for $r < \epsilon$, and asymptote to some finite values as $r \rightarrow \infty$. For $r > \epsilon$ the solution for A and B is given by

$$A(r) = -\frac{\kappa}{n-2} \ln \left(\frac{1+f(r)}{1-f(r)} \right), \quad r > \epsilon, \tag{2.14}$$

$$B(r) = -\frac{D-n}{n-2}A(r) + \frac{1}{n-2} \ln(1-f^2(r)), \quad r > \epsilon, \tag{2.15}$$

where $f(r) \equiv \left(\frac{r_*}{r}\right)^{n-2}$ and r_* is the integration constant, and where we have set other integrations constants such that $A_\infty = B_\infty = 0$. A pictorial representation of such setup is given in figure 1, where the gray disk describes the extra-dimensional shape of the inside bulk ($r < \epsilon$), the bell-shaped part is the asymptotically-flat outside bulk ($r > \epsilon$), the circle Σ is the fat brane and the "star" represents the would-be naked singularity $r = r_*$. Israel junction conditions (2.6) provide the equations at the location of the fat brane, $r = \epsilon$; including the contribution of the induced EH term

$$G^\mu{}_\nu = -\frac{(n-1)(n-2)}{2R_\epsilon^2} \delta^\mu{}_\nu \tag{2.16}$$

$$G^\alpha{}_\beta = -\frac{(n-3)(n-2)}{2R_\epsilon^2} \delta^\alpha{}_\beta \tag{2.17}$$

where $R_\epsilon \equiv \epsilon e^{B(\epsilon)}$ is the physical radius of the $(n-1)$ -sphere, we obtain

$$(n-2) \frac{2f^2(\epsilon)}{1-f^2(\epsilon)} + \frac{\epsilon L}{2} e^{B(\epsilon)} \left[\Lambda - \frac{(n-2)(n-3)}{R_\epsilon^2} \right] = 0 \quad (2.18)$$

$$(n-1) \frac{2f^2(\epsilon)}{1-f^2(\epsilon)} - \frac{D-2}{n-2} \frac{2\kappa f(\epsilon)}{1-f^2(\epsilon)} + \frac{\epsilon L}{2} e^{B(\epsilon)} \left[\Lambda - \frac{(n-1)(n-2)}{R_\epsilon^2} \right] = 0 \quad (2.19)$$

for the $(\alpha\beta)$ and $(\mu\nu)$ components respectively, with $L \equiv \frac{M^{D-3}}{M^{D-2}}$. We can rewrite the previous matching conditions in a more useful way as follows:

$$\frac{2f^2(\epsilon)}{1-f^2(\epsilon)} + \frac{L}{2R_\epsilon} (\lambda - n + 3) = 0, \quad (2.20)$$

$$\frac{D-2}{n-2} \frac{2\kappa f(\epsilon)}{1-f^2(\epsilon)} + \frac{L}{2R_\epsilon} (\lambda + n - 1) = 0. \quad (2.21)$$

where we have defined $\Lambda \equiv \lambda \frac{n-2}{R_\epsilon^2}$. Let us study possible solutions to these matching conditions with $r_* < \epsilon$ for which $0 < f(\epsilon) < 1$: they would be non-singular solutions as the would-be naked singularity $r = r_*$ is cut away. The second matching conditions can only be satisfied if $\Lambda < 0$. Hence λ must be a negative parameter; in other words there are no non-singular solutions of this type with positive tension. Moreover, from the first condition we have:

$$f(\epsilon) = \frac{-\lambda + n - 3}{-\lambda - n + 1} \sqrt{\frac{(n-1)(D-2)}{D-n}}. \quad (2.22)$$

For $n \geq 3$, the condition $0 < f(\epsilon) < 1$ admits no solutions with negative λ . Hence, the above matching conditions cannot be simultaneously satisfied within this class of solutions. For a different class of solutions that is curved both on the inside bulk and on the outside bulk it is possible to overcome the previous no-go theorem [24]. In [23] an upgraded version of the model [24], that suggested the use of brane form fields, was considered. In the next section we will see that changing the structure of the vacuum brane stress tensor, with the inclusion of higher-rank tensors is crucial also for type of geometry described above, as it allows smooth solutions. This type of geometry is the higher-codimensional version of that considered in [29]. Such type of regularization was studied in [40–45], in the context of compact codimension-two brane worlds, in order to obtain codimension-two effective actions. For the sake of generality we will thus consider in section 3 some higher-codimensional generalizations of the backgrounds considered in [40–45], that will require bulk higher-rank tensors as well as non-vanishing bulk cosmological constant or a bulk σ -model matter action [46, 47].

2.2 Adding the p -form field

In order to enrich the vacuum structure of our brane world we include a p -form field in the worldvolume of the blown-up brane Σ . We consider the case of a $(n-1)$ -form field strength but could equivalently consider its dual $(D-n)$ -form as in the string landscape [49]. We require its energy-momentum tensor to have the block-diagonal form

$$T_m{}^n(F) = \begin{pmatrix} T\delta_\mu{}^\nu & 0 \\ 0 & T'\delta_\alpha{}^\beta \end{pmatrix} \quad (2.23)$$

with T, T' constant. In order to achieve that let us use spherical coordinates

$$\begin{aligned} \gamma_{\alpha\beta} d\theta^\alpha d\theta^\beta &= d\theta_0^2 + \sin^2 \theta_0 d\theta_1^2 + \sin^2 \theta_0 \sin^2 \theta_1 d\theta_2^2 \\ &+ \dots + \sin^2 \theta_0 \dots \sin^2 \theta_{n-3} d\theta_{n-2}^2 \end{aligned} \quad (2.24)$$

to parameterize the $(n - 1)$ -sphere and let us consider the extended magnetic monopole field(s)

$$A_{[n-2]\pm} = \sqrt{2}\Phi R_\epsilon^{n-1} \left(\pm c + h(\theta_0) \right) E_{[n-2]} \quad (2.25)$$

with Φ constant, $E_{[n-2]}$ being the volume form of the equatorial $(n - 2)$ -sphere, and with $h'(\theta) = \sin^{n-2} \theta$, and $h(\pi) = -h(0)$; the field strength

$$F_{[n-1]} = \sqrt{2}\Phi R_\epsilon^{n-1} S_{[n-1]} \quad (2.26)$$

is thus proportional to the volume form of the unit $(n - 1)$ -sphere. The field configurations (2.25) are defined on the north (south) hemisphere of S^{n-1} and the integration constant c is fixed by regularity conditions at the poles [48]. From (2.26) one immediately obtains

$$T_m{}^n(F) = M^{D-3} \Phi^2 \begin{pmatrix} -\delta_\mu{}^\nu & 0 \\ 0 & +\delta_\alpha{}^\beta \end{pmatrix} \quad (2.27)$$

and

$$T_\mu{}^\nu = -M^{D-3} \frac{n-2}{R_\epsilon^2} (\lambda + \varphi^2) \delta_\mu{}^\nu \quad (2.28)$$

$$T_\alpha{}^\beta = -M^{D-3} \frac{n-2}{R_\epsilon^2} (\lambda - \varphi^2) \delta_\alpha{}^\beta \quad (2.29)$$

where we have defined $\Phi^2 \equiv (n-2)\varphi^2/R_\epsilon^2$. Hence the boundary conditions (2.20) and (2.21) still hold with the replacements $\lambda \rightarrow \lambda - \varphi^2$ and $\lambda \rightarrow \lambda - (2n - 3)\varphi^2$ respectively. We thus have

$$\frac{2f^2(\epsilon)}{1 - f^2(\epsilon)} = \frac{L}{2R_\epsilon} \left(-\lambda + \varphi^2 + n - 3 \right), \quad (2.30)$$

$$\frac{D-2}{n-2} \frac{2\kappa f(\epsilon)}{1 - f^2(\epsilon)} = \frac{L}{2R_\epsilon} \left(-\lambda + (2n - 3)\varphi^2 - n + 1 \right) \quad (2.31)$$

so that λ can be either positive or negative, provided φ^2 is large enough. The second condition gives

$$R_\epsilon = L \sqrt{\frac{D-n}{(n-1)(D-2)} \frac{1 - f^2(\epsilon)}{4f(\epsilon)} \left(-\lambda + (2n - 3)\varphi^2 - n + 1 \right)} \quad (2.32)$$

that replaced into the first condition yields

$$f(\epsilon) = \sqrt{\frac{(n-1)(D-2)}{D-n}} \frac{-\lambda + \varphi^2 + n - 3}{-\lambda + (2n - 3)\varphi^2 - n + 1} \quad (2.33)$$

that is the equivalent of (2.22). Note however that now there are smooth solutions with $f(\epsilon) < 1$ regardless of the value of the brane tension (here parameterized by λ). For example let us consider $n = 3$, $D = 7$: in such a case we have $f(\epsilon) = \sqrt{\frac{5}{2}} \frac{-\lambda + \varphi^2}{-\lambda + 3\varphi^2 - 2}$ that can be smaller than one, provided φ^2 is large enough. The flux φ^2 increases the value of the physical four-dimensional vacuum energy density that can be obtained by integrating (2.28) over the brane profile

$$\mathcal{E}_{4d} = (n - 2)S_{(n-1)} (\lambda + \varphi^2) M^{1+n} R_\epsilon^{n-3} \tag{2.34}$$

where $S_{(n-1)}$ is the volume of the unit-radius $(n-1)$ -sphere. Coupling of the form potential to a localized extended object leads to a quantization condition for the flux [48–52]; we come back to this point in the next section.

Let us point out a crucial difference between our setup ($n > 2$) and previously considered codimension-two smooth solution. In the codimension two case [29] the smooth solution considered has $A = \text{constant}$ and $B \sim \ln r$ so that the junction condition coming from the $(\alpha\beta)$ equation of motion (2.11) is trivial (there are no second derivatives in B in such a case) and this can only be satisfied if we “tune” $\Lambda = \Phi^2$. In our case the only requirement for the “flux” is a lower bound. Note however that, once λ and φ are chosen, the value of the physical radius is fixed in terms of (2.32).

In the present 4d-Poincaré-invariant Ricci-flat setup the inclusion of bulk fluxes is problematic because of the no-go theorem [54–56]. In other words the present solutions avoid such no-go theorem in a trivial way: no bulk fluxes, and presence of discontinuities in the derivatives of the warp factors that are absorbed by localized fluxes and brane tension.

2.3 Comments

We comment here on the possible physical scales involved in the model; we focus on the case $d = 4$, $n = 3$ for it displays all the details of these models. There is a variety of scenarios that might appear according to the different values of tension and flux and it is beyond the scopes of the present manuscript to give a detailed study of such issues. Let us however point out a few interesting features. The present model has codimension larger than two and there is, a priori, no critical value for the tension. However, phenomenological constraints impose that the internal radius of the brane satisfies $R_\epsilon < (TeV)^{-1}$. Since the scale L will be related to the crossover scale after which brane gravity turns higher-dimensional and it must thus be taken to be enormously large, it is natural to assume that the internal radius of the brane be extremely smaller than L . Hence, noting that a tiny value in the round parenthesis of (2.32) would yield to an inconsistent value for $f(\epsilon)$, equation (2.32) implies

$$f(\epsilon) \lesssim 1 \tag{2.35}$$

that inserted into (2.33) yields a fine-tuning relation

$$\varphi^2 \gtrsim \frac{2 - (\sqrt{5/2} - 1) \lambda}{3 - \sqrt{5/2}} \tag{2.36}$$

that can be satisfied, provided

$$\lambda \leq \lambda_M \equiv 2/(\sqrt{5/2} - 1) \tag{2.37}$$

that yields a critical value for the brane tension.

Casting the bulk metric into the form $ds^2 = V^2(\rho)dx_\mu dx^\mu + d\rho^2 + W^2(\rho)d\Omega_2^2$ one can check that there exists an allowed configuration $\bar{\rho}$ at which $W(\rho)$ is critical. For such a value, corresponding to $\bar{r} \sim 2r_*$ the physical radius $W(\rho) = r e^{B(r)}$ assumes its minimum value, after which the bulk radius asymptotically approaches the flat limit $R(r) \sim r$: the shape of the extra-dimensional space thus looks like a "throat" ending on the brane, like depicted in figure 2 and this scenario is somewhat similar to the "near-critical" limit of [29] where the bulk looks like a thin cylindrical sliver that ends up on the brane and opens up non trivially at very large scales. Then gravity on the brane should behave $4d$ at distances shorter than $L_C = \frac{M_4^2}{M_5^3} = \frac{M^{1+n} R^{n-1}}{M^{2+n} R^{n-1}} = L$, then an intermediate $5d$ behavior should take over at distances $\gtrsim L$, till the bulk finally opens up at a scale L' related to \bar{r} , and brane gravity behaves seven-dimensionally, provided the scale at which sources on the brane feel the whole seven-dimensional bulk is larger than L . In other words the bulk scale \bar{r} must be seen as a very large scale from the point of view of an observer on the brane.² However regardless of the specific details of the crossover physics we see that brane tension must be bounded from above at least for this class of smooth solutions. Also, as mentioned above, the form potential may be coupled to a charged particle $eM \int_W A$, where W is the particle worldline (for $n = 4$ it would be a string worldsheet and so on). When W wraps the horizon of the two-sphere, single-valuedness of the amplitude leads to a quantization condition for the flux [48] $eM \int_{S_2} F = 2\pi k$, that yields

$$\varphi R_\epsilon = \frac{k}{2\sqrt{2}eM}, \quad k \in \mathbb{Z} . \tag{2.38}$$

Hence, the above fine-tuning relation (2.36) can be attained only by the portion of tension that is quantized accordingly, and the excess of tension $\delta\lambda \sim \frac{1}{eM}$ seems to either gravitate or blow the internal radius to an unacceptably large value. Notice also that flux conservation due to Bianchi identity sets the conservation of φR_ϵ , similarly to what discussed in [53] for the finite-volume rugby-ball model. In the present setup, unlike what happens in [53], the flux is not fixed in terms of bulk parameters and this would, *a priori*, allow an eventual phase transition that locally changes the value of the brane tension. However, since the internal radius of the brane must locally change in order to "absorb" the different tension and keep the four-dimensional part of the brane flat, this would lead to a scenario where different four-dimensional domains (characterized by different values of tension) have different Planck masses.

²One may worry about the fact that parallel directions are necessarily warped and it may happen that a RS-like localization [57] takes place at those scales. More precisely one might expect an interplay of effects between induced gravity and RS localization, such as the one described in [58]. However it is easy to see that $\text{sign}(V'(\bar{\rho})) = \text{sign}(Q(\epsilon)) > 0$ so that, in the near-brane limit the $5d$ "bulk" behaves as a brane-to-boundary chunk of AdS and the five-dimensional length is thus infinite. It is thus natural to expect that no RS localization takes place.

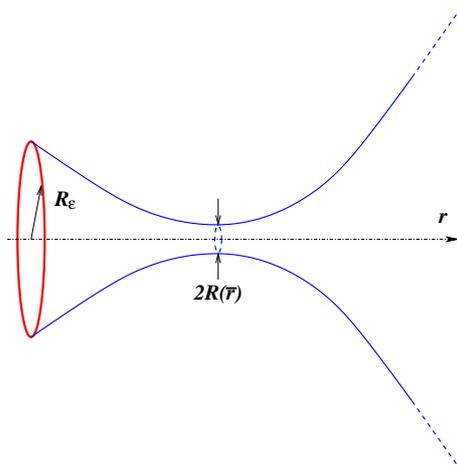


Figure 2. Pictorial representation of the extra-dimensional space.

Another issue concerns the stability of such solutions. Although such important point would require a detailed investigation let us here mention a few related results obtained in the past in similar models. In the absence of localized fluxes, instabilities were indeed found in the models discussed in [6, 23]. However, in [23] it was also shown that localized induced stress tensors of the form (2.23) do indeed lead to a stabilization and such effect is quite likely to take place in the present solutions as well.

To conclude this section, let us briefly mention that, for generic values of parameters, it seems plausible that the higher codimension resolved brane solutions discussed here behave more like the “subcritical” codimension-two cases [29], and the crossover scale from $4d$ gravity and $(4 + n)$ -gravity is expected to be given by $r_C^n \sim \frac{M_4^2}{M^{2+n}}$.

3 Non-vanishing bulk cosmological constant

In this section we consider some finite-volume counterparts of the solutions found in the previous section.³ What follows is to be understood as higher-codimension generalizations of the smooth codimension-two solution described in [27] of which we also use the conventions. The bulk part of the action (2.1) now gets generalized to

$$S_{\text{bulk}} = \widehat{M}^{D-2} \int d^D x \sqrt{-\widehat{g}} \left[\widehat{\mathcal{R}} - \widehat{\Lambda} - \frac{1}{2(p+1)!} \widehat{H}_{[p+1]}^2 \right] \quad (3.1)$$

whereas the brane part remains the same as before. We seek for a solution with spherical symmetry in the extra n -dimensional space and Poincaré invariance in the d parallel directions

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + R^2 \left(d\theta^2 + \cos^2 \theta d\Omega_{(n-1)}^2 \right), \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2} \quad (3.2)$$

³ Higher-codimensional brane solutions with bulk higher-rank tensors were considered in [50] where the regularization consisted in blowing up $n - 2$ directions of the brane, hence reducing its codimension from n to 2. Moreover, in [50], the “dual” form $\widetilde{H}_{[d]}$ was considered, instead of $H_{[n]}$.

and $d\Omega_{(n-1)}^2$ is the line elements of the $(n-1)$ -sphere, explicitly written in (2.24). We locate the smooth brane at a certain value of the azimuthal angle $\bar{\theta}$. In fact in general we might have more branes localized at different angles. For simplicity we will assume \mathbb{Z}_2 symmetry along θ and a pair of identical branes located at $\pm\bar{\theta}$: the symmetry allows us to concentrate only on the northern hemisphere $\theta > 0$. Again, we start considering the case where $p = n - 1$. Similarly to [27] we assume to have an “inside bulk” $\bar{\theta} < \theta < \pi/2$ and an “outside bulk” $0 < \theta < \bar{\theta}$ with different radii, $R_i = R\beta$ and $R_o = R$, and different cosmological constants, Λ_i and Λ_o respectively and we take the magnetic monopole ansatz for the n -form field strength, with

$$\widehat{H}_{[n]} = \begin{cases} \widehat{Q}_i (\beta R)^n S_{[n]}, & \text{inner bulk} \\ \widehat{Q}_o R^n W_{[n]}, & \text{outer bulk} \end{cases} \quad (3.3)$$

with $S_{[n]}$ and $W_{[n]}$ respectively being the volume forms of the unit-radius n -sphere and of the unit-radius wedged n -sphere whose line element is given by $d\theta^2 + \beta^2 \cos^2 \theta d\Omega_{(n-1)}^2$. The bulk equations of motion fix the value of the cosmological constants and magnetic fields in terms of the radii

$$R_a^{-2} = \frac{\widehat{\Lambda}_a}{(n-1)^2} = \frac{\widehat{Q}_a^2}{2(n-1)}, \quad a = i, o. \quad (3.4)$$

It is easy to see, that using (3.4), and redefining coordinates as

$$\theta(l) = \bar{\theta} - \left(\bar{\theta} - \frac{l}{R} \right) \left[\Theta(\bar{\theta}R - l) + \beta^{-1} \Theta(l - \bar{\theta}R) \right] \quad (3.5)$$

$$z^\alpha = \beta R \theta^\alpha, \quad \alpha = 1, \dots, n-1 \quad (3.6)$$

where Θ 's are Heaviside's step functions, the volume forms $(\beta R)^n S_{[n]}$ and $R^n W_{[n]}$ are both given by

$$V_{[n]} = (\cos \theta(l))^{n-1} dl \wedge \Omega_{[n-1]}(z), \quad ds^2 = dl^2 + \cos^2 \theta(l) d\Omega_{(n-1)}^2(z) \quad (3.7)$$

and

$$\widehat{H}_{[n]} = \sqrt{2(n-1)} \theta'(l) V_{[n]} \quad (3.8)$$

and $\theta'(l)$ is discontinuous at the location of the brane $\bar{l} = \bar{\theta}R$. The integration by parts associated to the equation of motion for the potential form field

$$\widehat{\omega}_{[n-1]} = \left[\pm c + f(\theta(l)) \right] \sqrt{2(n-1)} \Omega_{[n-1]}(z), \quad f'(\theta) = \cos^{n-1} \theta \quad (3.9)$$

whose field strength is \widehat{H} , will thus give rise to a jump condition at the location of the brane

$$\begin{aligned} \delta_\omega S(H) &\supseteq -\frac{\widehat{M}^{D-2}}{(n-1)!} \int_M d^D x \sqrt{-\widehat{g}} \nabla_{M_0} \delta \widehat{\omega}_{M_1 \dots M_{n-1}} \widehat{H}^{M_0 \dots M_{n-1}} \\ &= \frac{\widehat{M}^{D-2}}{(n-1)!} \int_{\partial M} d^{D-1} x \sqrt{-g} \delta \omega^{m_1 \dots m_{n-1}} \left\langle \widehat{H}_{l m_1 \dots m_{n-1}} \right\rangle_{\pm} \end{aligned} \quad (3.10)$$

where $\omega_{m_1 \dots m_{n-1}} = \widehat{\omega}_{m_1 \dots m_{n-1}}(\bar{\theta})$, and comes from the "-" branch of (3.9) as the brane sits inside the northern hemisphere. We thus need to ameliorate the $(n-1)$ -form field localized on the brane with the inclusion of a coupling to $\omega_{[n-1]}$, namely

$$\tilde{F}_{[n-1]} = F_{[n-1]} + eM\omega_{[n-1]} \quad (3.11)$$

and

$$\delta_\omega S(\tilde{F}) = -\frac{M^{D-2}}{(n-1)!} \int_\Sigma d^{D-1}x \sqrt{-g} \delta\omega^{m_1 \dots m_{n-1}} e \tilde{F}_{m_1 \dots m_{n-1}}, \quad (3.12)$$

so that

$$\left\langle \widehat{H}_{l m_1 \dots m_{n-1}} \right\rangle_\pm = L M e \tilde{F}_{m_1 \dots m_{n-1}} \quad (3.13)$$

is the jump condition for the form field, with

$$\left\langle \widehat{H}_{l \alpha_1 \dots \alpha_{n-1}} \right\rangle_\pm = \sqrt{2(n-1)} \frac{1-\beta}{R\beta} \cos^{n-1} \bar{\theta} \sqrt{\Omega} \epsilon_{\alpha_1 \dots \alpha_{n-1}}. \quad (3.14)$$

For the metric we have (2.6) instead, that using (3.7), simply yields the following non-vanishing components for the extrinsic curvature $K_{\alpha\beta}^\pm = \pm \frac{1}{2} \partial_l g_{\alpha\beta} = \mp \theta'(\bar{l} \pm) \tan \bar{\theta} g_{\alpha\beta}$. Then, choosing the brane magnetic field to be

$$F_{[n-1]} = \Phi \cos^{n-1} \bar{\theta} \Omega_{[n-1]} \quad \Rightarrow \quad \tilde{F}_{[n-1]} = \tilde{\Phi} \cos^{n-1} \bar{\theta} \Omega_{[n-1]} \quad (3.15)$$

with $\tilde{\Phi} = \Phi + eM \sqrt{2(n-1)} (f(\bar{\theta}) - c) (\cos \bar{\theta})^{1-n}$, we have the following junction conditions

$$\Lambda - \frac{1}{2} \tilde{\Phi}^2 = \frac{2(n-2)}{L} \frac{1-\beta}{R\beta} \tan \bar{\theta} \quad (3.16)$$

$$\Lambda + \frac{1}{2} \tilde{\Phi}^2 = \frac{2(n-1)}{L} \frac{1-\beta}{R\beta} \tan \bar{\theta} \quad (3.17)$$

$$e\tilde{\Phi} = \frac{\sqrt{2(n-1)}}{LM} \frac{1-\beta}{R\beta}. \quad (3.18)$$

where (3.16), (3.17) are the $(\alpha\beta)$ and $(\mu\nu)$ components of the junction condition for the metric and (3.18) is the junction condition for the form field. They can be solved to give

$$\Lambda = \frac{2n-3}{L} \frac{1-\beta}{R\beta} \tan \bar{\theta} = \frac{2n-3}{2} \tilde{\Phi}^2 \quad (3.19)$$

$$\tilde{\Phi} = \frac{2eM}{\sqrt{2(n-1)}} \tan \bar{\theta} \quad (3.20)$$

so that a brane of arbitrary tension can be accommodated on such a setup while maintaining $4d$ -Poincaré invariance. A few observations are in order. First, it is easy to see that, contrarily to what happens in [27], for fixed $4d$ vacuum energy density one cannot take the thin limit $\bar{\theta} \rightarrow \pi/2$. The vacuum energy density can be simply obtained from the integral of the l.h.s. of (3.17) over the internal profile of the brane. Up to irrelevant numerical constants it reads

$$T \sim \widehat{M}^{D-2} R^{n-2} (\beta \cos \bar{\theta})^{n-2} \sin \bar{\theta} (1-\beta). \quad (3.21)$$

For $n = 2$ one recovers the result of [11, 27]. For $n > 2$, holding T fixed, the aforementioned limit is impossible as β is bounded from above. In other words $T \rightarrow 0$ for $\bar{\theta} \rightarrow \frac{\pi}{2}$; it is thus difficult to imagine how to extend the approach of [40–45] to codimension higher than two, at least within this class of spherically symmetric regularizations. Also, coupling of the form fields to extended objects leads to quantization conditions [48, 50–52] for the fluxes that in turn yield a quantization condition for the brane tension.

Let us conclude by mentioning possible extensions of the latter solutions to the case of negative bulk cosmological constant. It is obvious that an unwarped solution like (3.2) with an internal AdS is prohibited by Maldacena-Nunez no-go theorem [54–56]. However, at least partial way-outs seem possible if, for instance, one allows the extra space to be non-compact. In fact, let us start from

$$ds^2 = R^2 \left(\frac{d\xi^2}{\xi^2} + \xi^2 \eta_{\mu\nu} dx^\mu dx^\nu \right) + \delta_{ab} dz^a dz^b, \quad z^a \cong z^a + 2\pi l^a \quad (3.22)$$

where the $(n-1)$ -torus parameterized by z^a is the internal profile of the brane localized at $\xi_0 = 1$. Bulk equation of motion in presence of negative cosmological constant and fluxes yields a similar fine-tuning condition like the one given in (3.4). Taking for simplicity a Z_2 -symmetry and setting $\xi = 1 + \epsilon|u|$ it is easy to see that (with the exception of codimension one, where there is no bulk flux and reduces to the RS2 model [57]) finite transverse volume ($\epsilon = -1$) implies positive brane tension, $\Lambda > 0$ but negative localized flux, $\tilde{\Phi}^2 < 0$, whereas infinite volume ($\epsilon = +1$) implies negative brane tension, $\Lambda < 0$ and positive flux, $\tilde{\Phi}^2 > 0$.

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