THE USE OF CONCRETE ARTEFACTS IN GEOMETRY TEACHER EDUCATION FOR SECONDARY SCHOOLS

Maria G. Bartolini Bussi[1], Francesca Martignone[2]

[1]Department of Education and Human Sciences – UNIMORE (Italy)

[2]Department of Science and Technology Innovation – UNIPMN (Italy)[i]

This presentation deals with the use of some concrete geometry artefacts (called Mathematical Machine)[ii] for the purpose of drawing curves and realizing geometric transformations within the MMLab-ER project developed by UNIMORE. In the ancient Greece, at the time of Euclid, some concrete artefacts (such as the straightedge and compass) were used in both practical and theoretical geometry. Other artefacts were known in the ancient age and were considered again by the most important European mathematicians as from the 16th century. This presentation reports today's use of working copies of those instruments (complementary to dynamic geometry system) in secondary school teacher education and development for the purpose of realizing laboratory activities in their own classrooms.

INTRODUCTION

The MMLab-ER project is being conducted by the Mathematical Machines Laboratory, of the University of Modena and Reggio Emilia, Italy[iii]. The Laboratory is a research centre for the teaching and learning of mathematics by means of instruments. The name comes from the most important collection of the Laboratory, the Mathematical Machines, which are working reconstructions of mathematical instruments taken from the history of mathematics. Most of the machines concern geometry: "a mathematical (geometrical) machine is a tool that forces a point to follow a trajectory or to be transformed according to a given law" (Bartolini Bussi & Maschietto, 2008). The simplest mathematical machine is the pair of compasses; there are also more complex curve-drawing devices and pantographs to represent geometric transformations.

The MMLab-ER project was founded in 2008 by the Emilia Romagna Region, in agreement with the Regional School Office. The project was designed to facilitate the implementation of a laboratory approach in the teaching and learning of mathematics, focusing mainly, but not exclusively, on geometry. The goal of the project was agreed by the steering committee, in which researchers, teachers, policy makers and school administrators were represented. This goal was twofold:

• to create a network of mathematical laboratories in different provinces of the Emilia-Romagna region with a selected collection of mathematical machines;

• to prepare a network of local groups of teachers to be able to implement a laboratory approach in their classrooms.

The participating teachers, who were selected by school principals, were mainly from secondary schools (from grade 6 to grade 13, that is, students aged 11-18 years). The pedagogical focus of the project was the introduction of the laboratory approach, the use of historical machines, and the analysis of exploration and proving processes (Antonini & Martignone, 2011). More information about this project are available at (Bartolini Bussi & Martignone, 2013).

The tasks faced by teachers during the MMLab-ER educational program were similar to classroom tasks, but designed for teachers. The tasks sequence focused on the proving processes, from conjecture and arguments production to proof construction. Teachers knew that these processes are fundamental in mathematics, but often found difficult to implement them in the classroom. In the following, we shall report about an example of exploration of a mathematical machines, as it was realized in the teacher education program.

AN EXAMPLE OF LABORATORY ACTIVITY WITH THE SCHEINER'S PANTOGRAPH

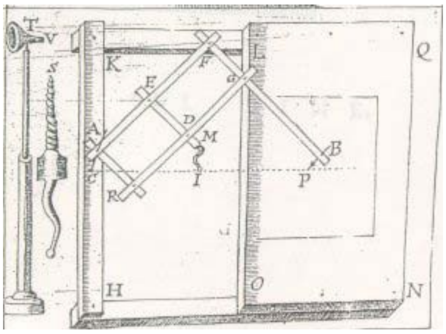


Fig.1: An historical picture of Scheiner's pantograph

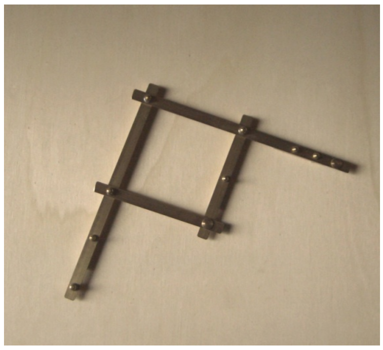


Fig.2: Photograph of MMLab-ER pantograph reconstruction

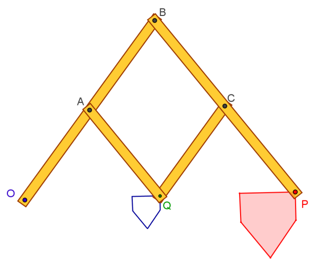


Fig.3: a virtual copy

One of the oldest artefacts for making geometrical transformation is the articulated parallelogram, that has been used in the past and is used also today, for drawing or engraving proportional figures. Since the end of the '500 this type of artefacts was practically used by painters and described by Scheiner in 1631 but the mathematical theory incorporated in the artefact was understood only in the ‘800 when geometric transformation theory developed, as the pantograph realizes a plane homothethy

In the development program, the teachers studied the history of these articulated systems and also explored and analysed their working reconstruction (figures 1, 2, 3). During the laboratory sessions, the teachers, divided in small groups, examined the pantographs following a *key questions script*. This script guides and scaffolds the processes of exploration, production of arguments and construction of proof about what the artefact does and why it does that. It is worthwhile to observe that the script is very similar to the one used in pre-school and primary school for the exploration of a whichever artefact (Bartolini Bussi, these proceedings) as both projects are framed by the theory of semiotic mediation after a Vygotskian approach (Bartolini Bussi & Mariotti, 2008).

Task 1: How is the artefact made?

This question focuses the attention on the pantograph physical components that is hence analyzed as an *artefact*, that is an object designed for answering a specific need (Rabardel, 1995). For example, what is in the foreground is the length of rods, the presence of pivot points and of fixed points. During the artefact examination there are two intertwined levels linked respectively to physical and to geometrical aspects. For instance, the teachers said: "the rods form a rhombus" and “if I imagine a line between the fixed point and the tracer points, I can see two equal triangles”. The teachers explored also the possible movements of the articulated system configurations and identified the region reachable by the tracer points. A common language to identify the components and characteristics of the artefact was created and shared: e.g. constraints, degrees of freedom of points, limit configurations, etc.

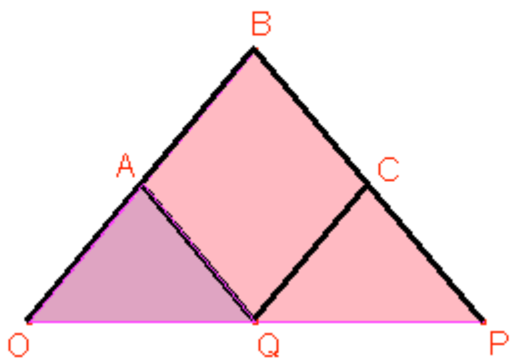
Task 2: What does the artefact do?

This task shifts the focus from the pantograph as *artefact* to the pantograph as *instrument* (Rabardel, 1995), that is the hybrid entity with an artefact type component and a cognitive component, called *utilization schemes* (Martignone & Antonini, 2009). These activities lead to the generation of conjectures about how the machine works and about the implemented mathematical law. In the production of conjecture about what the artefact does teachers analysed the relationship between the tracer points (see the fig. 3: "if it moves P, Q also moves" or " if the first tracer goes on a little bit, the other goes on the double") and identified the invariant properties of drawn figures (“this figure is an enlargement of the other", “They are similar!”). At this stage teachers developed the skill linked to the management of the artefact and analysed the possible products: for example they discussed the role of the choice of figure type (the figure to be enlarged or reduced) and position for the identification of the transformation proprieties.

Task 3: Why does it do that?

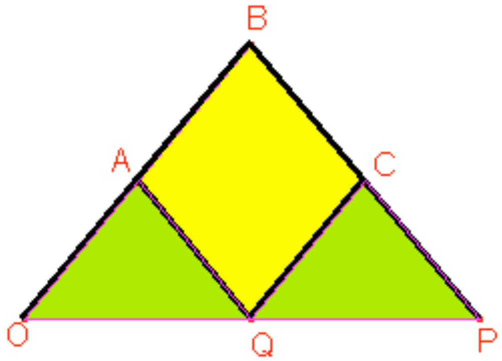
The question prompts the production of arguments for supporting the hypotheses produced about the geometric transformation realized by the pantograph. The arguments may be based on physical and mechanical considerations linked to the machine structure and movement, or on the identification of mathematical properties incorporated in the machine; as a matter of fact, the arguments may be different: "if I move one tracer also the other one moves, but twice because the distance from the fixed point is twice”. It is important to move quickly from physical arguments to the mathematical properties.

During the pantograph analysis the teachers found elements that characterize the structure: i.e. "the rods remain parallel during movement", "the rods form a rhombus." Moreover they identified the figures obtained by completing parts of the structure (additional elements): "we recognize two equal triangles" or "it's as if there were a large isosceles triangle and two similar ones" "connecting a tracer and the fixed point the other tracer is one the same straight line". These topics are crucial for the subsequent proof construction, in particular for showing that during the articulated system movements the tracer points remain aligned and with a proportional distance from the fixed point. Teachers constructed different proofs in order to validate their hypothesis about what the machine does. Examples follow.



OB=BP=2AQ, AQ=CQ and A and C are pivoted in the midpoints respectively of OB and BP. The isosceles triangle OBP is similar to the isosceles triangle OAQ because their angles are congruent (parallel lines intercepted by an intersecting line).The ratio is 1:2. In all the possible articulated system configurations the points O, P and Q remain aligned because the angle BOQ is equal to the angle BOP (correspondent angles of similar triangles).

Fig. 4: the first proof



In the pantograph OA=AB=BC=CP=AQ=QC. We can identify two isosceles triangles OAQ and QCP, which are congruent according to SAS (Side-Angle-Side) condition: OA = QA = QC = CP and the angles OAQ and QCP are congruent (AQ is parallel to BP and OB is parallel to CQ, because AQCB is a rhombus). For these reasons OQ=PQ and therefore the ratio OP/OQ is constant (1:2). In addition, O, P and Q are aligned because OQA + AQC + PQC is a straight angle congruent to the sum of the internal angles of the triangle OAQ: in fact CQP = OQA=AOQ and AQC = QAO.

Fig. 5: the second proof

Some crucial aspects emerged: the proof that the three points (O, P, Q) are aligned and the identification of the ratio of proportionality related to the articulated system structure. In the first proof the teachers work on similar triangles OAQ and OBP: it is usually produced by teachers who, during the examination of the machine, focus on the similar isosceles triangles (OAQ, OBP) visualized connecting the points O, Q and P. The core of the second proof is the identification of different figures in the structure of the machine: that is, the rhombus (AQCB) and congruent triangles (OQA and QPC).

The collective discussion following the production of these and other proofs dealt with the links between the figures and proprieties identified during the artefact examination and the arguments that support the proving chains. In this way teachers realized the close connection between the processes of conjectures, arguments production and the following proof construction. In particular *cognitive unity* (Mariotti, 1996) was in the foreground with continuity (and possible break) between the conjecture production and the proof construction.

During the teacher education program, and then also in the classroom teaching experiments, much time was devoted to think over the relationship between the conjectures genesis and the development of the argumentation processes because, as argued abovehs, the focus is on process, rather than only on the final mathematical product.

Task 3: What would happen if …?

The further task is focused on problem posing and solving activities. This task fosters the solution of open-ended problems and connects problem posing and problem solving (Watson & Sullivan, 2008). Teachers were asked to explore the variations of the parts of the artefact and the consequent variation of the parameters defining the homothety: *Is it possible to modify the artefact in order to get a ratio of 1:3? What could happen exchanging the fixed point O with the point Q in the fig. 3*?

At first teachers were asked to figure out how to change the pantograph structure, with regards to the figure formed by the articulated system rods, maintaining the geometric transformation implemented with a different ratio. In order to solve this problem teachers had to clearly understand how the transformation properties are embodied in the artefact structure and to be able to modify the artefact maintaining the main features of the transformation: the fixed and tracers points remain aligned and at a proportional distance during the movement. (Fig. 6).

In the second question the structure of the articulated system remained the same and another parameter is changed: the location of the fixed point (D instead of O). This artefact implements an homothety of ratio -1: the central symmetry. (Fig. 7)

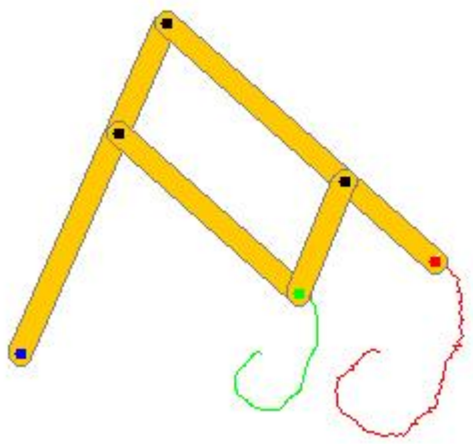


Fig. 6: Homothety of ratio 1:3

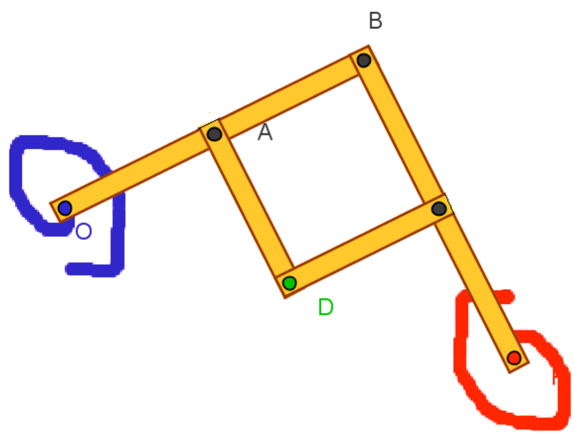


Fig. 7: Central symmetry

Teachers imagined the changes and then created the modified artefacts with dynamic geometry software. At last a Cartesian coordinate system was introduced in order to identify the equations of the transformations. Also the activities of finding the equation of a given law represented on plane (and not vice versa) were discussed with teachers highlighting the educational potential of the use of different theoretical tools and different processes to study the same mathematical objects.

DISCUSSION

The example about Scheiner's pantograph exploration shows how the teachers recognized and analysed the propriety of the homothety in a not conventional environment focusing on processes. The artefact was analysed first in a cultural perspective, as the teachers studied the historical developments of geometrical transformations exploring an instrument used in the history for different purposes, both practical and theoretical ones. Later, teachers manipulated the mathematical artefact and discovered the mathematical law incorporated. It is worthwhile to observe that their exploration was not much different from the one realized later by their students in the classroom experiments. They both were guided by specific tasks that focus on the development of mathematical processes, as problem posing and solving, production of conjectures and argumentation and proof construction. A strong attention was given to the proof construction and to the processes that lead to it. Despite the undeniable differences between "deductive organization of thinking" and "argumentative organization of thinking" (Duval, 1991), there is a fundamental link between the production, during the conjecturing phase, of the arguments that will be used later during the proving phase (Mariotti, 2006). In the example of Scheiner's pantograph, the first questions (*How is the artefact made? What does the artefact do*?) refer to different elements of the pantographs (the structure, the movement, the drawing traced by the artefact) and to different components (figural and conceptual) of the geometrical figures representing the linkages (Antonini & Martignone, 2011). The other questions exploit again the above elements. The argumentations that justify why the pantograph does a homothety are closely related to three elements: the drawings traced by the artefact, the structure of the artefact, and the artefact movements. The drawing help teachers to recognize the geometrical transformation and its proprieties, but in order to proof why the artefacts does that, they have to use arguments linked to the articulated system. In the case of argumentations referring to the artefact structure, there is cognitive unity between argumentation and proof: the arguments about the congruence or similarity of the triangle are used in the proof construction. On the other hand, the argumentations based on movement lead to further argumentations that explain the motion through the structure of the articulated system, and a cognitive unity may or may not occur. These types of arguments about the points movement and the relationship between different coordinated movements are not suitable for the proof constructions in the Euclidean geometry framework. The relationship between the tracers movements can be better studied using analytic geometry tools: for example choosing a Cartesian coordinate system and defining the homothety function.

As the Scheiner's pantograph played a role in the history of transformation geometry in the West, the delicate issue of *cultural transposition* comes to the foreground (Bartolini Bussi & Martignone, 2013): first, cultural artefacts may reveal valuable information about the society that made or used them; second, the exploration of the artefact draws on very deep systems of beliefs about activity in the mathematics classroom. Does is make sense to transpose this activity to other cultures?

References –

Antonini, S. & Martignone, F. (2011). Argumentation in exploring mathematical machines: a study on pantographs, *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education*, (Vol. 2, pp. 41-48), Ankara, Turkey.

Bartolini Bussi M. G. (2015), The use of concrete artefacts for calculations: a semiotic approach. these proceedings

Bartolini Bussi M. G & Martignone F. (2013), Cultural issues in the communication of research on mathematics education, For the Learning of Mathematics, 33 (1), 32-8.FLM

Bartolini Bussi M. G. & Mariotti M.A. (2008). Semiotic Mediation in the Mathematics Classroom: Artefacts and Signs after a Vygotskian Perspective. In: L. English, M. Bartolini, G. Jones, R. Lesh, B. Sriraman, D. Tirosh. *Handbook of International research in Mathematics education (2nd edition)*. p. 746-783, New York: Routledge Taylor & Francis Group,

Bartolini Bussi M. G. & Maschietto M. (2008). Machines as tools in teacher education. In Wood, B. Jaworski, K. Krainer, P. Sullivan, D. Tirosh (eds.), *International Handbook of Mathematics Teacher Education*, vol. 2, Rotterdam: Sense Publisher

Duval, R.(1992-93). Argumenter, demontrer, expliquer: continuité ou rupture cognitive?, Petit x 31, pp.37-61.

Mariotti A. (2006). Proof and proving in mathematics education. In A. Gutiérrez & P. Boero (Eds.), *Handbook of research on the Psyc. of Math. Educ. Past, present and future*, pp. 173-204. Rotterdam, Holland: SensePublishers.

Martignone, F., & Antonini, S. (2009). Exploring the mathematical machines for geometrical transformations: a cognitive analysis. In Tzekaki, M., Kaldrimidou, M., & Sakonidis, H. (Eds.), *Proceedings 33th Conf. of the Int. Group for the Psychology of Mathematics Education*. (Vol. 4, 105-112). Thessaloniki, Greece: PME.

Rabardel, P. (1995), *Les hommes et les technologies - Approche cognitive des instruments contemporains*. A. Colin, Paris.

Watson, A. & Sullivan, P. (2008). Teachers Learning about Tasks and Lessons. In Sullivan P. & Wood, T. (Eds.), *The International Handbook of Mathematics Teacher Education*. (Vol. 2, pp. 109-134). Purdue University, West Lafayette, USA: SensePublishers.

---

[i] The first author was the principal investigator. The second author was the responsible for teacher education program.

[ii] The word artefact is generally used in a very general way and encompasses oral and written forms of language, texts, physical tools used in the history of arithmetic (e.g., abaci and mechanical calculators) and geometry (e.g., straightedge and compasses), tools from ICT, manipulatives, and so on. In this paper the term artefact is used after Rabardel (1995) and distinguished from instruments.

[iii] www.mmlab.unimore.it