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A dialogue between cultures about task design for primary school / Bartolini, Maria Giuseppina; Xuhua, Sun; Ramploud, Alessandro. - ELETTRONICO. - (2013), pp. 551-560. (Intervento presentato al convegno Task Design in Mathematics Education tenutosi a Oxford nel July 22nd-26th 2013).

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09/04/2024 15:33

## A dialogue between cultures about task design for primary school

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**ABSTRACT.** The aim of this paper is to exploit intercultural dialogue to analyze two cases of task design (or, better, *re-design*) for word problems in different cultural traditions (the Eastern one, within the Confucian Heritage Culture, CHC, represented here by China and the Western one, represented here by Italy). By means of two paradigmatic examples, one developed in Italy and one in China, we aim at showing, on the one hand, the effects and advantages of intercultural dialogue and, on the other hand, the need to take into account and to respect culturally rooted pedagogies, avoiding uncritical transfer from one culture to another.

**Keywords:** word problems, problems with variation, primary school, intercultural dialogue, addition/subtraction

### Introduction.

Word problems are a special kind of tasks, presented all over the world, in mathematics textbooks to link numbers to real life objects and situations. They are the heirs to ancient traditions of mathematical texts, existing in all cultures. In this paper, we consider, as a paradigmatic case, *problems for early childhood classroom to be solved by addition and/or subtraction*, although some observations and examples are more general.

After reviewing some literature about word problems in mathematics education in China and in the West, we present a special kind of word problem taken from Chinese textbooks, i.e. *problems with variation* (*biànshì problems*).

Then we illustrate how the meeting with the Chinese tradition has suggested to a group of Italian researchers (including primary school teachers) to *re-design* (and test in the classrooms) a rich system of problems with variation. On the one hand this redesign is in order to meet needs emerging from Italian school practice, , and to tailor classroom activity to their system of beliefs, on the other hand. Conversely, the meeting with the Western tradition has modified some principles in the Chinese approach to problems with variation, enriching the original concept based structure and offering new criteria for task *re-design*.

Closing remarks summarize authors' contributions to the following questions (concerning themes C and E of the Study):

- C1) How do curriculum expectations influence authors' design principles?
- C2) How does an intention to promote change influence design?
- C3) How do cultural considerations about instruction and pedagogy influence design?
- E1) How does collaboration between teachers, or between researchers and teachers, influence design of tasks/sequences?
- E2) What is the effect of different cultural backgrounds on teachers' knowledge or belief on tasks and task design?

## Some literature about word problems.

### *A short review of Chinese literature*

Taoism has had profound influence on Chinese culture. The central Taoism idea of the evolution of events as a change process and the acceptance of the inevitability of change reveal the ideologies of “grasping ways beyond categories”; “categorize in order to unite categories” (以法通類, 以類相從). In China, for 5000 years, mathematics knowledge was elicited by word problems, which stems from the “Shu” (術) spirit (similar to “general methods”) in the problem-oriented tradition from Oriental mathematics, “... to produce new methods from word problems, promote them up to the level of general method, generalize them into ‘Shu,’ and deploy these ‘Shu’ to solve various similar problems which are more complicated, more important, and more abstruse” (Wu & Li, 1998). Impacted by the idea of “grasping ways beyond categories”; “categorize in order to unite categories”, word problems in ancient China were organized into different categories in terms of situations or algorithms. For example, *Jiǔzhāng Suànshù* (九章算術), the most classic literature of Chinese mathematics, used 246 word problems to spread mathematical knowledge.

The tradition of categorizing word problems did not go on when Chinese curriculum on mathematics knowledge was totally borrowed from the West in 1878, where word problems, labelled as “application problems” (應用題), played a role of knowledge application, not introducing original knowledge. In 1929, the first formal curriculum standard, since new China was founded, primary school arithmetic syllabus (draft) (《小学算术教学大纲(草案)》), claimed that “application problems are the important part and should be covered about half of the arithmetic contents” (Wang, 1996), which has remained unchanged as basic requirements of elementary curriculum for 80 years at least. In 1952, application problems in the first official textbook for whole country were categorized into simple application problems and complex application problems. Since 1958, application problems have not been organized by their categories but by their application content for the reason that knowledge system is required as a curriculum framework. Although application problems in textbooks were not organized by their categories again, their teaching organization stressed not a single problem but a group of problems with variation.

Following the thousands-years-traditions “categorize in order to unite categories”, one distinctive instruction feature of word problems is to develop the ability to identify the category of word problems (识类) belonging to and discern different categories (归类), namely, discern the invariant elements from the variant elements between problems and recognize the “class” every problem belong to. This pedagogy is generally called as *biànshì* (变式) in Chinese, where “biàn” stands for “changing” and “shì” means “form”, can be translated loosely as “variation” in English (Sun, 2011). Some categories of *biànshì* are the following (examples follow): OPMS (*One Problem Multiple Solutions*), where, for instance, the operation to solve the problem is carried out in different ways, with different grouping and ungrouping:

$$8 + 9 = (8+2) + 7; 8 + 9 = 7 + (1 + 9) \text{ and so on.}$$

OPMC (*One Problem Multiple Changes*), where in the same situation some changes are introduced.

MPOS (*Multiple Problem One Solution*), where the same operation can be used to solve different problems, as in summary exercises (Sun, 2011).

To sum up, in Chinese mathematics education problem variations aim to discern, compare the invariant feature of the relationship among concepts and

solutions and provide opportunities for making connections, since comparison is considered a pre-condition to perceive the structures, dependencies, and relationships that may lead to mathematical abstraction (Sun, Wong, & Lam, 2007).

### *A short review of Western literature*

Different strands (and a different pedagogy) emerge in the literature on word problems. In this paper we mention only some strands.

The *cognitive analysis* of word problems (see different contribution in Carpenter, Moser, & Romberg, 1982) focuses on the difficulties met by students in understanding the problem and looking for effective solution strategies and has produced well established categorizations likely to be employed in educational setting (e.g., the well agreed categorization of additive problems into *combine*, *change* and *compare*).

The *didactical analysis* of word problems (see different contributions in the recent volume by Verschaffel et al., 2009) criticizes the stereotyped and not realistic features, highlights the distance with modelling activity and focuses on the negative effects of word problems on students' sense making capabilities. Actually, there are examples of word problems which suggest uncritical applications of rules. There is the famous case (ibidem, p. xii) concerning the age of captain: "On a boat there are 20 sheep and 6 goats. How old is the captain?" The findings show that many students would respond to such question by adding the numbers of sheep and goats.

Some literature on word problems is strongly related to the early development of algebraic reasoning that may build a bridge between the two cultures (e.g. Ofir & Arcavi, 1992, Cai & Moyer, 2008).

### **Examples of Chinese problems with variation**

#### *The introduction of subtraction in the first grade by problems with variation*

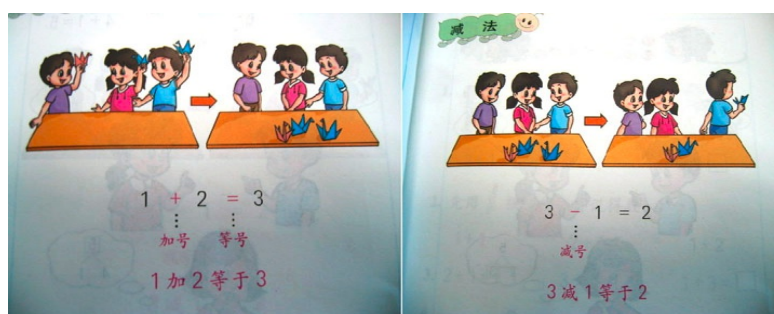


Fig 1. Mathematics Textbook Developer Group for Elementary School, 2005, vol. 1

Chinese textbook authors never separated the subtraction concept from addition.

Whenever there is addition there is subtraction (Yang Hui, 1274, in Siu, 2004, p. 164).

Figure 1 shows a paradigmatic example of problem variation: Xiao Ming folds a pink paper crane; xiao li and xiao hua fold two blue paper cranes. How many paper cranes do they fold? There are 3 paper cranes. Xiao ming takes a paper crane. How many paper cranes does he leave? The answers are:  $1+2=3$ ,  $2+1=3$ , and  $3-1=2$ . The drawing intends to help learners to recapitulate the relationship of addition and subtraction, and the meaning of "equal" from the problem set  $1+2=3$ ,  $2+1=3$ ,  $3-1=2$ . The problem

sets hinges on exemplifying relationships rather than objects and reflects the mathematical structure underlying the problems in this respect. The addition concept is different from that of subtraction, which belongs to a different category. Yet in the sense of part-part-whole we can combine these two concepts into one category and understand the ancient idea of “grasping ways beyond categories”; “categorize in order to unite categories.

***A summary system of problems with variation in second grade***

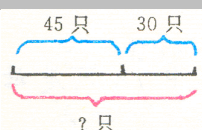
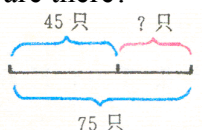
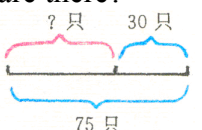
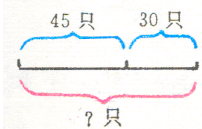
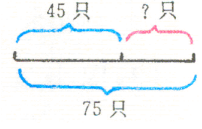
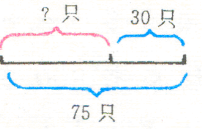
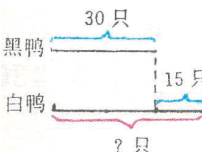
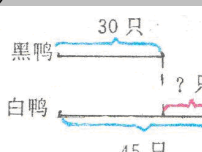
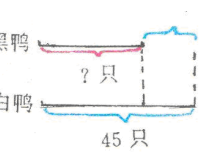
First solve the nine problems below. Then explain why they have been arranged in rows and columns in this way, finding relationships		
<p><b>(1) In the river there are 45 white ducks and 30 black ducks. All together how many ducks are there?</b></p> 	<p><b>(2) In the river there are white ducks and black ducks. All together there are 75 ducks. 45 are white ducks. How many black ducks are there?</b></p> 	<p><b>(3) In the river there are white ducks and black ducks. All together there are 75 ducks. 30 are black ducks. How many white ducks are there?</b></p> 
<p><b>(1) In the river there is a group of ducks. 30 ducks swim away. 45 ducks are still there. How many ducks are in the group (at the beginning)?</b></p> 	<p><b>(2) In the river there are 75 ducks. Some ducks swim away. There are still 45 ducks. How many ducks have swum away?</b></p> 	<p><b>(3) In the river there are 75 ducks. 30 ducks swim away. How many ducks are still there?</b></p> 
<p><b>(1) In the river there are 30 black ducks. White ducks are 15 more than black ducks (black ducks are 15 less than white ducks). How many white ducks are there?</b></p> 	<p><b>(2) In the river there are 30 black ducks and 45 white ducks. How many white ducks more than black ducks (How many black ducks less than white ducks)?</b></p> 	<p><b>(3) In the river there are 45 white ducks. Black ducks are 15 less than white ducks (white ducks are 15 more than black ducks). How many black ducks are there?</b></p> 

Table 1. Beijing education science research institute and Beijing instruction research center for basic education (1996), vol. 4, p. 88.

This is a system of nine problems concerning addition and subtraction, where the organization in rows refers to the already mentioned combine, change, compare categorization and the organization in column refers to the same arithmetic operation (either addition or subtraction, see MPOS above). In each row there is a problem (in the shaded cell) and two variations (see OPMC above). It is taken from a Chinese second grade textbook. The task is very complex and requires the students not only to

solve each problem but also to explain why the nine problems have been arranged in this way. Each problem is associated with a graphic scheme, that models on one or two lines the relationship between quantities. Such graphic schemes for both additive and multiplicative problems had been introduced systematically by Davydov (1966) in his algebraic approach to quantitative reasoning and relationships and are used in some countries of the CHC area, such as mainland China, Japan, Singapore. In a study carried out in Italy, we first used it in teacher education as a prompt to challenge teachers' beliefs (Bartolini Bussi et al., 2011; Bartolini Bussi et al., 2012); later a re-designed task was proposed to Italian students (several experiments from grade 2 to grade 5) in order to foster the approach to algebraic reasoning as soon as possible.

## **Task re-design in Italy and Hong Kong: hint at two case studies**

### ***A transposition of problems with variation in Italy***

#### *Needs in school practice*

In Italy, in spite of the different suggestions of the Standards and Programs (<http://www.mathunion.org/icmi/other-activities/database-project/introduction/italy/>) where the focus is rather on the sense making of a situation and on modelling (*mathematization*), it is very popular strategy among teachers to suggest “cues” in the problem text in order to detect the operation to be used. For instance, in a popular website (<http://www.lannaronca.it/>), for additive problems, teachers are instructed to invite students to underline words like “aggiungere” (adding), “in tutto” (in all), for addition, and words like “togliere” (remove), “restare” (remain), “in più” (more), “in meno” (less) and so on for subtraction. Without careful control, this cue might lead randomly to either right or wrong choices. Consider for instance the two texts with the same implicit question: “how many candies does John have?”.

“John has 5 candies and Anna gives him 2 more candies”

“Anna has 7 candies, 2 more than John”.

In spite of the same question and of the same cue, in the former case the direct reference is to addition and in the latter to subtraction. The confusion is reinforced by the fact that in Italian “più” is also the wording of “+” (plus), that is the special sign for addition. The two situations might be related to each other highlighting unknowns:

$$5 + 2 = ? \text{ and } 7 = ? + 2.$$

but teachers are not encouraged to link addition and subtraction. They are rather encouraged to follow the principle one-thing-at-a-time and to practice addition for months before introducing subtraction. A very popular series of exercise books (<http://www.erickson.it/Ricerca/Pagine/Results.aspx?k=matematicaimparo&start1=1>, *Matematicaimparo*) contains even two different booklets (with different authors), one for addition and one for subtraction problems. The above practice has the effect to produce very poor performances in the solution of arithmetic problems in the national assessment carried out at the end of the second and fifth grades.

#### *Task re-design*

Re-design concerns the task of the Table 1. Teachers-researchers who have collaborated in the pilot study have not implemented the same Chinese task, but have *re-designed* it to tailor it to the Italian tradition and to their individual teaching styles and systems of beliefs.

Three main changes were introduced:

- 1) the single task has been transformed into a set of several tasks;

2) classroom work was organized according to a sequence inspired by the theoretical framework of *semiotic mediation after a Vygotskian approach* (Bartolini Bussi & Mariotti, 2008):

a) individual or small group solution of each row of problems followed by the invention of three problems similar to the given ones, to foster the awareness of the problem structure;

b) collective discussion of the findings, with teacher's orchestration.

Moreover the solving graphic schemes (at the beginning) were removed and introduced later, after thorough exploration and solution of the problems, as students were not familiar with such schemes. In this way the use of a graphic scheme was acknowledged by students as meaningful and not perceived as an automatic answer to a given task. In this way the task (originally developed in China within a teacher centered and textbook centered tradition) was modified to fit a dialogic approach where teaching and learning are considered the two sides of the same collaborative process.

In the same way, multiplicative problems with variation were introduced as from the third grade in several classrooms. (Bartolini Bussi et al., 2011). The project is in progress and involves nearly 100 teachers from September 2012.

### ***A modified approach to problems with variation in Hong Kong***

#### *Needs in school practice*

Inspired by the Western tradition of phenomenography and pedagogy of variation (Marton & Booth, 1997) and theory on the process of mathematization (National Council of Teachers of Mathematics, 1989), Chinese researchers found that a focus on dealing with concepts and deriving a structure among these concepts is not sufficient; there is also a need to pay attention to two important processes of mathematization, *induction from real life contexts and application to different contexts*, that were introduced into the tradition of *biànshì* problems.

#### *Task re-design*

In a structured set of *biànshì* problems two types of “contextual variation” were proposed “inductive *biànshì*” and “application *biànshì*”. These two types of variation should play a role to extend and enrich the use of mathematic (structural) variation by appropriating relevant real life contexts. The use of inductive *biànshì* begins with a real life context in which the established concepts are carefully embedded and unfolded in a set of problems that leads to the new concept to be established (the deepening *biànshì*); and using application *biànshì* provides new contexts (even created by students) for students to connect (or apply) different acquired concepts (the widening *biànshì*). Examples associated with this approach to the division of fractions are reported by Sun (2007) and Wong, Lam, Sun, & Chan (2009).

### **Closing remarks**

The two cases of the last section have been developed independently by the authors in Italy and Hong Kong before having the occasion to discuss with each other. In both cases a task re-design had been started to answer needs coming from school practice and from theoretical and pedagogical backgrounds, on the one hand, and prompts from the other cultural tradition, on the other hand. In particular, in both cases the



needs for change (questions C1 and C2) were in the foreground, because of the awareness that existing practice did not meet the intended curriculum expectations. In both countries (question E1) collaborations between researchers and teachers were realized, although with different modalities (see below).

In general, the Chinese literature on word problems as tasks is focused on *teaching*, e.g. the preparation of good textbooks according to the intended curriculum and the study of effective lessons which are structured around textbooks, whilst the Western literature on word problems as tasks is focused on *learning*, i. e. the analysis of difficulties met by students to interpret the task or to make sense of the problem situations in mathematics. These differences have effects on the researchers' and teachers' conception of task design (and re-design). For instance, Chinese teachers generally have limited space to re-design tasks due to the fact that Chinese curriculum evaluation (exam) is unified by government and curriculum content is required to follow strictly the unified standards and the unified textbooks. In Italy teacher-researchers have more freedom to design tasks and to devote some sessions to pilot teaching experiments like the ones mentioned in this paper. In both cases, the culturally rooted pedagogies were essential to re-design tasks: in Italy teachers disassembled Chinese tasks to introduce collective discussions orchestrated by the teacher; in China induction from real life contexts was put within the variation scheme.

The questions C3 and E2 are surely the trickiest to be addressed. We give some elements to support this claim, although it is not the scope of this paper to address the general comparison of cultural traditions that encompasses the issue of word problems as tasks (for a general discussion, see Xie & Carspecken, 2008).

Word problems have been part of the historical development of mathematics in China and in other parts of the world as well. As Gerofsky (2009) claims, non-realistic word problems are a written and pedagogical genre that expressed generality through exemplification and have played an important role in all the pre-algebraic societies. From these shared roots two cultural traditions developed in different ways. In China the superb development of Algebra (maybe the main contribution of Chinese scholars to mathematics) was not accompanied by a parallel theoretical approach like Euclid's one; in the West the assumption of Euclid's *Elements* as the paradigm for mathematics development gave for many centuries a different status to practical mathematics (e.g. commercial arithmetic) from where the Western tradition of algebra was later nurtured.

*Mathematization* became a key point in both traditions, although in different periods. The focus on mathematization started in the West at the beginning of 19<sup>th</sup> century when the teaching of natural sciences was introduced systematically in schools and gave rise in 1983 to the International Community of Teachers of Mathematical Modelling and Applications that addressed criticisms of stereotyped word problems and influenced the development of mathematics curricula, (<http://www.icmihistory.unito.it/ictma.php>). The expansion to China is more recent although Chinese educators were exposed to Western influence for decades (e.g. Dewey, Smith, Freudenthal, see Wang, 2013, p. 38 ff.).

Hence, task re-design considered in this paper took place in different cultural traditions. It had in both cultures the advantage of respect the local pedagogical and theoretical roots and to exploit perspectives from the other culture. Our attitude was, in both cases, similar to the one described by Jullien (2008) in the case of philosophical dialogue between China and the West:



This is not about comparative philosophy, about paralleling different conceptions, but about a philosophical dialogue in which every thought, when coming towards the other, questions itself about its own unthought (Jullien, 2008, p. iii, our translation).

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