Mode III crack in couple stress elastic materials under harmonic loadings

Enrico Radi[1], Gennady Mishuris[2], Andrea Piccolroaz[2]
[1]*Dipartimento di Scienze e Metodi dell'Ingegneria, Università di Modena e Reggio Emilia, Italy*
E-mail: *eradi@unimore.it*

[2]*Institute of Mathematical and Physical Sciences, Aberystwyth University, UK*
E-mail: *ggm@aber.ac.uk, piccolro@ing.unitn.it*

Keywords: Antiplane crack, couple stress elasticity, harmonic loadings, Fourier Transform, Wiener-Hopf technique.

The problem of a rectilinear crack in an elastic solid with microstructures subject to harmonic loadings applied on the crack surfaces will be investigated in the proposed work. The material behavior is described by the indeterminate theory of couple stress elasticity developed by Koiter. This constitutive model includes the characteristic lengths in bending and torsion and thus it is able to account for the underlying microstructure of the material as well as for the strong size effects arising at small scales and observed when the representative scale of the deformation field becomes comparable to the length scale of the microstructure, such as the grain size in a polycrystalline or granular aggregate. It is sufficiently accurate to simulate the behavior of materials at the micron scale as well as the size effects occurring at distances to the crack tip comparable to characteristic lengths, but it is also simple enough to allow the achievement of closed-form solutions.
The stress and displacement fields near the tip of a Mode III crack are thus expected to be strongly influenced by the microstructural characteristic lengths. The quasistatic full-field solution [1], obtained by using Fourier transforms and Wiener-Hopf technique and the asymptotic analysis of the interface crack problem [2], showed that ahead of the crack tip within a zone smaller than the characteristic length in torsion, the total shear stress and reduced tractions occur with the opposite sign with respect to the classical LEFM solution, due to the relative rotation of the microstructural particles currently at the crack tip. However, this zone was found to have limited physical relevance and to become vanishing small for a characteristic length in torsion of zero. In this limit case, the solution recovers the classical K_{III} field with square root stress singularity. Outside the zone where the total shear stress is negative, the full field solution exhibits a bounded maximum for the total shear stress ahead of the crack tip, whose magnitude was adopted as a measure of the critical stress level for crack advancing. The corresponding fracture criterion defines a critical stress intensity factor, which increases with the characteristic length in torsion.
In the proposed research the previous analysis will be extended to the dynamic case in order to study the effects of inertia on the stress and deformation fields, as well as the variation of the fracture toughness due to the presence of microstructures.

References

[1] Radi, E., "Effects of the characteristic lengths ratio on mode III crack in couple stress elastic materials," *Int. J. Solids Struct.* **45**(10), 3033-3058 (2008).
[2] Piccolroaz, A., Mishuris, G., Radi, E., "Mode III interfacial crack in the presence of couple stress elastic materials," *Eng. Fracture Mech.* (2011).