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A Method Based on Bivariate Almost Stochastic Dominance for Multiple Criteria Group Decision Making With Probabilistic Dual Hesitant Fuzzy Information

QIAN ZHAO [1], YANBING JU [1], AND WITOLD PEDRYCZ [2], (Life Fellow, IEEE)

[1] School of Management and Economics, Beijing Institute of Technology, Beijing 100081, China

[2] Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB T6R 2V4, Canada

Corresponding author: Qian Zhao (qian_zhaoqz@163.com)

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ABSTRACT Probabilistic dual hesitant fuzzy sets (PDHFSs) are sound information granules to describe decision maker's aleatory and epistemic uncertainty in multiple criteria group decision making (MCGDM) process. In this paper, a bivariate almost stochastic dominance-based PROMETHEE-II method is presented to solve probabilistic dual hesitant fuzzy MCGDM problems which consider correlation averse behavior of decision makers. First, probabilistic dual hesitant fuzzy power Bonferroni mean (PDHFPBM) operator and probabilistic dual hesitant fuzzy power geometric Bonferroni mean (PDHFPGBM) operator are proposed to acquire collective preference information of decision makers. Second, based on the defined bivariate almost stochastic dominance (BASD) and BASD degree, qualitative and quantitative relationships between two probabilistic dual hesitant fuzzy elements (PDHFEs) with corresponding to all criteria are obtained. Third, distance-based correlation coefficient method for computing combined weight with respect to all criteria is proposed. Finally, a BASD-based PROMETHEE-II method is developed to determine the ranking results. Three illustrative examples followed by comparative analysis are included to show the practicality and effectiveness of the proposed method.

INDEX TERMS Probabilistic dual hesitant fuzzy sets (PDHFSs), bivariate almost stochastic dominance (BASD), probabilistic dual hesitant fuzzy power Bonferroni mean (PDHFPBM) operator, probabilistic dual hesitant fuzzy power geometric Bonferroni mean (PDHFPGBM) operator, PROMETHEE-II.

I. INTRODUCTION

Multiple criteria group decision making (MCGDM) aims to support decision makers (DMs) to select satisfactory alternatives from feasible alternatives according to multiple and often conflicting criteria [1]–[6]. Due to the uncertainty and vagueness existing in the problem at hand and the ambiguity of human thinking, it is challenging for DMs to provide precise values to characterize their judgements. To deal with this kind of epistemic uncertainty inherent in decision making problems, fuzzy sets are introduced to MCGDM and various studies on this topic have been reported in this area [7]–[9].

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As a generalization of fuzzy sets, probabilistic hesitant fuzzy sets (PHFSs) proposed by Xu and Zhou [10] exhibit advantages in characterizing simultaneous aleatory and epistemic uncertainties. For example, let $\{0.4|0.2, 0.7|0.5, 0.8|0.3\}$ be a probabilistic hesitant fuzzy element (PHFE), where the set $\{0.4, 0.7, 0.8\}$ represents the possible membership degrees of an object to a given set, and the set $\{0.2, 0.5, 0.3\}$ represents the corresponding probabilities of the three mentioned membership degrees. The example illustrates that a PHFE can capture the phenomenon that several possible membership degrees are uncertain. In fact, the uncertainty of those possible membership degrees can be captured by a discrete probability distribution. In other words, a PHFE can be regarded as a random variable following a certain probability distribution. More recently, Hao *et al.* [11] proposed the concept of probabilistic dual hesitant fuzzy sets (PDHFSs), which can capture more information than PHFSs by adding non-membership degrees with aleatory uncertainties. Similarly, a probabilistic dual hesitant fuzzy element (PDHFE) can be also regarded as two-dimensional random variable following a certain probability distribution.

Recently, much research has been focused to accommodate probabilistic hesitant fuzzy or probabilistic dual hesitant fuzzy decision contexts, and the studies on those fuzzy sets are becoming more visible. To deal with preference information fusion in the MCGDM problems, different kind of aggregation operators are proposed to collective preference information given by DMs, such as probabilistic hesitant fuzzy weighted average (PHFWA) operator, probabilistic hesitant fuzzy weighted geometric (PHFWG) operator, probabilistic dual hesitant fuzzy weighted average (PDHFWA) operator, probabilistic dual hesitant fuzzy weighted Einstein average (PDHFWEA) operator, probabilistic dual hesitant fuzzy ordered weighted Einstein average (PDHFOWEA) operator, probabilistic dual hesitant fuzzy weighted Einstein geometric (PDHFWEG) operator and probabilistic dual hesitant fuzzy ordered weighted Einstein geometric (PDHFOWEG) operator [10]–[13]. The above-mentioned aggregation operators are helpful in some ways. For example, those operators are good at integrating the importance of each preference information value and the importance of its location. However, they fail to consider the interrelationships between the input arguments and the relationships between the fused values [14]–[17]. Thus, based on the advantages of power average (PA) operator, power geometric (PG) operator, Bonferroni mean (BM) operator and geometric Bonferroni mean (GBM) operator, this study aims to propose novel aggregation operators which consider more information in the process of collecting probabilistic dual hesitant fuzzy preference information in group decision making. With extended multiple criteria decision making (MCDM) methods, a number of studies have been concentrated on making comparison between PHFSs or PDHFSs [10]–[12] [18]–[24]. For instance, Xu and Zhou [10] proposed a method of maximizing score and minimizing deviation to obtain DMs' weights and utilized aggregation operators to aggregate the information of all DMs, and then rank and optimize the alternatives. Zhang *et al.* [12] gave the definition of PHFEs with continuous form and proposed score functions to make comparison between the new form of PHFEs. Zhou and Xu [18] proposed hesitant values and expected hesitant values (i.e., score) at risk to measure PHFEs in the tail decision making. Combined on TOPSIS method, Wu *et al.* [19] proposed a novel closeness degree for PHFEs and applied it to dynamic emergency decision-making problem. After improving the consensus reaching process, Wu *et al.* [20] utilized score and deviation rules for comparing two PHFEs. Li and Wang [21] proposed Hausdorff distance between two PHFEs and constructed likelihood function to extend qualitative flexible multiple criteria method (QUALIFLEX) for solving probability hesitant fuzzy multiple criteria decision-making problems and then applied it for green suppliers' selection. Li and Wang [22] extended the traditional QUALIFLEX and the preference ranking organization method for enrichment evaluations II (PROMETHEE II) through possibility degree about two PHFEs to adapt hesitant probabilistic fuzzy inputs. Based on reference ideal method, He and Xu [23] proposed two distance measures for PHFEs and extended three different kind of MCDM methods to probabilistic hesitant fuzzy environment. Hao *et al.* [11] presented score and deviation rules to compare probabilistic dual hesitant fuzzy information and then employed it to Arctic geopolitics risk evaluation. Ren *et al.* [24] proposed an integrated (Analytic Hierarchy Process) AHP and VIKOR method for solving probability hesitant fuzzy MCDM problems. Ren *et al.* [25] proposed an extended TODIM method for solving probabilistic dual hesitant fuzzy MCDM problems. However, in the process of comparing two PDHFEs or two PHFEs, transferring probability information directly to real values may cause information loss, such as score and deviation rules. Most of the above-mentioned methods do not consider the behavior of DMs, i.e., the attitudes of DMs are risk neutral or completely rational. Risk attitudes or psychological behaviors of DMs may impose a significant impact on the result of decision making in real-world situations [26]–[31].

Stochastic dominance (SD) is an important tool for comparing two alternatives with stochastic information which considers different preferences of DMs. SD comes with less assumptions made with respect to the utility function, and as such has been widely applied in MCGDM [32]–[36]. Later, Leshno and Levy [37] introduced a relaxed version of SD, named univariate almost stochastic dominance (UASD), which relaxes the strict restrictions on distribution functions by eliminating some extreme utility functions. Considering the construction of bivariate case by two-attribute utility functions, bivariate stochastic dominance (BSD) is proposed by Levy and Paroush [38]. However, BSD has the same defect of SD because BSD is just the extension of dimension for SD. UASD has a limited effect to deal with the construction of bivariate case by two-attribute utility functions. Considering the construction of bivariate case by two-attribute utility functions, Denuit *et al.* [39] proposed bivariate almost stochastic dominance (BASD) corresponding to the preferences exhibiting confined correlation aversion and confined correlation loving which reflect the psychological behaviors of the DMs, respectively. Based on mentioned characteristics of BASD, it is interesting to utilize BASD for comparing PDHFEs, in which PDHFEs are expressed by the form of two-dimensional joint random distributions. Especially, it is more suitable for dealing with the probabilistic dual hesitant fuzzy MCGDM problems which consider psychological behaviors of DMs with correlation averse behavior.

On the basis of the analysis above, the disadvantages of the existing methods for comparing PDHFEs and aggregation operators will be emphasized in this paper. It further urges studies on novel aggregation operators to fuse probabilistic dual hesitant fuzzy preference information from different DMs. A novel comparison method which can properly deal with probability information is an urgent demand for PDHFEs. In order to close the application, it is better to consider the psychological behavior of DMs into the decision-making process. Here the innovation comes with a novel method based on BASD for probabilistic dual hesitant fuzzy MCGDM problems.

This paper proposes a BASD-based PROMETHEE-II method for solving probabilistic dual hesitant fuzzy MCGDM problems with the preferences of correlation averse DMs. Considering the uncertainty from randomness, the membership and non-membership values in PDHFEs can be considered as two-dimensional random variable. The main facet of originality is the presented BASD and BASD degree matrices formed with respect to all criteria are proposed to determine the qualitative and quantitative relationship between PDHFEs, respectively. There is no need to transfer the probability information to real values in the process of comparing PDHFEs. The psychological behavior of DMs can also be reflected by the utility functions which include the preferences of correlation averse preference in the BASD. For integrating preference given by DMs, probabilistic dual hesitant fuzzy power Bonferroni mean (PDHFPBM) operator and probabilistic dual hesitant fuzzy power geometric Bonferroni mean (PDHFPGBM) operator which consider much more information in the process of group decision making are proposed.

This paper is structured as follows. Section II briefly reviews some prerequisites of PDHFEs, and definitions of PA operator, PG operator BM operator and GBM operator. Section III builds two-dimensional random joint distribution by northwest corner rule and defines BASD corresponding to correlation aversion preference and BASD degree for the aim of comparing PDHFEs. Section IV proposes distance-based correlation coefficient method for combined weight of criteria, PDHFPBM operator and PDHFPGBM operator for collecting probabilistic dual hesitant fuzzy preference given by DMs. Section V proposes the BASD-based PROMETHEE-II method for solving MCGDM with probabilistic dual hesitant fuzzy preference information. Section VI provides three illustrative examples and comparative analysis to show the application and effectiveness of the proposed method. Finally, concluding remarks are given in Section VII.

II. PREREQUISITIES

This section briefly reviews the definitions which will be further used including definitions and operational laws of probabilistic dual hesitant fuzzy set (PDHFSs) and definitions of power average (PA) operator, power geometric (PG) operator, Bonferroni mean (BM) operator, and geometric Bonferroni mean (GBM) operator.

A. DEFINITION AND OPERATIONS OF PDHFSS

Definition 1 [11]: Let X be a fixed set. A PDHFS on X is defined as:

$$Q = \{\langle x, z(x)|p(x), v(x)|q(x)\rangle | x \in X\},$$

where $z(x)$ and $v(x)$ represent the possible membership and non-membership degrees of the variable $x \in X$, respectively. $p(x)$ and $q(x)$ represent the corresponding probabilistic information associated with these two types of degrees such that $p_i \in [0,1]$, $q_i \in [0,1]$, $\sum_{i=1}^{\#z} p_i = 1$, $\sum_{i=1}^{\#v} q_i = 1$, where $\#z$ and $\#v$ represent the number of elements in the components $z(x)|p(x)$ and $v(x)|q(x)$, respectively. In addition, the conditions $0 \le \mu, \upsilon \le 1$ and $0 \le \mu^+ + \upsilon^+ \le 1$ should be satisfied, where $\mu \in z(x)$, $\upsilon \in v(x)$, $\mu^+ = \max_{\mu\in z(x)}\{\mu\}$, $\upsilon^+ = \max_{\upsilon\in v(x)}\{\upsilon\}$, $p_i \in p(x)$ and $q_i \in q(x)$. For convenience, Hao *et al.* [11] called $\{\{z|p\},\{v|q\}\}$ as a probabilistic dual hesitant fuzzy element (PDHFE).

Usually, in real-world applications, we also provide with incomplete probabilistic information, that is, $\sum_{i=1}^{\#z} p_i \le 1$ and $\sum_{i=1}^{\#v} q_i \le 1$. Under this circumstance, the set Q is called a generalized PDHFS [11].

Definition 2 [11]: Let $A = \{\{z|p_z\},\{v|q_v\}\}$, $A_1 = \{\{z_1|p_{z_1}\},\{v_1|q_{v_1}\}\}$ and $A_2 = \{\{z_2|p_{z_2}\},\{v_2|q_{v_2}\}\}$ be three PDHFEs, then the operational rules of PDHFEs are given as follows:

(1)

$$A_1 \oplus A_2 = \cup_{\mu_1\in z_1,\upsilon_1\in v_1,\mu_2\in z_2,\upsilon_2\in v_2}\left\{\begin{array}{l}\{\mu_1+\mu_2-\mu_1\mu_2|p_{\mu_1}p_{\mu_2}\},\\ \{\upsilon_1\upsilon_2|q_{\upsilon_1}q_{\upsilon_2}\};\end{array}\right.$$

(2)

$$A_1 \otimes A_2 = \cup_{\mu_1\in z_1,\upsilon_1\in v_1,\mu_2\in z_2,\upsilon_2\in v_2}\left\{\begin{array}{l}\{\mu_1\mu_2|p_{\mu_1}p_{\mu_2}\},\\ \{\upsilon_1+\upsilon_2-\upsilon_1\upsilon_2|q_{\upsilon_1}q_{\upsilon_2}\};\end{array}\right.$$

(3) $\lambda A = \cup_{\mu\in z,\upsilon\in v}\left\{\left\{\left(1-(1-\mu)^\lambda\right)|p_\mu\right\},\left\{\upsilon^\lambda|q_\upsilon\right\}\right\}$, $\lambda \ge 0$;

(4) $A^\lambda = \cup_{\mu\in z,\upsilon\in v}\left\{\left\{\mu^\lambda|p_\mu\right\},\left\{\left(1-(1-\upsilon)^\lambda\right)|q_\upsilon\right\}\right\}$, $\lambda \ge 0$;

(5)

$$A^c = \begin{cases}\cup_{\mu\in z,\upsilon\in v}\left\{\{\upsilon|q_\upsilon\},\{\mu^\lambda|p_\mu\}\right\} & \text{if } z\neq\emptyset \text{ and } v\neq\emptyset,\\ \cup_{\mu\in z,\upsilon\in v}\left\{\{(1-\mu)|p_\mu\},\emptyset\right\} & \text{if } z\neq\emptyset \text{ and } v=\emptyset,\\ \cup_{\mu\in z,\upsilon\in v}\{\emptyset,\{(1-\upsilon)|q_\upsilon\}\} & \text{if } z=\emptyset \text{ and } v\neq\emptyset.\end{cases}$$

where A^c represents the complement of $A = \{\{z|p_z\},\{v|q_v\}\}$.

B. AGGREGATION OPERATORS

As useful tools for combining all input individual data into a single representative one, power average (PA) operator [14], power geometric (PG) operator [15], Bonferroni mean (BM) operator [16] and geometric Bonferroni mean (GBM) operator [17] are defined as follows.

Definition 3: Let $A_1, A_2, \ldots, A_n$ be values to be aggregated. Then the power average (PA) operator is defined as follows:

$$PA(A_1, A_2, \ldots, A_n) = \frac{\sum_{i=1}^n(1+T(A_i))}{\sum_{i=1}^n(1+T(A_i))}A_i,$$

where $T(A_i) = \sum_{j=1,j\neq i}^n \sup(A_i, A_j)$, $(i = 1, 2, \ldots, n)$.

The $\sup(A_i, A_j)$ denotes the support for A_i from A_j, which typically satifies the following conditions:

(1) $\sup(A_i, A_j) \in [0, 1]$;
(2) $\sup(A_i, A_j) = \sup(A_j, A_i)$;
(3) $\sup(A_i, A_j) \geq \sup(M, N)$ if $|A_i - A_j| \geq |M - N|$.

Obviously, $\sup(A_i, A_j)$ is essentially a similarity index.

Definition 4: Let $A_1, A_2, \ldots, A_n$ be values to be aggregated, then the power geometric (PG) operator is defined as follows:

$$PG(A_1, A_2, \ldots, A_n) = \prod_{i=1}^{n} A_i^{\frac{(1+T(A_i))}{\sum_{i=1}^{n}(1+T(A_i))}},$$

where $T(A_i) = \sum_{j=1, j\neq i}^{n} \sup(A_i, A_j)$, $(i = 1, 2, \ldots, n)$.

The $\sup(A_i, A_j)$ denotes the support for A_i coming from A_j, which typically satifies the following conditions:

(1) $\sup(A_i, A_j) \in [0, 1]$;
(2) $\sup(A_i, A_j) = \sup(A_j, A_i)$;
(3) $\sup(A_i, A_j) \geq \sup(M, N)$ if $|A_i - A_j| \geq |M - N|$.

Obviously, $\sup(A_i, A_j)$ is essentially a similarity index.

Definition 5: Let $A_1, A_2, \ldots, A_n$ be values to be aggregated, and $p, q \geq 0$. Then the Bonferroni mean (BM) operator is defined as follows:

$$BM^{p,q}(A_1, A_2, \ldots, A_n) = \left(\frac{1}{n(n-1)} \sum_{i,j=1, i\neq j}^{n} A_i^p A_j^q \right)^{\frac{1}{p+q}}.$$

Definition 6: Let $A_1, A_2, \ldots, A_n$ be values to be aggregated, and $p, q \geq 0$. Then the geometric Bonferroni mean (GBM) operator is defined as follows:

$$GBM^{p,q}(A_1, A_2, \ldots, A_n) = \frac{1}{p+q} \prod_{i,j=1, i\neq j}^{n} (pA_i + qA_j)^{\frac{1}{n(n-1)}}.$$

III. BIVARIATE ALMOST STOCHASTIC DOMINANCE AND BIVARIATE ALMOST STOCHASTIC DOMINANCE DEGREE FOR PDHFES

In this section, we first construct cumulative distribution functions (CDFs) of two-dimensional random variable with respect to PDHFEs through the principle of northwest corner rule. Then, bivariate almost stochastic dominance (BASD) and bivariate almost stochastic dominance (BASD) degree are presented to facilitate the comparison of two PDHFEs.

A. BUILDING TWO-DIMENSIONAL RANDOM JOINT DISTRIBUTION FOR PDHFES BY NORTHWEST CORNER RULE

Northwest corner rule is used to obtained to the expected mean of two PHFEs or two PDHFEs with the assumption that PHFEs or PDHFEs are not mutual independent [40]. Inspired by this idea, we form a CDF of two-dimensional random variable corresponding to PDHFE and replace PDHFE by the constructed CDF in the next subsection.

Let $A = \{\{z^{\delta(i)}|p_{z^{\delta(i)}}\}, \{v^{\delta(j)}|q_{v^{\delta(j)}}\}\}$ be a PDHFE, where $z^{\delta(i)}$ $(i = 1, 2, \ldots, \#z)$ is the ith smallest element in z, and $v^{\delta(j)}$ $(j = 1, 2, \ldots, \#v)$ is the jth smallest element in v. The membership degree of an object to X is a random variable ξ. Similarly, the non-membership degree of an object to X can be considered as a random variable ζ. Northwest corner rule is used to calculate the joint probabilities between membership values and non-membership values in PDHFE. When PDHFE A is considered as two-dimensional random variable, denoted as (ξ, ζ), the joint probabilities of (ξ, ζ) is given as shown in Table 1.

TABLE 1. The joint probabilities calculation for (ξ, ζ).

	$\mu^{\delta(1)}$	$\mu^{\delta(2)}$	$\cdots$	$\mu^{\delta(\#z)}$	
$\upsilon^{\delta(1)}$	f_{11}	f_{12}	$\cdots$	$f_{1\delta(\#z)}$	$q_{\upsilon^{\delta(1)}}$
$\upsilon^{\delta(2)}$	f_{21}	f_{22}	$\cdots$	$f_{2\delta(\#z)}$	$q_{\upsilon^{\delta(2)}}$
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$
$\upsilon^{\delta(\#v)}$	$f_{\delta(\#v)1}$	$f_{\delta(\#v)2}$	$\cdots$	$f_{\delta(\#v)\delta(\#z)}$	$q_{\upsilon^{\delta(\#v)}}$
	$p_{\mu^{(1)}}$	$p_{\mu^{(2)}}$	$\cdots$	$p_{\mu^{\delta(\#z)}}$	1

Considering the dual relationships expressed by PDHFE, epically the part of non-membership, we acquire the opposite number of non-membership degree values, i.e., $\varsigma = -\zeta$. The CDF of two-dimensional random variable between membership values and non-memberships values in the PDHFE is denoted by (ξ, ς). The joint probabilistic of (ξ, ς) are given as shown in Table 2.

TABLE 2. The joint probabilities calculation for (ξ, ς).

	$\mu^{\delta(1)}$	$\mu^{\delta(2)}$	$\cdots$	$\mu^{\delta(\#z)}$	
$-\upsilon^{\delta(1)}$	g_{11}	g_{12}	$\cdots$	$g_{1\delta(\#z)}$	$q_{\upsilon^{\delta(1)}}$
$-\upsilon^{\delta(2)}$	g_{21}	g_{22}	$\cdots$	$g_{2\delta(\#z)}$	$q_{\upsilon^{\delta(2)}}$
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$
$-\upsilon^{\delta(\#v)}$	$g_{\delta(\#v)1}$	$g_{\delta(\#v)2}$	$\cdots$	$g_{\delta(\#v)\delta(\#z)}$	$q_{\upsilon^{\delta(\#v)}}$
	$p_{\mu^{(1)}}$	$p_{\mu^{(2)}}$	$\cdots$	$p_{\mu^{\delta(\#z)}}$	1

Note that when calculating the distance of two PDHFEs in the next section, we utilize the joint probabilities calculation method in Table 1. When making comparison of two PDHFEs in the next section, we utilize the joint probabilities calculation method in Table 2.

Example 1: Let $A = \{\{0.1|0.2, 0.2|0.1, 0.3|0.7\}, \{0.6|0.3, 0.7|0.7\}\}$ be a PDHFE on X, then the membership degree of an object to the set X is represented by set $\{0.1,0.2,0.3\}$, and the corresponding probabilistic values are denoted by the set $\{0.2,0.1,0.7\}$. The non-membership degree of an object to the set X is

TABLE 3. The joint probabilities calculation for (ξ, ζ).

	0.1	0.2	0.3	
0.6	0.2	0.1	0	0.3
0.7	0	0	0.7	0.7
	0.2	0.1	0.7	1

TABLE 4. The joint probabilities calculation for (ξ, ς).

	0.1	0.2	0.3	
-0.7	0.2	0.1	0.4	0.7
-0.6	0	0	0.3	0.3
	0.2	0.1	0.7	1

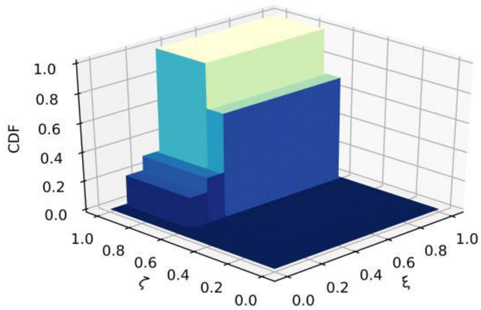

FIGURE 1. The CDF of two-dimensional random variable of (ξ, ζ).

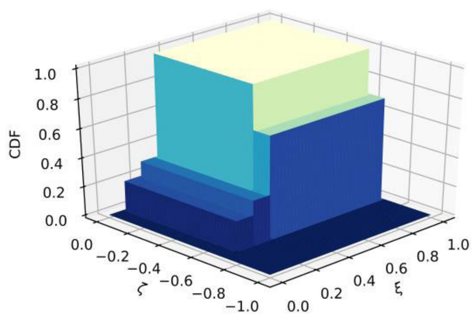

FIGURE 2. The CDF of two-dimensional random variable of (ξ, ς).

represented by set {0.6,0.7}, and the corresponding probabilistic values are denoted by the set {0.3,0.7}. Therefore, the joint probabilities of (ξ, ζ) and (ξ, ς) can be expressed as Tables 3 and 4, respectively.

And the CDF of two-dimensional random variable (ξ, ζ) and CDF of two-dimensional random variable (ξ, ς) are shown in Figs. 1 and 2, respectively.

B. BIVARIATE ALMOST STOCHASTIC DOMINANCE AND BIVARIATE ALMOST STOCHASTIC DOMINANCE DEGREE

After having confined correlation aversion to some acceptable, Denuit *et al.* [39] proposed a bivariate almost stochastic dominance (BASD) for preference exhibiting confined correlation aversion, which can be related to two-attribute utility functions excluding extreme preferences. However, BASD relationships are qualitative rather than quantitative. To overcome the disadvantage of BASD relationships, we propose a novel BASD degree based on BASD to facilitate the comparison of two PDHFEs.

Let $\Pr(X_1 \le x, Y_1 \le y)$ and $\Pr(X_2 \le x, Y_2 \le y)$ be the CDFs of two-dimensional random variable (X_1, Y_1) and two-dimensional random variable (X_2, Y_2), respectively, and they are finite within $[a, b] \times [-b, -a]$. Given a utility function u of two variables x and y $(0 \le x \le 1, -1 \le y \le 0)$, we denote the cross derivatives as $u^{(i,j)}(x, y) = \frac{\partial^{i+j} u(x,y)}{(\partial x)^i (\partial(y))^j} i, j \in \{0, 1\}$. Based on the definition of correlation aversion given by Eeckhoudt *et al.* [41], the class of all correlation aversion utility functions denoted by U, i.e., functions u satisfy the relationships

$$u \in U \Leftrightarrow \begin{cases} u^{(1,0)}(x, y) \ge 0, \\ u^{(0,1)}(x, y) \ge 0, \\ u^{(1,1)}(x, y) \le 0. \end{cases}$$

Suppose the bivariate almost utility functions $u(x, y)$ satisfy the following condition:

$$U^{\varepsilon} = \left\{ u \in U : 0 \le -u^{(1,1)}(x, y) \le \inf\left(-u^{(1,1)}(x, y)\right)\left(\frac{1}{\varepsilon} - 1\right) \right\}$$

for all x and y, where $0 < \varepsilon < 0.5$.

Based on the above utility function U^{ε}, the BASD with correlation aversion preference can be expressed as follows.

Definition 7: Let (ξ_1, ς_1) and (ξ_2, ς_2) be two two-dimensional random variables of PDHFE $A_1 = \left\{\left\{z_1 | p_{z_1}\right\}, \left\{v_1 | q_{v_1}\right\}\right\}$ and PDHFE $A_2 = \left\{\left\{z_2 | p_{z_2}\right\}, \left\{v_2 | q_{v_2}\right\}\right\}$, respectively. $A_1 \varepsilon$-bivariate almost stochastic dominance A_2 $(i.e., A_1 \varepsilon - BASD A_2)$ if $\Pr[\xi_2 \le x] > \Pr[\xi_1 \le x]$ for all x, $\Pr[\varsigma_2 \le y] > \Pr[\varsigma_1 \le y]$ for all y, and

$$\iint_S (\Pr[\xi_1 \le x, \varsigma_1 \le y] - \Pr[\xi_2 \le x, \varsigma_2 \le y]) dxdy$$

$$\le \varepsilon \int_0^1 \int_{-1}^0 |\Pr[\xi_2 \le x, \varsigma_2 \le y] - \Pr[\xi_1 \le x, \varsigma_1 \le y]| \, dxdy,$$

where $S = \{(x, y) | \Pr[\xi_1 \le x, \varsigma_1 \le y] > \Pr[\xi_2 \le x, \varsigma_2 \le y]\}$ for all x and y.

Example 2: Let $A_1 = \{\{0.2|0.2, 0.3|0.3, 0.7|0.5\}, \{0.1|0.3, 0.2|0.3, 0.3|0.4\}\}$ and $A_2 = \{\{0.1|0.3, 0.3|0.7\}, \{0.6|0.6, 0.7|0.4\}\}$ be two PDHFEs. According to section III, the joint probabilities calculations for (ξ_1, ς_1) and (ξ_2, ς_2) i.e., $\Pr[\xi_1 \le x, \varsigma_1 \le y]$ and $\Pr[\xi_2 \le x, \varsigma_2 \le y]$ can be obtained as shown in Tables 5 and 6, respectively.

The CDFs of four random variables ξ_1, ς_1, ξ_2 and ς_2 i.e., $\Pr[\xi_1 \le x]$, $\Pr[\varsigma_1 \le y]$, $\Pr[\xi_2 \le x]$ and $\Pr[\varsigma_2 \le y]$ can be expressed as follows:

$$\Pr[\xi_1 \le x] = \begin{cases} 0 & x < 0.2; \\ 0.2 & 0.2 \le x < 0.3; \\ 0.5 & 0.3 \le x < 0.7; \\ 1 & 0.7 \le x, \end{cases}$$

TABLE 5. The joint probabilities calculation for (ξ_1, ς_1).

	0.1	0.2	0.7	
-0.3	0.2	0.2	0	0.4
-0.2	0	0.1	0.2	0.3
-0.1	0	0	0.3	0.3
	0.2	0.3	0.5	1

TABLE 6. The joint probabilities calculation for (ξ_2, ς_2).

	0.1	0.3	
-0.7	0.3	0.1	0.4
-0.6	0	0.6	0.6
	0.3	0.7	1

$$\Pr[\varsigma_1 \le y] = \begin{cases} 0 & x < -0.3; \\ 0.4 & -0.3 \le x < -0.2; \\ 0.7 & -0.2 \le x < -0.1; \\ 1 & -0.1 \le x, \end{cases}$$

$$\Pr[\xi_2 \le x] = \begin{cases} 0 & x < 0.1; \\ 0.3 & 0.1 \le x < 0.3; \\ 1 & 0.3 \le x, \end{cases}$$

and

$$\Pr[\varsigma_2 \le y] = \begin{cases} 0 & x < -0.7; \\ 0.4 & -0.7 \le x < -0.6; \\ 1 & -0.6 \le x. \end{cases}$$

According to the above analysis, all these conditions are met, i.e., $\Pr[\xi_2 \le x] > \Pr[\xi_1 \le x]$ for all x, $\Pr[\varsigma_2 \le y] > \Pr[\varsigma_1 \le y]$ for all y, and $\int\int_S (\Pr[\xi_1 \le x, \varsigma_1 \le y] - \Pr[\xi_2 \le x, \varsigma_2 \le y])dxdy \le \varepsilon \int_0^1 \int_{-1}^0 |\Pr[\xi_2 \le x, \varsigma_2 \le y] - \Pr[\xi_1 \le x, \varsigma_1 \le y]|\, dxdy$.

At last, according to Definition 7, we have $A_1\varepsilon$-bivariate almost stochastic dominance A_2.

Following the above analysis, we obtain the qualitative relationship of one PDHFE over another one by utilizing BASD. To measure the quantitative relationship of two PDHFEs, BASD degree is presented as follows.

Definition 8: Let (ξ_1, ς_1) and (ξ_2, ς_2) be two-dimensional random variables of PDHFEs $A_1 = \left\{\left\{z_1|p_{z_1}\right\}, \left\{v_1|q_{v_1}\right\}\right\}$ and $A_2 = \left\{\left\{z_2|p_{z_2}\right\}, \left\{v_2|q_{v_2}\right\}\right\}$, respectively. If A_1 ε-bivariate almost stochastic dominance A_2 $(i.e., A_1\varepsilon - BASDA_2)$, then the BASD degree for A_1 over A_2 can be defined as follows:

$$\Phi\left(A_1\varepsilon - BASDA_2\right) = \frac{\int_0^1 \int_{-1}^0 \left(\Pr[\xi_2 \le x, \varsigma_2 \le y] - \Pr[\xi_1 \le x, \varsigma_1 \le y]\right) dxdy}{\int_0^1 \int_{-1}^0 \Pr[\xi_2 \le x, \varsigma_2 \le y]\, dxdy} \quad (1)$$

Example 3: Based on the given PDHFEs A_1 and A_2 in Example 2, we have $A_1\varepsilon$-bivariate almost stochastic dominance A_2. Then the BASD degree for A_1 over A_2 is as follows.

$$\begin{aligned}\Phi\left(A_1\varepsilon - BASDA_2\right) &= \frac{\int_0^1 \int_{-1}^0 \left(\Pr[\xi_2 \le x, \varsigma_2 \le y] - \Pr[\xi_1 \le x, \varsigma_1 \le y]\right) dxdy}{\int_0^1 \int_{-1}^0 \Pr[\xi_2 \le x, \varsigma_2 \le y]\, dxdy} \\ &= \frac{0.3772}{0.4864} \\ &= 0.7755.\end{aligned}$$

IV. THE DISTANCE-BASED METHODS FOR COLLECTING PROBABILISTIC DUAL HESITANT FUZZY INFORMATION

Under probabilistic dual hesitant fuzzy environment, Garg and Kaur [13] proposed two distance measures to describe the distances between PDHFEs. The first kind of distance measure has the requirement of equal lengths of two PDHFEs. For unequal situations, the experts repeat the least or greatest values among the shorter set of PDHFEs to obtain the same length according to different attitudes of experts. This method of adding artificial values is crude and unreasonable. Then, Garg and Kaur [13] provided another distance measure. Although the second distance measure overcame the above limitation, there are still many problems such as neglecting the interaction between membership and non-membership in PDHFE. To overcome above mentioned limitations, a new distance measure between two PDHFEs is proposed by joint probabilities of PDHFEs given in Table 1. Based the novel distance measure, a distance-based correlation coefficient method is proposed for computing combined weights of criteria to aggerate probabilistic dual hesitant fuzzy information. The probabilistic dual hesitant fuzzy power Bonferroni mean (PDHFPBM) operator and probabilistic dual hesitant fuzzy power geometric Bonferroni mean (PDHFPGBM) operator are presented for collecting probabilistic dual hesitant fuzzy preference given by DMs.

A. DISTANCE-BASED CORRELATION COEFFICIENT METHOD FOR COMPUTING COMBINED WEGHTS OF CRITERIA

The weights of criteria represent the relative importance in MCDM and they are used to collecting information. Weighting methods can be of two types: subjective methods and objective methods. The subjective weights are obtained by DMs' expertise and judgment, nevertheless the objective weights are obtained by mathematical computation [42]–[45]. In this section, we mainly discuss how to calculate the combined weight of criteria which include subjective weights and objective weights. For this purpose, a novel distance measure for PDHFEs by the comparison of corresponding joint CDFs is first defined. Then, a distance-based correlation coefficient measure between PDHFEs for computing objective weights is given as follows.

Let PDHFEs $A_{ij} = \{\{z_{ij}|p_{z_{ij}}\}, \{v_{ij}|q_{v_{ij}}\}\}$ and $A_{ij'} = \{\{z_{ij'}|p_{z_{ij'}}\}, \{v_{ij'}|q_{v_{ij'}}\}\}$ be evaluation values for alternative $P_i\,(i = 1, 2, \ldots, m)$ concerning criterion c_j and $c_{j'}$ $(j, j' = 1, 2, \ldots, n)$, respectively.

Definition 9: Let (ξ_{ij}, ζ_{ij}) and $(\xi_{ij'}, \zeta_{ij'})$ be two-dimensional random variables of PDHFEs $A_{ij} = \{\{z_{ij}|p_{z_{ij}}\}, \{v_{ij}|q_{v_{ij}}\}\}$ and $A_{ij'} = \{\{z_{ij'}|p_{z_{ij'}}\}, \{v_{ij'}|q_{v_{ij'}}\}\}$ $(i = 1, 2, \ldots, m; j, j' = 1, 2, \ldots, n)$, respectively, then the distance of A_{ij} and $A_{ij'}$ can be defined as follows:

$$d(A_{ij}, A_{ij'}) = \int_0^1\int_0^1 \left|\Pr[\xi_{ij} \le x, \zeta_{ij} \le y] - \Pr[\xi_{ij'} \le x, \zeta_{ij'} \le y]\right| \times dxdy, \quad j, j' = 1, 2, \ldots, n, \tag{2}$$

Property 1: For any two PDHFEs $A_{ij} = \{\{z_{ij}|p_{z_{ij}}\}, \{v_{ij}|q_{v_{ij}}\}\}$ and $A_{ij'} = \{\{z_{ij'}|p_{z_{ij'}}\}, \{v_{ij'}|q_{v_{ij'}}\}\}$, we have

(1) $0 \le d(A_{ij}, A_{ij'}) \le 1$;
(2) $d(A_{ij}, A_{ij'}) = 0$ if and only if $A_{ij} = A_{ij'}$;
(3) $d(A_{ij}, A_{ij'}) = d(A_{ij'}, A_{ij})$.

Definition 10: Let $R(A_{ij})_{m\times n} = (\{\{z_{ij}|p_{z_{ij}}\}, \{v_{ij}|q_{v_{ij}}\}\})_{m\times n}$ be probabilistic dual hesitant fuzzy decision matrix, where PDHFE A_{ij} represents the evaluation value of alternative P_i with respect to criteria $c_j\,(i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$, A_j^+ be the best PDHFE on criteria c_j, defined as follows:

$$A_j^+ = \left\{\max_i\{z_{ij}\cdot p_{z_{ij}}\}, \min_i\{v_{ij}\cdot q_{v_{ij}}\}\right\}, \quad j = 1, 2, \ldots, n. \tag{3}$$

Example 4: Give three probabilistic dual hesitant fuzzy PDHFEs $A_{11} = \{\{0.2|0.2, 0.3|0.3, , 0.7|0.5\}, \{0.1|0.3, 0.2|0.3, , 0.3|0.4\}\}$, $A_{21} = \{\{0.4|0.3, 0.5|0.7\}, \{0.2|0.1, 0.3|0.9\}\}$ and $A_{31} = \{\{0.6|0.5, 0.7|0.5\}, \{0.1|0.3, 0.2|0.7\}\}$. By the defined (3), we have

$$\begin{aligned}
&\max_i\{z_{i1}\cdot p_{z_{i1}}\}\\
&= \{z_{11}\cdot p_{z_{11}}, z_{12}\cdot p_{z_{12}}, z_{13}\cdot p_{z_{13}}\}\\
&= \max\left\{\begin{array}{l}0.2\times0.2+0.3\times0.3+0.7\times0.5,\\ 0.4\times0.3+0.5\times0.7,\\ 0.6\times0.5+0.7\times0.5\end{array}\right\}\\
&= \max\{0.48, 0.47, 0.65\}\\
&= 0.65
\end{aligned}$$

and

$$\begin{aligned}
&\min_i\{v_{i1}\cdot q_{v_{i1}}\}\\
&= \min\{v_{11}\cdot q_{v_{11}}, v_{21}\cdot q_{v_{21}}, v_{31}\cdot q_{v_{31}}\}\\
&= \min\left\{\begin{array}{l}0.1\times0.3+0.2\times0.3+0.3\times0.4,\\ 0.2\times0.1+0.3\times0.9,\\ 0.1\times0.3+0.2\times0.7\end{array}\right\}\\
&= 0.17.
\end{aligned}$$

From the above analysis, the maximum value is $z_{31}\cdot p_{z_{31}}$ and minimum value is $v_{31}\cdot q_{v_{31}}$. Thus, the best PDHFE on criteria c_1, i.e., $A_1^+ = \{\{0.6|0.5, 0.7|0.5\}, \{0.1|0.3, 0.2|0.7\}\}$.

Definition 11: Let (ξ_{ij}, ζ_{ij}) and $(\xi_{ij'}, \zeta_{ij'})$ be two two-dimensional random variables of PDHFEs $A_{ij} = \{\{z_{ij}|p_{z_{ij}}\}, \{v_{ij}|q_{v_{ij}}\}\}$ and $A_{ij'} = \{\{z_{ij'}|p_{z_{ij'}}\}, \{v_{ij'}|q_{v_{ij'}}\}\}$ $(i = 1, 2, \ldots, m; j, j' = 1, 2, \ldots, n)$, respectively, then the correlation coefficient of c_j and $c_{j'}$ is defined as (4), as shown at the bottom of the page, where $D_{ij} = d(A_{ij}, A_j^+)\big/\max_i d(A_{ij}, A_j^+)$, $D_{ij'} = d(A_{ij'}, A_{j'}^+)\big/\max_i d(A_{ij'}, A_{j'}^+)$ with A_j^+, $A_{j'}^+$ being the best PDHFEs for criterion c_j and $c_{j'}$, respectively.

Based on Definitions 9-11, the objective weight w_j^o can be obtained as follows:

$$w_j^o = \frac{\sum_{j'=1}^m (1 - r_{jj'})}{\sum_{j=1}^m\sum_{j'=1}^m (1 - r_{jj'})}, \quad j = 1, 2, \ldots, n. \tag{5}$$

Given the subjective weighted vector $(w_1^s, w_2^s, \ldots, w_n^s)$, where $\sum_{j=1}^n w_j^s = 1$ and $w_j^s \ge 0\,(j = 1, 2, \ldots, n)$, and the obtained objective weight vector $(w_1^o, w_2^o, \ldots, w_n^o)$ in defined (5), the combined weight vector $(w_1, w_2, \ldots, w_n)$ is:

$$w_j = \frac{\sqrt{w_j^o w_j^s}}{\sum_{j=1}^n\sqrt{w_j^o w_j^s}}, \quad j = 1, 2, \ldots, n. \tag{6}$$

We can calculate the combined weight including objective and subjective information by the non-linear weighted comprehensive method. Due to this combination of subjective and objective factor influence, it can well express both the subjective considerations of DMs and the objective information simultaneously.

$$r_{jj'} = \frac{\sum_{i=1}^m\left[\left(D_{ij} - \frac{1}{m}\sum_{i=1}^m D_{ij}\right)\left(D_{ij'} - \frac{1}{m}\sum_{i=1}^m D_{ij'}\right)\right]}{\sqrt{\sum_{i=1}^m\left(D_{ij} - \frac{1}{m}\sum_{i=1}^m D_{ij}\right)^2 \times \sum_{i=1}^m\left(D_{ij'} - \frac{1}{m}\sum_{i=1}^m D_{ij'}\right)^2}}, \quad j, j' = 1, 2, \ldots, n, \tag{4}$$

B. THE PROBABILISITIC DUAL HESITANT FUZZY POWER BONFERRONI MEAN OPERATOR AND PROBABILISTIC DUAL HESITANT FUZZY POWER GEMETRIC BONFERRONI MEAN OPERATOR

In order to collect preference given by DMs in the process of decision making, probabilistic dual hesitant fuzzy power Bonferroni mean (PDHFPBM) operator and probabilistic dual hesitant fuzzy power geometric Bonferroni mean (PDHFPGBM) operator are presented as follows.

Definition 12: Let $A_1, A_2, \ldots, A_n$ be a collection of PDHFEs and $p, q \geq 0$, then the PDHFPBM operator of dimension n is a mapping PDHFPBM: $PDHFEs^n \rightarrow PDHFEs$, is defined by

$$PDHFPBM^{p,q}(A_1, A_2, \ldots, A_n) = \left(\frac{1}{n(n-1)}\left(\underset{i,j,i\neq j}{\overset{n}{\oplus}}\left(\left(\frac{n(1+T(A_i))}{\sum_{t=1}^{n}(1+T(A_t))}A_i\right)^p \otimes\left(\frac{n\left(1+T\left(A_j\right)\right)}{\sum_{t=1}^{n}(1+T(A_t))}A_j\right)^q\right)\right)\right)^{\frac{1}{p+q}}$$

where $T(A_i) = \sum_{j=1,j\neq i}^{n} \sup\left(A_i, A_j\right)$, $\sup\left(A_i, A_j\right) = 1 - d\left(A_i, A_j\right)$ and $d\left(A_i, A_j\right)$ represents the distance measure between A_i and A_j.

Definition 13: Let $A_1, A_2, \ldots, A_n$ be a collection of PDHFEs and $p, q \geq 0$, then the PDHFPGBM operator of dimension n is a mapping PDHFPGBM:$PDHFEs^n \rightarrow PDHFEs$, is defined by

$$PDHFPGBM^{p,q}(A_1, A_2, \ldots, A_n) = \frac{1}{p+q}\left(\underset{i,j,i\neq j}{\overset{n}{\otimes}}\left(pA_i^{\frac{n(1+T(A_i))}{\sum_{t=1}^{n}(1+T(A_t))}} \oplus qA_j^{\frac{n\left(1+T\left(A_j\right)\right)}{\sum_{t=1}^{n}(1+T(A_t))}}\right)^{\frac{1}{n(n-1)}}\right)$$

where $T(A_i) = \sum_{j=1,j\neq i}^{n} \sup\left(A_i, A_j\right)$, $\sup\left(A_i, A_j\right) = 1 - d\left(A_i, A_j\right)$ and $d\left(A_i, A_j\right)$ represents the distance measure between A_i and A_j.

Theorem 1: Let $A_i = \left\{\left\{z_i|p_{z_i}\right\}, \left\{v_i|q_{v_i}\right\}\right\} (i = 1, 2, \ldots, n)$ be a collection of PDHFEs and $p, q \geq 0$, then the PDHFPBM operator of dimension n is a mapping PDHFPBM: $PDHFEs^n \rightarrow PDHFEs$, is defined $PDHFPBM^{p,q}(A_1, A_2, \ldots, A_n)$, as shown at the bottom of the page.

Proof: for the membership and non-membership values,by the operations of PDHFEs, we have

$$\underset{j=1,j\neq i}{\overset{n}{\otimes}}\left(\frac{n\left(1+T\left(A_j\right)\right)}{\sum_{t=1}^{n}(1+T(A_t))}A_j\right)^q = \bigcup_{\substack{\mu_1\in z_1,\ldots,\mu_n\in z_n\\ \upsilon_1\in v_1,\ldots,\upsilon_n\in v_n}} \times\left\{\begin{array}{l}\prod_{j=1,j\neq i}^{n}\left(1-\left(1-\mu_j\right)^{n\left(1+T\left(A_j\right)\right)/\sum_{i=1}^{n}(1+T(A_i))}\right)^q, \\ 1-\prod_{j=1,j\neq i}^{n}\left(1-\upsilon_j^{n\left(1+T\left(A_j\right)\right)/\sum_{i=1}^{n}(1+T(A_i))}\right)^q\end{array}\right\}$$

and

$$\left(\frac{n(1+T(A_i))}{\sum_{t=1}^{n}(1+T(A_t))}A_i\right)^p \otimes\left(\underset{j=1,j\neq i}{\overset{n}{\otimes}}\left(\frac{n\left(1+T\left(A_j\right)\right)}{\sum_{t=1}^{n}(1+T(A_t))}A_j\right)^q\right) = \bigcup_{\substack{\mu_1\in z_1,\ldots,\mu_n\in z_n\\ \upsilon_1\in v_1,\ldots,\upsilon_n\in v_n}} \times\left\{\begin{array}{l}\left(1-(1-\mu_i)^{n(1+T(A_i))/\sum_{i=1}^{n}(1+T(A_i))}\right)^p \\ \times\prod_{j=1,i\neq j}^{n}\left(1-\left(1-\mu_j\right)^{n\left(1+T\left(A_j\right)\right)/\sum_{i=1}^{n}(1+T(A_i))}\right)^q, \\ 1-\left(1-\upsilon_i^{n(1+T(A_i))/\sum_{i=1}^{n}(1+T(A_i))}\right)^p \\ \times\prod_{j=1,i\neq j}^{n}\left(1-\upsilon_j^{n\left(1+T\left(A_j\right)\right)/\sum_{i=1}^{n}(1+T(A_i))}\right)^q\end{array}\right\}$$

$$PDHFPBM^{p,q}(A_1, A_2, \ldots, A_n) = \bigcup_{\substack{\mu_1\in z_1,\ldots,\mu_n\in z_n\\ \upsilon_1\in v_1,\ldots,\upsilon_n\in v_n}} \times\left\{\begin{array}{l}\left\{\left(1-\prod_{i,j=1,i\neq j}^{n}\left(\begin{array}{l}1-\left(1-(1-\mu_i)^{n(1+T(A_i))/\sum_{i=1}^{n}(1+T(A_i))}\right)^p \\ \left(1-\left(1-\mu_j\right)^{n\left(1+T\left(A_j\right)\right)/\sum_{i=1}^{n}(1+T(A_i))}\right)^q\end{array}\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{p+q}}\right\}\Bigg|\prod_{i=1}^{n}p_{\mu_i}, \\ \left\{1-\left(1-\prod_{i,j=1,i\neq j}^{n}\left(\begin{array}{l}1-\left(1-\upsilon_i^{n(1+T(A_i))/\sum_{i=1}^{n}(1+T(A_i))}\right)^p \\ \left(1-\upsilon_j^{n\left(1+T\left(A_j\right)\right)/\sum_{i=1}^{n}(1+T(A_i))}\right)^q\end{array}\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{p+q}}\right\}\Bigg|\prod_{i=1}^{n}q_{\upsilon_i}\end{array}\right\}$$

From the above equations, we have $\frac{1}{n(n-1)} \underset{i,j,i\neq j}{\overset{n}{\oplus}} \left(\left(\frac{n(1+T(A_i))}{\sum_{t=1}^{n}(1+T(A_t))} A_i \right)^p \otimes \left(\frac{n(1+T(A_j))}{\sum_{t=1}^{n}(1+T(A_t))} A_j \right)^q \right)$, as shown at the bottom of the page. For the collective probabilistic values, multiply these corresponding probabilities. Therefore, by the above calucations, we have the Theorem 1.

Theorem 2: Let $A_i = \left\{ \left\{ z_i | p_{z_i} \right\}, \left\{ v_i | q_{v_i} \right\} \right\} (i = 1, 2, \ldots, n)$ be a collection of PDHFEs and $p, q \geq 0$, then the PDHFPGBM operator of dimension n is a mapping PDHFPGBM: $PDHFEs^n \rightarrow PDHFEs$, is defined by $PDHFPGBM^{p,q}(A_1, A_2, \ldots, A_n)$, as shown at the bottom of the page.

The proof of Theorem 2 is similar to the proof of Theorem 1 and can be skipped.

V. THE BASD-BASED PROMETHEE-II METHOD

To address probabilistic dual hesitant fuzzy MCGDM problems in which correlation averse behavior of DMs is considered, an BASD-based PROMETHEE-II method is proposed in this section. But before that, we first present the MCGDM framework, which is summarized as follows.

A. THE DESCRIPTION OF MCGDM PROBLEM

(1) A set of DMs is denoted by $D = \{D_1, D_2, \ldots, D_l\}$, where D_k is the kth DM with $k \in \{1, 2, \ldots, l\}$.

(2) A set of alternatives is denoted by $P = \{P_1, P_2, \ldots, P_m\}$, where P_i is the ith alternative with $i \in \{1, 2, \ldots, m\}$.

(3) A set of criteria is denoted by $C = \{c_1, c_2, \ldots, c_n\}$, where c_j is the jth criterion with $j \in \{1, 2, \ldots, n\}$. In this study, the n criteria are assumed to be independent and their associated weights are denoted by $W = \{w_1, w_2, \ldots, w_n\}$, such that $w_j \geq 0$ and $\sum_{j=1}^{n} w_j = 1$.

(4) Assume that the rating of an alternative P_i with respect to a criterion c_j provide by D_k is represented by is represented by $A_{ijk} = \left\{ \left\{ z_{ijk} | p_{z_{ijk}} \right\}, \left\{ v_{ijk} | q_{v_{ijk}} \right\} \right\}$, which takes the form of probabilistic dual hesitant fuzzy information. Thus, each DM corresponds to a decision matrix denoted by $R_k(A_{ijk})_{m\times n} = \left(\left\{ \left\{ z_{ijk} | p_{z_{ijk}} \right\}, \left\{ v_{ijk} | q_{v_{ijk}} \right\} \right\} \right)_{m\times n}$.

B. THE EXTENDED PROMETHEE-II METHOD

We develop an BASD-based PROMETHEE-II method to solve the probabilistic dual hesitant fuzzy MCGDM problem with correlation averse DMs. The detailed procedure of the proposed method consists of the following steps:

Step 1: Identify the MCGDM problem at hand, including the alternative set P, criterion set C and a group of DMs D.

Step 2: Elicit probabilistic dual hesitant fuzzy decision matrix from DMs, and each decision matrix is denoted by $R_k = \left(A_{ijk} \right)_{m\times n}, k = 1, 2, \ldots, l$.

Step 3: In general, the criteria set C can be divided into two sets, i.e., C_b and C_c, in which C_b denotes the set of benefit criteria and C_c denotes the set of cost criteria, such that $C_b \cap C_c = \emptyset$ and $C_b \cup C_c = C$. In practice, the criteria values should be normalized to ensure that they are of benefit type, and defined (7) can be used to transform the cost criteria to

$$\frac{1}{n(n-1)} \underset{i,j,i\neq j}{\overset{n}{\oplus}} \left(\left(\frac{n(1+T(A_i))}{\sum_{t=1}^{n}(1+T(A_t))} A_i \right)^p \otimes \left(\frac{n(1+T(A_j))}{\sum_{t=1}^{n}(1+T(A_t))} A_j \right)^q \right)$$
$$= \bigcup_{\substack{\mu_1\in z_1,\ldots,\mu_n\in z_n \\ \upsilon_1\in v_1,\ldots,\upsilon_n\in v_n}} \left\{ \begin{array}{l} 1 - \prod\limits_{i,j=1,i\neq j}^{n} \left(1 - \left(1 - (1-\mu_i)^{n(1+T(A_i))/\sum_{i=1}^{n}(1+T(A_i))} \right)^p \left(1 - (1-\mu_j)^{n(1+T(A_j))/\sum_{i=1}^{n}(1+T(A_i))} \right)^q \right)^{\frac{1}{n(n-1)}} \\ \prod\limits_{i,j=1,i\neq j}^{n} \left(1 - \left(1 - \upsilon_i^{n(1+T(A_i))/\sum_{i=1}^{n}(1+T(A_i))} \right)^p \left(1 - \upsilon_j^{n(1+T(A_j))/\sum_{i=1}^{n}(1+T(A_i))} \right)^q \right)^{\frac{1}{n(n-1)}} \end{array} \right\}$$

$$PDHFPGBM^{p,q}(A_1, A_2, \ldots, A_n)$$
$$= \bigcup_{\substack{\mu_1\in z_1,\ldots,\mu_n\in z_n \\ \upsilon_1\in v_1,\ldots,\upsilon_n\in v_n}} \times \left\{ \begin{array}{l} \left\{ 1 - \left(1 - \prod\limits_{i,j=1,i\neq j}^{n} \left(\begin{array}{l} 1 - \left(1 - \mu_i^{n(1+T(A_i))/\sum_{i=1}^{n}(1+T(A_i))} \right)^p \\ \left(1 - \mu_j^{n(1+T(A_j))/\sum_{i=1}^{n}(1+T(A_i))} \right)^q \end{array} \right)^{\frac{1}{n(n-1)}} \right)^{\frac{1}{p+q}} \right\} \left| \prod\limits_{i=1}^{n} p_{\mu_i}, \right. \\ \left\{ \left(1 - \prod\limits_{i,j=1,i\neq j}^{n} \left(\begin{array}{l} 1 - \left(1 - (1-\upsilon_i)^{n(1+T(A_i))/\sum_{i=1}^{n}(1+T(A_i))} \right)^p \\ \left(1 - (1-\upsilon_j)^{n(1+T(A_j))/\sum_{i=1}^{n}(1+T(A_i))} \right)^q \end{array} \right)^{\frac{1}{n(n-1)}} \right)^{\frac{1}{p+q}} \right\} \left| \prod\limits_{i=1}^{n} q_{\upsilon_i} \right. \end{array} \right\}$$

the benefit one.

$$\overline{A}_{ijk} = \left\{ \left\{ \overline{z}_{ijk} | \overline{p}_{\overline{z}_{ijk}} \right\}, \left\{ \overline{v}_{ijk} | \overline{q}_{\overline{v}_{ijk}} \right\} \right\} = \begin{cases} \left\{ \left\{ z_{ijk} | p_{z_{ijk}} \right\}, \left\{ v_{ijk} | q_{v_{ijk}} \right\} \right\}, & if\ c_j \in C_b, \\ \left\{ \left\{ v_{ijk} | q_{v_{ijk}} \right\}, \left\{ z_{ijk} | p_{z_{ijk}} \right\} \right\}, & if\ c_j \in C_c. \end{cases} \quad (7)$$

After that, the original decision matrix $R_k = (A_{ijk})_{m\times n}$ is transformed to the normalized decision matrix $\overline{R}_k = (\overline{A}_{ijk})_{m\times n}$.

Step 4: Aggerate all the individual probabilistic dual hesitant fuzzy decision matrices $\overline{R}_k = (\overline{A}_{ijk})_{m\times n}$ into the collective decision matrix $\overline{R} = (\overline{A}_{ij})_{m\times n}$.

Step 4.1: Translate the normalized decision matrix $\overline{R}_k = (\overline{A}_{ijk})_{m\times n}$ into two-dimensional random variables by the following formula $\overline{R}_k = ((\overline{\xi}_{ijk}, \overline{\zeta}_{ijk},))_{m\times n}$;

Step 4.2: Obtain collective decision matrix $\overline{R} = (\overline{A}_{ij})_{m\times n}$ by PDHFPBM operator or PDHFPGBM operator. Eq. (8), as shown at the bottom of the page or (9), as shown at the bottom of the page, where $T(\overline{A}_{ijk}) = \sum_{k'=1,k\neq k'}^{l} \sup(\overline{A}_{ijk}, \overline{A}_{ijk'})$, $\sup(\overline{A}_{ijk}, \overline{A}_{ijk'}) = 1 - d(\overline{A}_{ijk}, \overline{A}_{ijk'})$ and $d(\overline{A}_{ijk}, \overline{A}_{ijk'}) = \int_0^1 \int_0^1 \left| \Pr\left[\overline{\xi}_{ijk} \le x, \overline{\zeta}_{ijk} \le y\right] - \Pr\left[\overline{\xi}_{ijk'} \le x, \overline{\zeta}_{ijk'} \le y\right] \right| dxdy$.

Step 5: Calculate the combined weight vector for the criteria.

Step 5.1: Choose the best PDHFE on criteria c_j $(j = 1, 2, \ldots, n)$ by the defined (3), i.e.,

$$\overline{A}_j^+ = \bigcup_{\substack{\mu_{ij}^{(1)} \in \overline{z}_{ij}^{(1)}, \ldots, \mu_{ij}^{(t)} \in \overline{z}_{ij}^{(t)} \\ \upsilon_{ij}^{(1)} \in \overline{v}_{ij}^{(1)}, \ldots, \upsilon_{ij}^{(e)} \in \overline{v}_{ij}^{(e)}}} \left\{ \begin{array}{l} \left\{ \max\limits_i \left\{ \mu_{ij,t} \cdot p_{\mu_{ij,t}} \right\} \right\}, \\ \left\{ \min\limits_i \left\{ \upsilon_{ij,e} \cdot p_{\upsilon_{ij,e}} \right\} \right\} \end{array} \right\}$$

where $t = 1, 2, \ldots, \#\overline{z}_{ij}$, $e = 1, 2, \ldots, \#\overline{v}_{ij}$;

Step 5.2: Translate the above collective decision matrix $\overline{R} = (\overline{A}_{ij})_{m\times n} = \left(\left\{ \left\{ \overline{z}_{ij} | \overline{p}_{\overline{z}_{ij}} \right\}, \left\{ \overline{v}_{ij} | \overline{q}_{\overline{v}_{ij}} \right\} \right\} \right)_{m\times n}$ into decision matrix with two-dimensional random variables by the

$$\begin{aligned} \overline{A}_{ij} &= PDHFPBM^{p,q}\left(\overline{A}_{ij1}, \overline{A}_{ij2}, \ldots, \overline{A}_{ijl}\right) \\ &= \bigcup_{\substack{\mu_{ij1} \in \overline{z}_{ij1}, \ldots, \mu_{ijl} \in \overline{z}_{ijl} \\ \upsilon_{ij1} \in \overline{v}_{ij2}, \ldots, \upsilon_{ijl} \in \overline{v}_{ijl}}} \\ &\times \left\{ \begin{array}{l} \left\{ \left(1 - \prod\limits_{k,k'=1,k\neq k'}^{l} \left(1 - \left(1 - \left(1 - \mu_{ijk}\right)^{l\left(1+T\left(\overline{A}_{ijk}\right)\right) / \sum_{k=1}^{l}\left(1+T\left(\overline{A}_{ijk}\right)\right)}\right)^{p} \left(1 - \left(1 - \mu_{ijk'}\right)^{n\left(1+T\left(\overline{A}_{ijk'}\right)\right) / \sum_{k=1}^{l}\left(1+T\left(\overline{A}_{ijk}\right)\right)}\right)^{q} \right)^{\frac{1}{l(l-1)}} \right)^{\frac{1}{p+q}} \right\} \left| \prod\limits_{k=1}^{l} p_{\mu_{ijk}}, \right. \\ \left\{ 1 - \left(1 - \prod\limits_{k,k'=1,k\neq k'}^{l} \left(1 - \left(1 - \upsilon_{ijk}^{l\left(1+T\left(\overline{A}_{ijk}\right)\right) / \sum_{i=1}^{n}\left(1+T\left(\overline{A}_{ijk}\right)\right)}\right)^{p} \left(1 - \upsilon_{ijk'}^{n\left(1+T\left(\overline{A}_{ijk'}\right)\right) / \sum_{k=1}^{l}\left(1+T\left(\overline{A}_{ijk}\right)\right)}\right)^{q} \right)^{\frac{1}{l(l-1)}} \right)^{\frac{1}{p+q}} \right\} \left| \prod\limits_{k=1}^{n} q_{\upsilon_{ijk}} \right. \end{array} \right\} \end{aligned} \quad (8)$$

$$\begin{aligned} \overline{A}_{ij} &= PDHFPGBM^{p,q}\left(\overline{A}_{ij1}, \overline{A}_{ij2}, \ldots, \overline{A}_{ijl}\right) \\ &= \bigcup_{\substack{\mu_{ij1} \in \overline{z}_{ij1}, \ldots, \mu_{ijl} \in \overline{z}_{ijl} \\ \upsilon_{ij1} \in \overline{v}_{ij1}, \ldots, \upsilon_{ijl} \in \overline{v}_{ijl}}} \\ &\times \left\{ \begin{array}{l} \left\{ 1 - \left(1 - \prod\limits_{k,k'=1,k\neq k'}^{l} \left(1 - \left(1 - \mu_{ijk}^{n\left(1+T\left(A_{ijk}\right)\right) / \sum_{k=1}^{l}\left(1+T\left(A_{ijk}\right)\right)}\right)^{p} \left(1 - \mu_{ijk'}^{n\left(1+T\left(A_{ijk'}\right)\right) / \sum_{k=1}^{l}\left(1+T\left(A_{ijk}\right)\right)}\right)^{q} \right)^{\frac{1}{n(n-1)}} \right)^{\frac{1}{p+q}} \right\} \left| \prod\limits_{k=1}^{l} p_{\mu_{ijk}}, \right. \\ \left\{ \left(1 - \prod\limits_{k,k'=1,k\neq k'}^{l} \left(1 - \left(1 - \left(1 - \upsilon_{ijk}\right)^{n\left(1+T\left(A_{ijk}\right)\right) / \sum_{k=1}^{l}\left(1+T\left(A_{ijk}\right)\right)}\right)^{p} \left(1 - \left(1 - \upsilon_{ijk'}\right)^{n\left(1+T\left(A_{ijk'}\right)\right) / \sum_{k=1}^{l}\left(1+T\left(A_{ijk}\right)\right)}\right)^{q} \right)^{\frac{1}{n(n-1)}} \right)^{\frac{1}{p+q}} \right\} \left| \prod\limits_{k=1}^{l} q_{\upsilon_{ijk}} \right. \end{array} \right\} \end{aligned} \quad (9)$$

TABLE 7. Probabilistic dual hesitant fuzzy decision matrix given by the DM D_1.

Criteria	c_1	c_2	c_3	c_4
P_1	{{0.7|0.2, 0.6|0.2, 0.5|0.6}, {0.2|1}}	{{0.7|1}, {0.25|1}}	{{0.2|1}, {0.2|1}}	{{0.7|0.5, 0.6|0.5}, {0.3|1}}
P_2	{{0.1|1}, {0.4|1}}	{{0.3|1}, {0.7|1}}	{{0.7|1}, {0.3|0.5, 0.2|0.5}}	{{0.3|1}, {0.3|1}}
P_3	{{0.6|1}, {0.35|1}}	{{0.56|1}, {0.2|1}}	{{0.1|1}, {0.7|1}}	{{0.2|0.6, 0.4|0.4}, {0.4|1}}
P_4	{{0.05|0.7,0.2|0.3}, {0.5|1}}	{{0.3|0.5,0.2|0.5}, {0.6|0.5, 0.5|0.5}}	{{0.8|1}, {0.15|1}}	{{0.2|1}, {0.6|1}}
P_5	{{0.15|1}, {0.8|1}}	{{0.5|1}, {0.5|1}}	{{0.8|0.6, 0.6|0.4}, {0.15|1}}	{{0.12|1}, {0.7|0.9, 0.6|0.1}}
P_6	{{0.08|1}, {0.6|1}}	{{0.1|0.6,0.3|0.4}, {0.7|1}}	{{0.3|1}, {0.65|1}}	{{0.5|1}, {0.2|0.3, 0.4|0.7}}

following formula $\overline{R} = (\overline{A}_{ij})_{m\times n} = ((\overline{\xi}_{ij}, \overline{\zeta}_{ij}))_{m\times n}$ and translate the PDHFE $\overline{A}_j^+$ into two-dimensional random variables by the following formula $(\overline{\xi}_j, \overline{\zeta}_j)$;

Step 5.3: Obtain combined weight criteria are obtained by the defined (4)-(6).

Step 6: Establish the BASD degree matrix for each criterion.

Step 6.1: Translate the above collective decision matrix $\overline{R} = (\overline{A}_{ij})_{m\times n} = \left(\left\{\left\{\overline{z}_{ij}|\overline{p}_{\overline{z}_{ij}}\right\}, \left\{\overline{v}_{ij}|\overline{q}_{\overline{v}_{ij}}\right\}\right\}\right)_{m\times n}$ into decision matrix with two-dimensional random variables by the following formula $\overline{R} = (\overline{A}_{ij})_{m\times n} = ((\overline{\xi}_{ij}, \overline{\varsigma}_{ij}))_{m\times n}$.

Step 6.2: Validate BASD relations of decision matrix by means of pairwise comparisons of alternatives with each criterion according to Definition 7.

Step 6.3: Calculate the BASD degree by the defined (1) and the BASD degree matrix $H^j = \left(H^j_{\alpha\beta}\right)_{m\times n}$ for each criterion is given by the defined (10):

$$H^j_{\alpha\beta} = \begin{cases} \Phi\left(\overline{A}_{\alpha j}\varepsilon - BASD\overline{A}_{\beta j}\right), & if\ \overline{A}_{\alpha j}\varepsilon - BASD\overline{A}_{\beta j} \\ 0, & otherwise \end{cases}$$
$$\alpha, \beta = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n. \tag{10}$$

where $H^j_{\alpha\beta}$ denotes the BASD degree for alternative P_α dominates P_β corresponding to the criterion c_j.

Step 7: Determine the overall BASD degree matrix of m alternatives by the defined (11):

$$H = (H_{\alpha\beta})_{m\times m}, \tag{11}$$

where $H_{\alpha\beta}$ represents the BASD degree of the αth alternative P_α over the βth alternative P_β, and it can be calculated by the defined (12):

$$H_{\alpha\beta} = \sum_{j=1}^{n} w_j H^j_{\alpha\beta}, \alpha, \quad \beta = 1, 2, \ldots, m. \tag{12}$$

Step 8: Calculate the overall BASD degree $H^+(P_\alpha)$ for alternative P_α dominates the other alternatives and the overall BASD degree $H^-(P_\alpha)$ for other alternatives dominate alternative P_β, where

$$H^+(P_\alpha) = \sum_{\beta=1}^{m} H_{\alpha\beta}, \quad \alpha = 1, 2, \ldots, m, \tag{13}$$
$$H^-(P_\alpha) = \sum_{\beta=1}^{m} H_{\beta\alpha}, \quad \alpha = 1, 2, \ldots, m. \tag{14}$$

Step 9: Calculate the net BASD degree for the alternative P_k over other alternatives, where

$$H(P_\alpha) = H^+(P_\alpha) - H^-(P_\alpha), \quad \alpha = 1, 2, \ldots, m. \tag{15}$$

Step 10: Rank the alternatives by the descending order of $H(P_\alpha), \alpha = 1, 2, \ldots, m$.

VI. ILLUSTRATIVE APPLICATIONS AND COMPARATIVE ANALYSIS

This section implements the BASD-based PROMETHEE-II method to three pre-existent probabilistic dual hesitant fuzzy decision problems which include Arctic geopolitics risk evaluations [11], prospective candidate selection for software company [40] and strategy selection for software company about sustainable development [25] to show the use of the proposed method and complete some comparative analysis.

A. APPLICATION TO ARCTIC GEOPOLITICS RISK EVALUATIONS

Following Hao *et al.* [11], we help investors from six countries to grasp investing opportunity and finish a geopolitical risk evaluation for Arctic area. The countries adjacent to the Arctic are taken into considerations, such as the USA (P_1), Canada (P_2), Russia (P_3), Denmark (P_4), China (P_5). and Norway (P_6). The committee assesses the risk of Arctic area based on four criteria which follow: potential military conflicts (c_1), diplomatic disputes (c_2), dependence on energy imports (c_3) and control over marine routes (c_4). The subjective weight of all criteria given by experts is $W^s = (0.3, 0.2, 0.3, 0.2)^T$. A group of three DMs from who have different preference about these factors, denoted

TABLE 8. Probabilistic dual hesitant fuzzy decision matrix given by the DM D_2.

Criteria	c_1	c_2	c_3	c_4
P_1	{{0.5|1}, {0.5|1}}	{{0.2|1}, {0.4|0.8, 0.6|0.2}}	{{0.7|0.4, 0.4|0.6}, {0.3|0.7, 0.2|0.3}}	{{0.6|0.7, 0.7|0.3}, {0.25|1}}
P_2	{{0.3|0.5, 0.5|0.5}, {0.4|1}}	{{0.1|1}, {0.6|0.6, 0.8|0.4}}	{{0.4|0.8, 0.3|0.2}, {0.5|0.3,0.4|0.7}}	{{0.2|0.3, 0.3|0.7}, {0.6|1}}
P_3	{{0.1|0.1, 0.2|0.9}, {0.5|1}}	{{0.2|0.5, 0.3|0.5}, {0.3|0.5, 0.2|0.5}}	{{0.2|1}, {0.7|0.6, 0.5|0.4}}	{{0.5|1}, {0.4|1}}
P_4	{{0.2|1}, {0.6|0.9, 0.7|0.1}}	{{0.1|1}, {0.7|1}}	{{0.2|1}, {0.6|1}}	{{0.1|0.2, 0.2|0.8}, {0.2|0.6, 0.3|0.4}}
P_5	{{0.2|1}, {0.7|1}}	{{0.45|1}, {0.5|1}}	{{0.8|0.9, 0.6|0.1}, {0.11|1}}	{{0.3|1}, {0.2|1}}
P_6	{{0.4|0.4, 0.5|0.6}, {0.5|1}}	{{0.3|0.4, 0.4|0.6}, {0.5|1}}	{{0.3|1}, {0.6|1}}	{{0.2|1}, {0.6|1}}

TABLE 9. Probabilistic dual hesitant fuzzy decision matrix given by the DM D_3.

Criteria	c_1	c_2	c_3	c_4
P_1	{{0.4|1}, {0.5|1}}	{{0.9|1}, {0.1|1}}	{{0.3|1}, {0.5|0.4, 0.6|0.6}}	{{0.6|1}, {0.3|1}}
P_2	{{0.75|1}, {0.2|1}}	{{0.4|1}, {0.6|1}}	{{0.2|0.7, 0.4|0.3}, {0.2|1}}	{{0.3|1}, {0.6|1}}
P_3	{{0.6|0.6, 0.8|0.4}, {0.1|1}}	{{0.5|1}, {0.2|1}}	{{0.1|1}, {0.8|1}}	{{0.2|0.7, 0.4|0.3}, {0.6|1}}
P_4	{{0.2|1}, {0.7|1}}	{{0.5|0.6, 0.7|0.4}, {0.1|1}}	{{0.3|0.3, 0.5|0.7}, {0.2|0.5, 0.5|0.5}}	{{0.1|0.6,0.3|0.4}, {0.6|1}}
P_5	{{0.3|0.7, 0.4|0.3}, {0.4|0.6, 0.5|0.4}}	{{0.6|1}, {0.1|0.5, 0.2|0.5}}	{{0.7|1}, {0.2|1}}	{{0.1|0.45, 0.3|0.55}, {0.5|0.5, 0.65|0.5}}
P_6	{{0.2|0.2, 0.1|0.8}, {0.7|1}}	{{0.2|1}, {0.8|1}}	{{0.2|0.8, 0.3|0.2}, {0.6|1}}	{{0.35|1}, {0.5|0.5, 0.6|0.5}}

by $D = \{D_1, D_2, D_3\}$, are invited to provide intelligent support. Tables 7-9 represent the probabilistic dual hesitant fuzzy decision matrices given by the DMs. After obtaining all the information needed for this evaluation, the BASD-based PROMETHEE-II method is utilized to elicit the highest risk alternative and the detailed steps are shown below.

Step 1: Since all the considered criteria are of common type, then there is no need to implement the normalization process.

Step 2: By PDHFPBM in the defined (8) (or PDHFPGBM operator in the defined (9)), without the loss of generality, taking $p = q = 1$, we can obtain a collective assessment matrix for six countries about four criteria through aggregating the opinion of three decision makers.

Step 3: Calculate the BASD degree matrices with respect to each criterion. According to the defined (10), the BASD degree matrices H^1, H^2, H^3 and H^4 for each criterion can be obtained as follows:

$$H^1 = \begin{pmatrix} 0 & 0 & 0.6271 & 0 & 0 & 0 \\ 0.2755 & 0 & 0.7298 & 0 & 0 & 0.6271 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.6750 & 0 & 0 & 0.5513 \\ 0.8239 & 0.7569 & 0.9343 & 0.7979 & 0 & 0.9093 \\ 0 & 0 & 0.2756 & 0 & 0 & 0 \end{pmatrix},$$

$$H^2 = \begin{pmatrix} 0 & 0.7068 & 0.6600 & 0.7360 & 0.7384 & 0.6802 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2234 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$H^3 = \begin{pmatrix} 0 & 0 & 0 & 0.5913 & 0.5966 & 0.5838 \\ 0 & 0 & 0 & 0.5969 & 0.6020 & 0.5894 \\ 0 & 0.1988 & 0 & 0.6770 & 0.6811 & 0.6710 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$H^4 = \begin{pmatrix} 0 & 0.7152 & 0 & 0.6980 & 0.4555 & 0.8016 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.6565 & 0 & 0.6358 & 0 & 0.7607 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.4770 & 0.4454 & 0 & 0 & 0.6357 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Step 4: Calculate the combined weight of the criteria. Based on DMs' assessment, the subjective weight is given by $W^s = (0.3, 0.2, 0.3, 0.2)^T$. The objective weight can be obtained by the defined (2)-(5) in subsection IV, i.e., $W^b = (0.3023, 0.1994, 0.2440, 0.2543)^T$. Then, by the

TABLE 10. Probabilistic dual hesitant fuzzy decision matrix given by the DM D_1.

Criteria	c_1	c_2	c_3	c_4
P_1	{{0.7|0.7, 0.5|0.3}, {0.3|0.5, 0.2|0.5}}	{{0.3|1/3, 0.2|1/3, 0.1|1/3}, {0.45|0.5, 0.4|0.5}}	{{0.4|1/3, 0.3|1/3, 0.1|1/3}, {0.2|1}}	{{0.1|1}, {0.4|0.5, 0.3|0.5}}
P_2	{{0.4|0.5, 0.35|0.5}, {0.2|0.5, 0.1|0.5}}	{{0.5|0.5, 0.4|0.5}, {0.3|0.5, 0.2|0.5}}	{{0.4|0.5, 0.2|0.5}, {0.1|1}}	{{0.55|1/3, 0.5|1/3, 0.4|1/3}, {0.2|0.5, 0.1|0.5}}
P_3	{{0.5|0.5, 0.4|0.5}, {0.2|0.5, 0.1|0.5}}	{{0.4|0.5, 0.1|0.5}, {0.4|0.5, 0.25|0.5}}	{{0.4|0.5, 0.3|0.5}, {0.1|1}}	{{0.2|0.5, 0.1|0.5}, {0.4|0.5, 0.3|0.5}}
P_4	{{0.6|1/3, 0.5|1/3, 0.4|1/3}, {0.3|0.5, 0.1|0.5}}	{{0.3|0.25, 0.1|0.75}, {0.5|0.5, 0.4|0.5}}	{{0.5|0.5, 0.4|0.5}, {0.25|0.5, 0.2|0.5}}	{{0.25|0.5, 0.15|0.5}, {0.3|1}}

TABLE 11. Probabilistic dual hesitant fuzzy decision matrix given by the DM D_2.

Criteria	c_1	c_2	c_3	c_4
P_1	{{0.4|1}, {0.15|1}}	{{0.4|0.5, 0.2|0.5}, {0.1|1}}	{{0.2|0.5, 0.1|0.5}, {0.3|1}}	{{0.3|0.5, 0.1|0.5}, {0.5|1}}
P_2	{{0.3|0.5, 0.1|0.5}, {0.6|1}}	{{0.6|0.5, 0.2|0.5}, {0.15|0.5, 0.1|0.5}}	{{0.25|0.5, 0.15|0.5}, {0.1|1}}	{{0.75|1/3, 0.65|1/3, 0.6|1/3}, {0.25|0.5, 0.1|0.5}}
P_3	{{0.4|0.5, 0.3|0.5}, {0.1|1}}	{{0.5|1}, {0.3|0.5,0.2|0.5}}	{{0.2|0.5, 0.15|0.5}, {0.4|0.5, 0.3|0.5}}	{{0.35|0.5, 0.3|0.5}, {0.2|1}}
P_4	{{0.3|0.5, 0.1|0.5}, {0.4|1}}	{{0.3|0.5,0.1|0.5}, {0.4|1}}	{{0.2|1/3, 0.15|1/3, 0.1|1/3}, {0.05|1}}	{{0.3|1}, {0.2|0.5, 0.1|0.5}}

defined (6), the combined weight is given as follows: $W = (0.3021, 0.2003, 0.2714, 0.2262)^T$.

Step 5: Determine the overall BASD degree matrix of alternative P_α dominates P_β $(\alpha, \beta = 1, 2, 3, 4)$ corresponding to the criterion c_j $(j = 1, 2, 3, 4)$ using the defined (11) and (12).

$$H = \begin{pmatrix} 0 & 0.3031 & 0.3195 & 0.4849 & 0.4385 & 0.4908 \\ 0.0748 & 0 & 0.1981 & 0.1803 & 0.1819 & 0.3483 \\ 0 & 0.1916 & 0 & 0.3824 & 0.2058 & 0.3551 \\ 0 & 0 & 0.1832 & 0 & 0 & 0.1496 \\ 0.2236 & 0.3010 & 0.3428 & 0.2166 & 0 & 0.3741 \\ 0 & 0 & 0.0748 & 0 & 0 & 0 \end{pmatrix}$$

Step 6: Calculate the overall BASD degree $H^+(P_\alpha)$ for alternative P_α dominates the other alternatives and the overall BASD degree $H^-(P_\alpha)$ for other alternatives dominate alternative P_α $(\alpha = 1, 2, \ldots, 6)$ as defined (13) and (14).

$H^+(P_1) = 2.0368$, $H^+(P_2) = 0.9834$, $H^+(P_3) = 1.1349$, $H^+(P_4) = 0.3328$, $H^+(P_5) = 1.4581$, $H^+(P_6) = 0.0748$; $H^-(P_1) = 0.2984$, $H^-(P_2) = 0.7957$, $H^-(P_3) = 1.1184$, $H^-(P_4) = 1.2642$, $H^-(P_5) = 0.8262$, $H^-(P_6) = 1.7179$.

Step 7: Calculate the net BASD degree for the alternative P_α $(\alpha = 1, 2, \ldots, 6)$ over other alternatives by the defined (15). The finale results are shown as follows: $H(P_1) = 1.7384$, $H(P_2) = 0.1877$, $H(P_3) = 0.0165$, $H(P_4) = -0.9314$, $H(P_5) = 0.6319$, $H(P_6) = -1.6432$. Thus, the rankings of alternatives are $P_1 > P_5 > P_2 > P_3 > P_4 > P_6$.

B. APPLICATION TO SELECTING A PROSPECTIVE CANDIATE

Following Garg and Kaur [40], the proposed method help a software company to hire prospective projector manager who is the best suit for job. There are four prospective candidates for the personal interview. The committee provides the evaluation on four candidates $P = \{P_1, P_2, P_3, P_4\}$ based on four criteria which include educational qualification (c_1), technical knowledge (c_2), communication skills (c_3) and working experience (c_4). The subjective weight of all criteria given by experts is $W^s = (0.3, 0.4, 0.2, 0.1)^T$. A group of three domain DMs from who have different preference about these factors, denoted by $D = \{D_1, D_2, D_3\}$, are invited to provide intelligent support. Tables 10-12 represent the probabilistic dual hesitant fuzzy decision matrices given by the DMs. After obtaining all the information needed for this evaluation, the BASD-based PROMETHEE-II method is utilized to elicit the best candidate and the detailed steps are shown below.

Step 1: Since all the considered criteria are of common type, then there is no need to implement the normalization process.

Step 2: By PDHFPBM in the defined (8) (or PDHFPGBM operator in the defined (9)), without the loss of generality, taking $p = q = 1$, we can obtain a collective assessment matrix for four countries about four criteria through aggregating the opinion of three decision makers.

Step 3: Calculate the BASD degree matrices with respect to each criterion. According to the defined (10), the BASD degree matrices H^1, H^2, H^3 and H^4 for each criterion can be

TABLE 12. Probabilistic dual hesitant fuzzy decision matrix given by the DM D_3.

Criteria	c_1	c_2	c_3	c_4
P_1	{{0.4|1}, {0.2|0.5, 0.1|0.5}}	{{0.2|0.5, 0.1|0.5}, {0.3|1}}	{{0.25|0.5, 0.2|0.5}, {0.1|1}}	{{0.5|1/3, 0.4|1/3, 0.3|1/3}, {0.2|0.5, 0.1|0.5}}
P_2	{{0.3|0.1, 0.2|0.9}, {0.1|1}}	{{0.5|0.5, 0.4|0.5}, {0.3|1}}	{{0.3|1}, {0.5|0.5, 0.4|0.5}}	{{0.4|1/3, 0.3|1/3, 0.1|1/3}, {0.15|1}}
P_3	{{0.4|1}, {0.2|0.5, 0.1|0.5}}	{{0.3|1/3, 0.2|1/3, 0.1|1/3}, {0.4|1}}	{{0.2|0.5, 0.1|0.5}, {0.4|1}}	{{0.3|0.5, 0.2|0.5}, {0.15|1}}
P_4	{{0.45|0.5, 0.3|0.5}, {0.25|0.5, 0.2|0.5}}	{{0.3|0.5,0.2|0.5}, {0.1|1}}	{{0.7|1/3, 0.6|1/3, 0.5|1/3}, {0.3|1}}	{{0.35|0.5, 0.2|0.5}, {0.1|1}}

TABLE 13. Probabilistic dual hesitant fuzzy decision matrix.

Criteria	c_1	c_2	c_3	c_4
P_1	{{0.2|0.3, 0.3|0.4,}, {0.4|0.3, 0.5|0.6}}	{{0.7|0.5, 0.8|0.4}, {0.1|0.2, 0.2|0.8}}	{{0.5|0.3, 0.6|0.5}, {0.2|0.4, 0.3|0.5}}	{{0.3|0.3, 0.4|0.7}, {0.2|0.6, 0.3|0.4}}
P_2	{{0.3|0.4, 0.4|0.3}, {0.4|0.2, 0.5|0.7}}	{{0.6|0.7, 0.7|0.3}, {0.2|0.2, 0.3|0.4}}	{{0.5|0.5, 0.6|0.4}, {0.1|0.3, 0.2|0.4}}	{{0.4|0.2, 0.5|0.8}, {0.1|0.3, 0.2|0.7}}
P_3	{{0.1|0.2, 0.2|0.4, 0.3|0.3}, {0.3|0.6, 0.4|0.4}}	{{0.6|0.3, 0.8|0.7}, {0.1|0.4,0.2|0.5}}	{{0.4|0.3, 0.5|0.7}, {0.2|0.3, 0.3|0.7}}	{{0.2|0.2, 0.3|0.8}, {0.1|0.7, 0.2|0.2}}
P_4	{{0.3|0.7, 0.4|0.3}, {0.3|0.2, 0.4|0.7}}	{{0.7|0.6,0.8|0.3}, {0.1|0.3, 0.2|0.5}}	{{0.3|0.2, 0.4|0.8}, {0.2|0.6, 0.3|0.4}}	{{0.5|0.7, 0.6|0.3}, {0.2|0.8, 0.3|0.2}}

obtained as follows:

$$H^1 = \begin{pmatrix} 0 & 0.5125 & 0 & 0.4643 \\ 0 & 0 & 0 & 0 \\ 0 & 0.5958 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$H^2 = \begin{pmatrix} 0 & 0 & 0 & 0.1678 \\ 0 & 0 & 0.4418 & 0.4630 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$H^3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.2833 & 0.2453 & 0.4900 & 0 \end{pmatrix},$$

$$H^4 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0.6779 & 0 & 0.4756 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1773 & 0 \end{pmatrix}.$$

Step 4: Calculate the combined weight of the criteria. Based on experts' assessment, the subjective weight is given by $W^s = (0.3, 0.4, 0.2, 0.1)^T$. The objective weight can be obtained by the defined (2)-(5) in subsection IV, i.e., $W^b = (0.3540, 0.2034, 0.2500, 0.1926)^T$. Then, by the defined (6), the combined weight is given as follows: $W = (0.3348, 0.2949, 0.2297, 0.1425)^T$.

Step 5: Determine the overall BASD degree matrix of alternative P_α dominates P_β $(\alpha, \beta = 1, 2, 3, 4)$ corresponding to the criterion c_j $(j = 1, 2, 3, 4)$ using the defined (11) and (12).

$$H = \begin{pmatrix} 0 & 0.1716 & 0 & 0.2046 \\ 0.0966 & 0 & 0.1972 & 0.1357 \\ 0 & 0.1995 & 0 & 0 \\ 0.0651 & 0.0563 & 0.1378 & 0 \end{pmatrix}$$

Step 6: Calculate the overall BASD degree $H^+(P_\alpha)$ for alternative P_α dominates the other alternatives and the overall BASD degree $H^-(P_\alpha)$ for other alternatives dominate alternative P_α $(\alpha = 1, 2, 3, 4)$ by the defined (13) and (14).

$H^+(P_1) = 0.3762$, $H^+(P_2) = 0.4295$, $H^+(P_3) = 0.1995$, $H^+(P_4) = 0.2592$;$H^-(P_1) = 0.1617$, $H^-(P_2) = 0.4274$, $H^-(P_3) = 0.3350$, $H^-(P_4) = 0.3403$.

Step 7: Calculate the net BASD degree for the alternative P_α $(\alpha = 1, 2, 3, 4)$ over other alternatives by the defined (15). The finale results are shown as follows: $H(P_1) = 0.2145$, $H(P_2) = 0.0021$, $H(P_3) = -0.1355$, $H(P_4) = -0.0811$. Thus, the rankings of alternatives are $P_1 > P_2 > P_4 > P_3$.

C. APPLICATION TO A LONG-RANGE STRATEGIC PLANNING

Following Ren *et al.* [25], we utilize the approach to help a software company choose one from four strategy alternatives $P = \{P_1, P_2, P_3, P_4\}$ considering four main criteria: management(c_1), market (c_2), product development (c_3) and customers (c_4). The given weighting vector of the criteria is $W^s = (0.2, 0.15, 0.15, 0.5)^T$. Table 13 represents the probabilistic dual hesitant fuzzy decision matrix. After obtaining all the information needed for this evaluation, the BASD-based

PROMETHEE-II method is utilized to elicit the best strategy and the detailed steps are shown below.

Step 1: Since all the considered criteria are of common type, then there is no need to implement the normalization process.

Step 2: Calculate the BASD degree matrices with respect to each criterion. According to the defined (10), the BASD degree matrices H^1, H^2, H^3 and H^4 for each criterion can be obtained as follows:

$$H^1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.2687 & 0 & 0 & 0 \end{pmatrix},$$

$$H^2 = \begin{pmatrix} 0 & 0.5203 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0.8476 & 0 & 0 \\ 0 & 0.5381 & 0 & 0 \end{pmatrix},$$

$$H^3 = \begin{pmatrix} 0 & 0 & 0.2108 & 0 \\ 0 & 0 & 0.4861 & 0.5067 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$H^4 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0.4183 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.3203 & 0 & 0 & 0 \end{pmatrix}.$$

Step 3: Calculate the combined weight of the criteria. Based on experts' assessment, the subjective weight is given by $W^s = (0.2, 0.15, 0.15, 0.5)^T$. The objective weight can be obtained by the defined (2)-(5) in subsection IV, i.e., $W^b = (0.2215, 0.2649, 0.3189, 0.1947)^T$. Then, by the defined (6), the combined weight is given as follows: $W = (0.2238, 0.2119, 0.2325, 0.3318)^T$.

Step 4: Determine the overall BASD degree matrix of alternative P_α dominates P_β $(\alpha, \beta = 1, 2, 3, 4)$ corresponding to the criterion c_j $(j = 1, 2, 3, 4)$ using the defined (11) and (12).

$$H = \begin{pmatrix} 0 & 0.1103 & 0.0490 & 0 \\ 0.1388 & 0 & 0.1130 & 0.1178 \\ 0 & 0.1796 & 0 & 0 \\ 0.1664 & 0.1140 & 0 & 0 \end{pmatrix}$$

Step 5: Calculate the overall BASD degree $H^+(P_\alpha)$ for alternative P_α dominates the other alternatives and the overall BASD degree $H^-(P_\alpha)$ for other alternatives dominate alternative P_α $(\alpha = 1, 2, 3, 4)$the defined (13) and (14).

$H^+(P_1) = 0.1593$, $H^+(P_2) = 0.3696$, $H^+(P_3) = 0.1796$, $H^+(P_4) = 0.2804$;$H^-(P_1) = 0.3052$, $H^-(P_2) = 0.4039$, $H^-(P_3) = 0.1620$, $H^-(P_4) = 0.1178$.

Step 6: Calculate the net BASD degree for the alternative P_α $(\alpha = 1, 2, 3, 4)$ over other alternatives by the defined (15). The finale results are shown as follows: $H(P_1) = -0.1459$, $H(P_2) = -0.0343$, $H(P_3) = 0.0176$, $H(P_4) = 0.1626$. Thus, the rankings of alternatives are $P_4 > P_3 > P_2 > P_1$.

TABLE 14. Comparison analysis of the obtained results.

Research source	Comparative method	Ranking results
The proposed	BASD-based PROMETHEE-II	$P_1 > P_5 > P_2 > P_3 > P_4 > P_6$
Hao et al. [11]	Extended TODIM Method	$P_1 > P_4 > P_5 > P_2 > P_3 > P_6$
The proposed	BASD-based PROMETHEE-II	$P_4 > P_3 > P_2 > P_1$
Garg and Kaur [40]	Robust correlation coefficient	$P_2 > P_3 > P_4 > P_1$
The proposed	BASD-based PROMETHEE-II	$P_4 > P_3 > P_2 > P_1$
Ren et al. [25]	Score and deviation rules	$P_4 > P_3 > P_2 > P_1$

TABLE 15. Characteristic comparison of the proposed approach with different methods.

Methods	Characteristic i	Characteristic ii	Characteristic iii	Characteristic iv
Hao *et al.* [11]	yes	no	no	no
Garg and Kaur[40]	yes	yes	no	no
Ren *et al.*[25]	no	yes	no	no
The proposed method	yes	yes	yes	yes

D. COMPARATIVE ANALYSIS AND DISCUSSION

To demonstrate the advantages of the developed BASD-based PROMETHEE-II method, comparison analyses with existing methods in the literature [11], [25], [40] are performed.

All the rankings of alternatives are shown in Table 14. From Table 14, some observations can be summarized as follows:

Besides P_1 and P_6, the ranking results obtained by the proposed method have a different with the ranking results obtained by Hao *et al.* [11]. The reasons behind the differences can be considered from the following two aspects. On one hand, the proposed method considers the combined weight, which can simultaneously reflect subjective considerations of DMs and the objective information, while the existing method [11] has not taken this factor into consideration. On the other hand, the proposed BASD-based PROMETHEE-II considers psychological behavior of the DMs at the stage of eliciting the final ranking.

The ranking results obtained by the proposed method are significantly different from that obtained by Garg and Kaur [40]. The main reason of leading to the differences are the aggregation operator and weight of criteria. For aggerating the PDHFEs, the method of Garg and Kaur [40] has some problems, such as the aggerated PDHFEs could not be satisfied with the properties of PDHFEs. For that reason, our proposed method not only considers the combined weight of criteria which includes subjective considerations of DMs and objective information simultaneously but also the PDHFPBM (or PDHFPGBM) operator. After aggerating PDHFEs, they are still PDHFEs.

The ranking results obtained by the proposed method are the same as provided by Ren *et al.* [25]. It once again emphasizes that taking the psychological behavior of DMs is important and necessary in real-world decision-making problems. However, the method proposed by Ren *et al.* [25] haven't provided efficient aggregation operators to deal with the probabilistic dual hesitant fuzzy MCGDM problems.

Comparing the characteristics of the proposed approach with those of the existing methods, we can make further comparison analysis as shown in Table 15. The characteristics include whether consider more than one DM (Characteristic i), whether consider the psychological behavior of DMs (Characteristic ii), whether consider combined weight of criteria (Characteristic iii) and whether the aggregation operators reflect more information (Characteristic iv). It can be seen that the method proposed by Ren *et al.* [25] has not been suitable for probabilistic dual hesitant fuzzy MCGDM problems. Hao *et al.* [11] has not considered the psychological behavior of DMs. The exiting methods in Table 15 did not propose an effective way which reflect more information in process of aggregation in the MCGDM problems.

The advantages of the proposed method are as follows.

(1) From a probabilistic perspective, we construct two-dimensional joint random distribution with respect to PDHFEs by northwest rule. Then BASD and BASD degree which are considered as effective comparison tools are proposed. The qualitative and quantitative relationships of PDHFEs can be obtained by BASD and BASD degree. This avoid information loss as much as possible.

(2) Before comparing the PDHFEs by BASD, utility function should be selected. The different options of utility function can reflect different psychological behavior of DMs.

(3) In the process of PDHFEs, for weights of criteria, combined weights of criteria are proposed through a new kind of nonlinear combination which include subjective considerations of DMs and objective information simultaneously. For aggerating PDHFEs, PDHFPBM operator and PDHFPGBM operator are proposed which not only consider the importance of the DMs but also the support and reinforcement relationships among them.

In this paper, we only discuss the kind of DMs who have the attitude of risk aversion. We have not discussed too much DMs who have the attitude of risk seeking. This could be done by selecting utility function reflecting the attitude of DMs.

VII. CONCLUSION

PDHFS as a powerful tool in managing simultaneous aleatory and epistemic uncertainties is emerging and many problems on this decision-making context are to be addressed. In this paper, we propose a BASD-based PROMETHEE-II to probabilistic dual hesitant fuzzy MCGDM. The significant characteristics of BASD-based PROMETHEE-II are summarized as follows: (1) The presented method considers not only the support and reinforcement relationships among DMs but also the importance of the group members in the process of aggregating opinions of DMs, while the existing method for aggregating the individual opinions only considers the importance of the DMs, neglecting the support and reinforcement relationships among them. (2) From a stochastic perspective on PDHFEs, BASD and BASD degree are considered as comparison tools for PDHFEs. Then an BASD-based PROMETHEE-II method is presented to obtain the final ranking of alternatives. This paper relaxes the traditional assumption where the attitudes of DMs are complete rationality and avoids losing information when making comparisons. (3) The proposed method utilizes combined weights information over criteria can well express both the subjective considerations of DMs and the objective information simultaneously.

In the future, several further investigations could be carried out.

(1) Combined BASD rule, more classical MCDM method can be proposed to deal with real-world MCDM problems [21], [24].

(2) The proposed method can apply to deal with other forms of uncertainty in the MCGDM problems, such as probabilistic linguistic dual hesitant fuzzy sets, dual interval-valued hesitant fuzzy sets and dual extended hesitant fuzzy sets [46]–[50].

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QIAN ZHAO received the M.S. degree from Qufu Normal University, Rizhao, Shandong, China, in 2016. She is currently pursuing the Ph.D. degree with the School of Management and Economics, Beijing Institute of Technology, Beijing, China. Her current research interests include stochastic multi-attribute decision making, fuzzy measurement, and fuzzy information fusion.

YANBING JU received the Ph.D. degree from Beihang University, Beijing, China, in 2002. He is currently a Professor with the School of Management and Economics, Beijing Institute of Technology, Beijing. His main research interests include multiple attribute decision making, simulation and optimization of complex systems, supply chain management, and emergency management of emergencies.

WITOLD PEDRYCZ (Life Fellow, IEEE) received the M.Sc., Ph.D., and D.Sc. degrees from the Silesian University of Technology, Gliwice, Poland. He is currently a Professor and the Canada Research Chair (CRC-computational intelligence) with the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB, Canada. He is also with the Systems Research Institute, Polish Academy of Sciences, Warsaw, Poland. He has published numerous articles in these areas. His main research interests include computational intelligence, fuzzy modeling and granular computing, knowledge discovery and data mining, fuzzy control, pattern recognition, knowledge-based neural networks, relational computing, and software engineering. He is a member of a number of editorial boards of other international journals. He is intensively involved in editorial activities. He is an Editor-in-Chief of *Information Sciences*. He currently serves as an Associate Editor for the IEEE TRANSACTIONS ON FUZZY SYSTEMS.