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Collision avoidance for multiple Lagrangian dynamical systems with gyroscopic forces

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Abstract

This article introduces a novel methodology for dealing with collision avoidance for groups of mobile robots. In particular, full dynamics are considered, since each robot is modeled as a Lagrangian dynamical system moving in a three-dimensional environment. Gyroscopic forces are utilized for defining the collision avoidance control strategy: This kind of forces leads to avoiding collisions, without interfering with the convergence properties of the multi-robot system's desired control law. Collision avoidance introduces, in fact, a perturbation on the nominal behavior of the system: We define a method for choosing the direction of the gyroscopic force in an optimal manner, in such a way that perturbation is minimized. Collision avoidance and convergence properties are analytically demonstrated, and simulation results are provided for validation purpose.

Keywords

Collision avoidance, gyroscopic forces, multi-robot systems

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Introduction

This article describes a collision avoidance control strategy for a group of mobile robots whose dynamics are described according to the Lagrangian model.[1]

In the field of mobile robotics, collision avoidance is a primary issue that has thus been widely addressed in the past. When driving a robot to converge to the desired configuration, it is necessary to ensure that the interaction with the environment, as well as with static and dynamic obstacles, is sufficiently safe. This implies that the mobile robot's trajectory needs to be computed in such a way that collisions are always avoided.

Even though providing a comprehensive review of the literature on this topic is out of the scope of this article, we will briefly describe some of the main approaches that can be found in the literature, without claiming completeness, with the purpose of highlighting the motivation for the proposed methodology.

Typically, the *primary task* of a mobile robot is defined with the objective of reaching the desired configuration (possibly optimizing some cost function). Nevertheless, appropriate strategies for collision avoidance need to be defined, when dealing with realistic applications. Moreover, it is often desirable to introduce a *reactive behavior*, which allows the robots to handle unforeseen situations. A remarkable example of application where a reactive behavior is needed in the case where unknown obstacles may appear and may be identified by means of onboard sensors.

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In this case, the trajectory of the mobile robots must be modified and adapted online, as an obstacle is acquired, in order to ensure safety.

However, the introduction of *additional* control strategies, defined for collision avoidance purposes, typically generates interference with the primary task of the mobile robots. Consider, for instance, *artificial potential fields* (see the works of Rimon and Koditschek,[2] Bouraine et al.,[3] Hokayem et al.,[4] Leonard and Fiorelli,[5] Lindhe et al., [6] Sabattini et al.,[7] Su et al.,[8] and Tanner et al.[9]), which are a widely exploited and very effective method for avoiding collisions. Exploiting these strategies, robots are driven to perform the gradient descent of an opportunely designed artificial potential field, whose gradient can be computed in a decentralized manner, and produces a repulsive force that drives each robot to move away from obstacles or other robots. These strategies are very attractive because of their effectiveness in creating a reactive behavior, which provably avoids collisions with unforeseen obstacles. Moreover, these strategies are effective in coordinated multi-robot scenarios as well.[4–8] Furthermore, as shown in the work of Stastny et al.,[10] artificial potential fields can be combined with advanced additional nonlinear control strategies, such as model predictive control, for achieving collision avoidance in groups of multiple robotic systems characterized by complex dynamical models.

However, their main drawback is in the well-known local minima problem[11]: Interacting with the primary task of the mobile robots (e.g. convergence to a common position, creating a formation), collision avoidance artificial potential fields can create undesired asymptotically stable configurations that prevent the robots from reaching the desired configuration.

This article aims at defining a collision avoidance control strategy that allows a group of cooperative mobile robots to avoid collisions among each other and with environmental obstacles. The objective is to introduce the smallest possible interference with the multi-robot system's primary task. In order to create a provably safe collision avoidance reactive behavior, the dynamics of each robotic system are explicitly taken into account. Therefore, the collision avoidance action is defined as a *gyroscopic force.*

Several works on the use of gyroscopic forces for obstacle avoidance can be found in the literature,[12–17] typically for mobile robots moving on a two-dimensional environment (i.e. the ground floor). Roughly speaking, a gyroscopic force is always perpendicular to the velocity of the robot: This implies that these forces do not do any work. Hence, this is the main motivation in using gyroscopic forces: In fact, this property guarantees that they do not modify the convergence characteristics of desired control laws defined as the gradient of an artificial potential field.

The article is organized as follows. Related works are analyzed in "Related work and main contribution" section. "Problem statement and system definition" section provides a description of the model of the system and formally introduces the problem analyzed in this article. The collision avoidance control law is then introduced in "Definition of the collision avoidance control law" section. Avoidance of collisions is then demonstrated in "Collision avoidance and convergence" section. As an example application, in "Application: Rendezvous for fully actuated spacecraft vehicles, with global connectivity maintenance" section, we consider a group of six degree-of-freedom fully actuated vehicles that are required to perform rendezvous in a cluttered environment, while maintaining connectivity. Simulation results are described in "Simulations" section. Finally, "Conclusions" section contains some concluding remarks.

Related work and main contribution

In this section, we will analyze the main works that can be found in the literature on the use of gyroscopic forces for collision avoidance purposes. Subsequently, the main contribution of this article will be highlighted.

When planning the path for a mobile robot, it is desirable to ensure both convergence to the desired configuration and avoidance of collisions with environmental obstacles and other robots.

In the works presented by De Medio and Oriolo[15] and De Luca and Oriolo,[16] the authors consider a path planning problem for ground mobile robots. Specifically, the path for each robot is computed exploiting the artificial potential field method[18]: The path for each robot is computed according to the negative gradient of a global artificial potential field, whose minimum is in the desired configuration. Instead of introducing repulsive potential fields (as in the standard artificial potential field method), De Medio and Oriolo[15] and De Luca and Oriolo[16] introduce the so-called *vortex fields* that are distortions of the global artificial potential field which make the robots turn around the obstacles. This strategy can be extended considering nonholonomic constraints while planning the path.[19] This strategy is formally guaranteed to avoid the creation of local minima, which are undesired blocking points.

A similar result is obtained in the work of Antonelli et al.[17] and Antonelli et al.,[20,21] exploiting the *null-space-based* (NSB) behavioral control. In these strategies, several tasks are considered to be simultaneously accomplished by the robots. This approach can encode the necessity of reaching the desired configuration while avoiding collisions. Roughly speaking, the lowest priority task (i.e. convergence to the desired configuration) is performed as desired only if it does not interfere with the highest priority task (i.e. collision avoidance). If there is an interference between the two tasks, the highest priority task is always fulfilled, while only the projection of the lowest priority task on the null-space of the highest priority task is accomplished. Therefore, collision avoidance is always guaranteed, while convergence to the desired configuration is only partially fulfilled, in such a way that does not interfere with collision avoidance.

Both the vortex field and the NSB are very effective strategies for collision-free path planning. Considering the (possibly nonholonomic) kinematic model of a mobile robot, these strategies ensure the definition of a trajectory that drives the robot to the desired configuration while avoiding collisions with obstacles. However, it is worth noting that these strategies solve a kinematic problem: Dynamics of the robots are, in fact, not considered. Even though path planning is an inherently kinematic problem, the dynamic behavior of a mobile robot cannot always be neglected, when solving the collision avoidance problem. In fact, considering the presence of unpredictable obstacles, whose position is acquired by limited range sensors, it is necessary to ensure avoidance of collision, regardless of the velocity of the robot when the obstacle is identified. Therefore, is it important to explicitly consider the dynamic behavior of the mobile robots, when defining the collision avoidance control strategy?

Along these lines, in the study of Arogeti and Ailon,[22] the authors consider a group of quadrotors, modeled as nonlinear systems. Around each quadrotor, a forbidden region is then defined: Path planning methods are then used for ensuring that each quadrotor does not enter the forbidden region related to other quadrotors. A similar strategy is defined in the study of Jin et al.,[23] where obstacle-free regions in the environment are computed, taking into account the obstacles' velocities as well. The motion of the robots is then constrained within these regions.

In the studies of Chang et al.,[12] Chang and Marsden,[13] and Mi et al.,[24,25] the authors model the mobile robots as *double integrator* systems and develop a collision avoidance strategy based on the combination of a gyroscopic force and a braking force. In particular, the braking force is an appropriately defined damping force that ensures avoidance of collisions. Conversely, the gyroscopic force is in charge of making the robot move around the obstacles, thus ensuring convergence to the desired configuration. Inspired by this works, in this article we design a collision avoidance control strategy based on the use of gyroscopic forces. Unlike previous approaches, we explicitly consider the complete dynamics of the mobile robots, which are modeled as Lagrangian dynamical systems. Moreover, we consider the case where the robots move in a three-dimensional environment. While a few preliminary attempts on defining three-dimensional gyroscopic forces for obstacle avoidance can be found in the literature,[13,14] to the best of the authors' knowledge optimality in the choice of the gyroscopic force has never been considered. In fact, given the vector describing each robot's velocity, there are infinitely many directions that define a gyroscopic force, namely, all the forces laying onto the plane that is perpendicular to the velocity itself. Therefore, in this article, we define a method to select the optimal direction for the gyroscopic force, in order to introduce the smallest possible interference with the desired control law.

Hence, the main contribution of this article can be summarized as follows:

1. Gyroscopic forces are defined for obstacle avoidance, considering the motion of the robots in a three-dimensional environment.
2. The dynamics of the mobile robots are explicitly considered, describing the robots by means of the Lagrangian dynamical model.
3. An optimality criterion is defined to select the direction of the gyroscopic force.
4. Collision avoidance and convergence to the desired configuration are analytically proven, explicitly considering the dynamics of a system of multiple mobile robots.

This article extends the preliminary results presented by Sabattini et al.,[26] providing formal demonstration of all the presented results for the multi-robot application. Moreover, the braking force is redefined, providing a less conservative definition.

Problem statement and system definition

Consider a group of N homogeneous mobile robots, and define $q_i \in \mathbb{R}^m$, $i = 1, \ldots, N$ as the position vector of the i th robot. Assume then that the dynamics of each robots can be described by the Lagrangian dynamical model:

$$M(q_i)\ddot{q}_i + C(q_i, \dot{q}_i)\dot{q}_i + D\dot{q}_i + g(q_i) = u_i \tag{1}$$

where $u_i \in \mathbb{R}^m$ is the control input, the matrix $M(q_i) \in \mathbb{R}^{m\times m}$ is the symmetric positive definite mass matrix, the matrix $C(q_i, \dot{q}_i) \in \mathbb{R}^{m\times m}$ represents the Coriolis effects, the matrix $D \in \mathbb{R}^{m\times m}$ is a positive definite matrix that represents a dissipative term due to friction, and $g(q_i) \in \mathbb{R}^m$ is the gravity term. Further details can be obtained from the study of Ortega et al.[1]

Let $\chi = [q_1^T \ \ldots \ q_n^T]^T \in \mathbb{R}^{Nm}$ and $\dot{\chi} = [\dot{q}_1^T \ \ldots \ \dot{q}_n^T]^T \in \mathbb{R}^{Nm}$ be the position vector and the velocity vector of the multi-robot system, respectively. The multi-robot system can be considered, globally, as a unique Lagrangian system,[27] defining the following quantities

$$\begin{aligned}
\mathscr{M}(\chi) &= \operatorname{diag}(M(q_i)) \\
\mathscr{C}(\chi, \dot{\chi}) &= \operatorname{diag}(C(q_i, \dot{q}_i)) \\
\mathscr{D} &= \operatorname{diag}(D) \\
\mathscr{G}(\chi) &= \left[g(q_1)^T \ \ldots \ g(q_N)^T\right]^T
\end{aligned} \tag{2}$$

where the operator $\operatorname{diag}(\cdot)$ defines a block-diagonal matrix. Hence, the matrices in equation (2) define the multi-robot Lagrangian system. Namely, the matrix $\mathscr{M}(\chi) \in \mathbb{R}^{Nm\times Nm}$ is the symmetric positive definite mass matrix, the matrix $\mathscr{C}(\chi, \dot{\chi}) \in \mathbb{R}^{Nm\times Nm}$ represents the Coriolis effects, the matrix $\mathscr{D} \in \mathbb{R}^{Nm\times Nm}$ is a positive definite matrix representing dissipation due to friction, and $\mathscr{G}(\chi)$ is the gravity term.

Then, defining the multi-robot control input $\mathscr{U} = [u_1^T \ \dots \ u_N^T]^T \in \mathbb{R}^{Nm}$, it is possible to write the overall dynamics of the multi-robot Lagrangian system as follows:

$$\mathscr{M}(\chi)\ddot{\chi} + \mathscr{C}(\chi, \dot{\chi})\dot{\chi} + \mathscr{D}\dot{\chi} + \mathscr{G}(\chi) = \mathscr{U} \qquad (3)$$

Moreover, we make the following assumptions on the robots considered in this article.

Assumption 1.

- i. The translational degrees-of-freedom of each robot are fully actuated.
- ii. The shape of each robot can be bounded within a sphere.
- iii. Robots are homogeneous, that is, they have the same shape, and they are controlled by means of the same control strategy.
- iv. Each robot can identify the presence of an obstacle and measure its relative position and the distance from its boundary, within the detection range $R > 0$.

The collision avoidance problem will now be formally defined. We will hereafter make the following assumptions on the obstacles in the environment.

Assumption 2.

- i. The obstacles are convex and static.
- ii. The distance between two obstacles is greater than the size of a robot.

Therefore, we are assuming that the only moving obstacles are the robots themselves. Assumptions on the convexity and separation of the obstacles can be relaxed bounding each nonconvex obstacle (or each group of close obstacles) within a convex shape.

We will take into account two kinds of collisions:

1. Collision between a robot and an obstacle.
2. Inter-robot collisions, that is, a collision between two robots.

Considering assumptions 1 and 2, only translational dynamics will be hereafter taken into account, as rotational motion does not cause collision.

Hence, let:

$$q_i = [x_i^T \ \vartheta_i^T]^T \qquad (4)$$

where $x_i \in \mathbb{R}^3$ is the Cartesian position of the robot, and $\vartheta_i \in \mathbb{R}^{m-3}$ is the rotation of the robot, expressed with respect to any parametrization.

Moreover, we will hereafter use the term *obstacle* to indicate a generic entity with which a collision can happen. Conversely, when explicitly referring to a static object in the environment, we will use the term *environmental obstacle*.

According to assumption 1 (iv), we now introduce the definition of *active obstacle*.

Definition 1. An obstacle is *active* from the i th robot's perspective, if the obstacle is within the detection range of the robot, and if the robot's velocity has a nonzero component that points toward the obstacle.

According to assumption 1, it is possible to conclude that this relationship is *mutual*, when referring to inter-robot collision: Namely, if robot j is an active obstacle for robot i, then robot i is an active obstacle for robot j.

Consider, without loss generality, the case where Ψ_i obstacles are within the detection range of the i th robot. Subsequently, define $n_{i,j} \in \mathbb{R}^3$ as the vector from the i th robot's position to the nearest point of the j th obstacle, $\forall j = 1, \dots, \Psi_i$: It is worth noting that, being the obstacles convex, this vector is well-defined.

Hence, we introduce the function $\sigma_{i,j} \in \{0, 1\}$, defined as follows

$$\sigma_{i,j} = \begin{cases} 1 & \text{if} \|n_{i,j}\| \leq R \ \text{ and } \ \dot{x}_i{}^T n_{i,j} \geq 0 \\ 0 & \text{otherwise} \end{cases} \qquad (5)$$

Namely, $\sigma_{i,j} = 1$ if the j th obstacle is *active*, from the i th robot's perspective, according to Definition 1.

The set of active obstacles $\mathcal{A}_i$ for the i th robot is then defined as follows

$$\mathcal{A}_i = \{j \in [1, \Psi_i] \ \text{such that} \ \sigma_{i,j} = 1\} \qquad (6)$$

It is then possible to define $\sigma_i \in \{0, 1\}$ as follows

$$\sigma_i = \begin{cases} 1 & \text{if} |\mathcal{A}_i| > 1 \\ 0 & \text{otherwise} \end{cases} \qquad (7)$$

where $|\mathcal{A}_i|$ is the cardinality of set $\mathcal{A}_i$. Therefore, $\sigma_i = 1$ if an *active obstacle* exists, according to definition 1. In other words, σ_i indicates the presence of an obstacle within the detection range R and that the i th robot is moving toward it.

Throughout the article, we will consider the following situation: The objective of the multi-robot system is defined as the gradient descent of an appropriately defined artificial potential field. Specifically, let

$$\mathscr{U} = \mathscr{U}^d + \mathscr{U}^o \qquad (8)$$

where $\mathscr{U}^d \in \mathbb{R}^{Nm}$ is the desired control input for the multi-robot system, and $\mathscr{U}^o \in \mathbb{R}^{Nm}$ will be subsequently defined for collision avoidance purposes. Moreover, $\mathscr{U}^d = [u_1^{d\,T} \ \dots \ u_N^{d\,T}]^T$, with $u_i^d \in \mathbb{R}^m$ and $\mathscr{U}^o = [u_1^{o\,T} \ \dots \ u_N^{o\,T}]^T$, with $u_i^o \in \mathbb{R}^m$, $\forall i = 1, \dots, N$. Therefore, the control input u_i is defined as follows

$$\begin{aligned} u_i &= u_i^d + u_i^o \\ \forall i &= 1, \dots, N \end{aligned} \qquad (9)$$

Defining then an artificial potential field $\mathcal{U}^d(\chi) : \mathbb{R}^{Nm} \mapsto \mathbb{R}^+$, the desired control input is defined as follows

$$\mathscr{U}^d = -\nabla_\chi \mathcal{U}^d(\chi) \qquad (10)$$

Therefore, the objective of this control law is to drive the multi-robot system to the following desired configuration

$$\begin{cases} \chi &= \chi^d \\ \dot{\chi} &= 0_{Nm} \end{cases} \tag{11}$$

where $\chi^d \in \mathbb{R}^{Nm}$ is defined such that $\mathcal{U}^d(\chi^d)$ is a local minimum of $\mathcal{U}^d(\chi)$, and $0_{Nm} \in \mathbb{R}^{Nm}$ is a zero vector.

We make the following assumption on the desired configuration.

Assumption 3.

i. When $\chi = \chi^d$, the distance between each pair of robots is larger than the size of a robot.
ii. The distance between each obstacle and each position a robot will take when $\chi = \chi^d$ is larger than the size of a robot.

This technical assumptions ensure that obstacles do not prevent robots from reaching their desired position, and that robots do not interfere with each other, when some of them have already reached the desired position.

We also assume that the control law in equation (10) can be computed in a decentralized manner from each robot. Specifically, define the artificial potential field $\mathcal{U}_i^d(q_i) : \mathbb{R}^m \mapsto \mathbb{R}^+$, $\forall i = 1, \ldots, N$, such that

$$\mathcal{U}^d(\chi) = \sum_{i=1}^{N} \mathcal{U}_i^d(q_i) \tag{12}$$

On these lines, we assume that the desired control term u_i^d is defined as follows

$$u_i^d = -\frac{\partial \mathcal{U}^d(\chi)}{\partial q_i} = -\frac{\partial \mathcal{U}_i^d(q_i)}{\partial q_i} \tag{13}$$

$\forall i = 1, \ldots, N$. Moreover, let $u_i^t \in \mathbb{R}^3$ be the translational part of u_i^d.

Definition of the collision avoidance control law

According to the definitions and assumptions introduced in Problem statement and system definition section, in this section we will define the collision avoidance control law. Specifically, for each robot, we define the control input u_i^o, introduced in equation (9), as follows

$$u_i^o = \sigma_i \left(u_i^a + b(\dot{x}_i) \right) \tag{14}$$

$\forall i = 1, \ldots, N$. Then, define the following quantities

$$\begin{aligned} \mathscr{U}^a(\chi, \dot{\chi}) &= [\sigma_1 u_1^{a\,T} \ldots \sigma_N u_N^{a\,T}]^T \\ \mathscr{B}(\chi, \dot{\chi}) &= [\sigma_1 b^T(\dot{x}_1) \ldots \sigma_N b^T(\dot{x}_N)]^T \end{aligned} \tag{15}$$

where $\mathscr{U}^a(\chi, \dot{\chi}) \in \mathbb{R}^{Nm}$ and $\mathscr{B}(\chi, \dot{\chi}) \in \mathbb{R}^{Nm}$. Then, the collision avoidance control action $\mathscr{U}^o$ introduced in equation (8) can be defined as follows

$$\mathscr{U}^o = \mathscr{U}^a(\chi, \dot{\chi}) + \mathscr{B}(\chi, \dot{\chi}) \tag{16}$$

We will hereafter define the quantities introduced in equation (14).

Braking force

The term $b(\dot{x}_i) \in \mathbb{R}^m$ in equation (14) introduces a braking force, which acts as a selective energy dissipation along the $n_{i,j}$ direction, for each obstacle $j = 1, \ldots, \Psi_i$. As in the previously defined terms, we consider only the translational motion: Hence, the force $b(\dot{x}_i)$ is defined as follows

$$b(\dot{x}_i) = [b^{t\,T}(\dot{x}_i) \; 0_{m-3}^T]^T \tag{17}$$

where $b^{t\,T}(\dot{x}_i) \in \mathbb{R}^3$ represents the translational component of $b(\dot{x}_i)$, while $0_{m-3} \in \mathbb{R}^{m-3}$ represents a zero vector, $\forall i = 1, \ldots, N$. The term $b^t(\dot{x}_i)$ is defined as follows

$$b^t(\dot{x}_i) = \sum_{j=1,\ldots,\Psi_i} b_{i,j}^t(\dot{x}_i) \tag{18}$$

Then, let $v_{i,j} = \dot{x}_i^T n_{i,j} \in \mathbb{R}$ be the projection of the i th robot's velocity along $n_{i,j}$ and define $\beta(v_{i,j}) : \mathbb{R} \mapsto \mathbb{R}$ as follows

$$\beta(v_{i,j}) = -\,\mathrm{sgn}(v_{i,j}) \gamma_{i,j} \left(|v_{i,j}| + e^{-|v_{i,j}|} \right) \tag{19}$$

where $\mathrm{sgn}(\cdot)$ represents the signum function, and the parameter $\gamma_{i,j} > 0$ will be defined hereafter. Hence, we define each component of the braking force as follows

$$b_{i,j}^t(\dot{x}_i) = \beta(v_{i,j}) \frac{n_{i,j}}{\|n_{i,j}\|} \tag{20}$$

It is worth remarking that the braking force is defined only with respect to active obstacles: Therefore, according to definition 1, we can consider only the case in which $v_{i,j} > 0$. Subsequently, equation (19) can be simplified as follows

$$\beta(v_{i,j}) = -\gamma_{i,j}(v_{i,j} + e^{-v_{i,j}}) \tag{21}$$

Moreover, it is worth remarking that $\dot{\chi}^T \mathscr{B}(\chi, \dot{\chi}) \leq 0$. In fact, considering the definition of $\mathscr{B}(\chi, \dot{\chi})$ given in equation (15), and considering the definition of $b(\dot{x}_i)$ given in equation (17), it is possible to obtain the following equality

$$\dot{\chi}^T \mathscr{B}(\chi, \dot{\chi}) = \sum_{i=1}^{N} \sigma_i \dot{x}_i^T b^t(x_i) = \sum_{i=1}^{N} \sum_{j=1}^{\Psi_j} \sigma_i \dot{x}_i^T b_{i,j}^t(x_i) \tag{22}$$

Considering then the definition of $b_{i,j}^t(x_i)$ given in equations (20) and (21), equation (22) can be rewritten as follows

$$\dot{\chi}^T \mathscr{B}(\chi, \dot{\chi}) = \sum_{i=1}^{N} \sum_{j=1}^{\Psi_j} -\sigma_i \dot{x}_i^T \frac{n_{i,j}}{\|n_{i,j}\|} \gamma_{i,j}(v_{i,j} + e^{-v_{i,j}}) \tag{23}$$

Since $v_{i,j} = \dot{x}_i^T n_{i,j}$, and considering the definition of σ_i given in equation (7), we can conclude that all the terms in the summation in equation (23) are nonpositive, which implies $\dot{\chi}^T \mathscr{B}(\chi, \dot{\chi}) \leq 0$.

Obstacle avoidance gyroscopic force

The term $u_i^a \in \mathbb{R}^m$ in equation (14) defines the obstacle avoidance action, which makes the i th robot escape from the obstacles. As stated before, we consider only the translational motion: Hence, the force u_i^a is defined as follows

$$u_i^a = [u_i^{gT} \; 0_{m-3}^T]^T \tag{24}$$

where $u_i^g \in \mathbb{R}^3$ represents the translational component of u_i^a and is defined as a gyroscopic force, while $0_{m-3} \in \mathbb{R}^{m-3}$ represents a zero vector, $\forall i = 1, \ldots, N$. We define the gyroscopic force u_i^g as follows

$$\begin{aligned} w_i &= u_i^t - \left(u_i^{tT} \frac{\dot{x}_i}{\|\dot{x}_i\|} \right) \frac{\dot{x}_i}{\|\dot{x}_i\|} \\ u_i^g &= K_i^g \frac{w_i}{\|w_i\|} \end{aligned} \tag{25}$$

where $K_i^g > 0$ is a constant.

It is worth noting that equation (25) defines a gyroscopic force. In fact,

- The vector w_i is perpendicular to $\dot{x}_i$, as it is obtained by subtraction from u_i^t the projection of u_i^t itself along $\dot{x}_i$. Hence, w_i is the *orthogonal projection* of u_i^t on the plane that is perpendicular to $\dot{x}_i$.
- The vector w_i is then normalized and multiplied by the constant value $K_i^g > 0$. Hence, the force u_i^g has constant magnitude and is always perpendicular to $\dot{x}_i$.

Since u_i^g is always perpendicular to $\dot{x}_i$, then

$$u_i^{gT} \dot{x}_i = \dot{x}_i^T u_i^g = 0 \tag{26}$$

According to equation (24), a similar property holds for u_i^a, namely

$$u_i^{aT} \dot{q}_i = \dot{q}_i^T u_i^a = 0 \tag{27}$$

Hence, u_i^a represents a gyroscopic force, which does not do any work. On the same lines, it is possible to show that $\mathscr{U}^a(\chi, \dot{\chi})$ represents a gyroscopic force. In fact, according to equations (27) and (15)

$$\dot{\chi}^T \mathscr{U}^a(\chi, \dot{\chi}) = \sigma_1(\dot{q}_1^T u_1^a) + \ldots + \sigma_N(\dot{q}_n^T u_N^a) = 0 \tag{28}$$

As will be clarified later on, this feature ensures that the presence of this force does not modify the convergence properties of the desired artificial potential-based control law.

It is worth noting that this property is verified for any choice of the gyroscopic force, that is, any force u_i^a that is perpendicular to the velocity vector $\dot{q}_i$. However, even though the introduction of a gyroscopic force does not influence the convergence properties, it clearly modifies the transient behavior, with respect to the ideal situation, which is in the absence of obstacles.

As is well known, given a vector $\xi \in \mathbb{R}^\phi$, and a $(\phi - 1)$-dimensional subspace Φ, the best approximation of ξ along Φ is the orthogonal projection of ξ onto Φ. Hence, the definition of the gyroscopic force given in equation (25) represents an optimal choice: In fact, it introduces *the smallest possible perturbation* to the desired control law, since it is defined as the orthogonal projection.

It is worth noting that, according to equation (25), the force u_i^g is not always well defined. It is possible to identify different pathological cases:

1. The force u_i^g is not well defined when $\|\dot{x}_i\| = 0$, that is, when the translational velocity of the i th robot is zero. However, in this case, any random force exhibits the same property of a gyroscopic force, as described in equation (26).
2. The force u_i^g is not well defined when $\|w_i\| = 0$. This can happen in two different cases:
   (a) If $\|u_i^t\| = 0$, which means that the i th robot is in the desired configuration. Hence, in this case, the robot is no longer required to move, and it is possible to set u_i^g to zero.
   (b) If $u_i^t = (u_i^{tT}\dot{x}_i)(\dot{x}_i/\|\dot{x}_i\|)$, that is, u_i^t and $\dot{x}_i$ are aligned. Then, a small random perturbation can be added to u_i^t in equation (25). However, in real applications, the possibility that those two vectors are perfectly aligned is practically zero.

Hence, in order to take into account these cases, the definition of u_i^g given in equation (25) needs to be modified. Define vector $n_i \in \mathbb{R}^3$ as follows

$$n_i = \sum_{j=1}^{\Psi_i} \gamma_{i,j} \frac{n_{i,j}}{\|n_{i,j}\|} \tag{29}$$

Subsequently, define $\varphi_i \in \mathbb{R}^3$ as follows

$$\varphi_i = \begin{cases} \dot{x}_i & \text{if } \|\dot{x}_i\| \neq 0 \\ n_i & \text{otherwise} \end{cases} \tag{30}$$

$\forall i = 1, \ldots, N$. Note that the choice of the vector n_i will be clarified in the Proof of Corollary 2.

Subsequently, let $\psi_i \in \mathbb{R}^3$ be defined as follows

$$\psi_i = u_i^t - \left(u_i^{tT} \frac{\varphi_i}{\|\varphi_i\|} \right) \frac{\varphi_i}{\|\varphi_i\|} \tag{31}$$

$\forall i = 1, \ldots, N$. Hence, the definition of w_i given in equation (25) is modified as follows

$$w_i = \begin{cases} \psi_i & \text{if } \|\psi_i\| \neq 0 \\ \rho_i - \left(\rho_i^T \frac{\varphi_i}{\|\varphi_i\|} \right) \frac{\varphi_i}{\|\varphi_i\|} & \text{otherwise} \end{cases} \tag{32}$$

where $\rho_i \in \mathbb{R}^3$ is obtained adding a small random perturbation to u_i^t.

Finally, the force u_i^g is defined as follows

$$u_i^g = \begin{cases} K_i^g \dfrac{w_i}{\|w_i\|} & \text{if } \|u_i^t\| \neq 0 \\ 0_{m-3} & \text{otherwise} \end{cases} \tag{33}$$

where $0_{m-3} \in \mathbb{R}^{m-3}$ is a zero vector.

Collision avoidance and convergence

In this section, we will show that the control action introduced in Definition of the collision avoidance control law section ensures both collision avoidance and convergence to the desired configuration defined as the minimum of the potential function $\mathcal{U}^d(\chi)$.

For this purpose, define $\mathcal{E}(\chi, \dot{\chi}) : \mathbb{R}^{Nm} \times \mathbb{R}^{Nm} \mapsto \mathbb{R}^+$ as the total energy of the system, that is, the sum of potential and kinetic energy. Without loss of generality, we will hereafter consider the case where the gravity term $\mathscr{G}(\chi)$ is compensated by the control law. Therefore, in this case, the potential energy of the system is represented by the artificial potential field $\mathcal{U}^d(\chi)$. Moreover, let $\mathcal{K}_i(\dot{q}_i, q_i) : \mathbb{R}^m \times \mathbb{R}^m \mapsto \mathbb{R}^+$ be the kinetic energy of the i th robot, defined as follows

$$\mathcal{K}_i(\dot{q}_i, q_i) = \frac{1}{2}\dot{q}_i^T M(q_i)\dot{q}_i \tag{34}$$

$\forall i = 1, \ldots, N$. Subsequently, the kinetic energy of the multi-robot system $\mathcal{K}(\chi, \dot{\chi}) : \mathbb{R}^{Nm} \times \mathbb{R}^{Nm} \mapsto \mathbb{R}^+$ can be defined as follows

$$\mathcal{K}(\chi, \dot{\chi}) = \sum_{i=1}^{N} \mathcal{K}_i(\dot{q}_i, q_i) = \frac{1}{2}\dot{\chi}^T \mathscr{M}(\chi)\dot{\chi} \tag{35}$$

Therefore, the total energy of the multi-robot system $\mathcal{E}(\chi, \dot{\chi})$ can be defined as follows

$$\mathcal{E}(\chi, \dot{\chi}) = \mathcal{U}^d(\chi) + \mathcal{K}(\chi, \dot{\chi}) \tag{36}$$

According to the definition of the artificial potential field $\mathcal{U}^d(\chi)$ in equation (12), the total energy of the i th robot $\mathcal{E}_i(q_i, \dot{q}_i)$ can then be defined as follows

$$\begin{gathered} \mathcal{E}_i(q_i, \dot{q}_i) = \mathcal{U}_i^d(q_i) + \mathcal{K}_i(\dot{q}_i, q_i) \\ \forall i = 1, \ldots, N \end{gathered} \tag{37}$$

The following theorem shows that the proposed control law ensures that the total energy of the multi-robot system does not increase, as the system evolves. This result will be instrumental for proving collision avoidance and convergence to the desired configuration.

Theorem 1. Consider the dynamical system described in equation (3) and the control law defined in equation (8). Consider also the case where the gravity term $\mathscr{G}(\chi)$ is compensated by the control law. Then, the total energy of the multi-robot system $\mathcal{E}(\chi, \dot{\chi})$ defined as in equation (36) does not increase, as the system evolves.

Proof. Consider the total energy of the multi-robot system $\mathcal{E}(\chi, \dot{\chi})$, defined in equation (36). The time derivative of the total energy can be computed as follows

$$\dot{\mathcal{E}}(\chi, \dot{\chi}) = \dot{\chi}^T \left(\nabla_\chi \mathcal{U}^d + \mathscr{M}(\chi)\ddot{\chi} + \frac{1}{2}\dot{\mathscr{M}}(\chi)\dot{\chi} \right) \tag{38}$$

As the gravity term $\mathscr{G}(\chi)$ is compensated by the control law, according to equations (3), (8), and (16), the dynamics of the system can be rewritten as follows

$$\mathscr{M}(\chi)\ddot{\chi} + \mathscr{C}(\chi, \dot{\chi})\dot{\chi} + \mathscr{D}\dot{\chi} = -\nabla_\chi \mathcal{U}^d + \mathscr{U}^a(\chi, \dot{\chi}) + \mathscr{B}(\chi, \dot{\chi}) \tag{39}$$

Then, from equations (38) and (39), the time derivative of the total energy of the multi-robot system can be rewritten as follows

$$\dot{\mathcal{E}}(\chi, \dot{\chi}) = \dot{\chi}^T \left(-\mathscr{C}(\chi, \dot{\chi})\dot{\chi} - \mathscr{D}\dot{\chi} + \mathscr{U}^a(\chi, \dot{\chi}) + \mathscr{B}(\chi, \dot{\chi}) + \frac{1}{2}\dot{\mathscr{M}}(\chi)\dot{\chi} \right) \tag{40}$$

This may be rewritten as follows

$$\dot{\mathcal{E}}(\chi, \dot{\chi}) = \dot{\chi}^T \left(\frac{1}{2}\left(\dot{\mathscr{M}}(\chi) - 2\mathscr{C}(\chi, \dot{\chi}) \right)\dot{\chi} - \mathscr{D}\dot{\chi} + \mathscr{U}^a(\chi, \dot{\chi}) + \mathscr{B}(\chi, \dot{\chi}) \right) \tag{41}$$

Let

$$\mathscr{N}(\chi, \dot{\chi}) = \dot{\mathscr{M}}(\chi) - 2\mathscr{C}(\chi, \dot{\chi}) \tag{42}$$

As is well known,[23] the matrix $\mathscr{C}(\chi, \dot{\chi})$ can be defined such that $\mathscr{N}(\chi, \dot{\chi})$ is skew symmetric. Hence, equation (41) can be rewritten as follows

$$\dot{\mathcal{E}}(\chi, \dot{\chi}) = \dot{\chi}^T \left(-\mathscr{D}\dot{\chi} + \mathscr{U}^a(\chi, \dot{\chi}) + \mathscr{B}(\chi, \dot{\chi}) \right) \tag{43}$$

As $\mathscr{U}^a(\chi, \dot{\chi})$ is a gyroscopic force, the condition in equation (28) holds. Moreover, as shown in equations (22) and (23), $\dot{\chi}^T \mathscr{B}(\chi, \dot{\chi}) \leq 0$. As we have assumed $\mathscr{D}$ to be positive definite, we can then conclude that $\dot{\mathcal{E}}(\chi, \dot{\chi}) \leq 0$. This ensures that the total energy of the multi-robot system function $\mathcal{E}(\chi, \dot{\chi})$ does not increase as the system evolves.

Using similar arguments, it is possible to demonstrate that the total energy of the i th robot, that is, $\mathcal{E}_i(q_i, \dot{q}_i)$, does not increase as the system evolves.

Corollary 1. Consider the dynamical system described in equation (1), and the control law defined in equation (9), for the generic i th robot. Consider also the case where the gravity term $g(q_i)$ is compensated by the control law. Then the total energy of the i th robot $\mathcal{E}_i(q_i, \dot{q}_i)$ defined as in equation (37) does not increase, as the system evolves.

Proof. The proof is analogous to that of theorem 1 and is then omitted.

We will now exploit the results of theorem 1 and corollary 1 to demonstrate that the proposed control strategy ensures collision avoidance. For this purpose, consider the case where the j th obstacle becomes active, for robot i, at time $t = t_0$. Let $n_{i,j}(t)$ be the vector connecting the i th robot's position to the position of the j th obstacle, at time t. Then, define $n_{i,j}^0$ as

$$n_{i,j}^0 = n_{i,j}(t_0) \tag{44}$$

Moreover, with a slight abuse of notation, let $\mathcal{U}^d(t)$ be the value of $\mathcal{U}^d(\chi)$ at time t. Then define $\bar{\mathcal{U}}_i > 0$ as

$$\bar{\mathcal{U}}_i = \mathcal{U}^d(t_0) \tag{45}$$

In order to completely define the collision avoidance control action, the parameter $\gamma_{i,j} > 0$ introduced in equation (19) has to be defined. For this purpose, assume that the velocity of the robots is bounded, and that the upper bound on the velocity is known. Then assuming the mass matrix of each robot to be bounded, the kinetic energy is upper bounded as well. Namely, $\exists \bar{\mathcal{K}} > 0$ such that

$$\mathcal{K}_i(q_i, \dot{q}_i) \leq \bar{\mathcal{K}} \quad \forall i = 1, \ldots, N \tag{46}$$

where $\mathcal{K}_i(q_i, \dot{q}_i)$ is defined according to equation (34). Specifically, $\bar{\mathcal{K}}$ may be defined as follows

$$\bar{\mathcal{K}} = \max_{(q_i, \dot{q}_i)} \frac{1}{2} \dot{q}_i^T \mathcal{M}(q_i) \dot{q}_i \tag{47}$$

Then, the parameter $\gamma_{i,j}$ can be defined as follows:

$$\gamma_{i,j} = 2 \frac{\bar{\mathcal{K}} + \bar{\mathcal{U}}_i}{\|n_{i,j}^0\|} \tag{48}$$

Based on the results of theorem 1, the following theorem shows that this definition of $\gamma_{i,j}$ ensures collision avoidance.

Theorem 2. Consider the dynamical system described in equation (1) and the control law defined in equation (14). Then, the braking force in equation (20), with the parameter $\gamma_{i,j}$ defined as in equation (48), guarantees:

i. avoidance of collisions with environmental obstacles and
ii. avoidance of inter-robot collisions.

Proof. We show that the braking force is able to dissipate a sufficient amount of energy, in order to avoid collisions.

i. Consider the j th active obstacle with respect to the i th robot and consider the translational motion of the i th robot. We will now take into account only the component of the robot's motion toward the obstacle, that is, the component of the motion along $n_{i,j}$. Specifically, let $s_{i,j} \in \mathbb{R}$ represent the displacement of the robot along $n_{i,j}$. Then

$$\dot{x}_i = \dot{s}_{i,j} \frac{n_{i,j}}{\|n_{i,j}\|} \tag{49}$$

According to definition 1, in the presence of an active obstacle, $v_{i,j} = \dot{x}_i^T n_{i,j} \geq 0$. Moreover, from equation (49), $v_{i,j} = \dot{s}_{i,j}$. Define $\mathcal{E}_i^d(q_i) : \mathbb{R}^m \mapsto \mathbb{R}^+$ as the amount of energy dissipated by the i th robot. Considering equations (17) and (19), the amount of energy $\mathcal{E}^d(q_i)$ that can be dissipated by the braking force can be computed as the absolute value of the integral of the dissipated power, namely

$$\begin{aligned} \mathcal{E}_i^d(q_i) &= \left| \int \left(\gamma_{i,j} {v_{i,j}}^2(\tau) + \gamma_{i,j} e^{-v_{i,j}(\tau)} v_{i,j}(\tau) \right) \mathrm{d}\tau \right| \\ &\geq \left| \int \gamma_{i,j} e^{-v_{i,j}(\tau)} v_{i,j}(\tau) \ \mathrm{d}\tau \right| \end{aligned} \tag{50}$$

Considering the fact that, as stated before, when the j th obstacle is active, $v_{i,j} = \dot{s}_{i,j} \geq 0$, we can conclude that

$$\mathcal{E}_i^d(q_i) \geq \gamma_{i,j} \int \dot{s}_{i,j}(\tau) \ \mathrm{d}\tau = \gamma_{i,j} |s_{i,j}| \tag{51}$$

with $s_{i,j}$ being the i th robot's displacement. Without loss of generality, we considered $s_{i,j} = 0$ at time $t = t_0$, and we have dropped the dependence on time.

Then, assuming the available energy to the i th robot to be equal to $\mathcal{E}_i(q_i, \dot{q}_i)$, and letting all the energy to be dissipated by the braking force, the length of the i th robot's displacement is upper bounded as follows

$$|s_{i,j}| \leq \frac{\mathcal{E}_i^d(q_i)}{\gamma_{i,j}} = \frac{\mathcal{E}_i(q_i, \dot{q}_i)}{\gamma_{i,j}} \tag{52}$$

Hence, in order to ensure collision avoidance, it is sufficient to guarantee that $|s_{i,j}| \leq \|n_{i,j}^0\|$. According to equation (52), this implies

$$\frac{\mathcal{E}_i(q_i, \dot{q}_i)}{\gamma_{i,j}} \leq \|n_{i,j}^0\| \tag{53}$$

Consider then the definition of $\gamma_{i,j}$ given in equation (48), the condition in equation (53) can be rewritten as follows

$$\frac{\mathcal{E}_i(q_i, \dot{q}_i)}{\bar{\mathcal{K}} + \bar{\mathcal{U}}_i} \frac{\|n_{i,j}^0\|}{2} \leq \|n_{i,j}^0\| \tag{54}$$

With a slight abuse of notation, let $\mathcal{E}_i(t)$ be the value of $\mathcal{E}_i(q_i, \dot{q}_i)$ at time t. Then, the condition in equation (54) can be rewritten as follows

$$\frac{\mathcal{E}_i(t)}{\bar{\mathcal{K}} + \bar{\mathcal{U}}_i} \frac{\|n_{i,j}^0\|}{2} \leq \|n_{i,j}^0\| \tag{55}$$

for time $t > t_0$.

Then, according to equations (45) and (47), it is possible to conclude that

$$\mathcal{E}_i(t_0) \leq \bar{\mathcal{K}} + \bar{\mathcal{U}}_i \tag{56}$$

From corollary 1, we know that the total energy of the i th robot does not increase as the system evolves. Therefore

$$\mathcal{E}_i(t) \leq \mathcal{E}_i(t_0) \tag{57}$$

for all time $t > t_0$.

According to equations (56) and (57), it is possible to conclude that

$$\frac{\mathcal{E}_i(t)}{\bar{\mathcal{K}} + \bar{\mathcal{U}}_i} \leq 1 \tag{58}$$

$\forall t > t_0$. Therefore, the condition in equation (55) is satisfied $\forall t > t_0$, which proves the statement.

ii. Following the same arguments, it is possible to demonstrate that the proposed control law ensures avoidance of collisions between the i th robot and the j th robot. In particular, according to corollary 1, both $\mathcal{E}_i(q_i, \dot{q}_i)$ and $\mathcal{E}_j(q_j, \dot{q}_j)$ do not increase, as the system evolves.

Therefore, letting all the energy to be dissipated by the braking force, the lengths of the robots' displacements can be upper bounded as follows

$$\begin{cases} |s_{i,j}| \leq \dfrac{\mathcal{E}_i^d(q_i)}{\gamma_{i,j}} = \dfrac{\mathcal{E}_i(q_i, \dot{q}_i)}{\gamma_{i,j}} \\ |s_{j,i}| \leq \dfrac{\mathcal{E}_j^d(q_j)}{\gamma_{j,i}} = \dfrac{\mathcal{E}_j(q_j, \dot{q}_j)}{\gamma_{j,i}} \end{cases} \tag{59}$$

In this case, the i th robot is an active obstacle for the j th one and vice versa. Therefore, $\|n_{i,j}^0\| = \|n_{j,i}^0\|$. Hence, in order to ensure inter-robot collision avoidance, it is sufficient to guarantee that $|s_{i,j}|$ and $|s_{j,i}|$ are smaller than $(\|n_{i,j}^0\|)/2$.

With a slight abuse of notation, let $\mathcal{E}_i(t)$ and $\mathcal{E}_j(t)$ be the values of $\mathcal{E}_i(q_i, \dot{q}_i)$ and $\mathcal{E}_j(q_j, \dot{q}_j)$ at time t, respectively. Then, equation (59) can be rewritten as follows

$$\begin{cases} |s_{i,j}| \leq \dfrac{\mathcal{E}_i(t)}{\gamma_{i,j}} \\ |s_{j,i}| \leq \dfrac{\mathcal{E}_j(t)}{\gamma_{j,i}} \end{cases} \tag{60}$$

for all time $t > t_0$.

Consider then the definition of $\gamma_{i,j}$, $\gamma_{j,i}$ given in equation (48), and consider the inequalities given in equations (56) and (57). Therefore, equation (60) can be rewritten as follows

$$\begin{cases} |s_{i,j}| \leq \dfrac{\mathcal{E}_i(t)}{\bar{\mathcal{K}} + \bar{\mathcal{U}}_i} \dfrac{\|n_{i,j}^0\|}{2} \\ |s_{j,i}| \leq \dfrac{\mathcal{E}_j(t)}{\bar{\mathcal{K}} + \bar{\mathcal{U}}_j} \dfrac{\|n_{j,i}^0\|}{2} \end{cases} \tag{61}$$

According to equations (56) and (57), it is possible to conclude that

$$\frac{\mathcal{E}_i(t)}{\bar{\mathcal{K}} + \bar{\mathcal{U}}_i} \leq 1, \qquad \frac{\mathcal{E}_j(t)}{\bar{\mathcal{K}} + \bar{\mathcal{U}}_j} \leq 1 \tag{62}$$

$\forall t > t_0$. Therefore, the condition in equation (61) is satisfied $\forall t > t_0$, which proves the statement.

In theorem 2, we have demonstrated that, thanks to the braking force, collisions with environmental obstacles, and among robots, are always avoided. However, the introduction of the gyroscopic force is necessary for correctly performing obstacle avoidance and converging to the desired configuration.

In fact, using only the braking force would ensure collision avoidance but might cause deadlock situations. Namely, consider the case where the braking force $\mathscr{B}(\chi, \dot{\chi})$ and the desired control input $\mathscr{U}^d$ are such that

$$\mathscr{B}(\chi, \dot{\chi}) = -\mathscr{U}^d \tag{63}$$

In this case, the robots would be forced to remain in their current positions, even though they are not in the desired configuration (i.e. $\|\mathscr{U}^d\| \neq 0$). This is the reason why the introduction of the gyroscopic force is necessary.

In theorem 1, we have demonstrated that the total energy of the multi-robot system does not increase, as the system evolves. The following corollary shows that, thanks to the gyroscopic force, deadlocks are avoided, and the multi-robot system eventually converges to the desired configuration.

Corollary 2. Consider the dynamical system described in equation (3) and the control law defined in equation (8). Consider also the case where the gravity term $\mathcal{G}(\chi)$ is compensated by the control law. Then, the system evolves to the desired configuration defined as in equation (11), namely

$$\begin{cases} \chi & = \chi^d \\ \dot{\chi} & = 0_{Nm} \end{cases}$$

Proof. Consider the total energy of the system $\mathcal{E}(\chi, \dot{\chi})$, defined in equation (36), as a Lyapunov function. Exploiting the results in theorem 1, it is possible to conclude that the Lyapunov function does not increase as the system evolves.

We will now show that the only steady-state configurations are local minima of $\mathcal{U}^d(\chi)$. For this purpose, it is possible to invoke LaSalle's principle,[28] to show that the only configurations where $\dot{\mathcal{E}}(\chi, \dot{\chi}) = 0$ correspond to the desired configuration, defined as in equation (11).

According to equation (43), $\dot{\mathcal{E}}(\chi, \dot{\chi}) = 0$ if and only if $\|\dot{\chi}\| = 0$, that is, $\|\dot{q}_i\| = 0$, $\forall i = 1, \ldots, N$. Consider now two different cases: without obstacles and with obstacles.

1. In case there are no obstacles, when the multi-robot system is not in the desired configuration, it is always subject to a force $\mathscr{U}^d$ such that $\|\mathscr{U}^d\| \neq 0$, which makes the system accelerate. Hence, the only steady-state configurations correspond to local minima of $\mathcal{U}^d(\chi, \dot{\chi})$.
2. Referring, without loss of generality, to the i th robot, consider the presence of Ψ_i active obstacles.

As stated before, we are considering the case $\|\dot{q}_i\| = 0$, which implies also $\|\dot{x}_i\| = 0$. Subsequently, it is possible to state that

$$v_{i,j} = \dot{x}_i^T n_{i,j} = 0$$

Therefore, according to equation (21), $\beta(v_{i,j}) = \gamma_{i,j}$. Subsequently, in this case, according to equation (20)

$$u_i = u_i^d + u_i^a - \sum_{j=1}^{\Psi_i} \gamma_{i,j} \frac{n_{i,j}}{\|n_{i,j}\|} \tag{64}$$

Consider the vector n_i defined as in equation (29), namely

$$n_i = \sum_{j=1}^{\Psi_i} \gamma_{i,j} \frac{n_{i,j}}{\|n_{i,j}\|}$$

Subsequently, let $p_i, r_i \in \mathbb{R}^m$ define a right-handed coordinate system together with n_i: namely, the (p_i, r_i) plane is the plane orthogonal to n_i. Hence, it is possible to state that

$$u_i^d = \left(u_i^{dT} \frac{p_i}{\|p_i\|}\right) \frac{p_i}{\|p_i\|} + \left(u_i^{dT} \frac{r_i}{\|r_i\|}\right) \frac{r_i}{\|r_i\|} + \left(u_i^{dT} \frac{n_i}{\|n_i\|}\right) \frac{n_i}{\|n_i\|} \tag{65}$$

According to equations (24) and (33), it is possible to decompose u_a as follows

$$u_i^a = K_i' \left[\left(u_i^{dT} \frac{p_i}{\|p_i\|}\right) \frac{p_i}{\|p_i\|} + \left(u_i^{dT} \frac{r_i}{\|r_i\|}\right) \frac{r_i}{\|r_i\|} + \left(u_i^{dT} \frac{n_i}{\|n_i\|}\right) \frac{n_i}{\|n_i\|}\right] \tag{66}$$

where, according to equation (33), $K_i' = [K_i^g / (\|w_i\|)] > 0$.

Hence, equation (64) may be rewritten as follows

$$u_i = \alpha_i^p \frac{p_i}{\|p_i\|} + \alpha_i^r \frac{r_i}{\|r_i\|} + \alpha_i^n \frac{n_i}{\|n_i\|} \tag{67}$$

where

$$\begin{cases} \alpha_i^p = (1 + K_i') u_i^{dT} \frac{p_i}{\|p_i\|} \\ \alpha_i^r = (1 + K_i') u_i^{dT} \frac{r_i}{\|r_i\|} \\ \alpha_i^n = (1 + K_i') u_i^{dT} \frac{n_i}{\|n_i\|} - \gamma_i \end{cases} \tag{68}$$

Hence, unless u_i^d is exactly aligned with n_i, then α_i^p or α_i^r are guaranteed to be nonzero. Subsequently, it is possible to conclude that the i th robot is subject to a nonzero force that makes it accelerate.

Conversely, in case u_i^d is exactly aligned with n_i, as described in Obstacle avoidance gyroscopic force section, the definition of u_i^a is modified such that it can be rewritten as follows

$$u_i^a = K_i' \left[\left(\zeta_i^T \frac{p_i}{\|p_i\|}\right) \frac{p_i}{\|p_i\|} + \left(\zeta_i^T \frac{r_i}{\|r_i\|}\right) \frac{r_i}{\|r_i\|} + \left(\zeta_i^T \frac{n_i}{\|n_i\|}\right) \frac{n_i}{\|n_i\|}\right] \tag{69}$$

where, according to equation (32), $\zeta_i \in \mathbb{R}^m$ is a random vector.

Hence, as in the previous case, it is possible to conclude that the i th robot is subject to a nonzero force that makes it accelerate.

It is then possible to conclude that the only steady-state configurations are represented by the ones described as in equation (11), namely

$$\begin{cases} \chi &= \chi^d \\ \dot{\chi} &= 0_{Nm} \end{cases}$$

where $\chi^d \in \mathbb{R}^{Nm}$ is defined such that $\mathcal{U}^d(\chi^d)$ is a local minimum of $\mathcal{U}^d(\chi)$, and $0_{Nm} \in \mathbb{R}^{Nm}$ is a zero vector.

Application: Rendezvous for fully actuated spacecraft vehicles, with global connectivity maintenance

In this section, we apply the previously described obstacle avoidance strategy to a group of fully actuated spacecraft vehicles performing rendezvous while keeping connectivity.

A decentralized strategy for global connectivity maintenance for groups of Lagrangian systems, based on the gradient descend of an artificial potential field, was introduced by Sabattini and coworkers[29–31] and will now be briefly described, as well as the dynamical model of the spacecraft vehicles.

Spacecraft vehicles dynamical model

We consider a group of six degree-of-freedom spacecraft vehicles, whose dynamics are described by Kristiansen et al.[32]

Specifically, the configuration of these vehicles is described by the following state vectors

$$q_i = [x_i^T \;\; \vartheta_i^T]^T \quad \dot{q}_i = [\dot{x}_i^T \;\; \omega_i^T] \tag{70}$$

where $x_i \in \mathbb{R}^3$ represents the Cartesian position of the i th robot, and ϑ_i represents the rotation of the i th robot, expressed in terms of Euler parameters.[33] $\dot{x}_i \in \mathbb{R}^3$ and $\omega_i \in \mathbb{R}^3$ are the linear and angular velocity of the i th robot, respectively.

The following relationship holds

$$\dot{\theta}_i = \mathcal{T}(q_i)\omega_i \tag{71}$$

where $\mathcal{T}(q_i)$ is a properly defined transformation matrix.

Referring to equation (1), the matrices that describe the dynamics of each spacecraft vehicle are defined as follows

$$\begin{aligned} M(q_i) &= \begin{bmatrix} m_s I_3 & 0_{3\times3} \\ 0_{3\times3} & J_s(q_i) \end{bmatrix} \\ C(q_i, \dot{q}_i) &= \begin{bmatrix} C_t(x_i, \dot{x}_i) & 0_{3\times3} \\ 0_{3\times3} & C_r(\vartheta_i, \omega_i) \end{bmatrix} \\ g(q_i) &= \begin{bmatrix} g_t(x_i) \\ 0_{3\times1} \end{bmatrix} \\ D &= 0_{3\times3} \end{aligned} \tag{72}$$

where $0_{\zeta\times\xi} \in \mathbb{R}^{\zeta\times\xi}$ is a zero matrix, and $I_\xi \in \mathbb{R}^{\xi\times\xi}$ is the identity matrix. The value m_s represents the mass of the spacecraft, while $J_s(q_i)$ is the matrix representing the moments of inertia.

From equation (72), it is easy to see that translations and rotations are decoupled and can be independently controlled. Hence, hereafter we will consider only the translational dynamics of the system, as in the previous sections. The matrix $C_t(x_i, \dot{x}_i)$ is a Coriolis-like skew-symmetric matrix and is defined as follows

$$C_t(x_i, \dot{x}_i) = 2m_s\ddot{x}_i \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{73}$$

The gravity term $g_t(x_i)$ is defined as follows

$$g_t(x_i) = m_f \begin{bmatrix} \frac{\mu}{r_s^3} - \ddot{x}_i^2 & -\ddot{x}_i & 0 \\ \ddot{x}_i & \frac{\mu}{r_s^3} - \ddot{x}_i^2 & 0 \\ 0 & 0 & \frac{\mu}{r_s^3} \end{bmatrix} x_i \tag{74}$$

where r_s is the average radius of the orbit of the spacecraft. Let G be the universal constant of gravity, and let M_e be the mass of the Earth, then $\mu \approx GM_e$.

Connectivity maintenance

The communication architecture among a group of robot can be effectively modeled as graph, which is usually referred to as the communication graph.[34] As is well known, considering an undirected graph, the communication graph is connected if and only if the second-smallest eigenvalue of its Laplacian matrix L is positive. For this reason, this eigenvalue, which will be hereafter referred to as λ_2, is known as the algebraic connectivity of the graph.

We consider the following connectivity model: Two robots can communicate if their Euclidean distance is less than or equal to the communication radius $R_c > 0$. As a consequence, as in the works of Sabattini and coworkers,[29–31] we define a weighted communication graph, whose edge weights are defined as follows

$$a_{ij} = \begin{cases} e^{-\frac{\|x_i - x_j\|^2}{2\nu^2}} & \text{if } \|x_i - x_j\|^2 \leq R_s^2 \\ 0 & \text{otherwise} \end{cases} \tag{75}$$

The scalar parameter ν is chosen to satisfy the threshold condition $e^{-(R_s^2)/(2\nu^2)} = \Delta$, where Δ is a small predefined threshold. Hence, $a_{ij} \geq 0$ represents the weight of the edge connecting the i th and the j th robots: It is positive if the robots are connected, zero otherwise.

This definition of the edge weights is motivated by the fact that λ_2 is a nonincreasing function of each edge weight[34]: Hence, as two connected robots increase their distance, the value of λ_2 decreases, until they disconnect.

The control law defined by Sabattini and coworkers[29,30] drives the robot to perform a gradient descent of an appropriately designed function of λ_2, namely, $\mathcal{U}_c(\lambda_2)$. Defining $\epsilon > 0$ to be the desired lower bound for λ_2, the function $\mathcal{U}_c(\lambda_2) : (\epsilon, \infty) \mapsto \mathbb{R}^2$ is defined as a nonincreasing function, which goes to infinity as λ_2 approaches ϵ and goes to a constant value as λ_2 increases. As an example, in the work of Sabattini and coworkers,[29,30] the following function was used

$$\mathcal{U}_c(\lambda_2) = \coth(\lambda_2 - \epsilon) \tag{76}$$

Robots are then driven to perform a gradient descent of $\mathcal{U}_c(\lambda_2)$.

It is worth noting that, even though the algebraic connectivity of the communication graph is a global quantity, the connectivity maintenance control action can be implemented exploiting an estimate of λ_2 computed by means of the bounded-error decentralized estimation procedure introduced by Sabattini et al.[35–37]

Rendezvous

As an example, we consider the following cooperative task to be completed by the group of robots: meeting at some common point, exploiting only local information. This task is known in the literature as rendezvous.

It is easy to prove that, as long as the communication graph is connected, rendezvous can be obtained making the robots perform a gradient descent of the artificial potential function $U(\chi) : \mathbb{R}^{Nm} \mapsto \mathbb{R}^+$ defined as follows

$$U(\chi) = \sum_{i=1}^{N} \sum_{j \in \mathcal{N}_i} K_r(x_i - x_j) \tag{77}$$

where $K_r > 0$ is a constant, and $\mathcal{N}_i$ is the neighborhood of the i th robot, that is, the set of robots that can communicate with the i th one. It is worth noting that the gradient of this potential field can be computed in a decentralized manner.

Simulations

In this section, we describe the results of the Matlab (version: R2015b) simulations performed to validate the control strategy presented in this article. Specifically, simulations have been performed in the scenario described in Application: Rendezvous for fully actuated spacecraft vehicles, with global connectivity maintenance section, that is, a group of fully actuated Lagrangian dynamical systems that, starting from randomly chosen initial positions, are driven to achieve rendezvous, performing a gradient descent of the artificial potential function defined in equation (77), while moving among randomly placed point obstacles.

The proposed obstacle avoidance control strategy has been compared with a standard artificial potential field-based

Table 1. Comparison between gyroscopic forces and artificial potential field-based collision avoidance strategy.

	Distortion	
	Mean	Standard deviation
Gyroscopic action	1.0851	0.1020
Artificial potential action	1.3650	0.4184

control law.[9] In particular, let $z_k \in \mathbb{R}^m$ represent the position of the k th point obstacle. Then inspired by the work of Leonard and Fiorelli,[5] the obstacle avoidance action can be defined as the gradient of the function $\mathcal{Q}_i(q) : \mathbb{R}^m \mapsto \mathbb{R}^+$ defined as follows

$$\mathcal{Q}_i(q) = \sum_{k \in \mathcal{S}_i} K_p \left(\frac{1}{3} \|q_i - z_k\|^3 - R_0 \ln \|q_i - z_k\| \right) \quad (78)$$

where $R_0 = 0.1R$ is the minimum allowed distance between a robot and an obstacle, and the set $\mathcal{S}_i$ is defined as the obstacles whose distance is smaller than the sensing radius R.

In order to compare the performance of the two different collision avoidance control strategies, we introduce a criterion to evaluate the *distortion* of the control law. Consider the total control law u in equation (1), and consider the definition of u_d as the desired control law, defined according to equation (14). Then, u can be rewritten as $u = u_d + u_{\text{obst}}$, where $u_{\text{obst}} \in \mathbb{R}^m$ is the obstacle avoidance control law. Hence, we define the distortion index $\delta > 0$ as follows

$$\delta = \frac{\|u\|}{\|u_d\|} = \frac{\|u_d + u_{\text{obst}}\|}{\|u_d\|} \quad (79)$$

Clearly, for the obstacle avoidance action to introduce small interference with the primary task, δ is required to be close to 1.

We will hereafter report the results related to a representative example, where four robots were utilized, moving in a three-dimensional environment (x, y, z) filled with 150 randomly placed point obstacles.

The data reported in Table 1 summarize the results obtained in 100 simulation runs (mean value and standard deviation), starting from randomly varying initial positions.

According to the simulation results, it is possible to conclude that the proposed control law introduces a much smaller distortion in the resulting control law, if compared with an artificial potential field-based control action: approximately 8.5% versus 36.5%.

The results of a typical simulation run are summarized hereafter. In particular, trajectories of the robots are depicted in Figure 1: In particular, Figure 1(a) depicts the trajectories obtained utilizing, for collision avoidance, the method based on gyroscopic forces proposed in this article, while Figure 1(b) depicts the trajectories obtained utilizing artificial potential fields. It is possible to see that, in the presented example, due to the presence of local minima, artificial potential fields prevent the robots from converging to a rendezvous configuration.

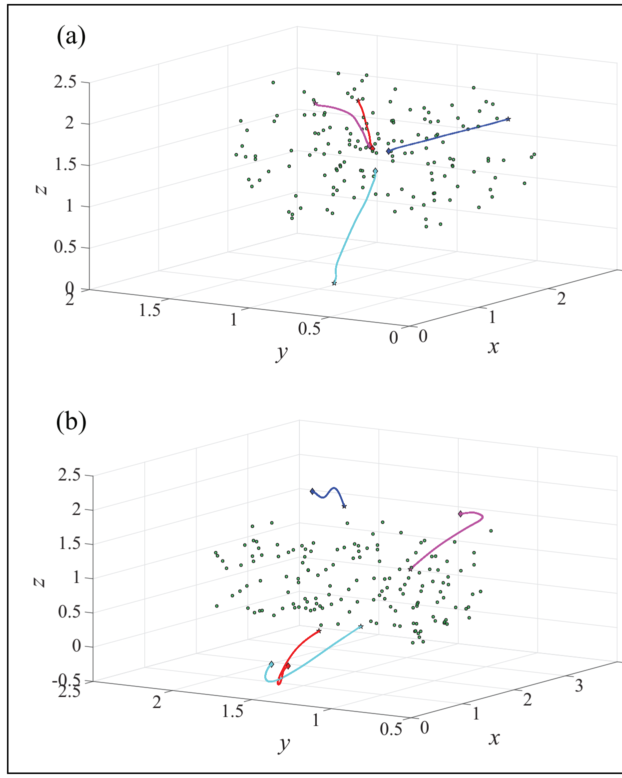

Figure 1. Trajectories of the robots: collision avoidance performed with (a) gyroscopic forces and (b) artificial potential field . Initial positions are represented with stars and final positions with diamonds. Green circles represent randomly placed point obstacles.

This fact is also highlighted comparing the distortion parameter. In particular, Figure 2 shows the value of the distortion index δ as the system evolves, with both collision avoidance control laws: Blue solid line represents the results obtained with the collision avoidance strategy presented in this article, while red dashed line is obtained with the artificial potential field control law in equation (78).

The two collision avoidance control laws were also compared in terms of amplitude of the control action itself. In particular, Figures 3 and 4 represent the maximum value (among the different robots) of the amplitude of the control action, along the three axes (x, y, z). It is possible to note that the amplitude of the proposed collision avoidance control action based on gyroscopic forces is generally smaller than (or comparable with) the amplitude of the corresponding artificial potential field-based control action.

Moreover, to validate the effectiveness of the collision avoidance strategy proposed in this article, Figure 5 shows the value of the minimum distance between a robot and an obstacle, as the system evolves, in a typical simulation run. As expected, it is always bounded away from zero.

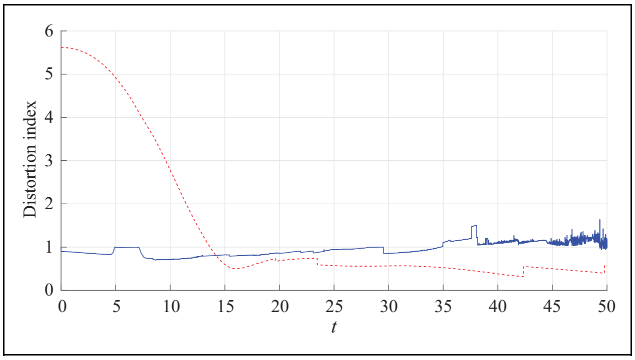

Figure 2. Value of the distortion index δ during a typical simulation run: Blue solid line represents the results obtained with the collision avoidance strategy presented in this article, while red dashed line is obtained with the artificial potential field control law in equation (78).

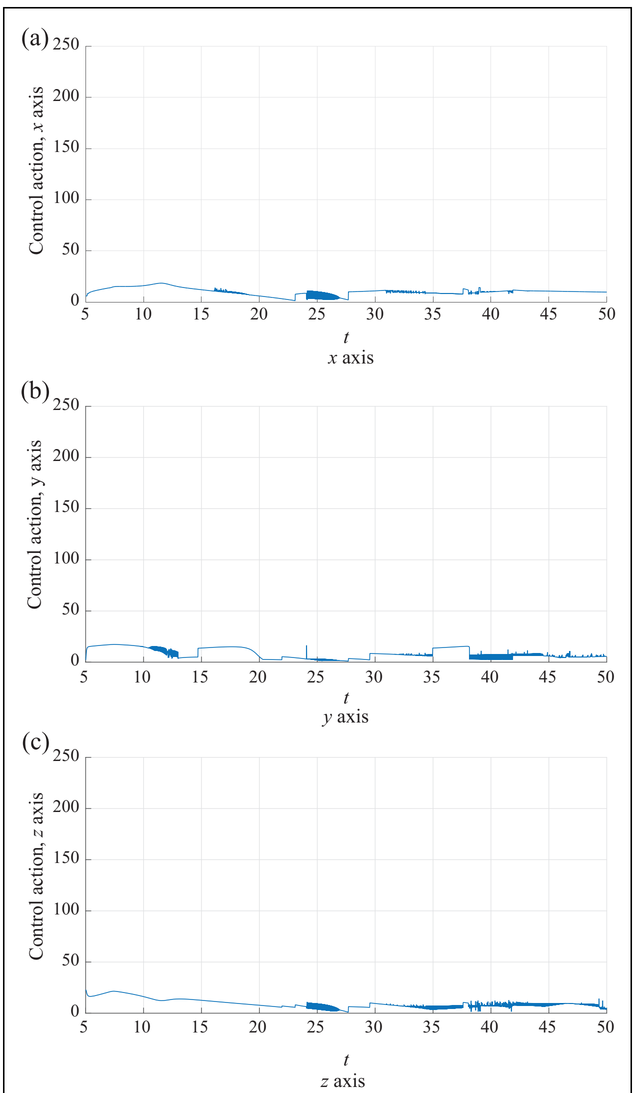

Figure 3. Maximum value of the amplitude of the collision avoidance control action along the three axes (x, y, z): gyroscopic forces.

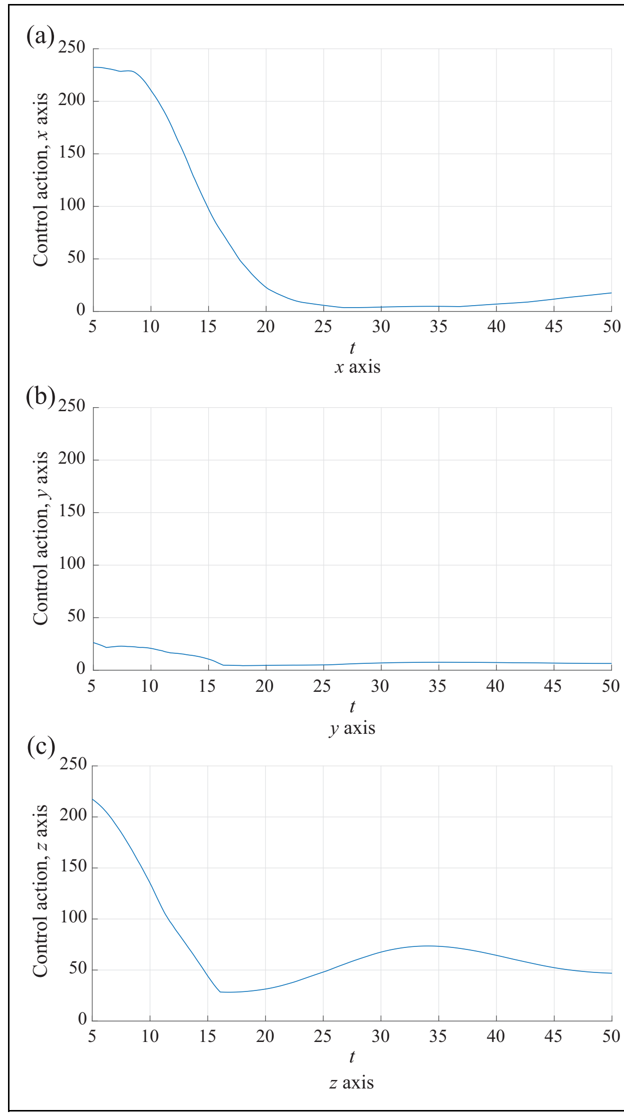

Figure 4. Maximum value of the amplitude of the collision avoidance control action along the three axes (x, y, z): artificial potential fields.

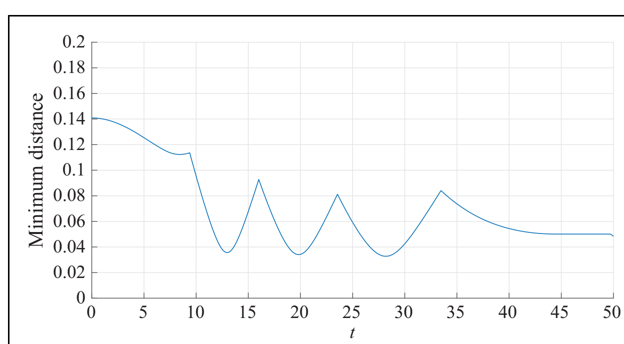

Figure 5. Minimum distance between a robot and an obstacle.

Conclusions

In this article, we propose a methodology for addressing collision avoidance for groups of mobile robots. The methodology is designed explicitly considering the full dynamics of the robots that are modeled as Lagrangian dynamical systems moving in a three-dimensional environment.

Collision avoidance is achieved by means of an appropriately defined gyroscopic force. This choice is motivated by the fact that, by definition, gyroscopic forces do not do any work. Therefore, considering a multi-robot system whose desired behavior is achieved with a potential-based control strategy, the introduction of a gyroscopic force does not modify the convergence properties. Moreover, the control law proposed in this article was defined in an optimized manner, in order to introduce the smallest possible perturbation with respect to the desired behavior of the system.

The proposed control strategy was analytically proven to guarantee collision avoidance and convergence to the desired configuration for multiple robotic systems. Simulation results were also provided for validation purpose.

Future work will aim at extending the proposed methodology to underactuated and nonholonomic systems, moving in the presence of nonconvex obstacles. Moreover, we aim at investigating the effect of the dynamics of realistic actuators on the performance of the collision avoidance control strategy. This will make it possible to perform experiments in realistic applications.

Authors' note

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