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Correction

Correction: Pavlačková, M.; Taddei, V. Mild Solutions of Second-Order Semilinear Impulsive Differential Inclusions in Banach Spaces. *Mathematics* 2022, *10*, 672

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The authors would like to correct parts of Step (*b*) and Step (*d*) in Theorem 1 in [1] since both Steps contain a mistake. In particular:

Step (b) Proving that Σ_m has a closed graph.

On page 11, the part of Step (b) from line 18 has to be corrected as follows:

Let us now prove that $f \in S^1_{G_m,q}$. Due to Mazur's convexity theorem, for each $k \in \mathbb{N}$, there exists $p_k \in \mathbb{N}$ and positive numbers $\beta_{k,i}$, $i = 0, \ldots, p_k$, such that $\sum_{i=0}^{p_k} \beta_{k,i} = 1$ and $r_k := \sum_{i=0}^{p_k} \beta_{k,i} f_{k+i} \to f$ in $L^1([a,b],E)$. From the sequence $\{r_k\}_k$, we extract a subsequence, denoted as the sequence as usual, such that $r_k(t) \to f(t)$, for all $t \in [a,b]\setminus N_1$ with $\lambda(N_1) = 0$. Moreover, for all $t \in [a,b]\setminus N_2$ with $\lambda(N_2) = 0$, $G_m(t, \cdot, \cdot)$ is weakly u.s.c.

Put $N = N_1 \cup N_2$ and consider $t_0 \in [a,b]\setminus N$. Then, for every weak neighborhood V of $G_m(t_0, q(t_0), \dot{q}(t_0))$, there exists a weak neighborhood W of $(q(t_0), \dot{q}(t_0))$ such that $G_m(t, x, y) \subset V$ when $(x,y) \in W$. Since the uniform convergence implies the weak pointwise convergence, it follows that $q_k(t_0) \rightharpoonup q(t_0)$ and $\dot{q}_k(t_0) \rightharpoonup \dot{q}(t_0)$. Thus, there exists $\bar{k}$ such that, for all $k \geq \bar{k}$, $(q_k(t_0), \dot{q}_k(t_0)) \in W$, yielding that $f_k(t_0) \in G_m(t_0, q_k(t_0), \dot{q}_k(t_0)) \subset V$, i.e., that $r_k(t_0) \in V$, because G_m is convex valued. Since $r_k(t_0) \to f(t_0)$, it follows that $f(t_0) \in \overline{V}$, for every weak neighborhood V of $G_m(t_0, q(t_0), \dot{q}(t_0))$. Since G_m is closed valued, the proof is complete.

Given $\mathit{\Phi} \in E^*$ and $t \in [a, b]$, consider the operator Φ: $L^1([a,t],E) \to \mathbb{R}$ defined by

$$\Phi(p) := \mathit{\Phi}\left(\int_a^t S(t-s)p(s)\, ds\right).$$

Since $S(t-s)$ is bounded and linear, for every t, s, Φ is clearly linear and bounded. Moreover, $f_k \rightharpoonup f$ also in $L^1([a,t],E)$, and hence, we have that

$$\mathit{\Phi}\left(\int_a^t S(t-s)f_k(s)\, ds\right) = \Phi(f_k) \to \Phi(f) = \mathit{\Phi}\left(\int_a^t S(t-s)f(s)\, ds\right).$$

By the arbitrariness of $\mathit{\Phi}$, we conclude that

$$\int_a^t S(t-s)f_k(s)\, ds \rightharpoonup \int_a^t S(t-s)f(s)\, ds.$$

Hence, since P_m is a linear and bounded operator taking values in the finite-dimensional space E_m, it holds that

$$\int_a^t P_m\, S(t-s)f_k(s)\, ds \to \int_a^t P_m\, S(t-s)f(s)\, ds.$$

check for updates

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The conclusion, then, can be completed like in last two lines of Part (b) [1].

Step (d) showing that Σ_m maps bounded sets into relatively compact sets.

The part of Step (d) starting twelve lines below formula (25) has to be corrected as follows:

According to Lemma 3, $(t, x) \to C(t)x$ is continuous in $[a,\ b] \times E$ and, hence, uniformly continuous in the pre-compact set $[a,\ b] \times I_1$.

According to Lemma 2 (h), since $S(s)$ is also linear and bounded, for every $s \in [a,\ b]$, we obtain by the same reasoning that

$$\left\{ S(s)f(s) : q \in\ nB_{m,}\ ,\ f \in S^1_{G_m,q} \right\} \subset E_m \cap\ X\ \subset X$$

is relatively compact, hence

$$I_2 = \left\{ \int_a^{t_0} S(s)f(s)\, ds : q \in\ nB_{m,}\ ,\ f \in S^1_{G_m,q} \right\}$$

is relatively compact in X.

According to (11), $(t, x) \to C'(t)x$ is continuous in $[a,\ b] \times X$ and, hence, uniformly continuous in the pre-compact set $[a,\ b] \times I_2$.

The conclusion, then, can be completed like in last five lines of Part (d) [1].

With these corrections, the order of some equations have been adjusted accordingly.

A correction has been made to Author Contributions, Funding and Acknowledgments. The correct information appears below as follows:

Author Contributions: Writing—original draft preparation, M.P. and V.T.; writing—review and editing, M.P. and V.T.; funding acquisition, M.P. and V.T. All authors have read and agreed to the published version of the manuscript.

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The authors state that the scientific conclusions are unaffected, and that this Correction does not influence the results contained in [1]. This correction was approved by the Academic Editor. The original publication has also been updated.

Reference

1. Pavlačková, M.; Taddei, V. Mild Solutions of Second-Order Semilinear Impulsive Differential Inclusions in Banach Spaces. *Mathematics* **2022**, *10*, 672. [CrossRef]