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INNOVATION

Essays on Applied Macroeconomics

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Abstract

This thesis consists of three essays on applied macroeconomics and macroeconometrics. In Chapter I, we challenge the claim of a recent authoritative study that identifies a unique shock as the main driver of business cycle fluctuations. We argue that the VAR used in that study is informationally insufficient, i.e., it is unable to recover the true structural shock driving business cycle fluctuations. Using a large-dimensional Structural Dynamic Factor model, we present an alternative view of the US business cycles, more in line with classical AD-AS theory. This underscores the multivariate nature of cycles and challenges the existence of a Main Business-Cycle shock. In Chapter II, we provide a few new empirical facts that theoretical models should feature in order to be consistent with US data. 1) There are two classes of shocks: demand and supply. Supply shocks have long-run effects on economic activity, demand shocks do not. 2) Both supply and demand shocks are important sources of business cycles fluctuations. 3) Supply shocks are the primary driver for consumption fluctuations, demand shocks for investment. 4) The demand shock is closely related to the credit spread, while the supply shock is essentially a news shock. The results are obtained using a novel approach which combines frequency domain identification and Dynamic Factor Model analysis. Chapter III delves into the asymmetric impact of demand shocks on the US economy using a Nonlinear Structural Dynamic Factor model. Our findings reveal that the effects of aggregate demand shocks are nonlinear, depending on their sign. Positive shocks are transitory, according to standard business cycle theory; conversely, negative shocks leave lasting scars on the economy. Recessions induced by demand-side shocks result in permanent declines in output, employment, and investment.

JEL subject classification: E32, C32.

Keywords: Business Cycle, Frequency Domain, Dynamic Factors Models, Non-linearity, Asymmetry.

Abstract

Questa tesi consiste in tre saggi di macroeconomica applicata e macroeconometria. Nel Capitolo I, mettiamo in discussione i risultati di un recente studio autorevole che identifica uno shock unico come il principale motore delle fluttuazioni del ciclo economico. Sosteniamo che il VAR utilizzato in tale studio sia carente dal punto di vista informativo, cioè non è in grado di recuperare il vero shock strutturale che guida le fluttuazioni del ciclo economico. Utilizzando un large-dimensional Structural Dynamic Factor model, presentiamo una visione alternativa del business cycle degli Stati Uniti, più in linea con la teoria AD-AS classica. Ciò sottolinea la natura multivariata dei cicli e mette in discussione l'esistenza di uno shock principale del ciclo economico. Nel Capitolo II, forniamo alcuni nuovi fatti empirici che i modelli teorici dovrebbero includere per essere coerenti con i dati statunitensi. 1) Ci sono due classi di shock: domanda e offerta. Gli shock di offerta hanno effetti a lungo termine sull'attività economica, gli shock di domanda no. 2) Sia gli shock di offerta che quelli di domanda sono importanti fonti di fluttuazioni dei cicli economici. 3) Gli shock di offerta sono il principale motore delle fluttuazioni del consumo, gli shock di domanda per gli investimenti. 4) Lo shock di domanda è strettamente legato al credit spread, mentre lo shock di offerta è essenzialmente un news shock. I risultati sono ottenuti utilizzando un approccio innovativo che combina l'identificazione nel dominio delle frequenze e l'analisi Dynamic Factor model. Nel Capitolo III, approfondiamo l'impatto asimmetrico degli shock di domanda sull'economia statunitense utilizzando un Nonlinear Structural Dynamic Factor model. I nostri risultati rivelano che gli effetti degli shock di domanda aggregata sono non lineari, a seconda del loro segno. Gli shock positivi sono transitori, secondo la teoria standard del ciclo economico; al contrario, gli shock negativi lasciano cicatrici durature sull'economia. Le recessioni indotte dagli shock di domanda producono declini permanenti nella produzione, nell'occupazione e negli investimenti.

JEL subject classification: E32, C32.

Keywords: Ciclo Economico, Dominio delle Frequenze, Modelli a Fattori Dinamici, Non linearità, Asimmetrie.

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Chapter 1

Two Main Business Cycle Shocks are Better than one^{*}

Abstract

This paper challenge the claim of a recent authoritative study that identifies a single shock as the main driver of business cycle fluctuations. We argue that the VAR used in that study is informationally deficient, thereby casting doubt on the causal interpretation of the identified shock. Using a large-dimensional Structural Dynamic Factor model, we present an alternative view of the US business cycles, more in line with classical AD-AS theory. This underscores the multivariate nature of cycles and challenges the existence of a Main Business-Cycle shock.

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1.1. INTRODUCTION

In their quest for a parsimonious explanation of business cycles, Kydland and Prescott (1982) put forward an appealing idea: cyclical fluctuations could be explained by a single shock.¹ In their model this shock is the technology shock. A recent authoritative paper, Angeletos et al. (2020), ACD henceforth, revives this idea but from a completely different perspective. The authors provide a comprehensive anatomy of the U.S. macroeconomy. The core of this anatomy is a set of five shocks, each of which accounts for the maximal volatility of a given macroeconomic variable (unemployment, output, hours worked, consumption and investment, respectively) over the business-cycle frequency band (6-32 quarters). These shocks share a common propagation mechanism, that is, they produce the same impulse-response functions (IRFs), and can be considered as the same shock, named the "Main Business Cycle" shock (MBC). The MBC shock accounts for the bulk of cyclical fluctuations in economic activity and has very special and well-defined features that challenge existing theories. On the one hand, it has no long-run effects; in that, it resembles a demand shock. On the other hand, it is disconnected from inflation; in that, it differs from a standard inflationary demand shock, of the New Keynesian variety. This is at odds with both the idea of news-driven cycles proposed by Beaudry and Portier (2006), and the view put forward by Christiano et al. (2014) that risk shocks are the dominant factors in cyclical fluctuations.² Most importantly, the picture that emerges is in sharp contrast with the standard AD-AS textbook theory, partly inspired by Blanchard and Quah (1989), where cyclical fluctuations are driven by two main shocks, a standard supply shock and a standard demand shock. This new perspective therefore lends support to theories that aspire to explain the bulk of the observed business cycles with a single demand shock, while posing a significant challenge to conventional New-Keynesian paradigm.

From a methodological point of view, ACD proposes a frequency-based identification method in the context of SVAR models. The method allows for the identification of the shock which maximizes the explained variance of a particular variable in a given frequency band.³ The approach of identifying dominant shocks in the frequency domain, starting from a VAR, is drawing increasing interest. This is because it allows isolating cyclical and long-run features without imposing economic conditions

¹This perspective is reminiscent of the original idea of Burns and Mitchell (1946), who argued that a reference cycle, or a one-dimensional latent cause of variation, drives the fluctuations of all macroeconomic variables.

 $^{^{2}}$ In general, it is incompatible with all those estimated New-Keynesian DSGE models that assume nominal rigidities or "sticky prices". In addition to Christiano et al. (2014), see also Smets and Wouters (2007) and Justiniano et al. (2010).

³This approach is essentially the frequency domain version of the max-share identification pionered by Uhlig (2004). See also Barsky and Sims (2011), Francis et al. (2014), among others.

that could invalidate any conclusions about the sources of fluctuations (DiCecio and Owyang, 2010; Giannone et al., 2019; Dieppe et al., 2021; Basu et al., 2021). However, this method is not immune to the well-known problems that affect VAR models. (1) Due to the so called "curse of dimensionality", VAR systems could be informationally deficient. This means that the variables used by the econometrician may not contain enough information to recover the structural shocks driving the economy and the related IRFs. This problem, known as "non-fundamentalness" or "non-invertibility" of Moving Average (MA) representations, is discussed among others in Sargent and Hansen (1991), Lippi and Reichlin (1993, 1994), Fernández-Villaverde et al. (2007), Alessi et al. (2011), Soccorsi (2016), Forni et al. (2019). (2) Many of the macroeconomic variables used in SVAR models are affected by measurement errors and/or small residuals of no economic interest. These can dynamically contaminate the estimated VAR shocks, potentially leading to misleading results even when information seems sufficient to correctly recover the IRFs (Lippi, 2019; Giannone et al., 2006; Forni et al., 2020). Due to the potential bias introduced by the lack of information and the presence of measurement errors, the results of SVAR analysis can be quite unstable, depending on the variables included in the vector.

This raises a fundamental question: Is ACD's VAR informationally sufficient to recover the MBC shock? It appears not. We test for sufficient information by using the "orthogonality" test proposed by Forni and Gambetti (2014).⁴ The test results suggest that informational sufficiency is rejected, as the estimated MBC shock can be predicted by the lags of the principal components (PCs) of a large dataset (the testing procedure is explained in Section 1.2). Since PCs reflect virtually all available macroeconomic information, it implies that the VAR used by ACD lacks some data that could be useful in predicting the shock of interest. The lack of information can lead to a misleading interpretation of what drives economic cycles, making it difficult to distinguish between "fact and fiction". This is our key insight.

Building upon this, our paper provides empirical evidence that challenges the existence of a single shock or a dominant propagation mechanism explaining the bulk of business cycle fluctuations, as suggested by ACD. We argue that this mechanism is not a robust feature of the data. Instead, it appears to be a product of the welldocumented instability of VAR results. Working in an environment that is free from the limitations of VARs enables us to draw an alternative anatomical template for the transmission mechanisms of the business cycles, that well fits into the traditional AD-AS narrative, contrary to what claimed by ACD. In this sense, large-dimensional Structural Dynamic Factor models (SDFM) offer a solution.

⁴Specifically, we investigate whether the 10-variable VAR considered by ACD contains enough information to recover the MBC shock obtained by targeting business cycle frequencies variation in unemployment, which is the baseline shock in ACD's anatomy.

Just like ACD, we use the frequency domain identification method described above to compile a collection of reduced-form shocks. Each shock maximizes the volatility of a different macroeconomic variable over either business cycle (6-32 quarters) or long run frequencies ($80-\infty$). In contrast to ACD, we assume that U.S. macroeconomic series follow a large-dimensional SDFM, as introduced by Forni et al. (2009) and Stock and Watson (2005). Our positive argument is that the availability of a large dataset, when combined with factor model techniques, helps in solving both problems affecting SVAR analysis. These models can be used for structural economic analysis in the same way as VAR models. However, unlike VARs, they include a large amount of information, so that insufficient information is unlikely.⁵ Moreover, they allow us to study the effect of structural shocks on the common components, which are the observed macroeconomic series cleaned of measurement error. To this end, we built a dataset for high dimensional macroeconomic analysis of 114 quarterly US time series, covering the period 1961-I to 2019-IV.

As suggested by factor model literature, we do not believe that a single shock is the sole driver of business cycles.⁶ Therefore, in our collection, we consider the possibility that there exist at least two important cyclical shocks. First, for each of the target variables, we identify the shock that has the largest contribution to the business-cycle volatility of that variable, which is equivalent to ACD's MBC. Next, we identify the shock that is orthogonal to the first one and has the second largest contribution, in order of importance, referred to as the "secondary" business cycle shock (SBC). This process generates ten reduced-form shocks (two for each variable, respectively) that target any of the following real activity quantities over the business cycle frequencies: output, unemployment, hours worked, consumption, and investment. This forms the core of our anatomy.⁷

Armed with this equipment, in the first part of our analysis, we examine the IRFs and the variance contribution of the five MBC shocks. Our goal is to determine whether our rich information setup provides evidence of a common propagation mechanism that supports the idea of a single, dominant business-cycle shock. However, our findings

⁵Large factor models, as shown in Forni et al. (2009), are generally unaffected by noninvertibility issues. Typically, the vector of the factors is singular, meaning it is driven by a number of shocks that is smaller than its dimension. In such cases, achieving fundamentalness becomes easier as it satisfies a less demandig condition.

⁶Studies by Sargent and Sims (1977), Giannone et al. (2005) and Watson (2004) show that two shocks account for a significant portion of US data volatility. Similarly, Onatski (2009) cannot reject the null hypothesis of two shocks against an alternative of 3 to 7 shocks. The subsequent factor literature has repeatedly confirmed this insight. More recently, Avarucci et al. (2021) proposed a new consistent estimator for the number of shocks, suggesting that the US business cycle is driven by two common shocks.

⁷Other important elements of our collection include shocks that target output, consumption, investment, TFP, and labor productivity over long-run frequencies, and the shock that targets inflation over business cycle frequencies.

suggest otherwise. Firstly, unlike ACD, a single shock that target any one of GDP, unemployment, consumption, investment, and hours worked is not sufficient to explain the bulk of business cycle fluctuations across all these variables. Most importantly, the shock that targets consumption is neither correlated nor interchangeable in terms of IRFs with the other identified shocks which, in contrast, appear to share roughly the same propagation mechanism. The former has significant permanent effects and accounts for only a quarter of the cyclical fluctuations in the remaining variables. It also induces a negative covariance between GDP growth and inflation changes. In terms of both IRFs and variance contributions, this reduced form shock closely resembles a standard deflationary supply shock. On the other hand, the remaining shocks are purely cyclical. They are disconnected from the long-run of real activity, and contribute minimally to consumption volatility. Specifically, the shocks obtained by targeting GDP, investment, and unemployment are highly correlated with each other and induce a positive covariance between GDP growth and inflation changes. In terms of both IRFs and variance contributions, each of these reduced form shocks closely resembles a standard inflationary demand shock.⁸ It follows that our results not only argue against the hypothesis of a single dominant business cycle shock, but also challenge the distinctive features of ACD's business cycle anatomy. Specifically, we question the two disconnects between the short and the long run, and between real activity and cyclical inflation. The shock that targets consumption is permanent and deflationary, interpretable as a standard supply shock, while the GDP, unemployment, and investment shocks are transitory and inflationary, interpretable as a standard demand shock.⁹

Building on these findings, we proceed to the second part of our study. Here, we enhance our anatomical analysis with the five "secondary" business cycle shocks and present our parsimonious representation of the observed business cycles. We find that, regardless of the target variable, two cyclical shocks are sufficient to account for most of the business cycle fluctuations in real activity variables and, to a somewhat lesser extent, in inflation. Then we look at the long-run. Our two shocks together also account for most of the long-run variance.¹⁰ Specifically, the SBC shock, obtained by targeting

⁸The hours worked shock is quite similar to the latter in terms of the IRFs it produces. However, it is quite different in terms of variance contribution: it turns out to be disconnected from inflation. This rules out the possibility of interpreting this shock as standard inflationary demand.

⁹As for inflation, the shock that targets unemployment (GDP) accounts for about 36% (20%) of the business cycle variation in inflation. Symmetrically, the shock that targets the cyclical variance of inflation explains 32% (22%) of the business cycle variation in unemployment (GDP), as against a scant 4% in ACD's template.

¹⁰Symmetrically, the shocks identified by targeting any one of GDP, investment, consumption, TFP or labor productivity at the long-run frequencies (referred to as main long run shock, MLR), make a non negligible contribution to the business cycle, particularly with respect to consumption cyclical fluctuations. This result is in sharp contrast to ACD, where the same

any one of GDP, unemployment, investment, and hours worked, explains more than half of the long-run volatility in real activity and induces a negative covariance between GDP growth and inflation changes. This shock behaves as an aggregate supply shock of the textbook-type. Thus, for each of these variables, while the MBC shock fits the profile of a demand shock, the SBC shock fits the profile of a supply shock.¹¹ On the other hand, the SBC shock, obtained by targeting consumption, accounts for a small fraction of long-run volatility and induces a positive covariance between GDP growth and inflation changes. For private consumption, while the MBC shock behaves as a generic supply shock, the SBC shock transmits a demand shock.

In essence, our empirical template of observed business cycles seems to fit the traditional AD-AS narrative. Two main forces are at play: demand shocks of the standard New Keynesian variety, which raise output and inflation, and supply shocks, which raise output but lower inflation and map to long-run movements in TFP. The business cycle of consumption is largely supply-driven, consistent with the permanent income hypothesis, while that of GDP, investment, and unemployment is mainly demand-driven, supporting the New Keynesian perspective. Hence, one may advance the concept of the "Two Main Business-Cycle" shocks as the main drivers of business cycle movements in real activity.

1.2. IS ACD'S VAR INFORMATIONALLY SUFFICIENT?

We address this question using the "orthogonality" test proposed by Forni and Gambetti (2014).¹² This test checks for the orthogonality of the estimated shock of interest with respect to the past of the PCs of a large macroeconomic dataset. The key insight is that the PCs encapsulate virtually all available information. Therefore, if the shock of interest is correlated with the past of available information (i.e., if orthogonality is rejected), it indicates that the VAR is informationally deficient. In this scenario, VAR results can be misleading: changing the variables may change the information set and therefore the estimated shock of interest.

The testing procedure unfolds as follows: First, we estimate the 10-variable VAR

shock has a small footprint on the business cycle.

¹¹Compared with the corresponding MBC, each of these SBC shocks significantly contributes to consumption volatility at business cycle frequencies, accounting for approximately half of fluctuations. Indeed, they are strongly correlated with the consumption-targeted MBC shock, reflecting shared supply dynamics.

¹²For our purposes, the relevant issue is not to establish whether the VAR is globally sufficient or not, but whether it can correctly recover a single shock of interest. Forni and Gambetti (2014) show that even if the VAR lacks sufficient information to capture all of the structural shocks (i.e., the MA representation of the variables in the vector is non-fundamental), it can still be informationally sufficient for a single shock. For this purpose, we use a less demanding test than the Granger causality test proposed in the same paper.

model, as proposed by ACD, with two lags, spanning the period from 1955:Q1 to 2017:Q4, and identify the shock that targets the unemployment rate at business-cycle frequencies.¹³ Second, we regress this shock on the past values of a set of variables that reflect agents' information, and perform an F-test for the significance of the regression. We use the first r = 6, 7, ..., 11, PCs of the variables in our large dataset as regressors, where $\hat{r} = 11$ is the number of factors driving the panel. This number is determined using the criterion of Bai and Ng (2002) with the penalty modification proposed in Alessi et al. (2010).¹⁴

The top panel of Table 1.1 shows the p-values of the test for different choices of PCs and number of lags. We find that informational sufficiency is rejected, since the estimated MBC shock is predicted by the lags of the PCs. This implies that by enlarging the information set, the estimated MBC shock could be different while maintaining the same identifying assumptions. In other words, the causal interpretation of the MBC shock is questioned. We perform the test for any of the other shocks that make up the main business cycle template (GDP, consumption, investment and hours worked) obtaining the same result.¹⁵

Orthogonality						
PRINCIPAL COMPONENTS	1 lags	2 lags	3 lags	4 lags		
r=6	0.00	0.00	0.00	0.03		
r=7	0.01	0.00	0.00	0.00		
r=8	0.01	0.00	0.00	0.01		
r=9	0.01	0.00	0.00	0.01		
r=10	0.02	0.01	0.00	0.04		
r=11	0.02	0.02	0.01	0.08		
VARIABLES	1 lags	2 lags	3 lags	4 lags		
Baa-GS10 spread	0.06	0.02	0.01	0.01		
S&P500	0.00	0.00	0.00	0.01		
JLN Uncertainty 3M	0.06	0.01	0.01	0.01		
BC12M	0.01	0.03	0.03	0.02		

Table 1.1: *p*-values of the orthogonality *F*-test, with 1, 2, 3 and 4 lags, for THE MBC SHOCK THAT TARGETS THE UNEMPLOYMENT RATE, ESTIMATED WITH ACD'S VAR specification. r = number of principal components used in the test.

We also try to provide some insight into the missing information. To do this, we

 $^{^{13}}$ As detailed in Section 2 of ACD, the data consist of quarterly observations on the following macroeconomic variables: the unemployment rate; the per-capita level of GDP, investment (inclusive of consumer durables), consumption (of non-durables and services), and total hours worked; labor productivity in the non-farm business sector; utilization-adjusted TFP; the labor share; the inflation rate (GDP deflator); and the federal funds rate.

¹⁴For the testing procedure, we adjusted the shock size by removing the initial six observations to align the start date with 1961:Q1 instead of 1955:Q1. This adjustment ensures that the period matches our sample for this specific exercise, which spans from 1961:Q1 to 2017:Q4, instead of extending to 2019:Q4 as in the rest of the analysis.

¹⁵Available upon request.

regress the estimated MBC shock on the past values of some forward-looking variables and on the past values of other variables that are widely used in business cycle analysis. These include the Shiller's real S&P stock price index (S&P500), the University of Michigan's confidence index on expected business conditions for the next year (BC12M),¹⁶ a measure of the risk spread (Baa-GS10 spread) and the Jurado et al. (2015)'s measure of macroeconomic uncertainty over a three-month horizon. The bottom panel of Table 1.1 shows that orthogonality of the estimated MBC shock with respect to the past of any of these variables is clearly rejected.

In conclusion, we believe that the potential lack of information in VAR analysis, can lead to a mis-characterization of the business cycle anatomy. In this sense, largedimensional SDFMs offer a solution. These models are free from this drawback by design; in fact, they use a large amount of data by enlarging the information set available to the econometrician.

1.3. Model and Method

1.3.1. Structural Dynamic Factor Model

Let x_t be a *n*-dimensional, stationary vector of observable economic variables. The vector x_t is part of an infinite-dimensional panel of time series. Each variable x_{it} , i = 1, ..., n, is decomposed into the sum of two mutually orthogonal unobservable components, the common component, χ_{it} , and the idiosyncratic component, ξ_{it} :

$$x_{it} = \chi_{it} + \xi_{it}.\tag{1.1}$$

The idiosyncratic components are interpreted as sources of variation that are specific to one or just a small group of variables, like regional or sectoral shocks, plus measurement error. In particular, for macroeconomic variables like GDP, investment or consumption, in which all local and sectoral shocks have been averaged out, the idiosyncratic part can be interpreted essentially as only containing measurement error. The ξ 's are allowed to be mildly cross-sectionally correlated, thus they have a covariance matrix which is not necessarily diagonal (see Forni et al., 2009, Assumption 5). The χ 's, on the contrary, account for the bulk of the co-movements among macroeconomic variables. This is because they are different linear combinations of the same r < n common factors, not depending on i, i.e. they span a r-dimensional vector space (see Stock and Watson,

¹⁶BC12M summarize responses to the following forward-looking question:"Turning to economic conditions in the country as a whole, do you expect that over the next year we will have mostly good times, or periods of widespread unemployment and depression, or what?". The anticipation properties of this variable on future movements in economic activity are widely discussed in Barsky and Sims (2012) and Beaudry and Portier (2006).

2002a,b; Bai and Ng, 2002). Then there exist an *r*-dimensional weakly stationary vector process $F_t = (F_{1t} \dots F_{rt})'$, orthogonal to $\xi_t = (\xi_{1t} \dots \xi_{nt})'$, and loadings λ_{ij} , $j = 1, \dots, r$, such that

$$\chi_{it} = \lambda_{i1}F_{1t} + \ldots + \lambda_{ir}F_{rt} \quad \text{or} \quad \chi_t = \Lambda F_t.$$
(1.2)

The unobservable coordinates of F_t are called the static factor and Λ , the factor loading matrix, is of size $n \times r$. We require the factors to be pervasive i.e. to have non-negligible effects on most of the variables x_{it} (see Forni et al., 2009, Assumption 4). Combining (1.1) and (1.2), we get a static equation linking the *n* observable variables x_{it} to the *r* factors and the idiosyncratic components

$$x_{it} = \lambda_{i1}F_{1t} + \ldots + \lambda_{ir}F_{rt} + \xi_{it} \quad \text{or} \quad x_t = \Lambda F_t + \xi_t.$$
(1.3)

Equation (1.3) is the static factor representation, where the factors have only contemporaneous effect on the common components. The dynamic nature of the model comes from the fact that the static factors F_t follow a VAR(p) driven by a q-dimensional vector of orthonormal structural white noise, or common shocks $u_t = (u_{1t}, \ldots, u_{qt})'$, with $q \leq r$. Precisely:

$$x_t = \Lambda F_t + \xi_t \tag{1.4a}$$

$$C(L)F_t = \epsilon_t \tag{1.4b}$$

$$\epsilon_t = R u_t \tag{1.4c}$$

where ϵ_t is the residual of the VAR on F_t , $E(\epsilon_t \epsilon'_t) = \Sigma_{\epsilon}$, C(L) is an $r \times r$, stable polynomial matrix and R is $r \times q$ and has maximum rank q. As a consequence, R has a left inverse and the vector u_t belongs to the space spanned by F_{t-s} , $s \ge 0$, that is, u_t is fundamental for F_t . By inverting the matrix C(L) we get $F_t = C(L)^{-1}\epsilon_t = C(L)^{-1}Ru_t$, so that the dynamic relationship between u_t and the common components is

$$\chi_t = \left[\Lambda C(L)^{-1} R\right] u_t = B(L) u_t.$$
(1.5)

Then, by merging (1.1) and (1.5), we have the structural dynamic representation

$$x_{it} = b_i(L)u_t + \xi_{it}$$
 or $x_t = B(L)u_t + \xi_t$, (1.6)

where the macroeconomic variables are represented as driven by a few pervasive structural shocks, loaded with the IRFs in B(L), plus measurement error. We are interested in the effect of structural shocks on the common components χ_t of some key series, i.e. on the variables obtained by removing idiosyncratic errors. Notice that representation (1.6) is not unique, since the IRFs are not identified. Forni et al. (2009) (Proposition 2), show that identification is achieved up to orthogonal rotations, just like in structural VAR models.

Let us consider the linear mapping in (1.4c), $\epsilon_t = Ru_t$. We define R = SH, where S is the Cholesky factor of Σ_{ϵ} , such that $SS' = \Sigma_{\epsilon}$, and H is an orthonormal matrix, namely a matrix such that $H^{-1} = H'$. We can then rewrite (1.5) as

$$\chi_t = \left[\Lambda C(L)^{-1}S\right] Hu_t = D(L)Hu_t = B(L)u_t$$
(1.7)

where $D(L) = \Lambda C(L)^{-1}S$ encapsulates the Cholesky IRFs and B(L) = D(L)H collects the structural IRFs. To identify the shocks, we must impose additional restrictions on the rotation matrix H. This is usually done as in standard SVAR analysis, which mainly employs an appropriate number of exclusion or sign restrictions motivated by specific economic theories. Here we implement an alternative approach: identification of dominant shocks in the frequency domain.

1.3.2. Identification of dominant shocks in the Frequency Domain

Our identification strategy follows ACD's spectral method. In this approach, a shock is identified as the one that explains the dominant fraction of the variance of a particular variable within a specific frequency band, such as the business cycle (6-32 quarters) or long-run (80- ∞ quarters) frequencies. In this section, we show how to use spectral decomposition to target the variance of a specific variable within a defined frequency domain. We also illustrate how to isolate the shocks that dominate this objective variance.

Consider representation (1.7). The effect of the *j*-th structural shock on the *k*-th common-component is given by the (k, j) element of the matrix B(L), that is $D^{[k]}(L)h$, where $D^{[k]}(L)$ is the *k*-th row of D(L) and *h* is the *j*-th column of *H*. On the other hand, the structural shocks are related to the VAR residuals by the relation $u_t = R^{-1}\epsilon_t = H'S^{-1}\epsilon_t = H'\eta_t$, η_t being the vector of the Cholesky shocks. Hence the *j*-th structural shock is given by the product of the *j*-th row of H' and η_t , that is $h'\eta_t$. Now, let $\left[\underline{\theta}, \overline{\theta}\right]$ be a band of frequencies, such that $0 \leq \underline{\theta} \leq \overline{\theta} \leq \pi$. In the frequency domain, the contribution of the *j*-th structural shock $h'\eta_t$ to the spectral density of the *k*-th variable over the frequency band $\left[\underline{\theta}, \overline{\theta}\right]$ is given by

$$\Psi\left(h;k,\underline{\theta},\overline{\theta}\right) = \int_{\underline{\theta}}^{\overline{\theta}} \left(D^{[k]}\left(e^{i\theta}\right)' h' D^{[k]}\left(e^{-i\theta}\right)h\right) d\theta$$

$$= h' \left[\int_{\underline{\theta}}^{\overline{\theta}} \left(D^{[k]}\left(e^{i\theta}\right)' D^{[k]}\left(e^{-i\theta}\right)\right) d\theta\right]h$$
(1.8)

where the matrix

$$V\left(k,\underline{\theta},\overline{\theta}\right) = \int_{\underline{\theta}}^{\overline{\theta}} \left(D^{[k]} \left(e^{i\theta}\right)' D^{[k]} \left(e^{-i\theta}\right) \right) d\theta$$

captures the entire frequency band variace of the k-th variable in terms of the contributions of all Cholesky shocks. The contribution of the j-th structural shock can then be re-written as

$$\Psi\left(h;k,\underline{\theta},\overline{\theta}\right) = h'V\left(k,\underline{\theta},\overline{\theta}\right)h.$$
(1.9)

Our approach is to identify the largest contributors to the variance of a particular variable k over a specific frequency band $\left[\underline{\theta}, \overline{\theta}\right]$, ordered in decreasing order of importance: First, the shock with the largest contribution to the target variance, then the shock orthogonal to the first with the second largest contribution, and so on. Suppose, without loss of generality, that the shocks $u_{1t}, u_{2t}, \ldots, u_{qt}$ have to be identified. The solution is given by the first q eigenvectors $h = [h_1, h_2, \ldots, h_q]$ corresponding to the q largest eigenvalues of the matrix $V\left(k, \underline{\theta}, \overline{\theta}\right)$ and provides the shocks $h'_1\eta_t, h'_2\eta_t \ldots, h'_q\eta_t$ ordered in terms of their contribution to the target. This strategy allows for the identification of a collection of shocks by systematically varying the target variable and/or frequency band.

We show below that two shocks are enough to explain the bulk of cyclical variance in the main macroeconomic aggregates, while the long run is explained by only one shock.

1.4. Empirical Analysis

1.4.1. Data

Coming to the empirical application, we collect a quarterly dataset for high dimensional macroeconomic analysis.

Our $N \times T$ dataset consists of 114 US quarterly series, spanning from 1961-I to 2019-IV. The majority of these series are sourced from FRED-QD.¹⁷ TFP data series are obtained from John Fernald's website (Fernald, 2012), while the Confidence data are accessible on the Michigan Survey of Consumer website.¹⁸ Lastly, the macroeconomic uncertainty series (Jurado et al., 2015) are retrieved from Sydney C. Ludvigson's website. Some series have been constructed by ourselves as transformation of the original ones. Following standard practice in macroeconomic analysis, consump-

 $^{^{17}\}mathrm{FRED}\text{-}\mathrm{QD}$ is a large quarterly macroeconomic database with 248 series, developed by McCracken and Ng (2020).

¹⁸http://www.sca.isr.umich.edu/

tion includes non-durables and services, while investment has been broadly defined to include consumer durables. Both measures are taken in real terms. Monthly data, like the macroeconomic uncertainty series, have been aggregated to get quarterly figures. Finally, it is worth noting that most series are expressed in per capita terms, dividing by population aged 16 years or more (civilian non-institutional population series) and stock market data have been deflated by the GDP deflator. We transform each series to reach stationarity. As for the transformations, we deviate from those suggested by McCracken and Ng (2020) for the interest rates, which are taken in levels rather than in differences; furthermore, we take prices and other nominal variables in log-differences, rather than in double differences of the logs. This avoids potential over-differentiation, which could enhance high frequencies of little interest for business cycle analysis. The complete list of variables and transformations is provided in Appendix (1.B).

To conclude this section, let us look at the common-idiosyncratic variance decomposition of the key variables above with $\hat{r} = 11$ static factors, shown in table 1.8. The common variance of the main macroeconomic aggregates like GDP, consumption, investment and unemployment rate are 94, 82, 90 and 94 percent of total variance, respectively. These numbers seem compatible with the measurement error interpretation of the idiosyncratic components.

1.4.2. Identification Strategy

We use the techniques discussed in Section (1.3.2) to compile a collection of shocks, in a way that is similar to, but somewhat distinct from, ACD. Just as in that paper, the core of our collection consists of shocks targeting any one of unemployment, output, consumption, investment, and hours worked over business cycle frequencies. The difference is that, as we show below, a single shock is not sufficient to provide an accurate description of business cycles in real activity variables. Therefore, we place a second business cycle shock at the center of our analysis, while ACD relegates it to the appendix.

Thus, for each of these five variables, we identify the q = 2 dominant shocks that explain the majority of business cycle fluctuations. They are selected in decreasing order of importance. First, we identify the shock with the largest contribution to the business cycle volatility, equivalent to ACD's shock. Then, we identify the shock orthogonal to the first one with the second-largest contribution, referred to as SBC shock. To do this, for $j = 1, \ldots, q$, we solve the maximization problem (1.3.2) in the frequency interval $[\underline{\theta}, \overline{\theta}] = [2\pi/32, 2\pi/6]$, corresponding to cycles with periodicity between 18 months and 8 years. Repeating this process for each of the aforementioned variables produces a collection of ten reduced-form shocks. Mimicking ACD's approach, in the first part of the analysis our focus is exclusively on the five MBC shocks. The aim is to determine whether our rich information setup provides evidence of a single, dominant business cycle shock. In the second part, we enhance our anatomical analysis with the five "secondary" business cycle shocks and present our parsimonious (two shocks) representation of the observed business cycles. A second, important but auxiliary subset comprises the shocks identified by targeting any one of GDP, investment, consumption, TFP, or labor productivity at long-run frequencies. For each of these variables, we find the shock that accounts for the bulk of long run fluctuations, referred to as MLR shock. In this case, we solve the maximization problem (1.3.2) by setting q = 1and in the frequency interval $[\underline{\theta}, \overline{\theta}] = [0, 2\pi/80]$, corresponding to periodicities greater than 20 years. These auxiliary shocks, along with other elements comprising our data anatomy, help us to characterize the properties of the business cycle picture we aim to provide. In doing so, we delve deeper into potential connections or disconnections with the nominal side, technology, and the long run.

1.5. Results

1.5.1. QUESTIONING THE EXISTENCE OF A "MAIN BUSINESS-CYCLE SHOCK"

The first part of the results aims to establish the existence of a single, dominant business-cycle shock. Following ACD's approach, we focus on the MBC shocks that target any one of GDP, unemployment, consumption, investment and hours worked. A key finding in ACD is that these shocks turn out to be interchangeable, in the sense that they produce essentially the same IRFs, or the same propagation mechanism. Moreover, any one of them not only explains approximately three-quarters of the business-cycle volatility in the targeted variable but also accounts for more than one half of the business-cycle volatility in the remaining variables. These findings serve as the foundation, outlining necessary requirements for establishing the existence of a singular driver of the business-cycle. Is there comparable evidence in our data?

Consider the first requirement: the interchangeability of these shocks in terms of IRFs. Figure 1.1, Panel A, compares the responses of selected variables to the shocks targeting output, unemployment, investment and hours worked. Meanwhile, Panel B reports the responses to the shock targeting consumption.¹⁹ It is clear from this figure that targeting consumption produces a shock with a different propagation mechanism from the others. The former (Panel B) has a large permanent effect on real economic activity variables and generates a temporary hump-shaped response of unemployment and hours worked (see Figure 1.3). GDP increases immediately by around 0.2%, peaks around the 10th quarter, and converges to 1.2% in the long run.

 $^{^{19}{\}rm For}$ a comprehensive view of the responses of all the variables to these shocks, please refer to Figure 1.3.

Unemployment behaves countercyclically, reaching a minimum of about -0.2% around the 8th quarter. This shock generates a negative comovement between the inflation rate and output growth. The former immediately falls by around 0.15%, after which the effect relatively quickly vanishes. Monetary policy, as proxied by the federal funds rate, reacts very weakly. The response of TFP follows an S shape, featuring a relatively small impact effect and a much larger long-run effect (about 1.1). In terms of IRFs, this shock is essentially an aggregate supply shock of the textbook-type.

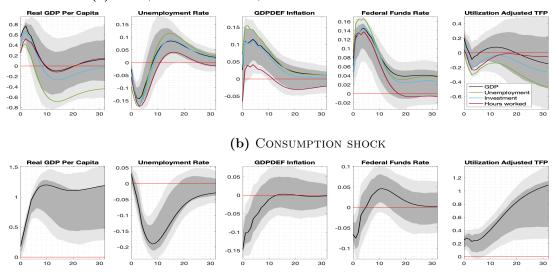




Figure 1.1: Impulse response functions of the MBC shock obtained by targeting different variables. The dark gray and light gray areas are the 68% and 90% confidence bands, respectively, for the shock that targets GDP (panel A) and consumption (Panel B).

Conversely, the other shocks are purely cyclical. Moreover, they seem to share roughly the same propagation mechanism. This is the first piece of evidence suggesting that at least two shocks with distinct propagation mechanisms are needed to explain business-cycle movements in real activity. Consider the shock that targets GDP (Panel A, black line). The responses of output and investment are temporary and humpshaped, peaking in the 2nd quarter. The effects are no longer statistically significant after about one year. GDP exhibits a positive impact effect of 0.6% and a peak of about 0.8%. For consumption the effect is positive but very short-lived, being barely significant only on impact (0.3%, see Figure 1.3). Unemployment reaches a minimum of around -0.15% after a few quarters. Then show a very short-lived rebound effect, with a peak of about 0.1%. This shock generates a positive comovement between the inflation rate and output growth. Inflation and the interest rate move in tandem. The former increases on impact, peaks just above 0.1%, and converges to zero afterward. TFP does not react, with the effect not being statistically significant. In terms of IRFs, this shock is essentially an aggregate demand shock of the textbook-type. As is evident from the figure, GDP, investment and unemployment shocks are highly interchangeable, suggesting that they represent multiple facets of the same inflationary demand shock.²⁰ The hours worked shock is very similar to the latter in terms of IRFs it produces. However, it differs in that the inflation response is nearly zero and lacks statistical significance.

Now, turning to the second necessary requirement: our goal is to assess whether each of these shocks can effectively explain the bulk of business-cycle fluctuations in real activity variables. Table 1.2 reports, for each variable, the percentage of variance explained by the MBC shocks at business-cycle frequencies (top panel) and in the long run (bottom panel). From the top panel of Table 1.2, it is evident that a single shock targeting any one of the aforementioned variables is not sufficient to explain the majority of business-cycle fluctuations in all of these variables. For example, the shock that targets GDP explains about 61% of the business-cycle volatility in GDP and only 36% of that in unemployment, compared to 80% and 56% in ACD. Similarly, the shock that targets unemployment explains 58% of the cyclical volatility in unemployment and only 39% of that in GDP, as opposed to 73% and 59% in ACD. Consistent with previous findings, the shock that targets consumption explains 66% of the cyclical volatility in consumption and only a quarter of that in the remaining variables. Symmetrically, the other shocks account for only 11% to 21% of the business-cycle fluctuations in consumption. This constitutes the second piece of evidence suggesting that a single shock is not sufficient to explain the bulk of fluctuations in real activity over the business cycle, contrary to what is claimed by ACD.

Finally, Table 1.3 corroborates the findings presented so far, showing the correlation coefficients between the identified shocks. The shock that targets consumption shows very weak or almost no correlations with the other shocks, ranging from 0.04 to 0.22. Conversely, the shock obtained by targeting any one of GDP, investment, and unemployment are highly correlated with each other, with coefficients ranging from 0.78 to 0.92.

²⁰The observed responses of both GDP and its components to the unemployment shock show a rebound effect followed by a long-run decline. Nevertheless, this effect is consistently not statistically significant. The transitory nature of this shock is confirmed in terms of its contribution to the long-run variance (see the next section). The IRFs of this shock with the corresponding confidence bands are available upon request.

	Main Bu	SINESS CY	CLE SHO	CK			
Percentage of Explained Cyclical Variance							
	GDP	Unemp	Cons	Invest	Hours		
GDP	60.6	39.3	26.9	52.3	29.5		
Unemployment	36.0	57.7	26.9	45.6	42.1		
Consumption	19.0	17.5	66.0	21.0	11.8		
Investment	52.8	48.1	24.8	61.6	35.6		
Hours Worked	26.7	40.1	29.9	33.5	57.5		
TFP	9.8	15.4	10.9	12.5	12.2		
Inflation	19.5	36.0	22.1	19.2	5.8		
\mathbf{FFR}	36.5	53.6	14.8	40.3	35.7		
S&P500	14.5	14.3	23.3	23.9	21.5		
Labor	45.9	31.3	18.6	37.4	22.5		
Percentage of Explained Long Run Variance							
	GDP	Unemp	Cons	Invest	Hours		
GDP	0.5	11.8	65.7	0.6	0.5		
Unemployment	5.3	9.6	58.1	4.2	5.5		
Consumption	1.1	10.5	68.9	2.3	0.2		
Investment	1.0	16.9	55.6	0.8	0.3		
Hours Worked	2.0	1.1	58.1	2.8	28.0		
TFP	2.2	10.3	55.7	3.5	0.2		
Inflation	10.3	14.8	2.2	7.3	1.0		
\mathbf{FFR}	22.1	23.6	0.6	17.0	7.4		
S&P500	1.3	3.2	22.6	0.3	3.4		
Labor	0.5	7.3	49.2	1.0	4.9		

Table 1.2: Percentage of variance explained by the main business cycle shock for a few selected variables, by frequency bands. The columns correspond to different targets in the construction of the shock.

Main Business Cycle Shock							
GDP UNEMP CONS INVEST HOURS							
GDP	1	0.78	0.17	0.92	0.67		
Unemployment	0.78	1	0.04	0.87	0.80		
Consumption	0.17	0.04	1	0.20	0.22		
Investment	0.92	0.87	0.20	1	0.75		
Hours Worked	0.67	0.80	0.22	0.75	1		

Table 1.3: Correlation between the shocks obtained by targeting GDP, Un-EMPLOYMENT, CONSUMPTION, INVESTMENT, AND HOURS WORKED

1.5.2. NATURE AND (DIS)CONNECTIONS OF OUR MBC SHOCKS

The results discussed so far exclude the existence of such thing as a "Main Business Cycle" shock. First of all, a single shock is not enough to explain most of the business-cycle fluctuations in real activity. More importantly, the shock that targets consumption is neither correlated nor interchangeable in terms of IRFs with the other identified shocks which, in contrast, appear to share roughly the same dynamic comovements. Therefore, our results do not support the hypothesis of a main, unifying propagation mechanism, as at least two distinct mechanisms are at play. Before moving on to the second part of the analysis, where we enhance our collection with the five "secondary" business-cycle drivers, we now examine more in detail the properties of our MBC shocks. Although the timing and magnitude of the responses in Figure 1.3 provide valuable insights into the nature of the shocks identified, they alone may not be exhaustive to offer a comprehensive interpretation. From this perspective, Table 1.2 provides additional information in terms of variance contributions, which help us to better understand potential connections or disconnections with the nominal side and the long run. This understanding is crucial to determine whether the nature of these shocks aligns with what the observed co-movements (IRFs) have previously suggested.

Let us turn our attention to the long run. From the lower panel of Table 1.2, we see that previous insights are confirmed. While the shocks that dominate the business cycle of GDP, unemployment, investment, and hours are largely disconnected from the long run of real economic activity, the shock that dominates the business cycle of consumption is far from being disconnected. Indeed, it explains over half of the long-run fluctuations in real activity variables, accounting for approximately 66% and 56% of the long-run variance in GDP and TFP, respectively. This is an early indication that what drives the long run of output and TFP has a non-negligible footprint on the business cycles. This point is further corroborated later. Conversely, the remaining shocks explain almost nothing, or very little (unemployment-shock), of the long-run variance in real activity, that is, they have a transient nature.

We now turn attention to the relation between inflation and real activity over the business cycle. First, as shown in Table 1.4 (which repeats a portion of the top panel of Table 1.2), all identified MBC shocks, except for the one that targets hours worked, have significant effects on the nominal side of the economy. In particular, differently from ACDs findings, the unemployment shock that we identify account for 36% of the business-cycle variation in inflation, as against a scant 7% in ACD. This is consistent with what has been observed in terms of co-movements, that is, the inflation rate seems to behave as suggested by the New-Keynesian framework: it increases when the unemployment rate is low (during expansion), and then converges to zero when the economy stabilizes. This result is largely in line with the figures reported in Bianchi et al. (2023).²¹ Shocks targeting GDP, investment and consumption also explain about 19% to 22%. These are relatively high shares, when considering that the identified shocks explain "only" about 60% of the business cycle fluctuations in the targeted variables. On the other hand, the hours worked shock turns out to be disconnected from inflation, in that it explains close to nothing of the business-cycle variation in

 $^{^{21}}$ In that paper, a Trend-Cycle VAR is used to identify the shock that explains most of the cyclical component of unemployment. This shock accounts for approximately 30% of the inflation cycle.

inflation (6%).

Inflation and the business cycle									
Percentage of Explained Cyclical Variance									
	GDP	Unemp	Cons	Invest	Hours	INFLATION			
GDP	60.6	39.3	26.9	52.3	29.5	21.8			
Unemployment	36.0	57.7	26.9	45.6	42.1	32.1			
Consumption	19.0	17.5	66.0	21.0	11.8	21.2			
Investment	52.8	48.1	24.8	61.6	35.6	23.7			
Hours Worked	26.7	40.1	29.9	33.5	57.5	12.3			
Inflation	19.5	36.0	22.1	19.2	5.8	86.6			
\mathbf{FFR}	36.5	53.6	14.8	40.3	35.7	40.1			

Table 1.4: Percentage of business cycle variance explained by the MBC shocks for a few selected variables. The columns correspond to different targets in the construction of the shock.

Second, the shock that targets the business cycle variance of inflation explains approximately 21% to 32% of the business cycle variation in unemployment, GDP, investment, and consumption. This result is in sharp contrast to ACD, where the same shock has a very small footprint on the business cycle of real economic activity (4 to 8%). Thus, business cycle fluctuations in inflation seem to co-move with real activity, at least to some extent. It follows that our results not only argue against the hypothesis of a single dominant business cycle shock, but also challenge the distinctive features of ACD's business cycle anatomy. To make a long story short, the interpretation of shocks obtained by targeting any one of GDP, unemployment and investment is in line with a demand shock in a textbook version of the New Keynesian model.²² Conversely, in terms of both IRFs and variance contributions, the interpretation of the shock that target consumption is in line with an aggregate supply shock, which raises output but lowers inflation, and maps to long run movements in productivity. However, neither categories of shock/mechanism alone is able to explain the bulk of the observed business cycles. In what follows, we enrich our collection with the five "secondary" business cycle shocks.

1.5.3. The two Main Business-Cycle Shocks

Are two shocks sufficient to explain the majority of business cycle fluctuations in real activity variables? And if yes, what are they and what are their effects? For each of the five macroeconomic quantities, we now identify two shocks. The first, already reported in the Part I, is the MBC shock of that specific variable. The second, referred to as the SBC shock, is identified by maximizing its contribution to the residual business

 $^{^{22}}$ As for the hours worked shock, despite its transitory nature, the disconnect with inflation rules out the possibility of interpreting this shock as a standard inflationary demand shock.

cycle volatility of that variable, after the effect of the MBC shock has been filtered out. Table 1.5 reports the percentage of variance jointly explained by the two shocks at business cycle frequencies and in the long run. Our hypothesis is broadly confirmed: two shocks are enough to provide an accurate description of the observed business cycle in real activity. Depending on the target variable, the percentage of cyclical volatility explained by the two shocks varies between 65 and 91 for GDP, 62 and 95 for unemployment, 66 and 95 for consumption, 68 and 96 for investment, 50 and 76 for hours worked. As for the relation between inflation and real activity at business cycle frequencies, while it is tenuous for the hours worked, it is still evident for the other variables, as the corresponding shocks explain between 44% and 62% of the variation in inflation. Then we look at the long-run. In principle, both shocks could be disconnected from long term real activity, since they are selected as those shocks maximizing cyclical variance. But this is not the case: our two shocks together account for most of the long-run variance in both output and productivity.

	The Two Main Business Cycle Shocks									
	Percentage of Business Cycle Variance					Percentage of Long Run Variance				
	GDP	Unemp	Cons	Inv	Hours	GDP	Unemp	Cons	Inv	Hours
GDP	91.5	69.7	65.2	83.0	58.3	60.8	63.7	76.6	47.8	52.6
Unemployment	73.0	95.0	62.1	82.4	76.0	76.8	77.5	74.2	72.8	63.1
Consumption	74.7	66.9	95.2	68.3	65.4	56.4	54.9	81.0	46.1	54.1
Investment	86.1	82.7	67.5	95.8	66.5	70.9	77.8	67.8	65.1	66.2
Hours	61.1	75.6	50.4	66.2	92.5	66.0	68.5	62.7	55.8	70.4
TFP	22.2	27.2	19.8	25.0	27.9	56.2	56.0	74.8	52.1	51.9
Inflation	51.5	61.9	43.7	51.9	33.1	18.9	22.2	7.5	18.1	6.2
FFR	56.0	67.7	50.0	62.2	59.7	22.7	23.8	7.7	19.4	8.0
S&P500	56.4	65.3	32.5	69.7	54.3	43.9	47.8	27.3	48.5	39.9
Labor	63.7	47.7	42.7	53.5	41.9	55.2	57.2	61.6	50.4	49.0

Table 1.5: Percentage of variance explained by the two main shocks for a few selected variables, by frequency band. The columns correspond to different targets in the construction of the shock.

The Two Main Business Cycle Shocks										
Percentage of Explained Cyclical Variance										
	GI	PР	Unemp		Cons		Invest		Hours	
	MBC	SBC	MBC	SBC	MBC	SBC	MBC	SBC	MBC	SBC
GDP	60.6	30.9	39.3	30.4	26.9	38.3	52.3	30.6	29.5	28.8
Unemployment	36.0	37.0	57.7	37.2	26.9	35.2	45.6	36.9	42.1	33.9
Consumption	19.0	55.7	17.5	49.4	66.0	29.3	21.0	47.2	11.8	53.6
Investment	52.8	33.4	48.1	34.6	24.8	42.7	61.6	34.2	35.6	30.9
Hours	26.7	34.4	40.1	35.5	29.9	20.5	33.5	32.7	57.5	35.0
TFP	9.8	12.4	15.4	11.8	10.9	8.9	12.5	12.5	12.2	15.7
Inflation	19.5	32.0	36.0	25.9	22.1	21.7	19.2	32.7	5.8	27.3
\mathbf{FFR}	36.5	19.4	53.6	14.1	14.8	35.2	40.3	21.9	35.7	24.0
S&P500	14.5	41.9	14.3	50.9	23.3	9.2	23.9	45.8	21.5	32.8
Labor	45.9	17.8	31.3	16.4	18.6	24.1	37.4	16.2	22.5	19.4
	Р	ERCENT	age of E	EXPLAIN	ed Long	-Run V	ARIANCE			
	GDP		Unemp		Cons		Invest		Hours	
	MBC	SBC	MBC	SBC	MBC	SBC	MBC	SBC	MBC	SBC
GDP	0.5	60.3	11.8	51.9	65.7	11.0	0.6	47.2	0.5	52.2
Unemployment	5.3	71.5	9.6	67.9	58.1	16.1	4.2	68.6	5.5	57.6
Consumption	1.1	55.4	10.5	44.4	68.9	12.1	2.3	43.8	0.2	53.9
Investment	1.0	69.9	16.9	60.9	55.6	12.2	0.8	64.3	0.3	65.9
Hours	2.0	63.9	1.1	67.4	58.1	4.7	2.8	53.0	28.0	42.4
TFP	2.2	54.0	10.3	45.7	55.7	19.1	3.5	48.6	0.2	51.7
Inflation	10.3	8.6	14.8	7.4	2.2	5.3	7.3	10.8	1.0	5.2
\mathbf{FFR}	22.1	0.6	23.6	0.2	0.6	7.1	17.0	2.4	7.4	0.7
S&P500	1.3	42.7	3.2	44.6	22.6	4.7	0.3	48.2	3.4	36.5
Labor	0.5	54.6	7.3	49.9	49.2	12.4	1.0	49.4	4.9	44.1

Table 1.6: PERCENTAGE OF VARIANCE EXPLAINED BY THE MBC SHOCK AND THE SBC SHOCK FOR A FEW SELECTED VARIABLES, BY FREQUENCY BAND. THE COLUMNS CORRESPOND TO DIFFERENT TARGETS IN THE CONSTRUCTION OF THE SHOCK.

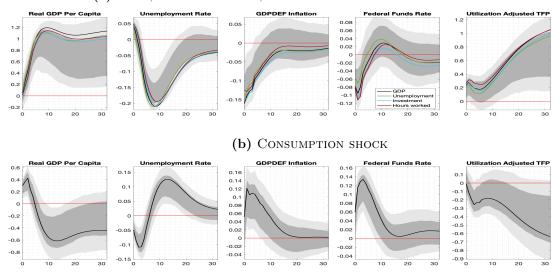
The key to understanding this representation lies in the SBC shock. Table 1.6 reports the percentage of variance explained by the MBC shock and the SBC shock at business cycle frequencies (top panel) and in the long run (bottom panel). From the bottom panel of the table, it can be observed that the SBC shock, obtained by targeting any one of GDP, unemployment, investment, or hours worked, accounts for 42 to 60% of the long-run fluctuations in GDP, and 46 to 54% in TFP, that is, it has long-lasting effects on economic activity.²³ When we turn our attention to the explained variance at business cycle frequencies (top panel), we find that the secondary shock, which targets any of these variables, is not only important in explaining long-run fluctuations, but also plays a crucial role in the business cycle. Its importance is almost comparable to that of the corresponding MBC shock. In particular, it is found to be dominant for consumption, in that it explains about one-half of its cyclical variance, while the corresponding MBC shock explains between 12 and 21%.²⁴ This finding reinforces the

 $^{^{23}}$ It also accounts for 58 to 72% of the fluctuations in unemployment, 44 to 55% in consumption, 61 to 70% in investment, and 42 to 67% in hours worked.

²⁴Note that the contribution of these shocks to the cyclical variance of consumption is left unrestricted.

previous insight: shocks that account for the long-run of output and productivity also make a significant contribution to the business cycle. Finally, depending on the target variable, this shock also accounts for about 26-33% of the fluctuations in inflation. Figure 1.2, Panel A, compares the IRFs of selected variables to the SBC shocks of output, unemployment, investment and hours worked.²⁵ As is evident from the figure, these shocks are nearly indistinguishable and share the typical features of a supply shock. The degree of matching is very high, with correlation coefficients ranging from 0.90 to 0.98 (Table 1.9). Therefore, for each of these variables (GDP, unemployment, investment and, to a somewhat lesser extent, hours worked), while the corresponding MBC shock fits the profile of an aggregate demand shock, the SBC shock fits the profile of an aggregate supply shock.

Building on the previous discussion, it's worth noting that each of these secondary shocks not only produces the same comovements/IRFs as the MBC shock of consumption, as detailed in section 1.5.1, but also exhibits a high correlation with it. This correlation is quantified by coefficients ranging from 0.80 to 0.90. Based on this, we can conclude that they represent interchangeable facets of the same aggregate supply shock.



(a) GDP, UNEMPLOYMENT, INVESTMENT AND HOURS WORKED SHOCKS

Figure 1.2: Impulse response functions of the SBC shock obtained by targeting different variables. The dark gray and light gray areas are the 68% and 90% confidence bands, respectively, for the shock that targets GDP (panel A) and consumption (Panel B).

What about the SBC shock obtained by targeting consumption? Unsurprisingly, while the corresponding MBC shock turns out to be a permanent supply shock, the

 $^{^{25}}$ For a comprehensive view of the responses of all the variables to these shocks and the shock that targets consumption, please refer to Figure 1.4.

SBC shock turns out to be a transitory shock, as the percentage of long-run fluctuations in GDP, consumption, and investment accounted for by this shock is negligible (Table 1.6, bottom panel).²⁶ Going back to the top panel of the table, it can be seen that this shock accounts for about 38, 35, and 43% of the business cycle fluctuations in GDP, unemployment, and investment, respectively, whereas the corresponding slowmoving MBC shock accounts for about 27% and 25%. This is consistent with previous results: transitory shocks are the most important factors in explaining the business cycle of output, unemployment and investment. Regarding cyclical inflation, this shock appears to be fairly connected with it, accounting for approximately 22%. Given these observations, it is very tempting to interpret this shock as a standard demand shock, in line with the interpretation given to the MBC shock that targets any one of GDP, unemployment, and investment. This interpretation finds some support in Panel B of Figure 1.2, which reports the IRFs of selected variables to this shock. The observed response of output growth shows a positive impact and a peak of about 0.4%at horizon 2, followed by a rebound and a substantial decline in the long run. However, this long-lasting effect is not statistically significant.²⁷ The inflation rate increases on impact, peaks at 0.12% in the second quarter, and converges to zero afterward. This is consistent with what we would expect from a demand shock. The interest rate follows a similar dynamic, increasing in a hump-shaped pattern and reaching a maximum of about 0.13%. Therefore, for private consumption, while the corresponding MBC shock fits the profile of an aggregate supply shock, the SBC shock fits the profile of an aggregate demand shock.

Once again, it's important to note that this shock is closely related to each of the MCB shocks that target GDP, unemployment, and investment, displaying correlation coefficients near 0.8.²⁸ This suggests that they share roughly the same propagation mechanism and represent different facets of the same aggregate demand shock.

Summing up, considering a sample from 1962 to 2019, the observed business cycles of GDP, investment, consumption, unemployment and, to a lesser extend hours worked, appears to be well described by two common factors/mechanisms of a different nature: a demand shock having only transitory effects (or vey small long-term impacts) and a generic supply shock having long-lasting effects on output and productivity. Hence, one may advance the concept of the "Two Main Business-Cycle shocks" as the main drivers of business cycle movements in real activity.

 $^{^{26}\}mathrm{However},$ note that this shock explains a non-negligible fraction of long-run fluctuations in TFP (about 19%).

²⁷The same applies to the response of TFP in the long run, which is never statistically significant. Note that these objects are still reduced-form shocks, the interpretation of which is inherently delicate, as also pointed out by ACD. This is the price of following an agnostic approach.

²⁸The correlation coefficient between the SBC shock of consumption and the MBC shock of hours worked is 0.51, indicating a slightly weaker connection.

1.5.4. The Main Long-Run Shock and the Business Cycle

One of the main findings emerging from our analysis is that, although the long-run is left unrestricted, our two main business cycle shocks togheter account for more than half of the long-run fluctuations in both output and productivity. This suggests that what drives the long run also leaves a nontrivial footprint on the business cycle. In what follows, we further corroborate this finding. A second, important subset of our anatomy indeed comprises the shocks identified by targeting any one of GDP, investment, consumption, TFP, or labor productivity at long-run frequencies. This subset allows us to answer two questions. First, is a single shock sufficient to account for the bulk of long-run fluctuations in real activity? If yes, how much of the business cycle variance in real activity is accounted for by this shock?

Table 1.7 and Figure 1.5 show that these shocks produce essentially the same results in terms of both IRFs and variance contributions, as well as being highly correlated with each other (Table 1.10). Furthermore, any one of them accounts for almost all of the long-run variance in the targeted variables and for more than one half of the long run variance in the remaining variables. For example, the shock that targets GDP explains about 97% of the long run volatility in GDP and 70% of that in TFP. Similarly, the shock that targets TFP explains 91% of the long run volatility in TFP and 74% of that in GDP. Therefore, we can assume the existence of a single main long-run shock. From the figure, it is evident that this shock has the typical features of a supply shock that reflects long-run movements in productivity: it has a large permanent effect on real activity variables, and induces a negative covariance between GDP growth and inflation changes. In response to this shock, unemployment and hours worked exhibit a temporary hump-shaped pattern. Note that TFP and labor productivity, after a relatively modest impact effect, slowly increase toward their new long run level, suggesting that the various facets of this shock include an important technological component related to news about future productivity.

Now, let us turn our attention to the explained variances at business cycle frequencies (Table 1.7, top panel). Despite the fact that the short-run is left unrestricted, we find that the main long-run shock has considerable effects on the business cycle. This shock, represented by the shock that targets TFP, explains 21% of the business cycle volatility for GDP and investment, 25% for unemployment, and 24% for hours worked. Moreover, consistent with previous findings, it accounts for approximately 46% of the business cycle volatility in consumption. These results support the thesis that private consumption fluctuations are mostly explained by supply shocks and therefore, to best describe the observed business cycles in real activity, both demand and supply shocks should be taken into account. Overall, our main long-run shock does not seem to be disconnected from short-term economic activity, just as our empirical template of the business cycle does not seem to be disconnected from the long run. The emerging picture stands in stark contrast to ACD, where the same shock presents a profound disconnect with the short run. At the same time, is clearly incompatible with both the standard RBC model and the view proposed by Beaudry and Portier (2006) that TFP news is the main driver of cyclical fluctuations in real activity.

To conclude, there is a nontrivial connection between the short run and the long run of real economic activity, which theoretical models should take into account.

Ν	Main Long Run Shock							
Percentage	Percentage of Explained Cyclical Variance							
	GDP	Cons	Invest	Labor	TFP			
GDP	21.8	19.8	23.1	19.2	20.5			
Unemployment Rate	27.1	23.7	33.0	24.3	25.1			
Consumption	48.2	51.5	41.3	38.5	46.2			
Investment	18.2	17.7	23.5	19.1	20.6			
Hours Worked	20.3	21.2	24.0	20.0	20.8			
Labor	24.6	14.7	24.5	24.3	26.0			
TFP	16.2	8.4	14.9	21.2	29.2			
Inflation	33.0	17.4	47.8	46.8	28.8			
FFR	17.1	19.5	24.4	15.1	23.8			
S&P500	9.0	5.9	15.3	11.9	7.3			
Percentage (of Expl	ained Lo	ng Run V	ARIANCE				
TARGET	GDP	Cons	Invest	Labor	TFP			
GDP	96.7	80.1	71.3	72.2	74.1			
Unemployment Rate	57.7	50.0	59.9	74.9	61.4			
Consumption	78.9	95.3	47.7	52.3	72.8			
Investment	70.8	47.2	96.4	66.7	51.9			
Hours Worked	64.1	59.0	45.4	66.9	53.1			
Labor	69.4	50.7	64.4	93.0	73.4			
TFP	69.9	69.5	49.8	72.2	90.8			
Inflation	5.5	1.0	23.2	23.1	2.8			
FFR	0.2	5.9	8.7	5.8	0.4			
S&P500	18.7	12.7	28.6	22.0	14.6			

Table 1.7: Percentage of variance explained by the main long run shock for a few selected variables, by frequency bands. The columns correspond to different targets in the construction of the shock.

1.5.5. Robustness

In this section we conduct a robustness check for the shocks that make up our business cycle anatomy. Specifically, we explore the robustness of our findings for the main and secondary shocks identified by targeting GDP or consumption. Firstly, we estimate the model with four lags instead of three. Secondly, we test the robustness to different numbers of static factors. Specifically, we compare the results of our baseline specification (r = 11) with two alternatives: r = 8, 12. The third robustness exercise serves a complementary objective. We take into account that economic expansions have become progressively longer, as suggested by Beaudry et al. (2020).²⁹ As a result, we adjust our approach to the business cycle by using a different frequency band. Instead of the conventional range of 18 months to 8 years, $[\underline{\theta}, \overline{\theta}] = [2\pi/32, 2\pi/6]$, we now consider cycles with periodicity between 18 month and 12 years, $[\underline{\theta}, \overline{\theta}] = [2\pi/48, 2\pi/6]$. Finally, we constrain the sample to 1961-2007, excluding the Great Recession and the Zero Lower Bound.

For each robustness exercise, Table 1.11 reports the contributions of both the MBC and SBC shocks, identified by targeting GDP growth, to the cyclical (top panel) and long-run (lower panel) variance. The first two columns correspond to our baseline specification, while the remaining are for the alternative specifications. Similarly, Table 1.12 provides the same information but for the main and secondary business cycle shocks identified by targeting consumption. As we move across specifications, we observe that the contribution of the identified shocks to the cyclical and long-run variances of the main macroeconomic aggregates remains almost unchanged. The main conclusions are all confirmed. Interestingly, in the third exercise, when considering cycles with a periodicity slightly longer than what is traditionally associated with business cycles, the relative importance of shocks with long-lasting effects/supply in explaining cyclical fluctuations in GDP are mostly explained by a transitory/demand shock.

Finally, the same robustness is found when considering the IRFs. Figures 1.6 and 1.7 plot the IRFs for the MBC and the SBC shocks that target GDP, respectively, for both the baseline and alternative specifications. The solid black lines and confidence bands are those obtained in the baseline. Likewise, Figures 1.8 and 1.9 display the IRFs for shocks targeting consumption. Although there are some differences when we set a lower number of static factors compared to the benchmark, or when we narrow the sample, the dynamic responses overall are reasonably similar to those obtained in the baseline exercise.

1.6. Discussion and Concluding Remarks

In this paper we challenge the ACD's hypothesis that most of the business cycle fluctuations in real economic activity can be explained by just one shock. We argue that the VAR used in their work is informationally deficient, rendering the causal interpretation of the "Main Business Cycle" shock untenable. By using a large-dimensional Struc-

²⁹The authors show that many macroeconomic aggregates appear to have a peak in their spectral densities at periodicities between 32 and 50 quarters and that the implied movements coincide with NBER cycle dating. For this reason, they argue that the traditional definition of business cycle may have become slightly too narrow and should be modified accordingly.

tural Dynamic Factor model along with ACD's frequency-domain method, we propose an alternative anatomical template for the transmission mechanisms of business cycles. The picture emerging from our empirical analysis is as follows. It is possible to account for the majority of the business-cycle fluctuations in GDP, investment, consumption, unemployment, and, to a somewhat lesser extent, hours worked, using a parsimonious two-shock model. These reduced-form shocks, which we refer to as the "Two Main Business Cycle" shocks, align with the traditional AD-AS narrative in terms of their characteristics. Both mechanisms are important factors of business cycle fluctations. Private consumption fluctuations are almost entirely explained by supply dynamics, whereas GDP growth fluctuations are mainly explained by demand dynamics. The last result is consistent with the standard New Keynesian narrative that the bulk of the business cycle in output is due to shifts in aggregate demand. The result on consumption can be explained in light of the Permanent Income Hypothesis: at the aggregate level, private consumption largely follows expectations about future income, and thus would be more responsive to permanent shocks than transitory ones (Quah, 1990).

Our conclusions are in line with those of Francis and Kindberg-Hanlon (2022), even though the model and the method used here are different. In that paper, a VAR is used, and the variance-maximizing method is coupled with additional theoretical constraints, to identify the dominant driver of US GDP at business cycle frequencies. In contrast, we use a SDFM and follow ACD's spectral method to identify a collection of reduced-form shocks, without imposing additional constraints. As noted above, our findings regarding the joint dynamics of inflation and real activity over the business cycle align with the evidence presented in Bianchi et al. (2023), which instead employs a Trend-Cycle VAR model. To conclude, the reduced-form shocks contained in our collection, the interpretation of which is inherently delicate, suggest that a simplified yet fairly complete representation of the US macroeconomy can be provided by only two shocks. In that, our paper can be regarded as complementary to Forni et al. (2023). In that paper, our evidence serves as a starting point to provide a comprehensive and stylized structural description of the US macroeconomy, focusing on both the business cycle and the long run.

TABLES

	χ	ξ
GDP	94.33	5.67
Unemployment Rate	94.17	5.83
Consumption	81.62	18.38
Investment	89.54	10.46
Hours Worked	83.53	16.47
TFP	80.91	19.09
Inflation	90.47	9.53
FFR	97.92	2.08
S&P500	94.47	5.53
Labor Productivity	89.31	10.69

Table 1.8: Percentage of the variance explained by the estimated common and idiosyncratic components. Baseline specification: r = 11 static factors.

Sec	ONDARY	BUSINESS	Cycle S	HOCK	
	GDP	Unemp	Cons	Invest	Hours
GDP	1	0.98	-0.26	0.98	0.95
Unemployment	0.98	1	-0.22	0.96	0.90
Consumption	-0.26	-0.22	1	-0.38	-0.30
Investment	0.98	0.96	-0.38	1	0.95
Hours Worked	0.95	0.90	-0.30	0.95	1

Table 1.9: Correlation between the SBC shocks obtained by targeting GDP, UNEMPLOYMENT, CONSUMPTION, INVESTMENT, AND HOURS WORKED

Main Long Run Shock								
	GDP	Unemp	Cons	Invest	Hours			
GDP	1	0.91	0.86	0.86	0.88			
Consumption	0.91	1	0.70	0.74	0.87			
Investment	0.86	0.70	1	0.83	0.73			
Labor	0.86	0.74	0.83	1	0.89			
TFP	0.88	0.87	0.73	0.89	1			

Table 1.10: Correlation between the MLR shocks obtained by targeting GDP,Consumption, Investment, Labor productivity and TFP

	Robustness: The Two Main Business Cycle Shocks (GDP)											
Percentage of Explained Cyclical Variance												
	BASE	ELINE	[1] :	P=4	[2] 1	R=8	[3] R	=12	[5] (6-48	[6] 196	61-2007
	MBC	SBC	MBC	SBC	MBC	SBC	MBC	SBC	MBC	SBC	MBC	SBC
GDP	60.6	30.9	63.5	28.9	59.5	35.0	61.2	29.9	55.4	34.4	51.4	35.7
Unemployment	36.0	37.0	39.1	35.2	35.6	42.3	37.1	36.2	33.9	39.1	39.5	34.9
Consumption	19.0	55.7	22.4	56.7	18.6	71.6	18.4	58.9	17.7	61.2	22.7	51.9
Investment	52.8	33.4	56.5	31.2	47.9	42.5	52.5	32.9	50.3	34.8	47.0	38.5
Hours	26.7	34.4	27.5	33.3	31.9	36.9	25.0	34.7	23.0	41.7	38.3	29.9
TFP	9.8	12.4	8.3	15.7	14.2	15.5	6.8	12.9	9.7	13.9	3.3	9.8
Inflation	19.5	32.0	25.8	27.8	18.7	32.6	21.0	29.8	24.2	30.0	20.7	40.6
\mathbf{FFR}	36.5	19.4	43.1	17.3	41.6	23.6	36.8	20.0	36.4	17.7	46.1	16.9
S&P500	14.5	41.9	19.2	34.1	17.0	39.1	16.5	37.2	14.1	36.4	18.0	37.2
Labor	45.9	17.8	46.0	19.7	42.9	18.9	45.7	19.6	42.7	20.8	34.8	26.8
		Per	CENTAG	e of E	XPLAINI	ed Lon	G-Run	VARIAN	CE			
	BASE	ELINE	[1] :	P=4	[2] 1	R=8	[3] R	=12	[5] (6-48	[6] 196	61-2007
	MBC	SBC	MBC	SBC	MBC	SBC	MBC	SBC	MBC	SBC	MBC	SBC
GDP	0.5	60.3	0.5	76.4	1.7	68.2	0.5	66.3	0.2	74.6	2.5	66.4
Unemployment	5.3	71.5	8.8	70.4	6.1	75.1	5.8	71.8	6.3	72.5	3.0	49.9
Consumption	1.1	55.4	1.2	68.4	1.4	71.5	1.2	62.4	1.6	65.3	0.3	70.7
Investment	1.0	69.9	1.8	78.7	2.4	78.3	1.9	69.8	1.5	76.2	0.4	69.4
Hours	2.0	63.9	1.7	65.6	5.1	71.4	0.8	60.2	1.5	65.1	0.4	17.1
TFP	2.2	54.0	2.7	66.0	2.5	59.1	2.2	62.4	2.9	63.3	1.8	44.3
Inflation	10.3	8.6	13.6	7.1	21.1	5.5	14.5	6.2	11.5	7.6	2.0	16.0
\mathbf{FFR}	22.1	0.6	30.2	0.2	36.8	0.3	24.9	0.2	23.2	0.4	20.2	21.3
S&P500	1.3	42.7	2.6	34.9	1.2	38.0	2.1	43.4	1.5	37.3	0.7	36.2
Labor	0.5	54.6	0.8	63.8	1.4	42.2	0.8	57.9	1.0	61.1	0.9	33.3

Table 1.11: Percentage of variance explained by the MBC shock and the SBC shock, obtained by targeting GDP, by frequency band. The columns correspond to different robustness exercises. Business cycle frequency band [5]: $[2\pi/48 \leq \omega \leq 2\pi/6]$ corresponding to cycles with periodicity between 18 months and 12 years.

ROBUSTNESS: THE TWO MAIN BUSINESS CYCLE SHOCKS (CONSUMPTION)												
Percentage of Explained Cyclical Variance												
	BASI	ELINE	[1] 1	P=4	[2] 1	R=8	[3] R	=12	[5] (6-48	[6] 196	61-2007
	MBC	SBC	MBC	SBC	MBC	SBC	MBC	SBC	MBC	SBC	MBC	\mathbf{SBC}
GDP	26.9	38.3	26.1	42.9	33.2	52.0	27.4	38.0	30.9	37.5	32.4	34.9
Unemployment	26.9	35.2	28.3	36.9	37.8	44.5	30.3	33.9	35.2	31.9	31.0	41.7
Consumption	66.0	29.3	62.6	33.8	75.7	21.4	64.5	29.6	68.4	26.5	57.4	34.7
Investment	24.8	42.7	24.6	47.8	37.7	53.0	27.2	42.6	28.1	42.9	33.0	38.0
Hours	29.9	20.5	30.9	22.2	33.0	34.9	30.5	18.3	36.8	19.3	27.5	37.5
TFP	10.9	8.9	13.9	5.3	14.4	17.9	9.3	7.9	11.0	8.8	13.7	4.7
Inflation	22.1	21.7	24.0	23.9	31.1	31.0	23.7	19.2	28.5	18.3	42.0	28.2
FFR	14.8	35.2	15.9	39.0	21.5	52.1	17.1	32.4	18.9	31.2	12.3	49.2
S&P500	23.3	9.2	21.2	14.5	26.0	23.4	23.1	9.9	20.6	13.9	23.3	14.9
Labor	18.6	24.1	19.3	26.7	20.5	32.8	17.5	25.2	18.2	25.5	24.9	24.7
		Per	CENTAG	e of E	XPLAINI	ed Lon	g-Run '	VARIAN	CE			
	BASI	ELINE	[1] 1	P=4	[2] 1	R=8	[3] R	=12	[5] (6-48	[6] 196	61-2007
	MBC	SBC	MBC	SBC	MBC	SBC	MBC	SBC	MBC	SBC	MBC	SBC
GDP	65.7	11.0	69.0	6.7	81.3	0.6	74.7	4.3	76.1	2.8	76.4	0.9
Unemployment	58.1	16.1	61.1	17.0	80.2	8.9	64.5	10.4	67.2	5.9	57.8	4.9
Consumption	68.9	12.1	71.5	7.2	86.5	2.3	77.1	5.3	80.2	3.0	88.8	0.6
Investment	55.6	12.2	55.6	12.4	78.1	4.2	63.1	8.0	64.5	3.7	65.1	0.7
Hours Worked	58.1	4.7	57.0	1.7	79.8	5.4	61.6	2.2	59.0	0.5	21.5	0.2
TFP	55.7	19.1	62.7	12.2	68.6	3.3	62.6	12.2	67.7	7.0	53.7	1.1
Inflation	2.2	5.3	3.9	7.2	4.7	39.6	2.2	8.2	4.4	4.9	20.8	2.3
\mathbf{FFR}	0.6	7.1	0.1	13.6	0.4	56.7	0.9	10.0	0.2	9.6	23.8	18.5
S&P500	22.6	4.7	22.2	6.3	25.7	1.3	27.3	3.4	24.1	0.9	27.5	1.4
Labor	49.2	12.4	53.3	7.1	57.4	9.0	54.9	6.2	58.2	4.0	37.8	0.1

Table 1.12: Percentage of variance explained by the MBC shock and the SBC shock, obtained by targeting Consumption, by frequency band. The columns correspond to different robustness exercises. Business cycle frequency band [5]: $[2\pi/48 \le \omega \le 2\pi/6]$ corresponding to cycles with periodicity between 18 months and 12 years.

FIGURES

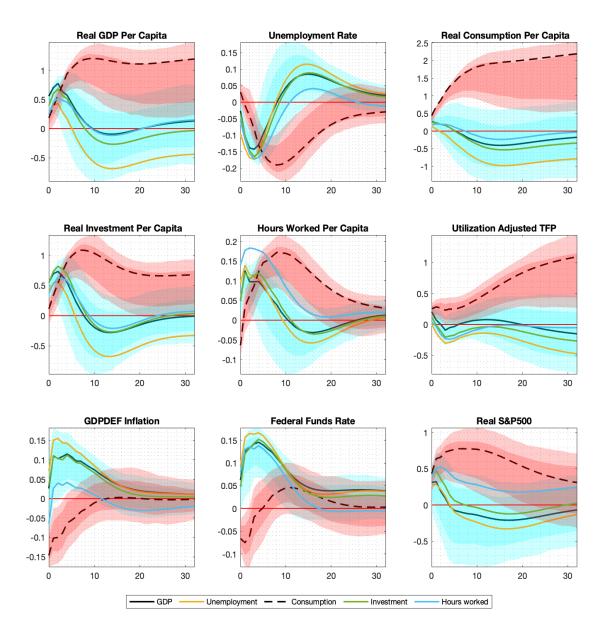


Figure 1.3: Impulse response functions of the MBC shock obtained by targeting different variables. The dark red (dark blue) and light red (light blue) areas are the 68% and 90% confidence bands, respectively, for the shock that targets GDP (consumption)

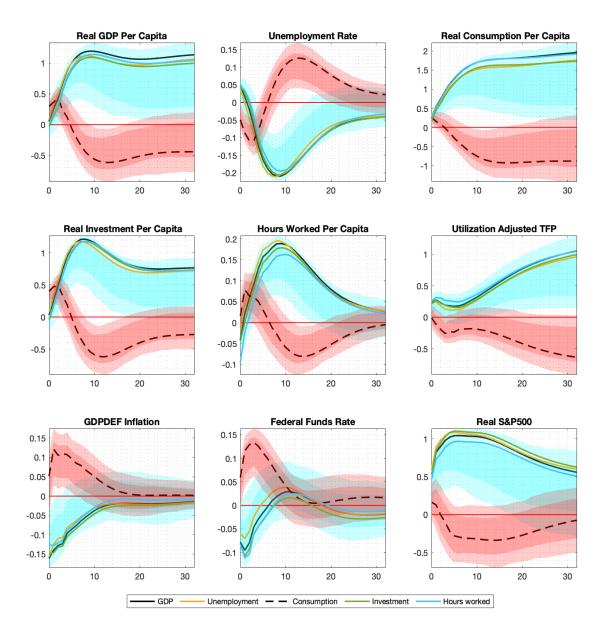


Figure 1.4: Impulse response functions of the SBC shock obtained by targeting different variables. The dark red (dark blue) and light red (light blue) areas are the 68% and 90% confidence bands, respectively, for the shock that targets GDP (consumption)

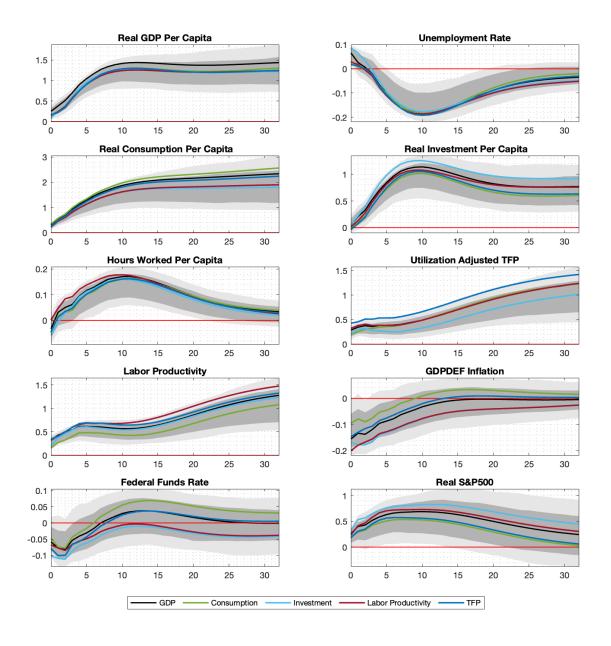


Figure 1.5: Impulse response functions of the MLR shock obtained by targeting different variables. The dark gray and light gray areas are the 68% and 90% confidence bands, respectively, for the shock that targets GDP.

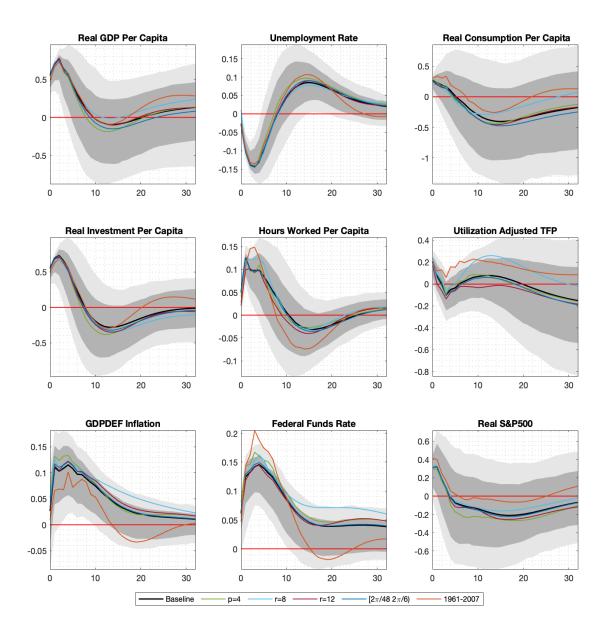


Figure 1.6: Impulse Response Functions of the MBC shock obtained by targeting GDP. The solid lines represent the point estimates for different robustness exercises. The dark gray and light gray areas are the 68% and 90% confidence bands, respectively. Black line and confidence bands: baseline specification.

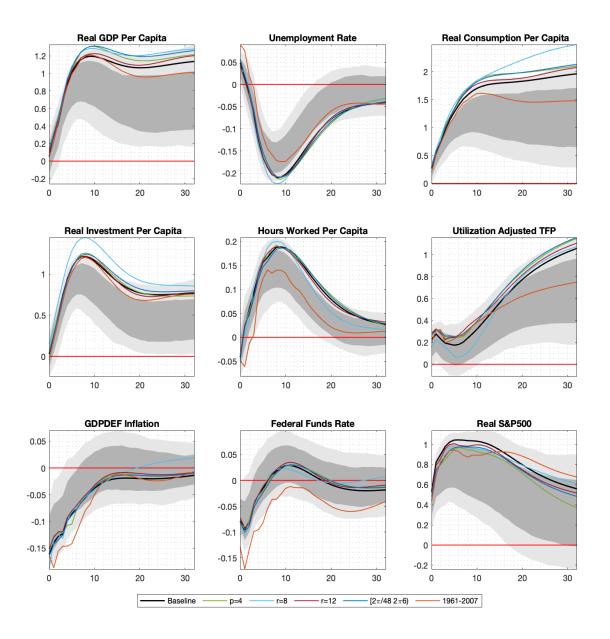


Figure 1.7: Impulse Response Functions of the SBC shock obtained by targeting GDP. The solid lines represent the point estimates for different robustness exercises. The dark gray and light gray areas are the 68% and 90% confidence bands, respectively. Black line and confidence bands: baseline specification.

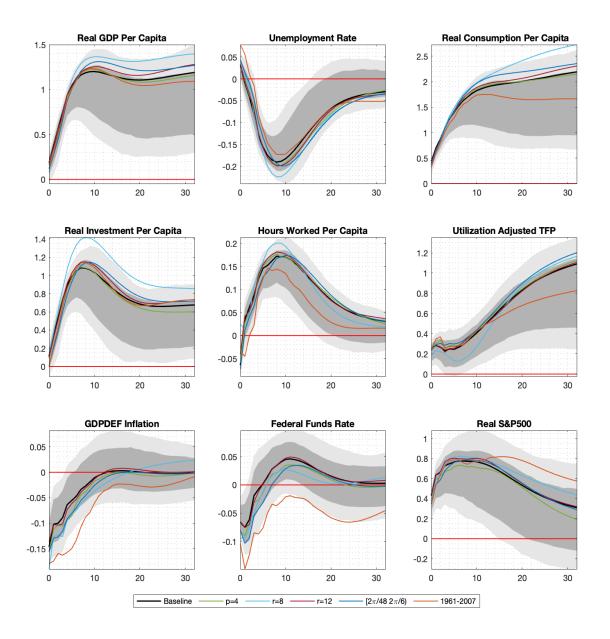


Figure 1.8: Impulse Response Functions of the MBC shock obtained targeting consumption. The solid lines represent the point estimates for different robustness exercises. The dark gray and light gray areas are the 68% and 90% confidence bands, respectively. Black line and confidence bands: baseline specification.

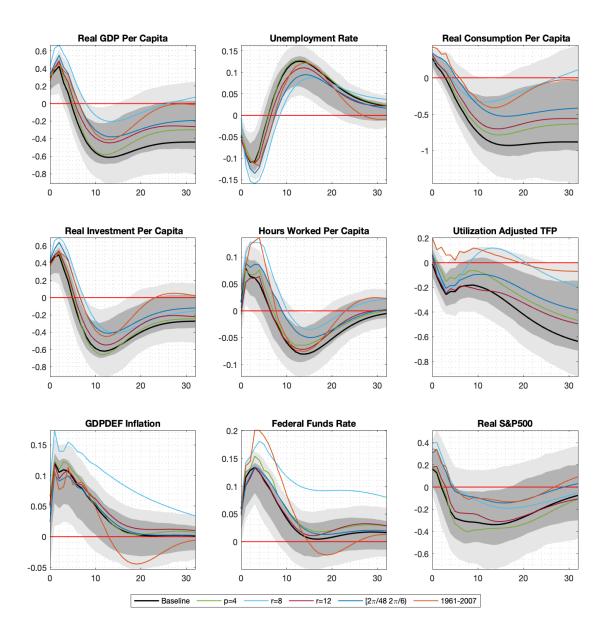


Figure 1.9: Impulse Response Functions of the SBC shock obtained by targeting consumption. The solid lines represent the point estimates for different robustness exercises. The dark gray and light gray areas are the 68% and 90% confidence bands, respectively. Black line and confidence bands: baseline specification.

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APPENDIX

Appendix

1.A. ESTIMATION PROCEDURE

In order to compute the spectra and the objective function we proceed as follows. We estimate the first two equations (1.4a)-(1.4b) using the two step estimation technique discussed in Forni et al. (2009), which we briefly review here. FIRST STEP. We set a value for the number r of the static factors, using the criterion by Bai and Ng (2002) with the penalty modification proposed in Alessi et al. (2010), finding a number of static factors $\hat{r} = 11$. In the robustness section, we take into account the uncertainty in estimating the number of static factors, and repeat the analysis with different specifications of \hat{r} . The static factors $F_t = (F_{1t} \dots F_{rt})'$ are estimated by the first \hat{r} principal components of the variables in our dataset, and the factor loadings, $\lambda_{ij}, j = 1 \dots r$, by the associated eigenvectors. Thus, the estimated loading matrix, $\hat{\Lambda}$, is the $n \times \hat{r}$ matrix having on the columns the normalized eigenvectors corresponding to the \hat{r} -largest eigenvalues of the sample covariance matrix of the data, $\hat{\Sigma}_x$. The estimated common component vector is given by $\hat{\chi}_t = \hat{\Lambda} \hat{F}_t$. Second Step. We run a VAR(p) for the estimated factors \hat{F}_t to get estimates $\hat{C}(L)$ and $\hat{\epsilon}_t$ of C(L) and the VAR innovations ϵ_t . The estimated MA representation is $\hat{F}_t = \hat{C}(L)^{-1}\hat{\epsilon}_t$. The number of lags p is determined according to the AIC criterion ($\hat{p}_{AIC} = 3$). The Cholesky IRFs of the common components are obtained according to (1.7) as $\hat{D}(L) = \hat{\Lambda}[\hat{C}(L)^{-1}\hat{S}]$. From this matrix we estimate the spectral density of the common components at the Fourier frequencies $\theta = 2\pi s/T$, $s = 1, \ldots, T$. Finally, we compute $V\left(k, \underline{\theta}, \overline{\theta}\right)$ by replacing the integral with the simple average of the spectral density matrix, across the frequencies belonging to the relevant interval. We do not apply the rank reduction step as this will be part of the identification strategy discussed below.

1.B. DATA DESCRIPTION AND DATA TREATMENT

For the description of each variable see McCracken and Ng (2020). For variables not in the FRED-QD dataset, refer to the Mnemonic and note. Treatment codes: 1 =no treatment; 2 = first difference, Δx_t ; $4 = \log(x_t)$; $5 = \log$ of the first difference, $\Delta \log(x_t)$.

The analysis presented in the main text focuses on a subset of 10 macroeconomic series of interest: (1) the log difference of the real per capita GDP [ID 1]; (2) the log difference of real per capita consumption [ID-21]; (3) the log difference of real per capita investment [ID-22]; (4) the unemployment rate [ID-37]; (5) the log of real per capita hours worked [ID-44]; (6) the inflation rate, defined as the log difference of the GDP deflator [ID-50]; (7) labour productivity [ID-62]; (8) the cumulated sum of the utility-adjusted total factor productivity [ID-68]; (9) the Federal Funds rate [ID-73] and the (10) Shiller's real S&P500 stock price index [ID-107].

ID	FRED-QD ID	Mnemonic	Treatment code	Note
1	1	GDPC1/CNP16OV	5	
2	2	PCECC96/CNP16OV	5	
3	3	PCDGx/CNP16OV	5	
4	4	PCESVx/CNP16OV	5	
5	5	PCNDx/CNP16OV	5	
6	6	GPDIC1/CNP16OV	5	
7	7	FPIx/CNP16OV	5	
8	8	Y033RC1Q027SBEAx/CNP16OV	5	
9	9	PNFIx/CNP16OV	5	
10	10	PRFIx/CNP16OV	5	
11	11	A014RE1Q156NBEA	1	
12	12	GCEC1/CNP16OV	5	
13	13	A823RL1Q225SBEA	1	
14	14	FGRECPTx/CNP16OV	5	
15	15	SLCEx/CNP16OV	5	
16	16	EXPGSC1/CNP16OV	5	
17	17	IMPGSC1/CNP16OV	5	
18	18	DPIC96/CNP16OV	5	
19	19	OUTNFB/CNP16OV	5	
20	20	OUTBS/CNP16OV	5	
21		(PCESVx+PCNDx)/CNP16OV	5	
22		(PCDGx+FPIx)/CNP16OV	5	
23	22	INDPRO/CNP16OV	5	
24	23	IPFINAL/CNP16OV	5	
25	24	IPCONGD/CNP16OV	5	
26	25	IPMAT/CNP16OV	5	
27	28	IPDCONGD/CNP16OV	5	
28	30	IPNCONGD/CNP16OV	5	
29	31	IPBUSEQ/CNP16OV	5	
30	35	PAYEMS/CNP16OV	2	
31	36	USPRIV/CNP16OV	2	
32	38	SRVPRD/CNP16OV	2	
33	39	USGOOD/CNP16OV	2	
34	51	USGOVT/CNP16OV	2	
35	57	CE16OV/CNP16OV (EMRATIO)	2	
36	58	CIVPART	2	
37	59	UNRATE	1	
38	60	UNRATESTx	1	
39	61	UNRATELTx	1	
40	62	LNS14000012	1	
41	63	LNS14000025	1	
42	64	LNS14000026	1	
43	74	HOABS/CNP16OV	4	

APPENDIX

ID	FRED-QD	MNEMONIC	Treatment	Norr
ID	ID	Mnemonic	CODE	Note
44	76	HOANBS/CNP16OV	4	
45	77	AWHMAN	1	
46	79	AWOTMAN	1	
$47 \\ 48$	81 95	HOUST/CNP160V PCECTPI	5 5	
49	95 96	PCEPILFE	5	
50		GDPDEF	5	GDP: Implicit Price Deflator
51	97	GDPCTPI	5	
52	98	GPDICTPI	5	
53	120	CPIAUCSL	5	
54	121	CPILFESL	5	
55	122	WPSFD49207	5	
$\frac{56}{57}$	123 124	PPIACO WPSFD49502	5 5	
58	124	PPIIDC	5	
59	129	WPU0561	5	
60	130	OILPRICEx	5	
61	135	COMPRNFB	5	
62	138	OPHNFB	5	
63	139	OPHPBS	5	
64	140	ULCBS	5	
65 66	142 143	ULCNFB UNLPNBS	5 5	
67	145	dtfp	1	Fernald's TFP growth
68		dtfp util	1	Fernald's TFP growth CU adjusted
69		dtfp I	1	Fernald's TFP growth - Inv
70		dtfp C	1	Fernald's TFP growth - Con
71		dtfp I util	1	Fernald's TFP growth CU - Inv
72		dtfp C util	1	Fernald's TFP growth CU - Con
73	144	FEDFUNDS	1	
74	145	TB3MS	1	
$75 \\ 76$	$146 \\ 147$	TB6MS GS1	1 1	
77	147	GS10	1	
78	150	AAA	1	
79	151	BAA	1	
80	152	BAA10YM	1	
81	156	GS10TB3Mx	1	
82		BAA-AAA	1	
83		GS10-FEDFUNDS	1	
$\frac{84}{85}$		GS1-FEDFUNDS BAA-FEDFUNDS	1 1	
86	158	BOGMBASEREALx/CNP16OV	5	
87	160	M1REAL/CNP16OV	5	
88	161	M2REAL/CNP16OV	5	
89	163	BUSLOANSx/CNP16OV	5	
90	164	CONSUMER _x /CNP16OV	5	
91	166	REALLNx/CNP16OV	5	
92	168	TOTALSLx/CNP16OV	5	
$93 \\ 94$	188	UMCSENTx Business Condition 12 Months	1 1	Michigan Consumer Survey
94 95		Business Condition 12 Months Business Condition 5 Years	1	Michigan Consumer Survey Michigan Consumer Survey
95 96		Current Index	1	Michigan Consumer Survey
97		Expected Index	1	Michigan Consumer Survey
98		News Index: Relative	1	Michigan Consumer Survey
99	197	UEMPMEAN	1	
100	201	GS5	1	
101	210	CUSR0000SAC	5	
102	211	CUSR0000SAD CUSR0000SAS	5	
$103 \\ 104$	212 213	CUSR0000SAS CPHILESL	5 5	
$104 \\ 105$	213 245	CPIULFSL S&P 500	5 5	
105	245 246	S&P: indust	5	
107		S&P 500/GDPDEF	5	
108		S&P: indust/GDPDEF	5	
109		JLN Macro Unc 1-month	1	Jurado, Ludvigson and Ng Uncertainty
110		JLN Macro Unc 3-month	1	Jurado, Ludvigson and Ng Uncertainty
$111 \\ 112$		JLN Macro Unc 12-month DPCCRC1Q027SBEAx/CNP16OV	1 5	Jurado, Ludvigson and Ng Uncertainty Real PCE Excluding food and energy

Continued on next page

APPENDIX

ID	FRED-QD ID	MNEMONIC	Treatment Code	Note
113		DFXARC1M027SBEAx/CNP16OV	5	Real PCE: Food
114		DNRGRC1Q027SBEAx/CNP16OV	5	Real PCE: Energy goods and services

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Chapter 2

An American Macroeconomic Picture: Supply and Demand Shocks in the Frequency Domain^{*}

with Mario Forni¹, Luca Gambetti², Luca Sala³, Stefano Soccorsi⁴

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Abstract

We provide a few new empirical facts that theoretical models should feature in order to be consistent with US data. 1) There are two classes of shocks: demand and supply. Supply shocks have long-run effects on economic activity, demand shocks do not. 2) Both supply and demand shocks are important sources of business cycles fluctuations. 3) Supply shocks are the primary driver for consumption fluctuations, demand shocks for investment. 4) The demand shock is closely related to the credit spread, while the supply shock is essentially a news shock. The results are obtained using a novel approach which combines frequency domain identification and Dynamic Factor Model analysis.

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2.1. INTRODUCTION

Figuring out what is the correct or most reliable theory underlying the data has always been the cornerstone of macroeconomic research. The empirical business cycle literature has tried to inform and support the theory by providing various stilized facts and representations of the macroeconomy.

At the origins of the modern empirical macroeconomic debate, Blanchard and Quah (1989) (BQ henceforth) draw a sketch of the macroeconomy as driven by two shocks, a permanent shock and a transitory one, interpreted as supply and demand, respectively. Both shocks are depicted as important sources of business cycle fluctuations.

In the following 30 years, empirical research moved away from the idea of a comprehensive representation of the macroeconomy, focusing mainly on partial identification and the study of single, more specific sources of fluctuation, such as technology shocks (Galì, 1999), news shocks (Beaudry and Portier, 2006), noise shocks (Blanchard et al., 2013), uncertainty shocks (Bloom, 2009), credit shocks (Gilchrist and Zakrajšek, 2012), to name just a few of the most important.

A couple of recent papers, however, departing from the widespread partial identification approach, go back to seeking a general and parsimonious representation of the macroeconomy. Angeletos et al. (2020) (ACD henceforth) look for the shock that most explains the business cycle —the so called "main business cycle shock" (MBC). The authors, using a frequency-domain identification method in the context of structural VARs, argue that the bulk of cyclical fluctuations in real economic activity can be explained by a single shock. This shock is not the technology shock of the RBC model (Kydland and Prescott, 1982), since it has no long run effects on output. However, it cannot be considered a standard demand shock either, because it has no effect on prices.

The second paper is Avarucci et al. (2021) (ACFZ henceforth). Within a large factor model framework, ACFZ find that just two statistically identified shocks are enough to describe all macroeconomic variables, thus confirming, albeit with a different method, a previous important result by Onatski (2009). Such reduced form shocks are found to be economically interpretable and could be a temporary demand shock and a permanent supply shock.

The present paper is close in spirit to BQ, ACD and ACFZ. What we do is to provide a general picture of the main forces driving the US macroeconomy, at both cyclical and long run frequencies, with the goal of identifying empirical regularities which theoretical models should feature in order to be consistent with the data.

Our working hypothesis is that there are two main shocks, as suggested by the above factor model literature, and that these can be identified as textbook-type demand and supply shocks. The former should move prices and quantities in the same direction and have only transitory effects on real activity variables, while the latter should move prices and quantities in the opposite directions and have permanent effects. What we have in mind is a simple AD-AS model, or a New Keynesian model where the macroeconomy is described in terms of an aggegate demand curve (AD) and a Phillips curve (NKPC) which we refer to as the "traditional view". In a nutshell, our main result is that this hypothesis is confirmed by the data.

We use a dataset of 114 quarterly US time series, covering the period 1961-I to 2019-IV and assume that the data follow a large-dimensional Structural Dynamic Factor model, as introduced by Stock and Watson (2005) and Forni et al. (2009), which is naturally designed to describe a large number of time series with a relatively small number of common shocks. Having a large dataset, we can study the impulse response functions of all relevant macroeconomic variables within a unified framework; moreover, the rich information environment enables us to avoid the well-known non-invertibility problem affecting SVAR analysis (Hansen and Sargent, 1991; Lippi and Reichlin, 1993, 1994). Last but not least, using High Dimensional Factor techniques, we estimate the common components and correct the observed macroeconomic variables for measurement error.

From a methodological point of view, we contribute to frequency domain analysis by providing a fairly comprehensive treatment of structural identification in the frequency domain. We extend the approach used in ACD^1 (see also Sarno et al., 2007; DiCecio and Owyang, 2010; Giannone et al., 2019) in several directions. In particular, in order to implement our identification scheme, we show how to jointly target variances of different variables and target covariances on a given frequency band.

Our identification strategy unfolds in two steps. In the first step, we select the two shocks maximizing the explained variance of the main macroeconomic variables, at all frequencies of macroeconomic interest, that is, excluding fluctuations with period of less than 18 months, of little interest for macroeconomic analysis. In so doing, we do not target a single variable at a time, as in ACD, but target jointly several variables. More specifically, we include in the target the variances of the main trending real activity variables (GDP, consumption, investment, TFP and labour productivity) as well as the variances of other important real and nominal variables (the unemployment rate, hours worked, the inflation rate, the federal funds rate and the S&P500 stock price index).

We find that these two shocks are successful in explaining the bulk of the variance of the main macroeconomic aggregates at both business cycle and long run frequencies,

¹ACD show how to identify the shock which maximizes the explained variance of a given variable on a specific frequency band. This method is the frequency domain version of Uhlig (2004), who identifies two shocks that maximize the majority of the k-step ahead prediction error variances in real GNP for horizons between 0 and 5 years.

providing a fairly complete picture of the US macroeconomy. Adding a third shock increases only marginally the explained variance of the main real and nominal variables.

In the second step, we rotate the two main shocks in order to give them an economic interpretation. We implement two different identification schemes. In the first one (Identification I) we define a demand shock and a supply shock with a completely novel criterion. The demand shock is obtained by maximizing the covariance of GDP and inflation at business-cycle frequencies. The supply shock is automatically identified, by the orthogonality condition, as the shock minimizing the above covariance. In the second scheme (Identification II) we define a permanent and a transitory shock. Precisely, we define the permanent shock as the one that explains most of the long run variance of trending real activity variables (i.e. GDP, TFP, consumption, investment and labor productivity). The transitory shock is automatically identified by the orthogonality condition as the one minimizing the explained long run variance of the above variables.

In a sense, this procedure is close in spirit to BQ. Just like BQ, we provide a general picture of the forces driving the macroeconomy. By reducing the number of shocks of interest in the first stage, and identifying all of these shocks in the second stage, our method can be regarded as a global identification exercise, as opposed to the prevailing partial identification approach.

Our main results are the following. First, the two identification schemes provide the same outcomes. The inflationary demand and the deflationary supply shocks of Identification I are almost identical to the transitory and permanent shocks of Identification II, respectively. Hence, we show empirically that demand shocks have transitory effect on real economic activity. Second, both shocks, demand and supply, explain sizable fractions of business cycle fluctuations. Third, the demand shock is the most important cyclical shock for output, investment and unemployment, while private consumption fluctuations are mostly explained by supply shocks. Finally, our demand shock is to a large extent a credit shock, since it explains almost all cyclical variance of the risk spread and is the main driver of interest rates at all frequencies; moreover, the supply shock has the features of a news technology shock. It accounts for almost all the long run and the long cycles (between 8 and 20 years) of real activity and is the main driver of the consumer confidence index.

The above findings are broadly consistent with BQ's ones, but complete BQ's scketch with a large body of new evidence about prices, interest rates, consumption, investment and other macroeconomic variables. Differently from BQ, where long run neutrality of demand shocks is assumed, here it shows up as a result. Several papers have shown that special demand side shocks, such as monetary policy shocks or financial shocks, have transitory effects on output. But no one, to our knowledge, have shown that shocks identified as standard demand shocks have no long run effects on

real activity.

By focusing on just two shocks, demand and supply, we do not want to deny that there is a plurality of sources of fluctuations, nor deny the importance of specific shocks analyzed in the literature. Rather, we think that such shocks can be grouped into the broader supply and demand categories: for instance, the technology shock is of course a supply shock, whereas uncertainty and credit shocks are best seen as transitory demand shocks. Our idea is that shocks having different nature but belonging to the same group, demand or supply, do have similar effects on the main macroeconomic aggregates, so that grouping them can produce meaningful results, in terms of impulse response functions and variance decomposition.

Our paper can be regarded as complimentary to ACFZ. In that paper, the focus is the criterion to estimate the number of shocks and the main empirical results is that there are two main shocks hitting US macroeconomy; in our paper we take this evidence as the starting point and go on by identifying the shocks on economic grounds and estimating the impulse-response functions.

Our results are partially at odds with the picture emerging from ACD. We agree that the demand shock is the most important cyclical shock and is disconnected with long run real activity. On the other hand, our demand shock is inflationary and our supply shock explains a sizable fraction of the cyclical variance of output. We explore all possible combinations of our two shocks, putting the ACD's MBC shock under the microscope (Subsection 2.4.6). The result is that in our rich information set up, there is no way to get a shock that is disconnected from both inflation and long-run real economic activity, as the ACD's MBC shock. Our explanation is that, as argued in Granese (2023), ACD's VAR is informationally deficient.

Our paper is also related to Furlanetto et al. (2021), since our identification scheme, albeit based on frequency domain techniques, is similar to theirs from a substantive economic point of view. In contrast with their findings, where the demand shock is found to have long run effects, our demand shock does not affect real per-capita GDP and labour market in the long run.

Finally, our results are largely in line with those of Francis and Kindberg-Hanlon (2022), even though the model and the method used here are different. In that paper a SVAR is used and variance maximization is coupled with additional identification constraints, whereas here we rely on a structural factor model and do not impose further constraints.

The paper is structured as follows. In Section 2 we present the factor model setup and a comprehensive treatment of frequency domain identification. In Section 3 we present the design of our empirical analysis, with special focus on our two-stage identification procedure. In Section 4 we present the results. Section 5 concludes.

2.2. Identification in the frequency domain

2.2.1. The Structural Dynamic Factor model

Let x_t be a *n*-dimensional, stationary vector of observable economic variables. The vector x_t is part of an infinite dimensional panel of time series. Each variable x_{it} , i = 1, ..., n, is decomposed into the sum of two mutually orthogonal unobservable components, the common component, χ_{it} , and the idiosyncratic component, ξ_{it} :

$$x_{it} = \chi_{it} + \xi_{it}.\tag{2.1}$$

The idiosyncratic components are interpreted as sources of variation that are specific to one or just a small group of variables, like regional or sectoral shocks, plus measurement error. In particular, for macroeconomic variables like GDP, investment or consumption, in which all local and sectoral shocks have been averaged out, the idiosyncratic part can be interpreted essentially as only containing measurement error. The idiosyncratic components are allowed to be mildly cross-sectionally correlated, thus they have a covariance matrix which is not necessarily diagonal (see Forni et al., 2009, Assumption 5).² The common components, on the contrary, account for the bulk of the co-movements among macroeconomic variables. This is because they are different linear combinations of the same r < n common factors, not depending on i, i.e. they span a r-dimensional vector space (see Stock and Watson, 2002a,b; Bai and Ng, 2002). Then there exist an r-dimensional weakly stationary vector process $F_t = (F_{1t} \dots F_{rt})'$, orthogonal to $\xi_t = (\xi_{1t} \dots \xi_{nt})'$, and loadings λ_{ij} , $j = 1, \dots, r$, such that

$$\chi_{it} = \lambda_{i1}F_{1t} + \ldots + \lambda_{ir}F_{rt} \quad \text{or} \quad \chi_t = \Lambda F_t. \tag{2.2}$$

The unobservable coordinates of F_t are called the static factor and Λ , the factor loading matrix, is of size $n \times r$. We require the factors to be pervasive i.e. to have non-negligible effects on most of the variables x_{it} (see Forni et al., 2009, Assumption 4). Combining (2.1) and (2.2), we get a static equation linking the *n* observable variables x_{it} to the *r* factors and the idiosyncratic components

$$x_{it} = \lambda_{i1}F_{1t} + \ldots + \lambda_{ir}F_{rt} + \xi_{it} \quad \text{or} \quad x_t = \Lambda F_t + \xi_t.$$

$$(2.3)$$

²A factor structure with mildly correlated idiosyncratic components is more realistic than a structure with orthogonal ones. However, in this case common and idiosyncratic component can be disentangled only as $n \to \infty$. This is what characterizes the large approximate dynamic factor model and motivates the assumption of an infinite number of variables. In the traditional dynamic factor model (Sargent and Sims, 1977; Geweke, 1977), on the other hand, the idiosyncratic components are orthogonal to each other; $\xi_t = (\xi_{1t} \dots \xi_{nt})'$ has no cross-sectional dependence, a more restrictive assumption but estimation is possible even if the cross-sectional dimension is finite.

Equation (2.3) is the static factor representation, where the factors have only contemporaneous effect on the common components. The dynamic nature of the model comes from the fact that the static factors F_t follow a VAR(p) driven by a q-dimensional vector of orthonormal structural white noise, or common shocks $u_t = (u_{1t}, \ldots, u_{qt})'$, with $q \leq r$. Precisely:

$$x_t = \Lambda F_t + \xi_t \tag{2.4a}$$

$$C(L)F_t = \epsilon_t \tag{2.4b}$$

$$\epsilon_t = R u_t \tag{2.4c}$$

where ϵ_t is the residual of the VAR on F_t , $E(\epsilon_t \epsilon'_t) = \Sigma_{\epsilon}$, C(L) is an $r \times r$, stable polynomial matrix and R is $r \times q$ and has maximum rank q. As a consequence, R has a left inverse and the vector u_t belongs to the space spanned by F_{t-s} , $s \ge 0$, that is, u_t is fundamental for F_t . By inverting the matrix C(L) we get $F_t = C(L)^{-1}\epsilon_t = C(L)^{-1}Ru_t$, so that the dynamic relationship between u_t and the common components is

$$\chi_t = \left[\Lambda C(L)^{-1} R\right] u_t = B(L) u_t.$$
(2.5)

Then, by merging (2.1) and (2.5), we have the structural dynamic representation

$$x_{it} = b_i(L)u_t + \xi_{it}$$
 or $x_t = B(L)u_t + \xi_t$, (2.6)

where the macroeconomic variables are represented as driven by a few pervasive structural shocks, loaded with the impulse response functions in B(L), plus measurement error. We are interested in the effect of structural shocks on the common components χ_t of some key series, i.e. on the variables obtained by removing measurement errors, so we are neglecting the idiosyncratic components. Notice that representation (2.6) is not unique, since the impulse response functions are not identified. Forni et al. (2009) (Proposition 2), show that identification is achieved up to orthogonal rotations, just like in structural VAR models.

Let us consider the linear mapping in (2.4c), $\epsilon_t = Ru_t$. We define R = SH, where S is the Cholesky factor of Σ_{ϵ} , such that $SS' = \Sigma_{\epsilon}$, and H is an orthonormal matrix, namely a matrix such that $H^{-1} = H'$. We can then rewrite (2.5) as

$$\chi_t = \left[\Lambda C(L)^{-1}S\right] Hu_t = D(L)Hu_t = B(L)u_t$$
(2.7)

where $D(L) = \Lambda C(L)^{-1}S$ encapsulates the Cholesky impulse response functions and B(L) = D(L)H collects the structural IRFs. Then, the effect of the *j*-th structural shock on the *k*-th variable is given by the (k, j) element of the matrix B(L) = D(L)H, that is, the product of the *k*-th row of D(L) and the *j*-th column of *H*. On the

other hand, the structural shocks are related to the VAR residuals by the relation $u_t = R^{-1}\epsilon_t = H'S^{-1}\epsilon_t = H'\eta_t$, η_t being the vector of the Cholesky shocks. Hence the *j*-th structural shock is given by the product of the *j*-th row of H' and η_t . Since we are interested in identifying the shocks, we deal with the choice of H. This is usually done as in standard SVAR analysis, which mainly employs an appropriate number of exclusion or sign restrictions motivated on economic grounds. Here we discuss an alternative approach: shock identification in the frequency domain.³

2.2.2. Frequency band targets

The identification approach is based on the maximization/minimization of the contribution of the structural shock to the variance or the comovements of a set of variables of interest in a given frequency band, which we refer to as *targeted frequency band covariances*. In this subsection we define the objects to be restricted to reach identification. In the two following subsections we show how to implement the identification.

Let us go back to representation (2.7). Letting $\left[\underline{\theta}, \overline{\theta}\right]$ be a band of frequencies such that $0 \leq \underline{\theta} \leq \overline{\theta} \leq \pi$, the comovements between the components of χ_t with period between $2\pi/\overline{\theta}$ and $2\pi/\underline{\theta}$ are measured by the *frequency band covariance matrix*

$$V\left(\underline{\theta},\overline{\theta}\right) = \int_{\underline{\theta}}^{\overline{\theta}} \Re\left(D\left(e^{-i\theta}\right)D\left(e^{i\theta}\right)'\right) \, d\theta$$

where $\Re(z)$ denotes the real part of z.⁴ The matrix $V(\underline{\theta}, \overline{\theta})$ captures the entire frequency band volatility of the variables. The variance (or covariance) contribution of any generic shock $h'\eta_t$, where h is such that h'h = 1, to $V(\underline{\theta}, \overline{\theta})$ is:

$$\Psi\left(\underline{\theta},\overline{\theta}\right) = \int_{\underline{\theta}}^{\overline{\theta}} \Re\left(D\left(e^{-i\theta}\right)hh'D\left(e^{i\theta}\right)'\right) d\theta.$$
(2.8)

Our identification approach consists of imposing restrictions on the contribution of the shock to the elements of the frequency band covariance matrix. The l, k element of $\Psi\left(\underline{\theta}, \overline{\theta}\right)$, is simply $\Psi_{lk}\left(\underline{\theta}, \overline{\theta}\right) = \mathcal{E}_{l}\Psi\left(\underline{\theta}, \overline{\theta}\right)\mathcal{E}'_{k}$ where \mathcal{E}_{l} is the *l*-th row of the *n*-

³This is not the first paper using frequency domain techniques to identify structural shocks —in addition to ACD, let us mention Christiano et al. (2006), Sarno et al. (2007), DiCecio and Owyang (2010), Giannone et al. (2019), Dieppe et al. (2021). It is however, to our knowledge, the first paper providing a comprehensive theory of identification in frequency domain.

⁴The diagonal elements of the spectral density matrix are real while the off-diagonal elements, the cross-spectra, are typically complex, with a real part, called co-spectrum, and an imaginary part. The integral of the co-spectrum of two variables over a given frequency band is the covariance of the two variables over that band, while the integral of the cross-spectrum is the cross covariance.

dimensional identity matrix. Using equation (2.8), we have⁵

$$\Psi_{lk}\left(\underline{\theta},\overline{\theta}\right) = h'\left[\int_{\underline{\theta}}^{\overline{\theta}} \Re\left(D\left(e^{-i\theta}\right)' \mathcal{E}'_{l} \mathcal{E}_{k} D\left(e^{i\theta}\right)\right) d\theta\right] h.$$

This is the objective function to be restricted to reach identification, in the case of a single target. The specification of the objective function can be properly defined for different targets (l, k) and/or frequency band, according to the identification scheme. For instance, if the interval $[\underline{\theta}, \overline{\theta}]$ is the cyclical band, the diagonal element $\Psi_{11}\left(\underline{\theta}, \overline{\theta}\right)$ is the cyclical variance of x_{1t} attributable to the combination $h'\eta_t$. This is the objective function used in ACD to identify the business cycle shock. The off-diagonal term $\Psi_{12}\left(\underline{\theta}, \overline{\theta}\right)$ is the cyclical covariance between variable x_{1t} and x_{2t} attributable to the same shock. In the empirical section below, one of our identification schemes targets the covariance between GDP growth and inflation.

It is also possible to target more than one element of $\Psi\left(\underline{\theta},\overline{\theta}\right)$. This multiple-target approach is a key point to implement the identification strategy used in the empirical section below. Letting $(M_1, N_1), (M_2, N_2), \ldots, (M_m, N_m)$ be the *m* entries of interest, we can target a weighted sum of such entries. For instance, we can take the simple sum of the variances of different variables, or a weighted sum, with weights equal to the reciprocals of the standard deviations (which is equivalent to taking the sum of the variances of the standardized variables). The contribution of the shock $h'\eta_t$ to a weighted sum is given by

$$h'\left[\int_{\underline{\theta}}^{\overline{\theta}} \Re\left(D\left(e^{-i\theta}\right)'\sum_{k=1}^{m} \omega_k \mathcal{E}'_{M_k} \mathcal{E}_{N_k} D\left(e^{-i\theta}\right)\right) d\theta\right] h$$

where ω_k are the weights, to be chosen by the researcher.

Finally, notice that $\sum_{k=1}^{m} \omega_k \mathcal{E}'_{M_k} \mathcal{E}_{N_k} = P'_M \Omega P_N$, where $P_M = \left(\mathcal{E}'_{M_1}, \mathcal{E}'_{M_2}, \dots, \mathcal{E}'_{M_m}\right)'$ and $P_N = \left(\mathcal{E}'_{N_1}, \mathcal{E}'_{N_2}, \dots, \mathcal{E}'_{N_m}\right)'$ are $m \times n$ matrices, and $\Omega = \text{diag}(\omega_1, \omega_2, \dots, \omega_m)$ is a $m \times m$ matrix. Hence the above equation can be re-written as

$$\sum_{k=1}^{m} \omega_k \Psi_{M_k N_k} \left(\underline{\theta}, \overline{\theta}\right) = h' O_{MN} \left(\underline{\theta}, \overline{\theta}\right) h$$
(2.9)

where

$$O_{MN}\left(\underline{\theta},\overline{\theta}\right) = \int_{\underline{\theta}}^{\theta} \Re\left(D\left(e^{i\theta}\right)' P_{M}' \Omega P_{N} D\left(e^{-i\theta}\right)\right) d\theta.$$

This is the objective function of our identification problem, in the case of multiple targets. Of course, this objective function reduces to the single target objective function

⁵To see this, notice that $\mathcal{E}_l D\left(e^{-i\theta}\right) h$ is a scalar so that it is equal to $h' D\left(e^{-i\theta}\right)' \mathcal{E}'_l$. The same reasoning applies to $h' D\left(e^{i\theta}\right)' \mathcal{E}'_k$

in the case m = 1.

An example of multiple-target identification is the cyclical variance of a set of real economic activity variables: one could jointly maximize the cyclical variance of GDP growth and unemployment. Assuming that GDP growth and unemployment are the first two variables in x_t , we have m = 2, $M_1 = N_1 = 1$ and $M_2 = N_2 = 2$,

$$P'_{M} = P'_{N} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{pmatrix}, \quad \Omega = \begin{pmatrix} \omega_{1} & 0 \\ 0 & \omega_{2} \end{pmatrix}.$$

In this case, a reasonable choice for the weights is to take the reciprocals of the cyclical variances of the variables, i.e. $\omega_1 = \frac{1}{V_{11}(\underline{\theta},\overline{\theta})}$ and $\omega_2 = \frac{1}{V_{22}(\underline{\theta},\overline{\theta})}$.

2.2.3. Identification constraints

The identification strategy pursued in this paper is based on quantitative restrictions. Qualitative constraints could also be considered and their implementation is similar to that in the time domain.⁶

Let us assume that the shock of interest is the first one, u_{1t} , and that such shock is the one maximizing $\Psi_{lk}\left(\underline{\theta},\overline{\theta}\right)$, in the case of a single target, or $\sum_{k=1}^{m} \omega_k \Psi_{M_k N_k}\left(\underline{\theta},\overline{\theta}\right)$, in the case of multiple target. In this case h_1 , the first column of the matrix H, is formally given by

$$h_1 = \underset{h \in \mathbb{R}^n}{\operatorname{arg\,max}} h' O_{MN}\left(\underline{\theta}, \overline{\theta}\right) h \qquad \text{s.t.} \quad h'h = 1.$$
(2.10)

It is easily seen that h_1 is equal to the eigenvector associated to the largest eigenvalue of the matrix $O_{MN}(\underline{\theta}, \overline{\theta})$ (Uhlig, 2004), and delivers the shock $u_{1t} = h'_1 \eta_t$. This is a generalization of the approach used in ACD to identify the business cycle shock. In that paper, a single target is used, with k = l, so that the objective function is $\Psi_{ll}(\underline{\theta}, \overline{\theta})$. We can then retrieve the corresponding structural IRFs as

$$B(L) = D(L)h_1 = \left[\Lambda C(L)^{-1}S\right]h_1.$$
 (2.11)

If the researcher is interested in identifying more than one shock, the procedure can be extended to identify multiple shocks sequentially: first, obtain the shock with the largest contribution to the frequency band covariance, then obtain the shock orthogonal

⁶That is, we could draw rotation matrices, or rotation vectors h, and then retain the draws satisfying the desired restrictions on the elements of interest of the frequency band covariance.

to the first, solving another maximization problem, and so on. Suppose, without loss of generality, that the shocks $u_{1t}, u_{2t}, ..., u_{qt}$, have to be identified. The vector h_1 is found according to equation (2.10). The vectors h_j with $1 < j \leq q$ are found solving the following maximization problem:

$$h_{j} = \underset{h \in \mathbb{R}^{n}}{\operatorname{argmax}} h' O_{MN}\left(\underline{\theta}, \overline{\theta}\right) h \qquad \text{s.t.} \quad \begin{cases} h'h = 1, \\ h'h_{\ell} = 0, \quad \ell < j. \end{cases}$$
(2.12)

Notice that the objective function can in principle be appropriately redefined for each shock by changing the targets (M, N) and/or the frequency band $\left[\underline{\theta}, \overline{\theta}\right]$, according to the identification scheme (even if for notational simplicity we avoid to explicit the possible dependence on j of $M, N, \underline{\theta}$ and $\overline{\theta}$).

Here are some examples.

For instance, we could identify the aggregate supply shock as the one maximizing the long run variance of GDP growth and then identify the aggregate demand shock as the shock orthogonal to the supply shock, which maximizes the cyclical variance of GDP growth. In this case, we change the frequency band of interest in the two maximization problems. Another example is the identification of a real and a nominal shock. We could first maximize the variance of GDP growth and then maximize the variance of inflation. In this case, the target would change in the two maximization problems. Moreover, we might be interested in identifying the two main business cycle shocks: first, the shock with the largest contribution to the frequency band covariance, then the shock orthogonal to the first with the second largest contribution. In this case, the target and the frequency band are assumed to be the same for all shocks.

It is also possible to use the sequential procedure just explained to nest two sets of quantitative constraints, i.e. two step procedure, by maximizing the appropriate target functions on the corresponding frequency band. For instance, in the first step, two main shocks are obtained by maximizing the appropriate target function on the band $[0 \ 2\pi/6]$, which excludes fluctuations of less than 18 months, of little interest for macroeconomic analysis. In the second step two structural shocks are found by combining the two shocks obtained in the first step. This is the route we follow in this paper and the specific approach will be discussed below.

Of course, in the above problems, the argmax can be replaced by the argmin. For instance if we want to identify a shock that has only transitory effects on a given variable, the long run variance of such a variable has to be minimized.

2.3. Empirical Approach

2.3.1. Data and estimation procedure

Coming to the empirical application, we use the quarterly dataset for high dimensional macroeconomic analysis recently developed by Granese (2023).

The $N \times T$ dataset is made up of 114 US quarterly series, covering the period 1961-I to 2019-IV. Most series are from the FRED-QD database.⁷ TFP data series are from John Fernald's website (Fernald, 2012) while the Confidence data are available on the Michigan survey of consumer website.⁸ Following standard practice, consumption includes non-durables and services, while investment has been broadly defined to include consumer durables. Both measures are deflated. Monthly data, like the macroeconomic uncertainty measure estimated by Jurado et al. (2015), have been aggregated to get quarterly figures. Finally, it is worth noting that most series are expressed in per capita terms, dividing by population aged 16 years or more (civilian non-institutional population series) and stock market data have been deflated by the GDP deflator. We transform each series to reach stationarity. The complete list of variables and transformations is provided in Appendix (2.B).

The analysis focuses on a subset of 13 macroeconomic series of interest: (1) the log difference of the real per capita GDP; (2) the log difference of real per capita consumption, defined as the sum of non-durable consumption and services; (3) the log difference of real per capita investment, computed as the sum of fixed investment and durable consumption; (4) the unemployment rate, (5) the log of real per capita hours worked; (6) the inflation rate, defined as the log difference of the GDP deflator; (7) labour productivity; (8) the cumulated sum of the utility-adjusted total factor productivity; (9) the Federal Funds rate; (10) the risk spread between Moody's Baa Corporate Bond Yeald and the 10-Year Treasury Constant Maturity Rate; (11) Shiller's real S&P500 stock price index; (12) the measure of macroeconomic uncertainty by Jurado et al. (2015) at the three-month horizon and (13) the Michigan University confidence index component concerning expected business conditions for the next five years (BC5Y).⁹

In order to compute the spectra and the objective function for our maximization

 $^{^{7}}$ The FRED-QD is a large (248 series) quarterly macroeconomic database developed by McCracken and Ng (2020).

⁸http://www.sca.isr.umich.edu/

⁹BC5Y summarizes responses to the following forward-looking question: "Turning to economic conditions in the country as a whole, do you expect that over the next five years we will have mostly good times, or periods of widespread unemployment and depression, or what?". The anticipation properties of this variable on future movements in economic activity in general and TFP in particular are widely discussed in Barsky and Sims (2012) and Beaudry and Portier (2006).

problems we proceed as follows. We estimate the first two equations (2.4a)-(2.4b) using the two step estimation technique discussed in Forni et al. (2009), which we briefly review here.

FIRST STEP. We set a value for the number r of the static factors, using the criterion by Bai and Ng (2002) with the penalty modification proposed in Alessi et al. (2010), finding a number of static factors $\hat{r} = 11$.¹⁰ The static factors $F_t = (F_{1t} \dots F_{rt})'$ are estimated by the first \hat{r} principal components of the variables in our dataset, and the factor loadings, λ_{ij} , $j = 1 \dots r$, by the associated eigenvectors. Thus, the estimated loading matrix, $\hat{\Lambda}$, is the $n \times \hat{r}$ matrix having on the columns the normalized eigenvectors corresponding to the \hat{r} -largest eigenvalues of the sample covariance matrix of the data, $\hat{\Sigma}_x$. The estimated common component vector is given by $\hat{\chi}_t = \hat{\Lambda}\hat{F}_t$.

SECOND STEP. We run a VAR(p) for the estimated factors \hat{F}_t to get estimates $\hat{C}(L)$ and $\hat{\epsilon}_t$ of C(L) and the VAR innovations ϵ_t . The estimated Moving Average representation is $\hat{F}_t = \hat{C}(L)^{-1}\hat{\epsilon}_t$. The number of lags p is determined according to the BIC criterion ($\hat{p}_{BIC} = 1$). In the robustness section we repeat the analysis with different lags order. To orthogonalize the shocks we use the Cholesky factor \hat{S} of $\hat{\Sigma}_{\epsilon}$. Therefore, the Cholesky IRFs of the common components are obtained according to (2.7) as

$$\hat{D}(L) = \hat{\Lambda}[\hat{C}(L)^{-1}\hat{S}].$$

From this matrix we estimate the spectral density of the common components at the Fourier frequencies $\theta = 2\pi s/T$, $s = 1, \ldots, T$, and take the real part, so that the resulting off-diagonal terms are co-spectra rather than cross-spectra. This is useful when we take an off-diagonal term as a target, since the integral of the co-spectrum of two variables over a given frequency band is the covariance of the two variables over that band. Finally, we compute $V\left(\underline{\theta}, \overline{\theta}\right)$ by replacing the integral with the simple average of the real part of the spectral density matrix, across the frequencies belonging to the relevant interval. $\Psi(\underline{\theta}, \overline{\theta})$ and $O_{MN}\left(\underline{\theta}, \overline{\theta}\right)$ are estimated in a similar way.

We do not apply the rank reduction step (see the on-line Appendix 2.A) as this will be part of the identification strategy discussed below.

To conclude this section, let us look at the common-idiosyncratic variance decomposition of the key variables above with $\hat{r} = 11$ static factors, shown in Table 2.1. The common variance of the main macroeconomic aggregates like GDP, consumption, investment and unemployment rate are 94, 82, 90 and 94 percent of total variance, respectively. These numbers seem compatible with the measurement error interpretation of the idiosyncratic components.

¹⁰In the robustness section, we take into account the uncertainty in estimating the number of static factors, and repeat the analysis with different specifications of \hat{r} .

2.3.2. Identification: A two-step procedure

Aim of this work is to provide a global and parsimonious description of the main forces driving the macroeconomy overall, at both cyclical and long run frequencies. There are two main questions we want to address. First, how many shocks are needed to explain the bulk of fluctuations in the main macroeconomic aggregates? Second, what are they and what are their effects? To address these two questions we develop a two-step strategy based on the econometric theory presented in the previous section.

FIRST STEP. First of all, we find the q shocks which explain the bulk of cyclical and long run variance of the main macroeconomic aggregates, both real and nominal. To do this, we solve maximization problems (2.10) and (2.12) with a multiple target and in the frequency interval $[\underline{\theta} \ \overline{\theta}] = [0 \ 2\pi/6]$ (the trend-cycle band henceforth), which corresponds to periodicities greater than 18 months, thus excluding high frequency fluctuations of less than 18 months, of little interest for macroeconomic analysis.¹¹ More specifically we include in the target the variances of the growth rates for trended real activity variables (i.e. GDP, consumption, investment, TFP, labour productivity) as well as the variances of other real and nominal variables (i.e. unemployment rate, hours worked, inflation rate, Federal Funds Rate and S&P500 stock price index). The weights are given by the reciprocals of the (frequency band) variances of the variables, computed as the average of the spectral densities in the relevant frequency interval. Let us set M_1 and N_1 equal to the position of GDP in the data set, M_2 and N_2 equal to the position of consuption, etc., and call q_j , for j = 1, ..., q, the q vectors solving the maximization problem $g_j = \arg \max g' O_{MN}\left(\underline{\theta}, \overline{\theta}\right) g$ subject to g'g = 1 and $g'g_l = 0$ for l < j; we obtain a matrix $G = [g_1 \ g_2 \dots g_q]$ of dimension $r \times q$. We show below that two shocks are enough to explain the bulk of cyclical and long run fluctuations in the main macroeconomic aggregates.

SECOND STEP. The shocks $g'_1\eta_t, ..., g'_q\eta_t$ lack of any economic interpretation: they are simply the largest contributors to the frequency band variances ordered in decreasing order of importance. We therefore move on to the second step and identify two structural shocks. We use two identification schemes.

IDENTIFICATION I. We identify a demand shock and a supply shock using a novel approach. The demand shock is obtained by maximizing the covariance of GDP growth and the inflation rate at business cycle frequencies. The supply shock is automatically identified by the orthogonality condition as the shock minimizing such covariance. This identification scheme is related to the one recently used

¹¹The band [0 $2\pi/6$] includes: business-cycle frequencies, [$2\pi/32 \ 2\pi/6$], corresponding to cycles between 18 months and 8 years, long cycles, [$2\pi/80 \ 2\pi/32$), which includes waves ranging from 8 and 20 years, and the long run, [0 $2\pi/80$), corresponding to cycles of 20 years or more, with quarterly data.

by Furlanetto et al. (2021), in that the demand shock is defined on the basis of the comovements of output and inflation and can in principle affect output in the long run.¹²

IDENTIFICATION II. We identify a permanent and a transitory shock. The permanent shock is identified as the one that explains most of the long run variance¹³ of trending real activity variables, i.e. GDP growth, TFP, consumption growth, investment growth and labor productivity. The transitory shock is automatically identified by the orthogonality condition as the one minimizing the explained long run variance of the above variables. The effects on cyclical variance are left unrestricted, so that the two shocks can explain whatever fraction of business cycle fluctuation in the real activity variables, as well as the cyclical volatility of inflation and interest rate.

To impose the identifying restrictions in the second step we solve a problem very similar to the one of equation (2.10). The only difference is that now we rotate just the q = 2 main shocks obtained from the first step rather than the \hat{r} Cholesky shocks. Formally, let $G = [g_1 \ g_2]$ and consider the $n \times q$ matrix $D^*(L) = D(L)G$. We combine the columns of $D^*(L)$ and the shocks $G'\eta_t$ by solving the following maximization problem:

$$h_{1}^{*} = \operatorname{argmax}_{h^{*} \in \mathbb{R}^{q}} h^{*'} O_{MN}^{*} \left(\underline{\theta}, \overline{\theta}\right) h^{*} \quad \text{s.t.} \quad h^{*'} h^{*} = 1$$

$$O_{MN}^{*} \left(\underline{\theta}, \overline{\theta}\right) = \int_{\underline{\theta}}^{\overline{\theta}} \Re \left(D^{*} \left(e^{-i\theta}\right)' P_{M}' \Omega P_{N} D^{*} \left(e^{-i\theta}\right) \right) d\theta \qquad (2.13)$$

where now h^* and h_1^* are 2-dimensional orthonormal vectors. In a context with two structural shocks, the solution to (2.13) is enough to identify simultaneously both $h_1 = Gh_1^*$ and, similarly, $h_2 = Gh_2^*$ since the vector h_2^* is pinned down by the orthogonality restrictions. The structural impulse-response function are the entries of B(L) = D(L)H, where $H = [h_1 h_2]$ and the structural shocks are $u_t = H'\eta_t$. For the two identifications, the specification of the objective function is the following:

IDENTIFICATION I: the frequency interval is $[\underline{\theta} \ \overline{\theta}] = [2\pi/32 \ 2\pi/6]$. *M* is the position of GDP and *N* the position of inflation in vector x_t .

IDENTIFICATION II: the frequency interval is $[\underline{\theta} \ \overline{\theta}] = [0 \ 2\pi/80]$, and M = N is the vector whose elements are the positions of the real variables in the vector x_t .

 $^{^{12}\}mathrm{Note}$ that unlike our identification scheme, the one used by Furlanetto et al. (2021) is implemented in the time domain.

¹³The long run is defined as frequencies in the interval $[0 \ 2\pi/80)$, corresponding to cycles of 20 years or more.

2.4. Results

2.4.1. Two shocks

As explained above, in the first step of our procedure we select the two shocks maximizing the explained variance of the main macroeconomic variables on the trend-cycle band, that is, on a frequency band that includes all the frequencies of main interest for macroeconomic analysis. Table 2.2 reports, for each variable, the percentage of variance jointly explained by the two shocks on the whole trend-cycle band, on the business-cycle frequencies and on the long run, along with the variance explained by the shock with the third largest contribution. The aim is to see how large is the explained variance when only two shocks are selected and how large is the variance we lose with respect to the specification with three shocks.

The percentage of cyclical variance jointly explained by the two shocks is about 76 for real per capita GDP growth, 70 for consumption, about 79 for investment and unemployment rate. We also see that two shocks are enough to capture about 86% of cyclical inflation fluctuations, about 76% of the federal funds rate and more than 82% of the risk spread, the JLN uncertainty measure and BC5Y. We conclude that two shocks are enough to provide an accurate description of the business cycle fluctuations in both real and nominal variables.

Turning to the long run, we see that the percentage of variance jointly explained by the two shocks is 81 for real per capita GDP growth, 82 for unemployment rate, about 76 for consumption and about 66 for TFP. Two shocks account for about 85% of inflation fluctuations, 86% of the FFR and risk spread, and about 91% of uncertainty. Thus, two shocks not only account for the bulk of business cycles fluctuations, but also explain the long run.

The variance that we lose by selecting two shock instead of three is negligible for almost all variables, so the third shock is not large or pervasive enough to be considered as a main driver of the US economy. The third shock capture essentially the cyclical fluctuations of TFP, which are of little interest for our analysis, because we are mainly interested in the long-run fluctuations of TFP.

All in all, our findings depict a picture of the US macroeconomy where two shocks provide a complete and parsimonious characterization at both cyclical and long run frequencies. This is in line with existing factor model evidence. As pointed out in the introduction, Onatski (2009), using his test for the number of shocks in a large dynamic factor model, cannot reject the null that there are 2 shocks against the alternative that there are from 3 to 7. ACFZ propose a new consistent estimator for the number of shocks, the "Dynamic eigenvalue Difference Ratio estimator" (DDR), that can be applied to single frequencies as well as to frequency bands, and finds that the US macroeconomy is well described by two major shocks. These results are in line with the evidence provided in papers such as Sargent and Sims (1977) and Giannone et al. (2005). To further corroborate our results, we apply the DDR estimator to our dataset on the whole interval $[0 \ \pi]$ and on the trend-cycle band. The criterion selects two shocks on both bands.¹⁴

2.4.2. Identification I: explained cyclical and long run variances

Table 2.3 presents the results for Identification I, where we identify a supply and a demand shock based on the cyclical covariance between the inflation rate and per capita GDP growth. The table reports the cyclical and long run variances explained by the identified shocks. Notice that under this identification scheme both the long run and cyclical variance contributions are left unrestricted. Thus, we can verify whether the supply shock is permanent or not and whether the demand shock is transitory or not.

A first key result is that the demand shock explains a negligible fraction of the long run variance of trending real activity variables. It account for about 3% of GDP growth, less than 9% of consumption and hours worked, about 5% of investment, 11% of unemployment and less than 1% of TFP. Hence, unlike Furlanetto et al. (2021), we do not find evidence of hysteresis effects on output and labor market. On the other hand, our demand shock explains most of the long run variance in the inflation rate (about 65%) and the federal funds rate (about 84%).

The supply shock explains the bulk of the long run variance of real activity variables. It explains 78% of output growth, about 70% of consumption, investment and unemployment, and 55% of hours worked. Note that the percentage of TFP long run variance explained by the supply shock is about 65%, in line with the view that supply shocks include an important technological component.

Turning to the explained variances at business cycle frequencies, we see that the demand shock is the main source of cyclical fluctuations in output growth. It accounts for about 49% of GDP fluctuations. Still, the supply shock explains a sizable fraction of GDP cyclical variance, about 27%. As for inflation fluctuations, both demand and supply shocks explain an important part of cyclical variance. The former captures about 44% while the latter explains 42%.

An interesting result emerges when comparing the importance of the two shocks for GDP, consumption, investment, unemployment and hours worked. The supply shock is dominant for consumption. It accounts for about 41% of business cycle fluctuations, whereas the demand shock explains less than 30%. This result can easily be explained

¹⁴To compute the DDR estimator, we set the bandwidth parameter $M_T = \lfloor a\sqrt{T} \rfloor$ with a = 0.5.

in the light of permanent income theory: consumption is largely driven by permanent income, and permanent shocks have larger effects on permanent income than transitory shocks (Quah, 1990).¹⁵

The demand shock is also dominant for unemployment and investment. The cyclical variance of unemployment explained by our demand shock is about 50%, whereas the variance due to the supply shock is 29%. This result is in line with the evidence in Blanchard and Quah (1989), where the aggregate demand shock, the transitory one, plays a major role for unemployment fluctuations. As for investment, the demand shock accounts for about 55% of the cyclical variance, whereas the permanent shock accounts for only 24%. A possible explanation is that private investment is closely related to credit market conditions, which in turn are largely driven by demand. Indeed the demand shock explains almost all cyclical variance of the risk spread – about 77%, as against a scanty 11% explained by the supply shock. These numbers suggest that our demand shock is to a large extent a credit shock.

A few additional observations are in order. First, the forward-looking measure of consumer confidence (BC5Y) is mostly explained by the supply shock, both at business cycle frequencies and in the long run. This finding seems consistent with Barsky and Sims (2012) and with the "news" interpretation of confidence indicators: consumer confidence is likely to reflect information about future productivity rather than animal spirits.

Second, the federal funds rate is explained almost exclusively by the demand shock, both at cyclical frequencies and in the long run. This is consistent with the idea that monetary policy follows a systematic rule according to which the nominal rate reacts positively to current inflation and real activity changes, in order to stabilize cyclical fluctuations. Supply shocks induce negative comovements of inflation and GDP growth, so that monetary policy react weakly to them.

Finally, both demand and supply have a sizable role in explaining JLN uncertainty at cyclical frequencies. Demand shocks explain 46% while supply shocks explain about 37%. If we interpret exogenous uncertainty shocks as demand shocks, we are left with a lower bound of approximately 40% of endogenous uncertainty fluctuations, induced by non-uncertainty shocks (that is, supply shocks and other demand-side shocks, such as credit or monetary policy shocks). Therefore, JLN macroeconomic uncertainty can be considered endogenous to a considerable extent. This finding is broadly consistent with Ludvigson et al. (2021).

Figure A.1 summarizes the above findings by reporting the variance decomposition

¹⁵Micro evidence suggests that individual choices of consumption and saving may differ from the predictions of the permanent income theory. In particular, theories of liquidity-constrained households are supported by empirical evidence. However, this does not preclude that at the aggregate level consumption largely follows expectations about future income that are mainly driven by permanent shocks.

for the variables of interest. The figure reports the percentage of explained variance of each shock, frequency by frequency. The pink area is the long run frequency band, the lilac area is the business cycle frequency band. The blue line refers to the permanent shock and the red line to the transitory shock. The yellow line is the sum of the two.

The figure also provides additional information about the "long cycles" frequency band, i.e. fluctuations of periodicity between eight and twenty years that fall in the white area between the long run and the business cycle frequency bands. The upperleft panel refers to GDP growth: long cycles are explained almost exclusively by the supply shock. The same result applies to all real activity variables but unemployment. It follows that if the business cycle were defined by including longer cycles, e.g. cycles with periodicity between 6 and 50 quarters as suggested by Beaudry et al. (2020), the importance of the supply shock in explaining real activity fluctuations would increase.¹⁶

2.4.3. Identification I: impulse response functions

Turning to the impulse response functions, Figure A.2 overlaps the responses to the supply shock of Identification I and the permanent shock of Identification II, whereas Figure A.3 overlaps the responses to the demand shock of Identification I and the transitory shock of Identification II. The solid black lines are the point estimates for Identification II and the dark and light gray areas are the 68% and 90% confidence band, respectively, relative to Identification I.¹⁷

Let us now focus on responses to the supply shock, Identification I (black lines, Figure A.2). The shock has a large positive permanent effect on GDP and its components and generates a temporary hump-shaped response of unemployment and hours worked. GDP increases immediately by around 0.2%, peaks around the 10th quarter and converges to 1.2% in the long run. The effect on consumption appears to be slightly larger and persistent, reaching a maximum of about 2%. Unemployment behaves counter-cyclically and reaches a minimum of about -0.2% around the 8th quarter. The supply shock generates a negative comovement between inflation rate and output growth. The former immediately falls by around -0.2% and the effect is relatively short lived. The response of stock prices is positive and persistent, peaking at 0.9 percent, while the risk premium, after a nearly zero impact effect, decreases with a temporary hump-shape, reaching a minimum of about -0.14%.

A few additional observations are in order. First, we see that systematic monetary

¹⁶Beaudry et al. (2020) show that many macroeconomic aggregates appear to have a peak in their spectral densities at periodicities between 32 and 50 quarters and that the implied movements coincide with NBER cycle dating. For this reason, they argue that the definition of the business cycle should be modified accordingly.

¹⁷The IRFs of Identification II with their confidence bands are reported in Appendix 2.C.

policy, as proxied by the federal funds rate, reacts negatively to the supply shock on impact, with an insignificant response after about one year. This suggests that systematic policy reacts more to inflation than real activity. However, the effect of the unit variance supply shock is really small, the maximum being about 10 basis points, as against the 21 basis points of the demand shock (black line, Figure A.3). Second, the response of TFP to the supply shock has an S shape which resembles the one typically found for the news technology shock, with a relatively small impact effect (about 0.4) and a much larger long run effect (about 1.2). This suggests that the supply shock includes an important news shock component as in Beaudry and Portier (2006). The significant positive impact effect of the supply shock on the consumer confidence component BC5Y, documented above, is in line with this interpretation, given the anticipation properties of this variable about future technology. Finally, JLN uncertainty decreases immediately in response to positive supply shocks, with a maximum effect at horizon one of about -0.25%. These movements in macro uncertainty persist for about two years after the shock.

Figure A.3 reports the impulse response functions to the demand shock, Identification I (solid black lines). The responses of real economic activity variables are temporary and hump-shaped, peaking at horizon 3 or 4 (one year after the shock). The effects are no longer statistically significant after about 2-3 years. GDP has a positive impact effect of 0.4% and a peak of about 0.8%. Unemployment falls at a minimum of around -0.2%, then shows a significant and short lived rebound effect between the 12th and the 20th quarter, with a peak of about 0.1%. Investment shows a similar, albeit less pronounced and not significant rebound effect.

The response of inflation and the interest rate are very similar, in terms of both shape and magnitude. The former increases on impact by about 0.15%, peaks at 0.2%and converges to zero afterward. The effect appears to be more persistent than that of the permanent shock. The interest rate increases in a hump-shaped pattern, reaching a maximum of about 0.23%. As noted above, this suggests a very active behavior of monetary policy, consistent with standard Taylor rules, implying a systematic policy reaction to inflation and output. As expected, TFP essentially does not react to the unit variance demand shock, the effect being not significant at all horizon. For stock prices the effect is positive but very short lived, being significant only on impact (about 0.5%). Thus, the stock market reacts more to supply shocks than demand shocks. The effects on the risk premium are much larger and short lived for demand shocks than for supply shocks. The shape of the impulse response function of the risk premium, with a maximum effect on impact and at lag 1 (about -0.35%), closely resembles the one of the excess bond premium obtained in Gilchrist and Zakrajšek (2012). This again suggests that shocks related to credit and financial conditions are an important component of the demand shock.

2.4.4. Identification II

Let us now turn to Identification II, where we identify a permanent and a transitory shock on real variables. Here the co-spectrum of inflation and GDP growth is left unrestricted, so that, looking at the impulse-response functions, we can verify whether the permanent shock is a supply shock and the transitory shock is a demand shock.

More importantly, the two identification schemes provide very similar outcomes. The matching is really striking: the correlation of the demand (supply) shock of identification I and the transitory (permanent) shock of Identification II is higher than 0.99.

Table 2.4 presents results for the variance decomposition. Notice first that Identification II is successful in isolating a transitory shock. Indeed, the percentage of GDP growth, consumption and TFP long run fluctuations accounted for by the transitory shock is negligible (1.7, 5.9 and 1.6% respectively). The variance decompositions in the table are very similar to the ones of Identification I. Once again, both shocks are important sources of business cycle fluctuations in real economic activity. The permanent shock is more important for consumption, while the transitory shock is more important for output growth, unemployment and investment. Concerning inflation, both transitory and permanent shocks explain a large percentage of cyclical fluctuations. In particular, the transitory shock is not disconnected from inflation, in that it accounts for about 49% of cyclical variance, contrary to what found in ACD. This result is not at all implied by our identification.

Turning to the impulse response functions, Figure A.2 and Figure A.3 compare results of Identification II (cyan dashed lines) with those of Identification I (solid black lines). The correspondence between the two identification schemes is striking. The key message is that our expansionary transitory shock raises inflation, whereas our expansionary permanent shock reduces inflation, in line with New Keynesian textbook models and thus supporting the traditional view.

2.4.5. Discussion

The general picture emerging from our empirical analysis is the following. US data are consistent with a view of the macroeconomy as driven by two main shocks: a deflationary supply shock having long-lasting effects on real economic activity and an inflationary demand shock having only transitory effects. Both shocks explain a sizable part of business-cycle fluctuations.

This picture is clearly incompatible with the standard RBC model and largely in line with BQ, where transitory shocks are found important in explaining the businesscycle fluctuations of economic activity. Our findings are also incompatible with the view put forward by Beaudry and Portier (2006) that news shocks capture the bulk of cyclical fluctuations in real activity. Rather, they are consistent with Barsky and Sims (2011) and Forni et al. (2014), where the news technology shock explains a minority, albeit sizable, part of business cycle fluctuations.

Our evidence, far from being at odds with the partial identification literature, provides evidence in favor of some of the studies cited in the Introduction. In particular, the response of TFP to the supply shock has an S shape which resembles the one typically found for the news technology shock (Beaudry and Portier, 2006), suggesting that news shocks are the dominant component of supply shocks. Moreover, the explained variance and the shape of the impulse response function of the risk premium to the demand shock are very much similar to the ones found in the credit shock literature (Gilchrist and Zakrajšek, 2012) consistently with the idea that credit shocks are the dominant component of demand shocks (even if they could include an exogenous uncertainty component).

As already observed, our results are partially at odds with the picture emerging from ACD. The finding that the bulk of cyclical fluctuations are not driven by a permanent shock is in line with ACD: the demand shock is the most important business cycle shock for output growth and is largely disconnected from the long run of real economic activity. On the other hand, the ACD's hypothesis that most of the business cycle fluctuations of real activity can be explained by just one shock, a non-inflationary demand shock affecting all real activity variables with the same dynamics, is rejected here: our supply shock explains a sizable part of cyclical fluctuations and is the main business-cycle driver for consumption, suggesting that at least two shocks are needed to explain the bulk of cyclical fluctuations in real economic activity variables. This important point is studied in detail in Granese (2023). Moreover, the demand shock is not disconnected from inflation at both cyclical and long run frequencies. These last two results are broadly in line with ACFZ.

2.4.6. ACD UNDER THE MICROSCOPE: INFLATION AND THE LONG-RUN

While the results discussed thus far exclude the existence of such thing as a main business cycle shock explaining most of the business cycle fluctuations of the real activity, it is still unclear whether our data supports a representation in which a shock presents the distinctive feature of ACD's MBC: the contemporaneous disconnection from both inflation and long-run real economic activity. Is there an orthogonalization of our two shocks such that both disconnections hold?

Let us start from the results of Identification II. Clearly, the temporary shock is, by construction, disconnected from the long-run real economic activity. Yet it is far from being disconnected from the inflation (see Table 2.4). So, imposing the longrun disconnection yields a shock with no inflation disconnection. What about the Finally, let us see what happens if we apply to our two shocks the same identification imposed in ACD, that is, maximizing the cyclical variance of a single real economic activity variable (here we use GDP growth). This identification yields a shock which is partially disconnected from the inflation but again not disconnected from the long-run — it explains 13.6% of the cyclical variance of inflation and 34.4% of GDP growth's long-run variance (see Table 2.5, Panel B).

(see Table 2.5, Panel A). Hence, imposing the inflation disconnection yields a shock

More details are found in Appendix C (Figure C.6) where we explore all possible linear combinations of our two shocks: given all rotation angles, we show that a shock explaining less than 20% of both cyclical inflation and long-run GDP cannot be obtained.

Why our result are different from ACD? A possible explanation is the following. It is well-known that, while large dimensional factor models are generally unaffected by non-invertibility issues, VAR systems could be *informationally deficient*. Granese (2023) investigates whether the 10-variable VAR considered by ACD contains enough information to recover the MBC shock obtained by targeting the unemployment rate. To do so, the author uses the invertibility test of Forni and Gambetti (2014), which tests for the orthogonality of the estimated shock with respect to the past of the principal components of a large macroeconomic dataset (the author uses the same data set used here). He finds that informational sufficiency is rejected, since the MBC shock is predicted by the lags of the principal components (the p-values are reported in Table C.2, online Appendix 2.C). In other words, the causal interpretation of ACD's MBC shock is untenable.

2.4.7. ROBUSTNESS

with no long-run disconnection.

In this subsection we conduct a few robustness exercises for Identification I. Robustness results for Identification II are similar and are reported in Appendix 2.C.

First, we test robustness to the inclusion of additional lags with respect to the one lag baseline specification. We estimate the model with two, three (as suggested by the AIC) and four lags, respectively. Table 2.6 reports the cyclical (top panel) and long run (bottom panel) variances accounted for by the identified supply and demand shocks. The first two columns correspond to our baseline specification, p = 1, while the remaining are for the alternative specifications, p = 2, 3, 4. In addition, Panel (a) of Table 2.8 summarizes the above findings by reporting, for each variable and shock, the maximum and minimum shares of explained variance, as the lag order changes.

As for the business cycle, baseline results appear to be quite robust with respect to changes in specification. The GDP growth variance explained by the supply shock ranges from a minimum of 27% (baseline) to a maximum of 30% (4 lags specification), while for the demand ranges from 47% (3 lags) to 51% (4 lags). The investment variance explained by the supply shock ranges from a minimum of 24% (baseline) to a maximum of 34% (4 lags specification), while for the demand ranges from 49% (4 lags) to 55% (baseline). The finding that consumption fluctuations are mostly explained by supply shocks is a fully robust result. In the 3 lags specification, it explains 51% of the consumption cyclical variance, while only 20% is explained by the demand shock, a difference of 31 percentage points. All in all, the demand shock is still the most important cyclical shock for real activity, but the increase in the number of lags seems to enhance the cyclical footprint of the supply shock, reinforcing our view that the business cycle is driven by two main shocks.

The only sensitivity analysis worth noting is the following. As lags increase, the demand shock appears less tightly connected, in terms of variance contributions, to inflation fluctuations. The cyclical variance explained by the demand shock ranges between a minimum of 17% (4 lags specification) to a maximum of 44% (baseline) while for the supply shock it ranges from 42% (baseline) to 63% (4 lags). The demand shock is partially disconnected from inflation only in the 4 lags specification in which, however, it accounts for 17% of inflation, as against the 7% found in ACD. For the transitory shock of Identification II, the percentage of explained variance of inflation is somewhat more robust across lag specifications, ranging between 29 and 49% (see Appendix 2.C).¹⁸

Turning to the long run, the variance decomposition displays figures fairly close to the baseline for most of the variables. For example, the output growth long run variance explained by the supply shock varies from 67% (4 lags) to 78% (baseline), while for the demand shock ranges from about 3% (3 lags and baseline cases) to 11% (4 lags). The main conclusions about the long run contribution of the two shocks are confirmed, except one: the finding that demand shock explains most of the long run fluctuations in inflation (64% vs. 20% of the supply shock) is not robust: for the 2, 3 and 4 lags specifications, demand explains 36, 21 and 13% percent, respectively, while supply explains 34, 26 and 36%.

Figures A.4 and A.5 display the impulse response functions to the supply and the demand shocks, respectively, for different lag specifications. The solid black lines (point estimates) and confidence bands are those obtained in the baseline exercise. All

¹⁸As suggested by a referee, in light of recent literature (Barnichon and Mesters, 2020) the divorce of demand shocks and inflation could be more pronounced in recent subsamples.

in all, the dynamic responses to supply shocks are similar to those obtained in the baseline exercise, most of them lying within the baseline confidence bands. As for the demand shock, the magnitude of responses is slightly smaller only for inflation and interest rate, with similar shapes.

Finally, we check the robustness of the results as the number of static factors increases. In particular, we compare the results of our baseline specification (r = 11)with four alternatives: r = 13, 15, 17, 20. Table 2.7 reports the cyclical (top panel) and long run (bottom panel) variances accounted for by the identified supply and demand shocks. As the number of static factors changes, the contribution of the identified shocks to the cyclical and long run variances of the main macroeconomic variables does not change much. As in the previous exercise, panel (b) of Table 2.8 summarizes the above findings by reporting, for each variable and shock, the maximum and minimum shares of explained variance obtained as the factor specification changes. For example, the percentage of cyclical variance explained by the demand shock varies between 49 and 52 for GDP, depending on the specification of r, 25 and 29 for consumption, 53 and 55 for investment, and so on. The results become slightly sensitive only when the number of static factors becomes very large with respect to the benchmark. For example, the consumption cyclical variance explained by the supply shock ranges between a minimum of about 26% (r = 17 and r = 20) to a maximum of 41% (baseline case): when r = 17 and r = 20, supply is no longer dominant for consumption, although demand alone still cannot explain most of the cyclical fluctuations.

The same robustness is found when considering the IRFs to the supply and the demand shocks obtained in this exercise and reported in Appendix 2.C. The responses are very much similar to the baseline. All in all the results are fairly robust to different specifications.

2.5. Summary and conclusions

In this paper we provide a comprehensive and stylized description of the U.S. macroeonomy and investigate whether the traditional view has support in the data. The evidence shows that this is the case.

The result is obtained assuming that data follow a Structural Dynamic Factor Model and using a novel identification technique in the frequency domain. Our identification strategy unfolds in two steps. In the first step, we select the two shocks with the largest contribution to the cyclical and long run variance of the main real and nominal macroeconomic variables. We show that adding a third shock would only marginally increase the explained variance. In the second step, we rotate the two main shocks in order to give them an economic interpretation. We implement two different

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identification schemes: in the first one we define a demand and a supply with a completely novel criterion based on the covariance between inflation and output, while in the second scheme we define a permanent shock on real activity and a transitory one in a way that is very close to BQ.

The two identification schemes provide strikingly similar outcomes in terms of both variance decomposition and impulse response functions. The US macroeconomy is driven by two main forces: a supply shock, which is permanent and generates a negative comovement between prices and quantities, and a demand shock, which is transitory and generates a positive comovement between prices and quantities. We show empirically that demand shocks have only transitory effect on real economic activity. Both demand and supply are important sources of business cycle fluctuations. The demand shock is closely related to credit market conditions and is the main business-cycle shock for output, investment and unemployment, while the supply shock is to a large extent a news technology shock and is the main business cycle shock for private consumption. Finally, supply shocks not only account for almost all the long run fluctuations of real activity, but also for long cycles (between 8 and 20 years).

All in all, the evidence strongly support the very standard view of the macroeconomy where fluctuations in real economic activity and prices arise from shifts in the aggregate demand and aggregate supply curves. From our perspective, theory should look at the U.S. macroeconomy through the lens of a two-shock, New Keynesian textbook framework, in order to be consistent with the data.

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TABLES

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VARIABLES	χ	ξ
GDP	94.33	5.67
Consumption	81.62	18.38
Investment	89.54	10.46
Unemployment Rate	94.17	5.83
Hours Worked	83.53	16.47
Inflation	90.47	9.53
Labor Productivity	89.31	10.69
TFP	80.91	19.09
\mathbf{FFR}	97.92	2.08
Baa-GS10 Spread	78.05	21.95
S&P500	94.47	5.53
JLN Uncertainty 3M	83.81	16.19
BC5Y	75.87	24.13

Table 2.1: Percentage of the variance explained by the estimated common and idiosyncratic components of selected variables. Baseline specification: r = 11 static factors. We run the test proposed by Alessi et al. (2010).

VARIABLES	TREND-CYC	LE BAND	Cyclical	BAND	Long Run band		
	First two	THIRD	First two	THIRD	First two	THIRD	
GDP	77.9	1.9	76.2	2.0	81.0	0.7	
Consumption	70.8	1.0	69.7	0.6	75.6	1.6	
Investment	79.9	0.5	78.9	0.6	72.3	0.2	
Unemployment Rate	83.7	3.9	78.5	1.6	82.0	7.3	
Hours Worked	65.3	14.6	58.1	12.6	63.5	16.6	
Inflation	85.5	6.3	86.1	7.2	85.4	5.8	
Labor Productivity	47.3	30.8	46.9	31.0	63.4	10.8	
TFP	31.6	54.0	27.4	58.0	66.1	20.0	
FFR	83.8	1.1	75.5	3.6	85.9	0.3	
Baa-GS10 spread	85.0	0.8	87.8	0.3	86.1	1.0	
S&P 500 real	55.0	2.0	57.1	1.3	30.9	6.0	
JLM uncertainty	85.4	1.2	82.9	1.3	91.8	2.0	
BC5Y	85.5	6.8	89.1	2.4	83.4	9.2	

Table 2.2: Percentage of variance explained by the first two main shocks and by the third for a few selected variables, by frequency band. Business cycle frequency band: $[2\pi/32 \le \omega \le 2\pi/6]$ corresponding to cycles with periodicity between 18 months and 8 years. Long run frequency band: $[0 \le \omega \le 2\pi/80]$, corresponding to periodicity greater than 20 years, with quarterly data.

VARIABLES	Cycli	CAL VARIAN	ICE	Long Run variance			
VIIIIIIIIII	SUPPLY	Demand	Sum	Supply	Demand	Sum	
GDP	27.1	49.1	76.2	77.7	3.3	81.0	
Consumption	40.6	29.2	69.7	66.9	8.7	75.6	
Investment	23.6	55.3	78.9	67.8	4.5	72.3	
Unemployment Rate	29.0	49.5	78.5	70.9	11.0	82.0	
Hours Worked	26.3	31.9	58.1	54.7	8.8	63.5	
Inflation	41.8	44.3	86.1	20.0	65.4	85.4	
Labor Productivity	22.5	24.4	46.9	60.1	3.3	63.4	
TFP	21.0	6.4	27.4	65.2	0.9	66.1	
FFR	13.3	62.2	75.5	2.3	83.6	85.9	
Baa-GS10	10.8	77.0	87.8	44.0	42.1	86.1	
S&P500	33.3	23.8	57.1	30.4	0.5	30.9	
JLN Uncertainty 3M	37.4	45.5	82.9	54.5	37.3	91.8	
BC5Y	69.1	20.1	89.1	74.8	8.6	83.4	

Table 2.3: Identification I. Percentage of variance explained by the supply (deflationary) shock and the demand (inflationary) shock for a few selected variables, by frequency band. Business cycle frequency band: $[2\pi/32 \le \omega \le 2\pi/6]$ corresponding to cycles with periodicity between 18 months and 8 years. Long run frequency band: $[0 \le \omega \le 2\pi/80]$, corresponding to periodicity greater than 20 years, with quarterly data.

VARIABLES	Cycli	CAL VARI	ANCE	Long Run variance			
,	Perm	TRANS	Sum	Perm	TRANS	SUM	
GDP	29.6	46.6	76.2	79.3	1.7	81.0	
Consumption	43.9	25.9	69.7	69.7	5.9	75.6	
Investment	25.5	53.4	78.9	67.2	5.1	72.3	
Unemployment Rate	30.1	48.4	78.5	68.7	13.2	82.0	
Hours Worked	29.2	29.0	58.1	57.3	6.3	63.5	
Inflation	37.2	48.8	86.1	15.5	69.9	85.4	
Labor Productivity	23.0	23.9	46.9	58.3	5.1	63.4	
TFP	20.7	6.7	27.4	64.5	1.6	66.1	
\mathbf{FFR}	10.9	64.5	75.5	0.9	85.0	85.9	
Baa-GS10	12.9	74.8	87.8	49.2	36.9	86.1	
S&P500	36.2	20.9	57.1	30.2	0.7	30.9	
JLN Uncertainty 3M	39.8	43.1	82.9	49.2	42.5	91.8	
BC5Y	71.2	17.9	89.1	71.5	11.9	83.4	

Table 2.4: Identification II. Percentage of variance explained by the permanent shock and the transitory shock for a few selected variables, by frequency band. Business cycle frequency band: $[2\pi/32 \le \omega \le 2\pi/6]$ corresponding to cycles with periodicity between 18 months and 8 years. Long run frequency band: $[0 \le \omega \le 2\pi/80]$, corresponding to periodicity greater than 20 years, with quarterly data.

VARIABLES	A: NO-IN	IFL SHOCK	B: GDP targeting shock		
	Cyclical v.	Long-Run v.	Cyclical v.	Long-Run v.	
GDP	56.1	57.0	59.5	34.4	
Consumption	60.9	62.4	54.1	43.0	
Investment	52.2	34.3	59.1	17.3	
Unemployment Rate	46.4	27.0	51.0	12.7	
Hours Worked	51.2	54.0	49.6	38.4	
Inflation	8.0	6.0	13.6	23.6	
Labor Productivity	26.9	20.8	27.0	6.9	
TFP	11.6	31.2	7.9	13.9	
FFR	17.0	28.0	33.3	52.3	
Baa-GS10	56.8	83.2	73.0	77.0	
S&P500	51.7	15.1	45.9	7.0	
JLN uncertainty 3M	59.5	6.3	59.2	7.3	
BC5Y	63.7	19.4	47.6	4.5	

Table 2.5: Identification exercises of Section 2.4.6. Percentage of cycli-CAL AND LONG-RUN VARIANCE EXPLAINED BY THE SHOCK DISCONNECTED FROM INFLATION (PANEL A), AND BY THE GDP TARGETING SHOCK (PANEL B) FOR A FEW SELECTED VARI-ABLES, BY FREQUENCY BAND. BUSINESS CYCLE FREQUENCY BAND: $[2\pi/32 \le \omega \le 2\pi/6]$ CORRESPONDING TO CYCLES WITH PERIODICITY BETWEEN 18 MONTHS AND 8 YEARS. LONG RUN FREQUENCY BAND: $[0 \le \omega \le 2\pi/80]$, CORRESPONDING TO PERIODICITY GREATER THAN 20 YEARS, WITH QUARTERLY DATA.

	P=	1	P=	2	P=	3	P=	4
VARIABLES		PERCEN	TAGE OF	Explai	NED CYCL	ICAL V	ARIANCE	
	Supply	DEM	SUPPLY	Dem	SUPPLY	DEM	SUPPLY	Dem
GDP	27.1	49.1	26.5	49.7	29.0	47.4	30.4	51.2
Consumption	40.6	29.2	45.6	21.2	50.7	20.1	46.5	25.9
Investment	23.6	55.3	25.2	53.3	30.4	49.9	34.2	49.2
Unemployment	29.0	49.5	31.8	51.4	37.3	44.2	41.8	40.1
Hours Worked	26.3	31.9	23.2	40.1	28.0	32.5	27.3	34.1
Inflation	41.8	44.3	54.2	33.2	57.9	23.1	62.8	16.5
Labor Productivity	22.5	24.4	25.1	30.6	21.9	38.5	15.9	40.7
TFP	21.0	6.4	20.5	8.7	16.6	14.0	12.7	10.9
FFR	13.3	62.2	21.6	55.6	27.0	41.0	32.2	36.8
Baa-GS10	10.8	77.0	14.0	72.9	22.1	60.1	23.4	55.9
S&P500	33.3	23.8	32.3	21.1	26.0	33.6	25.1	35.2
JLN Uncertainty	37.4	45.5	41.5	42.4	44.0	41.3	47.6	38.3
BC5Y	69.1	20.1	68.4	20.9	68.8	19.9	69.4	21.1
]	PERCEN	tage of 1	Explain	NED LONG	Run V.	ARIANCE	
	Supply	Dem	Supply	Dem	SUPPLY	Dem	SUPPLY	Dem
GDP	77.7	3.3	69.6	5.2	71.4	2.3	66.5	11.3
Consumption	66.9	8.7	52.8	10.9	57.9	2.8	52.0	9.8
Investment	67.8	4.5	74.5	1.1	77.3	1.1	76.7	4.2
Unemployment	70.9	11.0	81.2	6.0	84.6	4.6	85.7	4.9
Hours Worked	54.7	8.8	50.5	21.2	63.3	13.1	53.9	24.1
Inflation	20.0	65.4	33.6	36.3	26.3	20.8	36.3	13.2
Labor Productivity	60.1	3.3	65.1	0.5	76.4	0.2	74.0	5.0
TFP	65.2	0.9	60.3	0.7	67.1	1.3	63.8	5.5
FFR	2.3	83.6	12.4	66.5	9.2	42.6	18.8	39.0
Baa-GS10	44.0	42.1	23.6	35.3	27.9	14.7	21.2	18.0
S&P500	30.4	0.5	37.5	0.1	43.1	0.8	46.2	1.5
JLN Uncertainty	54.5	37.3	70.8	21.5	68.8	17.6	80.5	9.5
BC5Y	74.8	8.6	85.5	1.0	88.2	1.3	91.8	0.4

Table 2.6: Identification I: Percentage of variance explained by the supply shock and the demand shock for a few selected variables, by frequency band, according to different lags order: $p = [1 \ 2 \ 3 \ 4]$. Baseline specification: p = 1. Business cycle frequency band: $[2\pi/32 \le \omega \le 2\pi/6]$ corresponding to cycles with periodicity between 18 months and 8 years. Long run frequency band: $[0 \le \omega \le 2\pi/80]$, corresponding to periodicity greater than 20 years, with quarterly data.

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	R=	=11	R=	13	R=	=15	R=	=17	R=	=20
VARIABLES		Percentage of Explained Cyclical Variance								
	Supp	Dem	Supp	Dem	Supp	Dem	Supp	Dem	Supp	Dem
GDP	27.1	49.1	22.2	51.8	23.8	49.0	18.2	50.9	18.0	50.8
Consumption	40.6	29.2	30.4	28.4	31.0	25.4	26.1	28.6	26.6	27.5
Investment	23.6	55.3	24.3	54.1	25.6	52.8	22.1	55.2	21.6	53.7
Unemployment	29.0	49.5	30.6	46.7	30.9	44.6	27.4	51.8	28.2	49.9
Hours Worked	26.3	31.9	19.8	32.3	23.5	28.4	18.2	29.7	19.2	31.5
Inflation	41.8	44.3	45.0	30.6	40.8	29.1	43.5	30.7	44.5	28.9
Labor Productivity	22.5	24.4	18.1	29.0	20.3	27.2	16.6	32.5	17.8	33.8
TFP	21.0	6.4	14.5	5.7	16.4	5.8	20.4	3.9	17.9	3.8
FFR	13.3	62.2	24.9	52.3	23.8	52.9	15.7	47.6	17.0	45.7
Baa-GS10	10.8	77.0	13.0	72.1	13.1	67.4	12.7	49.1	12.6	49.3
S&P500	33.3	23.8	25.6	32.3	26.9	31.7	19.7	38.0	16.7	36.9
JLN Uncertainty 3M	37.4	45.5	43.8	36.7	43.5	36.9	43.2	33.7	42.9	33.7
BC5Y	69.1	20.1	54.1	20.2	47.8	15.5	45.3	14.4	43.7	13.3
		I	PERCENT	AGE OF	EXPLAIN		g Run V	ARIANC	E	
	Supp	Dem	Supp	Dem	Supp	Dem	Supp	Dem	Supp	Dem
GDP	77.7	3.3	74.7	6.1	75.4	5.6	67.9	4.8	69.1	6.5
Consumption	66.9	8.7	60.9	9.5	61.0	8.3	56.7	8.8	57.0	10.2
Investment	67.8	4.5	68.4	2.5	68.1	2.9	64.9	1.4	64.4	1.7
Unemployment	70.9	11.0	78.0	7.8	73.0	8.6	74.0	10.0	74.8	9.3
Hours Worked	54.7	8.8	52.8	12.6	50.9	11.0	55.8	10.3	53.8	10.9
Inflation	20.0	65.4	22.6	47.7	20.4	48.6	19.1	47.4	20.7	46.0
Labor Productivity	60.1	3.3	62.0	1.4	62.2	1.8	69.8	0.6	70.3	0.2
TFP	65.2	0.9	65.5	0.1	65.3	0.1	70.4	0.3	68.7	0.7
\mathbf{FFR}	2.3	83.6	6.0	70.5	5.0	71.6	3.8	69.8	4.9	67.9
Baa-GS10	44.0	42.1	38.5	33.4	37.2	34.1	28.8	23.6	27.6	25.6
S&P500	30.4	0.5	29.7	1.9	29.0	1.8	22.6	1.2	22.4	1.0
JLN Uncertainty 3M	54.5	37.3	67.0	23.7	61.5	25.3	54.3	29.3	57.7	26.5
BC5Y	74.8	8.6	82.7	4.2	79.9	4.8	80.8	4.5	79.8	3.5

Table 2.7: Identification I: Percentage of variance explained by the Demand shock and the Supply shock for a few selected variables, by frequency band, according to the number of static factors: $r = [11 \ 13 \ 15 \ 17 \ 20]$. Baseline specification: r = 11 static factors. Business cycle frequency band: $[2\pi/32 \le \omega \le 2\pi/6]$ corresponding to cycles with periodicity between 18 months and 8 years. Long run frequency band: $[0 \le \omega \le 2\pi/80]$, corresponding to periodicity greater than 20 years, with quarterly data.

TABLES

	Cyclical Variance				Long Run Variance				
VARIABLES	Sui	PPLY	Demand		SUPPLY		Dem	Demand	
	Min	MAX	Min	MAX	Min	MAX	Min	MAX	
GDP	26.5	30.4	47.4	51.2	66.5	77.7	2.3	11.3	
Consumption	40.6	50.7	20.1	29.2	52.0	66.9	2.8	10.9	
Investment	23.6	34.2	49.2	55.3	67.8	77.3	1.1	4.5	
Unemployment	29.0	41.8	40.1	51.4	70.9	85.7	4.6	11.0	
Hours Worked	23.2	28.0	31.9	40.1	50.5	63.3	8.8	24.1	
Inflation	41.8	62.8	16.5	44.3	20.0	36.3	13.2	65.4	
Labor Productivity	15.9	25.1	24.4	40.7	60.1	76.4	0.2	5.0	
TFP	12.7	21.0	6.4	14.0	60.3	67.1	0.7	5.5	
FFR	13.3	32.2	36.8	62.2	2.3	18.8	39.0	83.6	
Baa-GS10	10.8	23.4	55.9	77.0	21.2	44.0	14.7	42.1	
S&P500	25.1	33.3	21.1	35.2	30.4	46.2	0.1	1.5	
JLN Uncertainty 3M	37.4	47.6	38.3	45.5	54.5	80.5	9.5	37.3	
BC5Y	68.4	69.4	19.9	21.1	74.8	91.8	0.4	8.6	

(a) Robustness Identification I: Maximum and Minimum percentage value of explained variance according to different lags order: p = [1 2 3 4]. Baseline specification: p = 1 lag.

(b) Robustness Identification I: Maximum and minimum value of explained variance according to the number of static factors: $r = [11 \ 13 \ 15 \ 17 \ 20]$. Baseline specification: r = 11 static factors.

	Сү	CLICAL	VARIA	NCE	Long Run Variance			
VARIABLES	SUPPLY		Demand		SUPPLY		Demand	
	Min	Max	Min	Max	Min	Max	Min	MAX
GDP	18.0	27.1	49.0	51.8	67.9	77.7	3.3	6.5
Consumption	26.1	40.6	25.4	29.2	56.7	66.9	8.3	10.2
Investment	21.6	25.6	52.8	55.3	64.4	68.4	1.4	4.5
Unemployment	27.4	30.9	44.6	51.8	70.9	78.0	7.8	11.0
Hours Worked	18.2	26.3	28.4	32.3	50.9	55.8	8.8	12.6
Inflation	40.8	45.0	28.9	44.3	19.1	22.6	46.0	65.4
Labor Productivity	16.6	22.5	24.4	33.8	60.1	70.3	0.2	3.3
TFP	14.5	21.0	3.8	6.4	65.2	70.4	0.1	0.9
\mathbf{FFR}	13.3	24.9	45.7	62.2	2.3	6.0	67.9	83.6
Baa-GS10	10.8	13.1	49.1	77.0	27.6	44.0	23.6	42.1
S&P500	16.7	33.3	23.8	38.0	22.4	30.4	0.5	1.9
JLN Uncertainty 3M	37.4	43.8	33.7	45.5	54.3	67.0	23.7	37.3
BC5Y	43.7	69.1	13.3	20.2	74.8	82.7	3.5	8.6

Table 2.8: Percentage of variance explained by the supply shock and the demand shock (Identification I) for a few selected variables, by frequency band. Business cycle frequency band: $[2\pi/32 \le \omega \le 2\pi/6]$ corresponding to cycles with periodicity between 18 months and 8 years. Long run frequency band: $[0 \le \omega \le 2\pi/80]$, corresponding to periodicity greater than 20 years, with quarterly data.

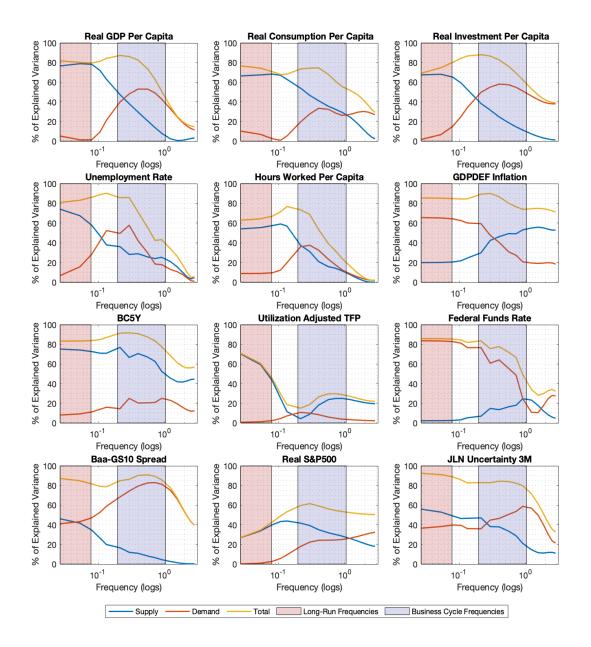


Figure A.1: Identification I: Spectral Decomposition for a few selected variables, frequency by frequency. The figure reports the percentage of explained variance. Blue line: Contribution of the supply shock; Red line: Contribution of the demand shock; Yellow line: sum. Pink shadowed area: Long run frequencies (>80 quarters); Lilac shadowed area: Business Cycle frequencies (6-32 quarters).

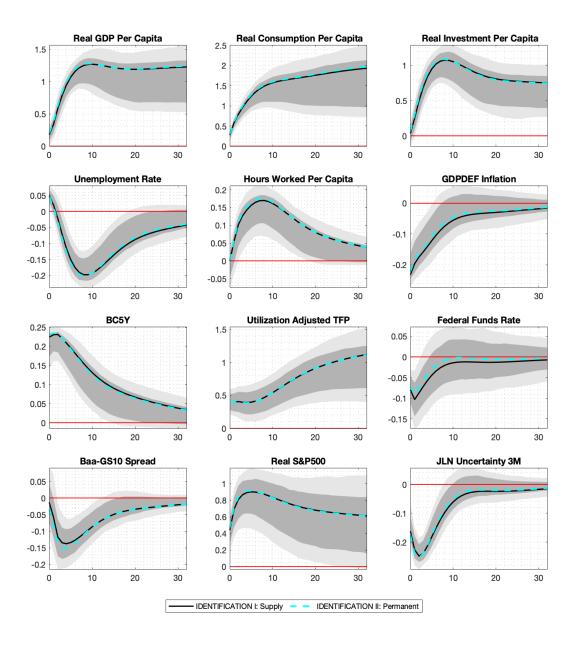


Figure A.2: IMPULSE RESPONSE FUNCTIONS OF THE SUPPLY SHOCK (IDENTIFI-CATION I, BLACK LINE) AND THE PERMANENT SHOCK (IDENTIFICATION II, CYAN DASHED LINE). THE DARK GRAY AND LIGHT GRAY AREAS ARE THE 68% AND 90% CONFIDENCE BANDS, RESPECTIVELY, FOR IDENTIFICATION I.

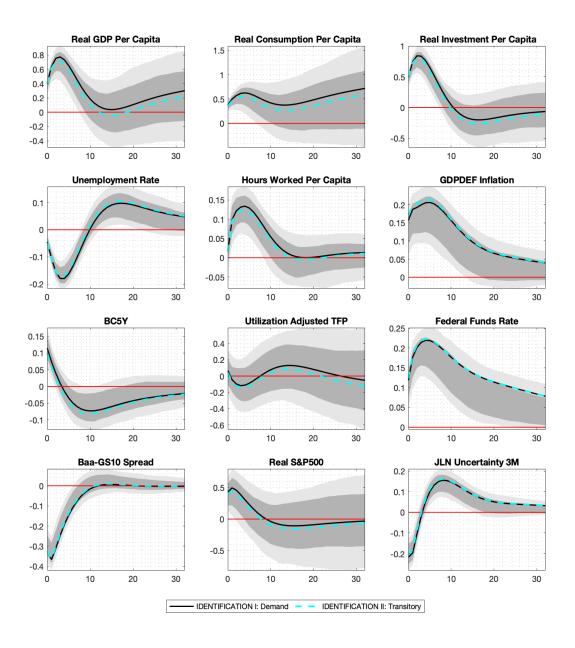


Figure A.3: IMPULSE RESPONSE FUNCTIONS OF THE DEMAND SHOCK (IDENTIFICATION I, BLACK LINE) AND THE TRANSITORY SHOCK (IDENTIFICATION II, CYAN DASHED LINE). THE DARK GRAY AND LIGHT GRAY AREAS ARE THE 68% AND 90% CONFIDENCE BANDS, RESPECTIVELY, FOR IDENTIFICATION I.

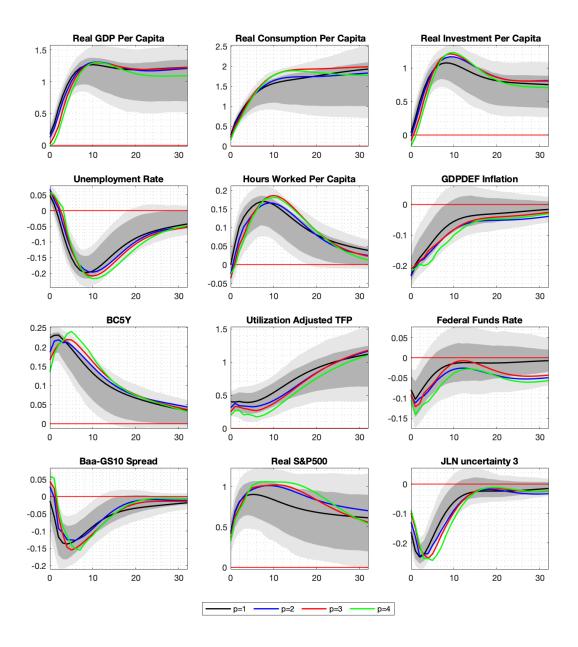


Figure A.4: Identification I: Impulse response functions of the Supply shock, according to different lags order: $p = [1 \ 2 \ 3 \ 4]$. Baseline specification: p = 1. The dark gray and light gray areas are the 68% and 90% confidence bands, respectively. Black line and confidence bands: baseline specification.

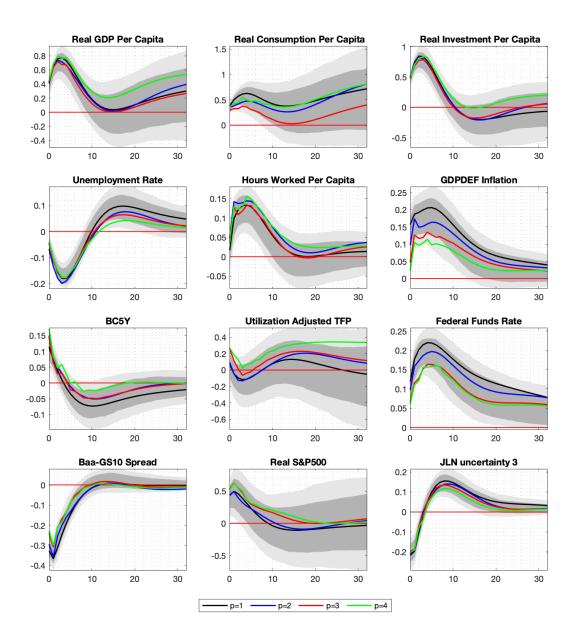


Figure A.5: Identification I: Impulse response functions of the Demand shock, according to different lags order: $p = [1 \ 2 \ 3 \ 4]$. Baseline specification: p = 1. The dark gray and light gray areas are the 68% and 90% confidence bands, respectively. Black line and confidence bands: baseline specification.

Appendices for Online Publication

2.A. RANK REDUCTION STEP

In the standard estimation procedure the identification techniques are applied to the residuals of the VAR estimated for F_t after estimating q, the number of common shocks, and the rank reduction. The estimated factors \hat{F}_t are not exactly singular, as they contain a residual of the idiosyncratic components that disappears completely only asymptotically. As a consequence, the vector $\hat{\epsilon}_t$ has rank r > q, although the last r-q eigenvalues of $\hat{\Sigma}_{\epsilon}$ are close to zero (Forni et al., 2020). In the standard procedure, singularity is forced on $\hat{\epsilon}_t$ by means of rank-reduction techniques. In Forni et al. (2009), the rank reduction is obtained by using the spectral decomposition of $\hat{\Sigma}_{\epsilon}$, so that the vector $\hat{\epsilon}_t$ is replaced by the \hat{q} -dimensional vector $V^{-1}\hat{\epsilon}_t$, where V^{-1} is the matrix whose rows are the normalised eigenvectors corresponding to the q-largest eigenvalues of the variance-covariance matrix of $\hat{\epsilon}_t$. This is equivalent to assume that the static rank of the common components is r, which is the rank of its covariance matrix, while the dynamic rank is q, which is the rank of its spectral density. In empirical situation, the number q of dynamic factors or common shocks is unknown and has to be determined by existing information criteria. For instance, the criterion proposed by Hallin and Liška (2007) is based on the properties of dynamic eigenvalues of the data and looks for the value q that minimizes the contribution of the idiosyncratic component. Alternative methods are proposed by Onatski (2009), Amengual and Watson (2007) and Bai and Ng (2007). Recently, Avarucci et al. (2021) introduce a novel consistent criterion to estimate the number of common shocks that can be applied to single frequencies as well as to frequency bands. Such criteria, albeit consistent, often give different results each other.

Forni et al. (2020) shown that the rank reduction step can be ignored with no consequences on the (IRFs) estimation accuracy. Since different information criteria often give different results, the estimation of q and the rank reduction can be a potential source of error, in particular whether \hat{q} underestimates the true q, leading to large estimation errors implied by a possible mis-specification of q. Therefore, we apply the identification techniques to the not exactly singular Cholesky-transformed residuals of the estimated VAR without reducing the rank.

Moreover, by reducing the number of shocks of interest in the first stage of our identification strategy, where we select the two shocks maximizing the explained variance of targeted variables on the band $[0 2\pi/6]$, rather than across all frequencies, we do not need to implement the rank reduction step in our estimation procedure.

2.B. DATA DESCRIPTION AND DATA TREATMENT

For the description of each variable see McCracken and Ng (2020). For variables not in the FRED-QD dataset, refer to the Mnemonic and note. Treatment codes: 1 =no treatment; 2 = first difference, Δx_t ; $4 = \log(x_t)$; $5 = \log$ of the first difference, $\Delta \log(x_t)$.

ID	FRED-QD ID	Mnemonic	Treatment Code	Note
1	1	GDPC1/CNP16OV	5	
2	2	PCECC96/CNP16OV	5	
3	3	PCDGx/CNP16OV	5	
4 5	4 5	PCESVx/CNP16OV PCNDx/CNP16OV	5 5	
6	6	GPDIC1/CNP16OV	5	
7	7	FPIx/CNP16OV	5	
8	8	Y033RC1Q027SBEAx/CNP16OV	5	
9	9	PNFIx/CNP16OV	5	
10	10	PRFIx/CNP16OV	5	
11 12	11 12	A014RE1Q156NBEA GCEC1/CNP16OV	1 5	
12	12	A823RL1Q225SBEA	1	
14	14	FGRECPTx/CNP16OV	5	
15	15	SLCEx/CNP16OV	5	
16	16	EXPGSC1/CNP16OV	5	
17	17	IMPGSC1/CNP16OV	5	
18	18	DPIC96/CNP16OV	5	
19 20	19 20	OUTNFB/CNP16OV OUTBS/CNP16OV	5 5	
20 21	20	(PCESVx+PCNDx)/CNP16OV	5	
22		(PCDGx+FPIx)/CNP16OV	5	
23	22	INDPRO/CNP16OV	5	
24	23	IPFINAL/CNP16OV	5	
25	24	IPCONGD/CNP16OV	5	
26 27	25 28	IPMAT/CNP16OV IPDCONGD/CNP16OV	5 5	
27	28 30	IPNCONGD/CNP16OV	5	
29	31	IPBUSEQ/CNP16OV	5	
30	35	PAYEMS/CNP16OV	2	
31	36	USPRIV/CNP16OV	2	
32	38	SRVPRD/CNP16OV	2	
33	39	USGOOD/CNP16OV	2 2	
$\frac{34}{35}$	$51 \\ 57$	USGOVT/CNP16OV CE16OV/CNP16OV (EMRATIO)	2	
36	58	CIVPART	2	
37	59	UNRATE	1	
38	60	UNRATESTx	1	
39	61	UNRATELTx	1	
40	62	LNS14000012	1	
41 42	63 64	LNS14000025 LNS14000026	1	
43	74	HOABS/CNP16OV	4	
44	76	HOANBS/CNP16OV	4	
45	77	AWHMAN	1	
46	79	AWOTMAN	1	
47	81	HOUST/CNP160V	5	
$\frac{48}{49}$	95 96	PCECTPI PCEPILFE	5 5	
49 50	30	GDPDEF	5	GDP: Implicit Price Deflator
51	97	GDPCTPI	5	
52	98	GPDICTPI	5	
53	120	CPIAUCSL	5	
54	121	CPILFESL	5	
$55 \\ 56$	122 123	WPSFD49207 PPIACO	5 5	
56 57	123 124	WPSFD49502	5 5	
58	124	PPIIDC	5	
59	129	WPU0561	5	
60	130	OILPRICEx	5	
61	135	COMPRNFB	5	
62	138	OPHNFB OPHDBS	5	
$63 \\ 64$	139 140	OPHPBS ULCBS	5 5	
65	140	ULCNFB	5	
66	143	UNLPNBS	5	
67		dtfp	1	Fernald's TFP growth

Continued on next page

ID	FRED-QD ID	Mnemonic	Treatment code	Note
68		dtfp util	1	Fernald's TFP growth CU adjusted
69		dtfp I	1	Fernald's TFP growth - Inv
70		dtfp C	1	Fernald's TFP growth - Con
71		dtfp I util	1	Fernald's TFP growth CU - Inv
72		dtfp C util	1	Fernald's TFP growth CU - Con
73	144	FEDFUNDS	1	-
74	145	TB3MS	1	
75	146	TB6MS	1	
76	147	GS1	1	
77	148	GS10	1	
78	150	AAA	1	
79	151	BAA	1	
80	152	BAA10YM	1	
81	156	GS10TB3Mx	1	
82		BAA-AAA	1	
83		GS10-FEDFUNDS	1	
84		GS1-FEDFUNDS	1	
85		BAA-FEDFUNDS	1	
86	158	BOGMBASEREALx/CNP16OV	5	
87	160	M1REAL/CNP16OV	5	
88	161	M2REAL/CNP16OV	5	
89	163	BUSLOANSx/CNP16OV	5	
90	164	CONSUMER _x /CNP16OV	5	
91	166	REALLNx/CNP16OV	5	
92	168	TOTALSLx/CNP16OV	5	
93	188	UMCSENTx	1	
94	100	Business Condition 12 Months	1	Michigan Consumer Survey
95		Business Condition 5 Years	1	Michigan Consumer Survey
96		Current Index	1	Michigan Consumer Survey
97		Expected Index	1	Michigan Consumer Survey
98		News Index: Relative	1	Michigan Consumer Survey
99	197	UEMPMEAN	1	intelligan consumer partoy
100	201	GS5	1	
101	210	CUSR0000SAC	5	
102	211	CUSR0000SAD	5	
103	212	CUSR0000SAS	5	
104	212	CPIULFSL	5	
105	245	S&P 500	5	
106	246	S&P: indust	5	
107	240	S&P 500/GDPDEF	5	
108		S&P: indust/GDPDEF	5	
109		JLN Macro Unc 1-month	1	Jurado Ludvigson and Ng Uncertaint
110		JLN Macro Unc 3-month	1	JLN Uncertainty
111		JLN Macro Unc 12-month	1	JLN Uncertainty
112		DPCCRC1Q027SBEAx/CNP16OV	5	Real PCE Excluding food and energy
112		DFXARC1M027SBEAx/CNP16OV	5	Real PCE: Food
113		DNRGRC1Q027SBEAx/CNP16OV	5	Real PCE: Energy goods
114		DINGRUIQU2/SBEAX/UNP160V	Э	near FUE: Energy goods

2.C. Additional Results and Robustness

TABLES

Frequencies	DDR	DGR	DER
$0 \le \omega \le 2\pi/6$	2	2	1
$0 \le \omega \le 2\pi/8$	2	2	1
$0 \le \omega \le \pi$	2	1	1

Table C.1: Number of estimated dynamic factors by DDR, DGR and DER evaluated at selected frequencies or frequency bands. The size of the spectral window - bandwidth parameter - is $M_T = \lfloor a\sqrt{T} \rfloor$ with a = 0.5. DDR: Dynamic Difference Ratio Estimator; DGR: Dynamic Growth Ratio Estimator; DER: Dynamic Eigenvalue Ratio Estimator.

Orthogonality									
PRINCIPAL COMPONENTS	1 lags	2 lags	3 lags	4 lags					
r=6	0.00	0.00	0.00	0.03					
r=7	0.01	0.00	0.00	0.00					
r=8	0.01	0.00	0.00	0.01					
r=9	0.01	0.00	0.00	0.01					
r=10	0.02	0.01	0.00	0.04					
r=11	0.02	0.02	0.01	0.08					

Table C.2: *p*-values of the orthogonality *F*-test (Forni and Gambetti, 2014), one to four lags, for the MBC shock, estimated with ACD's VAR specification. r is the number of principal components used in the test. Source: Granese (2023).

	Р	=1	Р	P=2		P=3		P=4				
VARIABLES		Perci	ENTAGE C	of Explai	NED CYC	lical Vaf	RIANCE					
	Perm	Trans	Perm	Trans	Perm	Trans	Perm	Trans				
GDP	29.6	46.6	29.6	46.6	28.8	47.7	27.1	54.5				
Consumption	43.9	25.9	50.4	16.5	52.7	18.1	51.3	21.0				
Investment	25.5	53.4	27.1	51.4	30.0	50.3	30.2	53.2				
Unemployment	30.1	48.4	31.8	51.4	36.4	45.1	36.4	45.5				
Hours Worked	29.2	29.0	27.7	35.6	29.6	31.0	31.7	29.6				
Inflation	37.2	48.8	44.0	43.4	52.1	28.9	43.6	35.7				
Labor Productivity	23.0	23.9	26.1	29.5	22.0	38.5	17.4	39.2				
TFP	20.7	6.7	19.1	10.0	17.3	13.3	16.3	7.3				
\mathbf{FFR}	10.9	64.5	15.0	62.2	23.1	44.9	17.9	51.2				
Baa-GS10	12.9	74.8	16.2	70.7	21.6	60.6	20.0	59.3				
S&P500	36.2	20.9	37.2	16.2	29.5	30.1	35.4	24.9				
JLN Uncertainty 3M	39.8	43.1	43.7	40.2	44.0	41.2	45.0	40.9				
BC5Y	71.2	17.9	72.5	16.8	70.6	18.1	71.0	19.5				
		Percentage of Explained Long Run Variance										
	Perm	Trans	Perm	Trans	Perm	TRANS	Perm	Trans				
GDP	79.3	1.7	72.9	2.0	73.0	0.7	76.5	1.4				
Consumption	69.7	5.9	57.6	6.1	59.2	1.5	59.4	2.4				
Investment	67.2	5.1	73.3	2.2	76.9	1.5	79.6	1.3				
Unemployment	68.7	13.2	80.1	7.1	83.9	5.3	83.9	6.7				
Hours Worked	57.3	6.3	58.8	12.9	68.0	8.4	71.3	6.7				
Inflation	15.5	69.9	24.2	45.8	22.0	25.0	22.3	27.2				
Labor Productivity	58.3	5.1	64.1	1.5	76.4	0.2	78.9	0.1				
TFP	64.5	1.6	60.2	0.7	67.4	0.9	68.3	1.1				
\mathbf{FFR}	0.9	85.0	5.6	73.3	5.9	45.8	5.6	52.2				
Baa-GS10	49.2	36.9	31.6	27.3	31.2	11.3	31.5	7.7				
S&P500	30.2	0.7	36.9	0.8	43.6	0.2	46.2	1.6				
JLN Uncertainty 3M	49.2	42.5	59.8	32.4	62.5	23.8	61.0	29.0				
BC5Y	71.5	11.9	81.6	4.9	85.7	3.8	86.8	5.4				

Table C.3: Identification II: Percentage of variance explained by the permanent shock and the transitory shock for a few selected variables, by frequency band, according to different lags order: $p = [1 \ 2 \ 3 \ 4]$. Baseline specification: p = 1. Business cycle frequency band: $[2\pi/32 \le \omega \le 2\pi/6]$ corresponding to cycles with periodicity between 18 months and 8 years. Long run frequency band: $[0 \le \omega \le 2\pi/80]$, corresponding to periodicity greater than 20 years, with quarterly data.

	R=	=11	R=	R=13		R=15		R=17		R=20	
VARIABLES			Perc	ENTAGE O	f Explai	NED CYCI	ICAL VAI	RIANCE			
	Perm	TRANS	Perm	Trans	Perm	TRANS	Perm	TRANS	Perm	TRANS	
GDP	29.6	46.6	24.0	50.1	25.7	47.1	18.7	50.4	19.0	49.9	
Consumption	43.9	25.9	33.3	25.4	33.3	23.1	27.1	27.6	28.3	25.8	
Investment	25.5	53.4	25.5	53.0	26.8	51.6	22.4	54.9	21.9	53.4	
Unemployment	30.1	48.4	30.7	46.5	31.3	44.1	26.9	52.3	27.3	50.8	
Hours Worked	29.2	29.0	23.3	28.8	26.2	25.6	20.5	27.5	21.9	28.8	
Inflation	37.2	48.8	36.9	38.6	34.4	35.5	36.7	37.5	34.5	38.8	
Labor	23.0	23.9	18.3	28.8	20.4	27.1	17.3	31.9	18.8	32.8	
TFP	20.7	6.7	13.6	6.6	15.4	6.8	19.9	4.5	17.3	4.4	
FFR	10.9	64.5	18.3	58.9	18.1	58.6	11.6	51.8	10.7	52.0	
Baa-GS10	12.9	74.8	14.4	70.7	14.2	66.3	13.7	48.2	13.7	48.2	
S&P500	36.2	20.9	31.1	26.8	31.9	26.7	23.3	34.3	21.8	31.9	
JLN Uncertainty	39.8	43.1	46.7	33.8	45.8	34.6	44.3	32.5	44.2	32.4	
BC5Y	71.2	17.9	56.1	18.1	50.1	13.1	47.7	12.1	46.8	10.2	
			Perci	ENTAGE OF	F EXPLAI	NED LONG	Run Va	RIANCE			
	Perm	Trans	Perm	Trans	Perm	Trans	Perm	Trans	Perm	Tran	
GDP	79.3	1.7	78.7	2.1	78.8	2.1	70.3	2.5	73.2	2.4	
Consumption	69.7	5.9	65.6	4.8	64.9	4.4	59.9	5.6	62.0	5.2	
Investment	67.2	5.1	68.3	2.5	68.0	3.0	64.6	1.7	64.8	1.4	
Unemployment	68.7	13.2	74.8	11.0	70.2	11.4	71.4	12.6	70.9	13.2	
Hours Worked	57.3	6.3	58.4	7.0	55.5	6.4	60.1	6.0	59.9	4.8	
Inflation	15.5	69.9	15.1	55.3	14.0	54.9	13.4	53.1	12.3	54.4	
Labor	58.3	5.1	59.4	4.0	59.8	4.2	68.1	2.3	68.0	2.5	
TFP	64.5	1.6	64.4	1.2	64.3	1.2	70.1	0.6	68.7	0.8	
FFR	0.9	85.0	2.1	74.5	1.8	74.8	1.3	72.3	1.0	71.8	
Baa-GS10	49.2	36.9	46.5	25.4	44.4	26.9	33.9	18.5	35.3	17.9	
S&P500	30.2	0.7	31.0	0.6	30.2	0.7	23.4	0.4	23.1	0.3	
JLN Uncertainty	49.2	42.5	57.9	32.8	53.8	33.1	47.1	36.5	47.1	37.2	
BC5Y	71.5	11.9	77.5	9.4	75.4	9.2	76.8	8.5	74.2	9.1	

Table C.4: IDENTIFICATION II: PERCENTAGE OF VARIANCE EXPLAINED BY THE TRANSITORY SHOCK AND THE PERMANNENT SHOCK FOR A FEW SELECTED VARIABLES, BY FREQUENCY BAND, ACCORDING TO THE NUMBER OF STATIC FACTORS: $r = [11 \ 13 \ 15 \ 17 \ 20]$. BASELINE SPECIFICATION: r = 11 STATIC FACTORS. BUSINESS CYCLE FREQUENCY BAND: $[2\pi/32 \le \omega \le 2\pi/6]$ CORRESPONDING TO CYCLES WITH PERIODICITY BETWEEN 18 MONTHS AND 8 YEARS. LONG RUN FREQUENCY BAND: $[0 \le \omega \le 2\pi/80]$, CORRESPONDING TO PERIODICITY GREATER THAN 20 YEARS, WITH QUARTERLY DATA.

	Сү	CLICAL	VARIA	NCE	Long Run Variance				
VARIABLES	Perm		Trans		Perm		Trans		
	Min	Max	Min	Max	Min	Max	Min	Max	
GDP	27.1	29.6	46.6	54.4	72.9	79.3	0.7	2.0	
Consumption	43.9	52.7	16.5	25.9	57.6	69.7	1.5	6.1	
Investment	25.5	30.2	50.3	53.4	67.2	79.6	1.3	5.1	
Unemployment	30.1	36.4	45.1	51.4	68.7	83.9	5.3	13.2	
Hours Worked	27.7	31.7	29.0	35.6	57.3	71.3	6.3	12.9	
Inflation	37.2	52.1	28.9	48.8	15.5	24.2	25.0	69.9	
Labor Productivity	17.4	26.1	23.9	39.2	58.3	78.9	0.1	5.1	
TFP	16.3	20.7	6.7	13.3	60.2	68.3	0.7	1.6	
FFR	10.9	23.1	44.9	64.5	0.9	5.9	45.8	85.0	
Baa-GS10	12.9	21.6	59.3	74.8	31.2	49.2	7.7	36.9	
S&P500	29.5	37.2	16.2	30.1	30.2	46.2	0.2	1.6	
JLN Uncertainty 3M	39.8	45.0	40.2	43.1	49.2	62.5	23.8	42.5	
BC5Y	70.6	72.5	16.8	19.5	71.5	86.8	3.8	11.9	

(a) Identification II: Maximum and Minimum percentage value of explained variance according to different lags order: $p = [1 \ 2 \ 3 \ 4]$. Baseline specification: p = 1 lag.

(b) Identification II: Maximum and minimum value of explained variance according to the number of static factors: $r = [11 \ 13 \ 15 \ 17 \ 20]$. Baseline specification: r = 11 static factors.

	Сү	CLICAL	VARIA	NCE	Long Run Variance				
VARIABLES	Perm		Tr	ANS	Pe	ERM	TRANS		
	Min	Max	Min	Max	Min	MAX	Min	Max	
GDP	18.7	29.6	46.6	50.4	70.3	79.3	1.7	2.5	
Consumption	27.1	43.9	23.1	27.6	59.9	69.7	4.4	5.9	
Investment	21.9	26.8	51.6	54.9	64.6	68.3	1.4	5.1	
Unemployment	26.9	31.3	44.1	52.3	68.7	74.8	11.0	13.2	
Hours Worked	20.5	29.2	25.6	29.0	55.5	60.1	4.8	7.0	
Inflation	34.4	36.9	35.5	48.8	12.3	15.5	53.1	69.9	
Labor Productivity	17.3	23.0	23.9	32.8	58.3	68.1	2.3	5.1	
TFP	13.6	20.7	4.4	6.8	64.3	70.1	0.6	1.6	
FFR	10.7	18.3	51.8	64.5	0.9	2.1	71.8	85.0	
Baa-GS10	12.9	14.4	48.2	74.8	33.9	49.2	17.9	36.9	
S&P500	21.8	36.2	20.9	34.3	23.1	31.0	0.3	0.7	
JLN Uncertainty 3M	39.8	46.7	32.4	43.1	47.1	57.9	42.5	32.8	
BC5Y	46.8	71.2	10.2	18.1	71.5	77.5	8.5	11.9	

Table C.5: PERCENTAGE OF VARIANCE EXPLAINED BY THE PERMANET SHOCK AND THE TRANSITORY SHOCK (IDENTIFICATION II) FOR A FEW SELECTED VARIABLES, BY FREQUENCY BAND. BUSINESS CYCLE FREQUENCY BAND: $[2\pi/32 \le \omega \le 2\pi/6]$ CORRESPONDING TO CYCLES WITH PERIODICITY BETWEEN 18 MONTHS AND 8 YEARS. LONG RUN FREQUENCY BAND: $[0 \le \omega \le 2\pi/80]$, CORRESPONDING TO PERIODICITY GREATER THAN 20 YEARS, WITH QUARTERLY DATA.

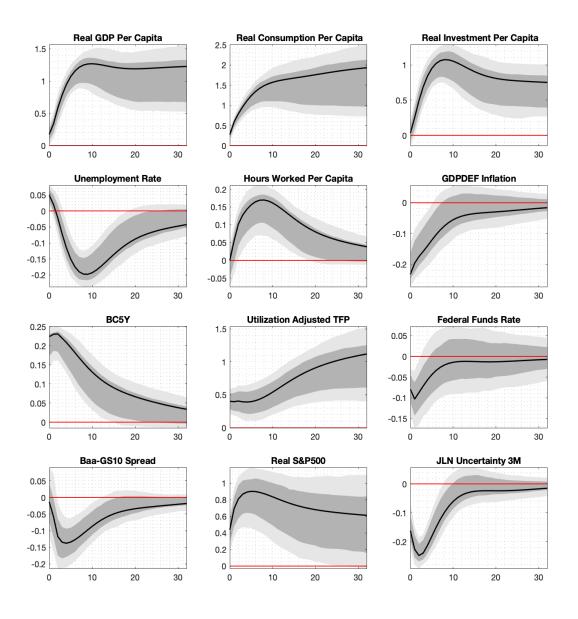


Figure C.1: Identification I: Point estimates of the Impulse Response Functions of the Supply Shock. The dark gray and light gray areas are the 68% and 90% confidence bands, respectively.

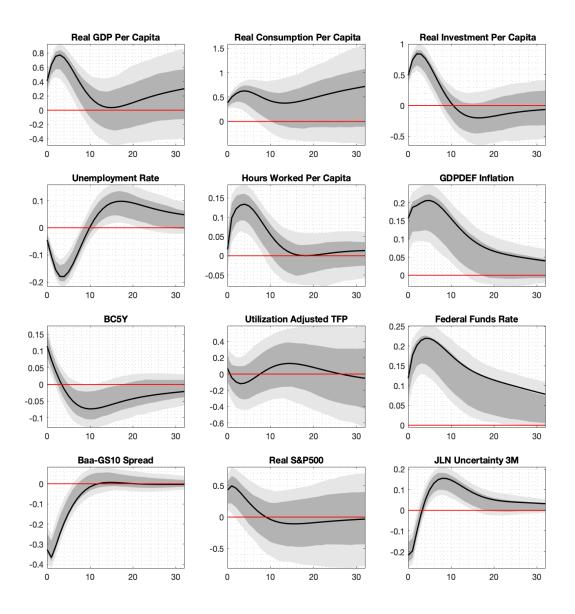


Figure C.2: Identification I: Point estimates of the Impulse Response Functions of the Demand Shock. The dark gray and light gray areas are the 68% and 90% confidence bands, respectively.

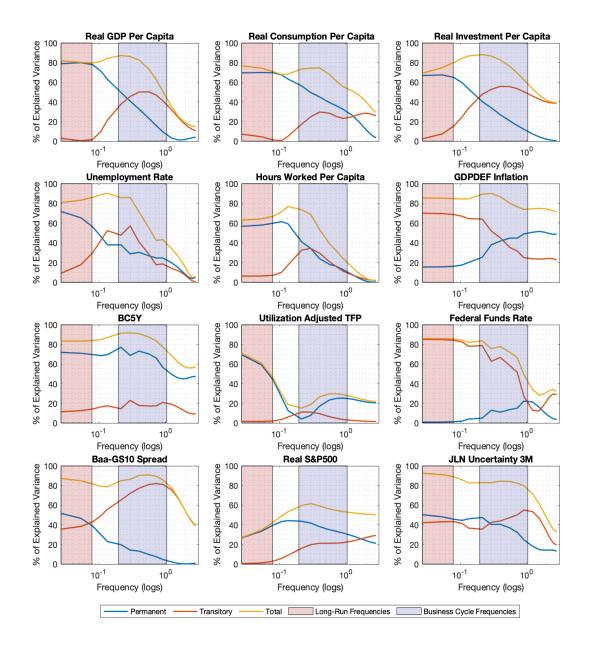


Figure C.3: Identification II: Spectral Decomposition for a few selected variables, frequency by frequency. The figure reports the percentage of explained variance. Blue line: Contribution of the permanent shock; Red line: Contribution of the transitory shock; Yellow line: sum. Pink shadowed area: Long run frequencies (>80 quarters); Lilac shadowed area: Business Cycle frequencies (6-32 quarters).

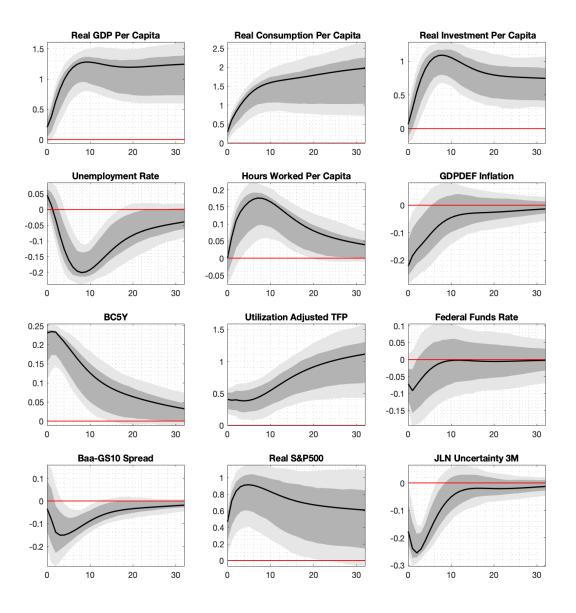


Figure C.4: Identification II: Point estimates of the Impulse Response Functions of the Permanent Shock. The dark gray and light gray areas are the 68% and 90% confidence bands, respectively.

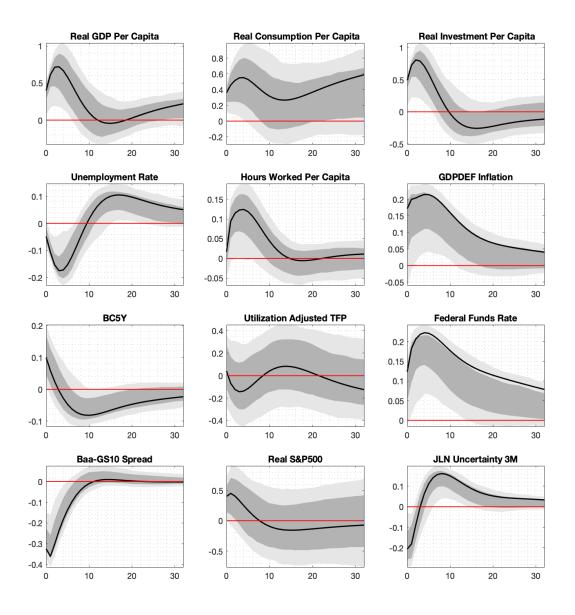


Figure C.5: Identification II: Point estimates of the Impulse Response Functions of the Transitory Shock. The dark gray and light gray areas are the 68% and 90% confidence bands, respectively.

APPENDIX

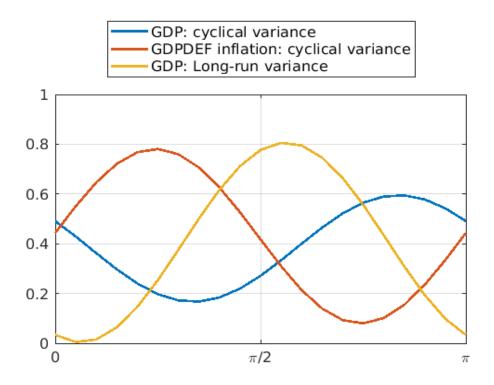


Figure C.6: Percentage of variances explained by all linear combination of the two shocks.

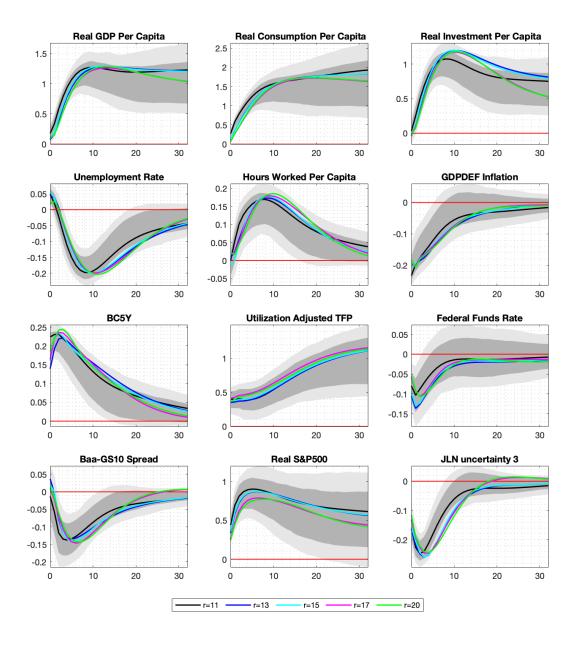


Figure C.7: Identification I: Impulse response functions of the Supply shock, according to different number of static factors: $r = [11 \ 6 \ 9 \ 13 \ 15]$. Baseline specification: r = 11. The dark gray and light gray areas are the 68% and 90% confidence bands, respectively. Black line and confidence bands: baseline specification.

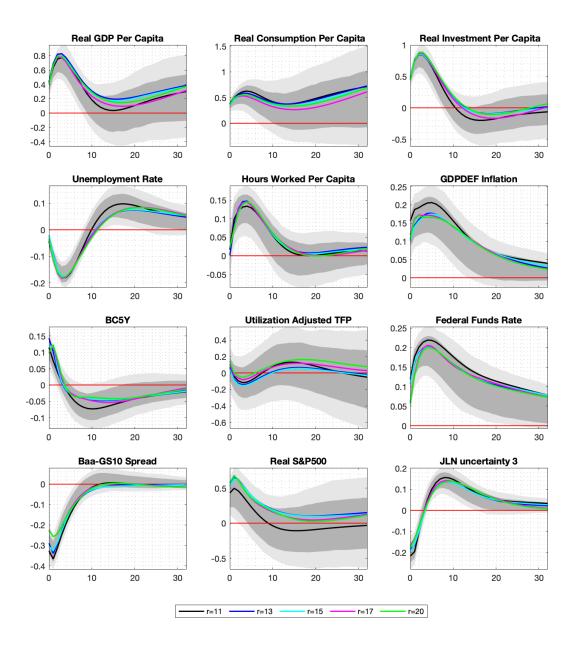


Figure C.8: Identification I: Impulse response functions of the Demand shock, according to different number of static factors: $r = [11 \ 6 \ 9 \ 13 \ 15]$. Baseline specification: r = 11. The dark gray and light gray areas are the 68% and 90% confidence bands, respectively. Black line and confidence bands: baseline specification.

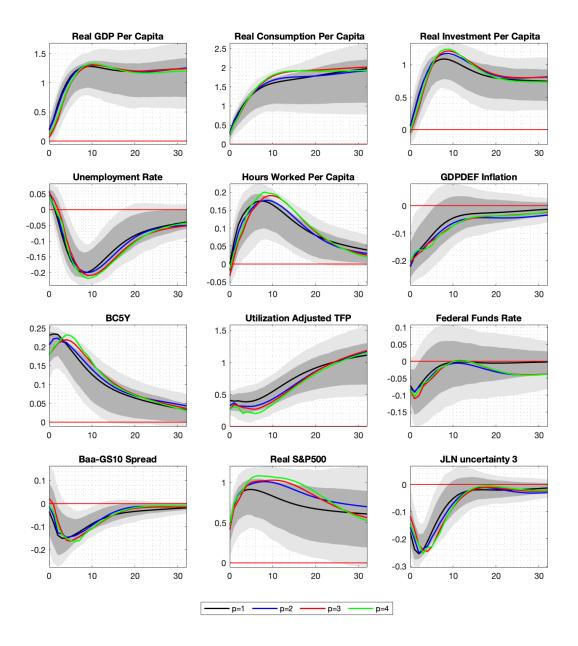


Figure C.9: Identification II: Impulse response functions of the Permanent shock, according to different lags order: $p = [1 \ 2 \ 3 \ 4]$. Baseline specification: p = 1. The dark gray and light gray areas are the 68% and 90% confidence bands, respectively. Black line and confidence bands: baseline specification.

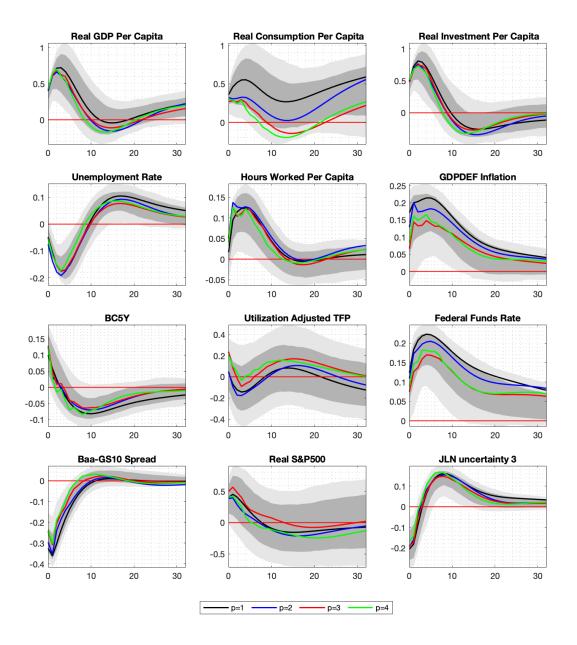


Figure C.10: Identification II: Impulse Response functions of the Transitory shock, according to different Lags order: $p = [1 \ 2 \ 3 \ 4]$. Baseline specification: p = 1. The dark gray and light gray areas are the 68% and 90% confidence bands, respectively. Black line and confidence bands: baseline specification.

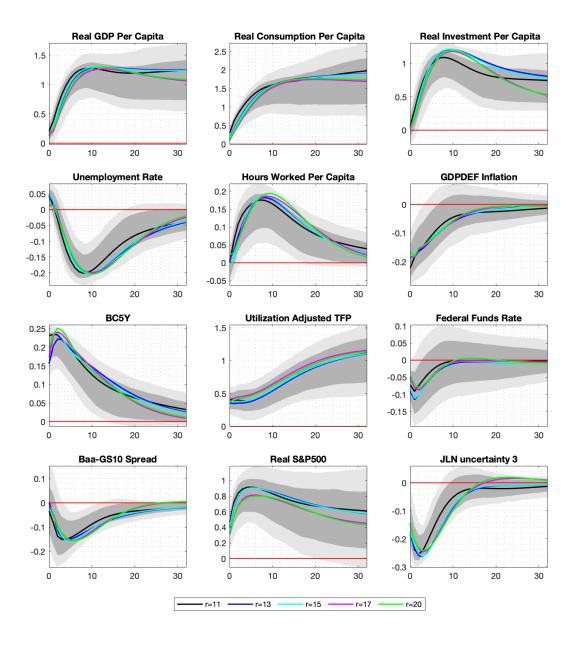


Figure C.11: Identification II: Impulse Response functions of the Per-MANENT SHOCK, ACCORDING TO DIFFERENT NUMBER OF STATIC FACTORS: $r = [11\ 6\ 9\ 13\ 15]$. Baseline specification: r = 11. The dark gray and light GRAY AREAS ARE THE 68% AND 90% CONFIDENCE BANDS, RESPECTIVELY. BLACK LINE AND CONFIDENCE BANDS: BASELINE SPECIFICATION.

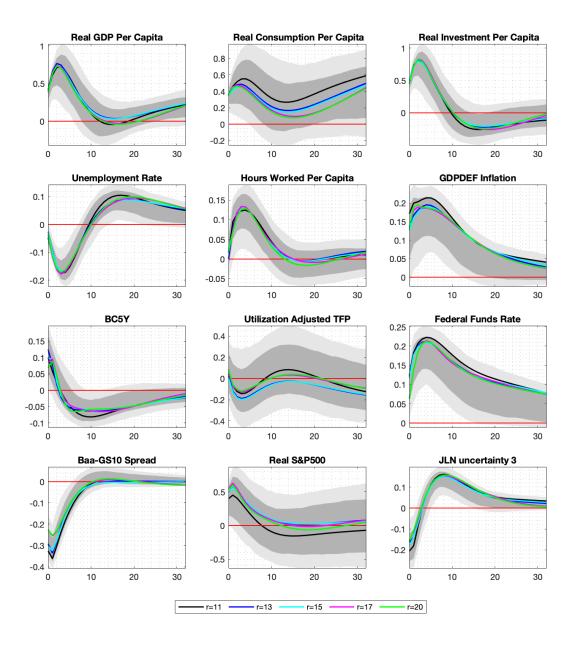


Figure C.12: Identification II: IMPULSE RESPONSE FUNCTIONS OF THE TRAN-SITORY SHOCK, ACCORDING TO DIFFERENT NUMBER OF STATIC FACTORS: $r = [11\ 6\ 9\ 13\ 15]$. Baseline specification: r = 11. The dark gray and light GRAY AREAS ARE THE 68% AND 90% CONFIDENCE BANDS, RESPECTIVELY. BLACK LINE AND CONFIDENCE BANDS: BASELINE SPECIFICATION.

Chapter 3

Asymmetric Transmission of Demand Shocks through the lens of a Nonlinear Structural Dynamic Factor Model[†]

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Abstract

This study explores the asymmetric impact of demand shocks on the economy using a nonlinear Structural Dynamic Factor Model. Our findings reveal that the effects of aggregate demand shocks are nonlinear, depending on their sign. Positive shocks are transitory, in line with standard business cycle theory; negative shocks, on the other hand, leave lasting scars on the economy. Recessions driven by demand-side shocks lead to permanent declines in output, employment and investment.

 $^{^\}dagger We$ thank Luca Gambetti for very useful comments and suggestions. The authors acknowledge the financial support of the Italian Ministry of Research and University, PRIN 2017, grant J44I20000180001.

3.1. INTRODUCTION

Conventional macroeconomic analysis typically suggests that demand shocks cause only temporary fluctuations (business cycles) around a stable, upward trend, with long-run changes being driven by supply-side factors. Blanchard and Quah (1989) first applied this assumption within the Structural Vector Autoregression (SVAR) framework aiming to distinguish between shocks that have a permanent effect on real economic activity, interpreted as supply disturbances, and those that do not, seen as demand disturbances. This method has since become a standard in macroeconomic analysis. Some empirical studies corroborate this perspective, showing that demandside shocks, such as monetary or financial disturbances, are cyclical in nature. In a recent paper, Forni et al. (2023) (FGGSS) lends strong support to the traditional view for the post-1960s US economy by showing that shocks identified as standard demand have no long-run effects on real activity. In their large factor model framework, the demand shock—defined on the basis of the short-run comovement between output and inflation—could theoretically impact GDP in the long run; however, empirical evidence suggests otherwise.

Yet, the long-lasting economic consequences of the 2008 financial crisis and the fears of a slow recovery from the the recession caused by the COVID-19 pandemic have challenged this perspective, sparking renewed interest in the potential persistent (or even permanent) effects of demand shocks. This has led economists to re-examine the concept of demand-driven hysteresis¹ (Cerra et al., 2023, for a recent survey). In this renewed debate, the early insights of Blanchard and Summers (1986) on how fluctuations in demand, especially those leading to significant recessions, may have longlasting impacts on potential output through hysteresis effects, gains new relevance.² Recent empirical works lend further support to this view. For example, Blanchard et al. (2015)'s study on demand-driven recessions in advanced economies, shows that a large share of such downturns are associated with a permanently lower level of output. Maffei-Faccioli (2021) through a Bayesian SVAR model with common trends and sign restrictions based on price and quantity co-movements, finds that while supply-side factors predominantly drove GDP growth before 2000, demand-side factors account for approximately half of the slowdown post-2000. This suggests a substantial role for demand forces in influencing long-term growth (super-hysteresis, see Ball, 2014). Regarding specific types of demand shocks, evidence supporting the long-term effects of monetary and fiscal shocks is presented in Jordà et al. (2020) and Fatás and Summers (2018).

Recently, Furlanetto et al. (2024) find convincing evidence of demand shock hysteresis for the post-1980s US data. They show that certain demand shocks not only lead to a permanent decrease in output but also play a quantitatively significant role in explaining GDP fluctuations across all horizons. Using a structural VAR framework

¹In this work, we refer to the concept of hysteresis as the presence of aggregate demand shocks with permanent effects on output.

 $^{^2{\}rm This}$ view was supported by developments in Europe during the 1980s, where unemployment stabilized at higher levels after recessions.

that combines long-run zero and short-run sign restrictions, they identify two shocks with potentially permanent effects on economic activity: a traditional supply shock and a more novel demand shock. Their findings suggest that recessions driven by demandside shocks lead to permanent (effects) declines in output, employment and investment, while labor productivity responds only to the permanent supply shock.³ Interestingly, the analysis reveals that these "negative permanent demand shocks" appear as significant drivers of output fluctuations in the baseline sample (1983:Q1-2019:Q4), but their relevance decreases when data from the 1970s are included. Results over this period are much more in line with the standard view of economic fluctuations.⁴

All in all, the potential presence of cyclical (demand) shocks with permanent effects, or demand-side hysteresis, appears to be related to an asymmetric or nonlinear transmission of demand shocks in the data. It is important to point out that the literature primarily focuses on this phenomenon in relation to shocks that lead to recessions, rather than economic booms, thus highlighting the importance of the direction of the shocks. Despite this, potential hysteresis effects are usually studied using linear models.

In this study, we aim to reconcile the conventional view, as supported by FGGSS, with the evidence of demand-driven hysteresis provided by Furlanetto et al. (2024), by exploring whether shocks identified as standard demand shocks have different effects on the US economy, depending on the sign of the shocks. Our hypothesis is that negative demand shocks may have more persistent and difficult-to-reverse effects than positive ones. If this hypothesis holds true, it would imply that a nonlinear model could detect demand-driven hysteresis where a linear model fails to do so.

To investigate the potential sign asymmetry, we employ the method developed by Forni et al. (2023), which allows us to extend the identification in FGGSS to a nonlinear setting where the economy is described by a structural Vector Moving Average, augmented with a nonlinear function of the shock of interest; here we use the absolute value of the demand shocks. In this model economy, macroeconomic variables react linearly to both the shock and its nonlinear function. Under suitable conditions, the model admits a VARX representation, in which the demand shock and its absolute value are exogenous variables. Thus, by combining the coefficient associated with the shock and its nonlinear function we can estimate the effects of positive and negative demand shocks on economic variables.

The identification and estimation procedure consists of two steps within a single nonlinear model. In the first step, we estimate the demand shock. In the second step, we use the shock and its nonlinear function as exogenous regressors in a VARX to estimate the nonlinear transmission mechanism (IRFs). It should be noted that this

³They also show that hysteresis mainly transmits through employment, with an increase in long-term unemployment and a decrease in labor force participation, with particular impact on the least productive workers.

⁴The authors conclude that to capture hysteresis effects when considering a longer sample, a nonlinear setting seems necessary. Benati and Lubik (2021), using an approach similar to but somewhat distinct from Furlanetto et al. (2024), find limited evidence of hysteresis in the post-WWII US data. Moreover, their findings suggest that aggregate demand shocks with a permanent effect are virtually absent for samples excluding the 2008 financial crisis.

procedure requires the invertibility of the shock estimated in the first step, that is, the data should be informationally sufficient for the demand shock. This implies that the shock of interest can be obtained as a simple linear combination of current and past data, as in a standard linear VAR where the nonlinear term is omitted, although the true model is a VARX. In other words, the nonlinear term in the model is not needed to estimate the demand shock. However, in a nonlinear context such as this, invertibility is a demanding property.

The novelty is that we implement this nonlinear setting in a large-dimensional Structural Dynamic Factor model, as introduced by Forni et al. (2009) and Stock and Watson (2005). Our positive argument is that large factor models, as shown in Forni et al. (2009), are generally unaffected by non-invertibility issues. Typically, the vector of the factors is singular, meaning it is driven by a number of common shocks that is much smaller than its dimension. This holds true even if one or more nonlinear functions are included among the sources of variation. In such cases, achieving invertibility becomes easier as it satisfies a less demandig condition. In the empirical application, we use a dataset of 114 quarterly US time series, covering the period 1961:Q1 to 2019:Q4.

In a nutshell, our results confirm the initial hypothesis of asymmetric—or nonlineartransmission of demand shocks. The sign of the shock, whether positive or negative, is a determinant of the dynamics and persistence of the effects. Specifically, negative shocks appear to be more persistent than positive ones: a contractionary demand shock leads to a permanent decline in output, investment and employment, proving to be an important driver of labor market fluctuations, at all horizons. Conversely, in line with conventional wisdom, positive demand shocks, display temporary effects on all real variables: over the long run, the economy revert to its equilibrium path once the shock loses its intensity. In line with Furlanetto et al. (2024) we find that hysteresis transmits through an increase in long-term unemployment. Contrary to their findings, we attribute a minor, yet not negligible, role to demand shocks in explaining long-term economic fluctuations.

The remainder of the paper is organized as follows. Section 2 outlines discusses the econometric approach, including the identification of the shock and the estimation of nonlinear effects. Section 3 presents the results. Section 4 concludes.

3.2. Econometric Approach

In this section, we present our empirical model and show how to estimate the nonlinear effects of demand shocks on the common components of macroeconomic variables.

3.2.1. Nonlinear Structural Dynamic Factor Model

Let x_t be a *n*-dimensional vector of stationary macroeconomic variables. The vector x_t is part of an infinite-dimensional panel of time series. Each variable x_{it} , i = 1, ..., n, is decomposed into the sum of two mutually orthogonal unobservable components, the common component, χ_{it} , and the idiosyncratic component, ξ_{it} . The ξ 's are interpreted

as sources of variation that are specific to one or just a small group of variables, like sectoral shocks or measurement error. In particular, for macroeconomic variables like GDP, investment or consumption, in which all local and sectoral shocks have been averaged out, the idiosyncratic part can be interpreted essentially as only containing measurement error. They are poorly correlated in the cross-sectional dimension (see Forni et al., 2009, Assumption 5). The common components, on the contrary, account for the bulk of the co-movements between macroeconomic variables. This is because they are different linear combinations of the same r < n common factors, $F_t = (F_{1t} \dots F_{rt})'$, not depending on *i* (see Stock and Watson, 2002a,b; Bai and Ng, 2002):

$$\chi_{it} = \lambda_{i1}F_{1t} + \ldots + \lambda_{ir}F_{rt} \quad \text{or} \quad \chi_t = \Lambda F_t. \tag{3.1}$$

The entries of F_t are called static factors and Λ is a $n \times r$ matrix of factor loadings. We require the factors to be pervasive i.e. to have non-negligible effects on most of the variables x_{it} (see Forni et al., 2009, Assumption 4). Equation (3.1) is the static factor representation, where the factors have only contemporaneous effect on the common components.

The dynamic nature of the model comes from the fact that static factors have the following Moving Average representation, augmented with a nonlinear function of the demand shock.

Structural representation. The *r*-dimensional vector F_t follow the singular MA process

$$F_t = \Psi(L)u_t \tag{3.2a}$$

$$F_t = \Gamma(L)v_t + \gamma(L)u_t^d + \beta(L)g(u_t^d)$$
(3.2b)

where $u_t = [v_t' \ u_t^d \ g(u_t^d)]'$ is a q-dimensional vector of common shocks, with q < r, so that the vector F_t is singular, and $\Psi(L) = [\Gamma(L) \ \gamma(L) \ \beta(L)]$ is an $r \times q$ matrix of rational IRFs with maximum rank $q < r \ll n$.

In equation (3.2b), we derive an equivalent representation by splitting the linear terms from the nonlinear one; below we further develop this representation to obtain the VARX to be estimated.

The vector u_t^d is the demand shock, $g(u_t^d)$ is a nonlinear function of the demand shock, and v_t is a *m*-dimensional vector of structural shocks other than the demand shock, with m < q. We further assume that the vector $[v_t' u_t^d]'$ is i.i.d. zero mean, with identity covariance matrix. The serial and mutual independence assumption implies that all structural shocks, including u_t^d , are uncorrelated with the lags of $g(u_t^d)$ and F_t . The vector $\gamma(L) = \gamma_0 + \gamma_1 L + \gamma_2 L^2 + \ldots$ is the column of $\Psi(L)$ including the IRFs to the demand shock; the vector $\beta(L) = \beta_0 + \beta_1 L + \beta_2 L^2 + \ldots$ includes the IRFs to the nonlinear function of the demand shock. Finally, $\Gamma(L) = \Gamma_0 + \Gamma_1 L + \Gamma_2 L^2 + \ldots$ is a $r \times m$ matrix of IRFs to the remaining structural shocks, with maximum rank m < q.

Equations (3.2a) and (3.2b) are different way to write the same singular MA representation, augmented with a non linear function of the demand shock, which in our application takes the functional form $g(u_t^d) = |u_t^d|$.

Combining the equations (3.1) and (3.2b) it can be seen that the common components themselves have the following structural dynamic representation

$$\chi_t = \Lambda[\Gamma(L)v_t + \gamma(L)u_t^d + \beta(L)g(u_t^d)]$$

or equivalently

$$\chi_t = \Gamma^*(L)v_t + \gamma^*(L)u_t^d + \beta^*(L)g(u_t^d)$$
(3.3)

The total effects of the demand shock $u_t^d = \bar{u}^d$ is nonlinear and can be found by combining the two terms $\gamma^*(L)$ and $\beta^*(L)$ as

$$IRF(u_t^d = \bar{u}^d) = \gamma^*(L)u_t^d + \beta^*(L)g(u_t^d).$$
(3.4)

We now further develop the model and derive the VARX representation, which is the one we will estimate.

In this sense, the singularity has very important consequences. Indeed, in singular systems, Anderson and Deistler (2008) show that, under mild assumptions,⁵ a finite VAR representation for the factors always exists.

Let us first consider equation (3.2a). Inverting the matrix $\Psi(L)$ we obtain the following VAR representation.

VAR Representation. The *r*-dimensional vector F_t admits the finite-order VAR representation

$$A(L)F_t = e_t$$

$$e_t = Ru_t$$
(3.5)

where A(L) is an $r \times r$, stable polynomial matrix and R is $r \times q$ and has maximum rank q. As a consequence, R has a left inverse and the vector u_t belongs to space spanned by current and past values of the vector F_t , that is, u_t is fundamental for F_t . Therefore, if an estimate of F_t is available, the shocks in the vector u_t can be estimated by means of a singular VAR for F_t .

Let us now consider equation (3.2b). Inverting only the linear term $\Gamma(L)$ we obtain the following VARX representation.

VARX Representation. The *r*-dimensional vector F_t admits the finite-order VARX representation

$$C(L)F_t = \epsilon_t + C(L)\gamma(L)u_t^d + C(L)\beta(L)g(u_t^d)$$

$$\epsilon_t = Sv_t$$
(3.6)

or equivalently

$$F_t = \tilde{C}(L)F_{t-1} + \tilde{\gamma}(L)u_t^d + \tilde{\beta}(L)g(u_t^d) + \epsilon_t$$
(3.7)

⁵The basic assumption is that the entries of $\Psi(z)$, or $\Gamma(z)$, are rational functions in the complex variable z and $\Psi(z)$, or $\Gamma(z)$, is zeroless, i.e. it has maximum rank q, orm, everywhere in the complex plane.

Model (3.7) is a VARX where the demand shock and its nonlinear function are the exogenous variables.

Note that if $\hat{\beta}(L) = 0$, the structural representation (3.2b) reduces to a linear model. We can test for linearity by testing either for the null of $\hat{\beta}(L) = 0$ in equation (3.7) or for the null of $\beta^*(L) = 0$ in the IRFs (3.4).

3.2.2. Identification and estimation

The estimation of the demand shock's total effects follows a two-step procedure. In the first step, an estimate of the demand shock is obtained. In the second step, the nonlinear effects of the shock are obtained using the estimated shock and its nonlinear function as regressors in equation (3.7).

STEP I. The demand shock is estimated using the FGGSS's frequency domain strategy. Let us briefly recall their approach. First, they obtain the two reduced form shocks that maximize the explained variance of key macroeconomic aggregates at frequencies larger than 18 months, thus excluding fluctuations of little interest for macroeconomic analysis. The authors show that q = 2 shocks are sufficient to explain most of the business cycle and long-run fluctuations in real activity. Once these two shocks have been obtained, they perform a rotation to provide structural identification to the shocks: a demand and a supply shock. The demand shock is obtained by maximizing the covariance between inflation and GDP growth at business cycle frequencies, with the supply shock then being "automatically" identified as the shock that minimizes such covariance.

As in that paper, here the identification is implemented in a large-dimensional SDFM. Therefore, in order to estimate the shock and move to the second stage, we first set a value for the number r of the static factors, using the criterion by Bai and Ng (2002) with the penalty modification proposed in Alessi et al. (2010), finding a number of static factors $\hat{r} = 11$. The static factors $F_t = (F_{1t} \dots F_{rt})'$ are estimated by the first \hat{r} principal components of the variables in our dataset, and the factor loadings, λ_{ij} , $j = 1 \dots r$, by the associated eigenvectors.⁶ The estimated common component vector is given by $\hat{\chi}_t = \hat{\Lambda} \hat{F}_t$.

STEP II. We use the estimates of the shock and its nonlinear functions, \hat{u}_t^d and $g(\hat{u}_t^d) = |\hat{u}_t^d|$, as regressors in model (3.7), obtaining $\widehat{C(L)}$, $\widehat{\gamma(L)}$ and $\widehat{\beta(L)}$. Finally, one can estimate the IRFs of the linear and nonlinear terms as $\widehat{\gamma^*(L)} = \widehat{\Lambda \widehat{\gamma(L)}}$ and $\widehat{\gamma^*(L)} = \widehat{\Lambda \widehat{\gamma(L)}}$. The total effects are obtained from equation (3.4).

Note also that, having a narrative measure of the shock of interest, one could adapt the nonlinear external Proxy-SVAR approach presented in Debortoli et al. (2023), to a large SDFM framework. In that paper, the authors show that if the VAR's variables are informationally sufficient for the shock of interest, the shock itself can be consistently

⁶Thus, the estimated loading matrix, $\hat{\Lambda}$, is the $n \times \hat{r}$ matrix having on the columns the normalized eigenvectors corresponding to the \hat{r} -largest eigenvalues of the sample covariance matrix of the data, $\hat{\Sigma}_x$.

estimated as the fitted value of the regression of the instrument onto the residuals of a standard linear VAR. Once an estimate of the shock is available, the VARX and the implied nonlinear IRFs can be estimated. Again, the condition of informational sufficiency is more easily satisfied in a large SDFM. This, however, is left for future research.

3.3. Empirical Analysis

In this section we present our main empirical results about the nonlinear transmission of demand shocks. In the empirical application, we use a dataset of 114 quarterly US time series, covering the period 1961:Q1 to 2019:Q4. A description of the data with the complete list of variables and transformations is provided in Appendix 3.A. Having a large dataset at our disposal, another advantage of the SDFM is that we can study the IRFs of all relevant macroeconomic variables within a unified framework.⁷ We set $g(u_t^d) = |u_t^d|$ to accommodate sign asymmetries and use four lags.

3.3.1. Are nonlinearities important?

Figure A.1 displays the Impulse Response Functions for both the linear and nonlinear models, obtained using the absolute value of the demand shock as the relevant nonlinear function. The first column compares the linear responses to the demand shock u_t^d , $\gamma^*(L)$, from the nonlinear model with those obtained from a standard linear model, while the second column reports the responses to the nonlinear term $|u_t^d|$, $\beta^*(L)$. The black solid lines and gray areas are the point estimates and the 90% confidence bands for the nonlinear model, respectively. The cyan dotted lines are the point estimates for the linear model, which do not take into account any nonlinear terms.

First of all, the responses to the linear term in the nonlinear setting closely mirror those obtained from the linear model itself. This confirms that the identification in the nonlinear model is the same as in the linear one and that the estimated demand shock is indeed a linear combination of the present and past variables, although the true model is nonlinear. In other words, in the first step, the nonlinear term of the model is not needed to estimate the demand shock.

The responses to the linear term (first column) are in line with previous findings (see FGGSS). Although there appear to be some positive long-term effects, especially in

⁷The analysis focuses on a subset of 12 macroeconomic series of interest: (1) the log difference of the real per capita GDP; (2) the log difference of real per capita consumption, defined as the sum of non-durable consumption and services; (3) the log difference of real per capita investment, computed as the sum of fixed investment and durable consumption; (4) the unemployment rate, (5) the log of real per capita hours worked; (6) the inflation rate, defined as the log difference of the GDP deflator; (7) labour productivity; (8) the first difference of employment-to-population ratio; (9) the long-term unemployment rate (for more than 27 weeks of unemployment); (10) the cumulated sum of the utility-adjusted total factor productivity; (11) the Federal Funds rate and the (12) the risk spread between Moody's Baa Corporate Bond Yeald and the 10-Year Treasury Constant Maturity Rate.

the employment-to-population ratio, these are never statistically significant, or barely so. The responses of output and investment are temporary and hump-shaped, peaking one year after the shock. The effects are no longer statistically significant after about two years. GDP has a positive impact effect of 0.5% and a peak of about 0.9%. Unemployment rates behave countercyclically, reaching a minimum of about -0.2%. The effect on long-term unemployment appears to be slightly more persistent. The shock generates a positive comovements between the inflation rate and output growth.

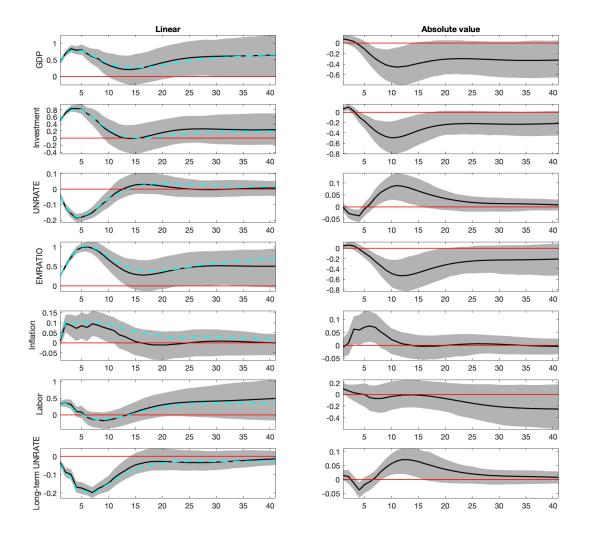


Figure A.1: Linear and nonlinear impulse responses to a demand shock u_t^d and to its absolute value $g(u_t^d) = |u_t^d|$ using a nonlinear model. Solid lines represent point estimates, the gray areas are the 90% confidence bands. The cyan dotted lines are the impulse responses to a demand shock estimated in the linear model.

Let us now focus on the responses to the nonlinear function of the demand shock (second column). Although the impact effects are almost zero, we can see that the absolute value has significant and lasting effects on all variables except labor productivity and, to a lesser extent, the inflation rate. This observation leads us to reject the null hypothesis of linearity, suggesting that the effects of demand shocks in the data are asymmetric, depending on their sign. We observe a persistent decline in output, investment, and employment, along with an increase in the unemployment rates. The negative effects are particularly strong on the employment-to-population ratio. This result suggests that the nonlinear term slightly dampens the short run expansionary effects of large demand shocks, while it amplifies the contractionary effects, in terms of both magnitude and, more importantly, duration. As we will see better in the next subsection, when the nonlinear part is taken into account, the effects of negative demand shocks on these variables are more persistent than the conventional view suggest. Conversely, in the case of prices, the nonlinear component modestly enhances the inflationary effects of expansionary shocks while softening the deflationary impacts of contractionary ones. Figure A.3 plots the demand shock (top panel) estimated in the first step and its absolute value (bottom panel).

Overall, the results point towards significant nonlinear effects of demand shocks: the sign of the shock - whether positive or negative - it is a crucial determinant of the dynamics and duration of the effects.

3.3.2. Asymmetric transmission of demand shocks

Let us now turn our attention to the cumulative effects of demand shocks by summing the linear and nonlinear terms, $\gamma(L)u_t^d$ and $\beta(L)|u_t^d|$, respectively, the individual effects of which have been previously discussed. Figure A.2 plots the IRFs of positive (second column) and negative (first column) demand shocks. The black solid lines and gray areas are the point estimates and the 90% confidence bands, respectively.

Our hypothesis is confirmed: negative demand shocks may indeed have more persistent and difficult-to-reverse effects compared to positive ones. While it can be seen that expansionary shocks are transitory, in line with conventional business cycle theory, we find that contractionary shocks produce persistent effects. A negative shock leads to a permanent decline in output and employment. GDP growth decreases immediately by around -0.4% and converges to -1.2% in the long run. Employment-to-population follows a similar patter, reaching a minimum of about 1.3% and converging to -0.8% at horizon 40. A negative shock has effects on employment that are not only more persistent but also larger than those of a positive shock. Hysteresis thus appears to be transmitted largely through employment.

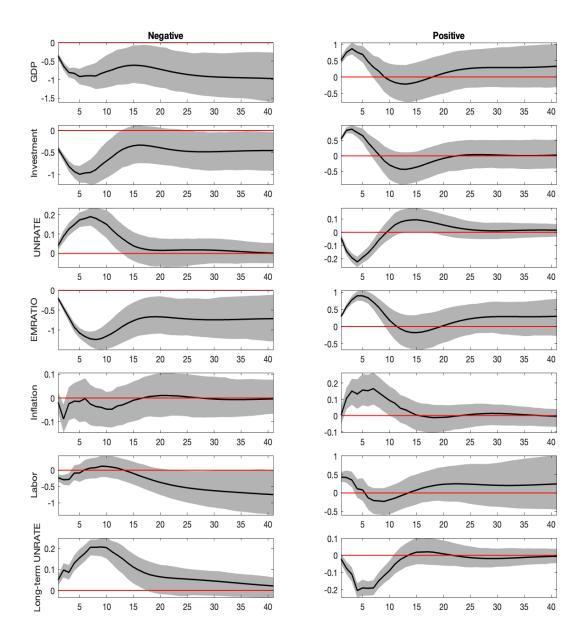


Figure A.2: Nonlinear impulse responses to a negative (first column) and a positive (second column) demand shock. Solid lines represent point estimates, the dark gray areas are the 90% confidence bands.

A few interesting results stand out. Negative demand shocks have a more lasting impact on long-term unemployment than positive shocks do. Furthermore, it is observed that after the economy experiencing a negative shock, although the overall unemployment rate begins to recover after approximately two years, the response of long-term unemployment remains pronounced for a significantly longer period, and is also marginally more severe. This aligns with Furlanetto et al. (2024)'s findings, where hysteresis in employment manifests through an increase in long-term unemployment and a decline in labor market participation (see also Blanchard, 2018).

We also found that investment declines permanently in response to contractionary shocks, converging to -0.5% (the entire 90 percent confidence band is below the zero line after the 25th quarter). The above result pairs well with the theoretical channel proposed by Benigno and Fornaro (2018). While Furlanetto et al. (2024) states that labor productivity, measured as output per worker, is barely affected by demand shocks, our results suggest that there may be a negative response of labor productivity in the long run. A persistent decline in investment, especially if in R&D, can slow labor productivity in the long run, potentially leading to a permanent impact on GDP (Anzoategui et al., 2019). However, this long-run effect on productivity is not statistically significant (the entire 90 percent confidence band is below the zero line after the 4th quarter), so it is to be taken with caution.

Finally, contractionary shocks lead to a modest and very short lived effect on inflation, while positive shocks lead to a much larger response. A possible explanation is that prices tend to be "stickier" downwards in response to negative demand shocks because of factors like "menu costs", wage rigidity and the unwillingness of firms to enter into price wars.

We can conclude that demand shocks can induce hysteresis effects, provided that asymmetries in the transmission of shocks are taken into account. Figures A.4 and A.5 present the outcomes of our analysis for other significant macroeconomic and financial indicators. It is evident that negative demand shocks exert permanent impacts on consumption and a strongly persistent effect on per capita hours worked, whereas long-term Total Factor Productivity (TFP) remains largely unaffected.

At this point, a natural question is how relevant demand shocks and their nonlinearities are in explaining long-term fluctuations in macoeconomic variables.

Table 3.1 reports, for each variable, the percentage of variance explained by both the linear and nonlinear terms, at business cycle (upper panel) and long-run (lower panel) frequencies. At long-run frequencies, the demand shock, including both the linear and nonlinear terms, accounts for approximately 19%, 23%, and 21% of the variance in GDP, investment, and the unemployment rate, respectively. More importantly, it accounts for approximately 26% of the long-run variance in the employmentto-population ratio and a significant 36% in that of hours worked per capita and the long-term unemployment rate. These numbers highlight a more significant role of the demand shock for real activity and labor market long-run dynamics, than is typically observed over the sample period from 1961:Q1 to 2019:Q4, in linear analyses.

However, these results are quantitatively far from those reported in Furlanetto et al. (2024). The differences can be probably attributed to the different sample used in the analysis. In their baseline exercise, the sample used is shorter than ours. With a longer sample, their results are become more in line with ours. Finally, it should be noted that the nonlinear term also explains a non-negligible fraction of the inflation business cycle variance (about 11 percent).

VARIANCE I	Decomposi	TION			
Percentage of Explaine	Percentage of Explained Business-Cycle Variance				
	LINEAR	Nonlinear	Sum		
GDP	58.1	7.9	65.9		
Consumption	31.8	13.0	44.8		
Investment	54.3	8.7	63.0		
UNRATE	47.7	10.4	58.1		
EMRATIO	50.8	7.7	58.5		
Inflation	17.6	10.8	28.4		
Labor	42.5	2.8	45.3		
Federal Funds Rate	45.2	9.1	54.2		
Hours Worked Per Capita	39.4	6.6	46.0		
Baa-GS10 Spread	54.6	3.4	57.9		
Long-term UNRATE	49.3	8.2	57.5		
TFP	6.5	1.5	8.0		
Percentage of Expla	ined Long	-Run Variano	CE		
	LINEAR	Nonlinear	Sum		
GDP	11.7	6.9	18.5		
Consumption	6.3	9.5	15.8		
Investment	14.4	8.7	23.1		
UNRATE	14.3	7.0	21.3		
EMRATIO	17.5	8.5	26.0		
Inflation	6.6	1.9	8.5		
Labor	12.1	2.7	14.9		
Federal Funds Rate	30.0	1.0	31.0		
Hours Worked Per Capita	30.3	5.2	35.5		
Baa-GS10 Spread	19.5	5.3	24.8		
Long-term UNRATE	31.0	5.5	36.4		
TFP	4.5	5.3	9.8		

Table 3.1: PERCENTAGE OF VARIANCE EXPLAINED BY THE DEMAND SHOCK IN THE NON-LINEAR MODEL, BY FREQUENCY BANDS.

In conclusion, the variance decomposition confirms the critical role of the nonlinear term and underscores the importance of nonlinearities in the transmission of demand shocks.

3.4. Concluding remarks

We have shown that when sign dependence is taken into account, a nonlinear model detects demand-driven hysteresis where a linear model fails to do so. This finding suggests hysteresis is deeply connected to the asymmetric effects of positive and negative demand shocks on the economy: the former leading to temporary fluctuations, while the latter can cause lasting structural changes. In this way, we reconcile the conventional wisdom with the perspective on hysteresis, revealing the long-run non-neutrality of negative demand shocks. This nonlinearity is obtained through the application of the econometric procedure developed by Forni et al. (2023). This procedure relies on estimating a Moving Average (MA) representation that incorporates a nonlinear function—the absolute value—of the demand shock. The demand shock is identified following the FGGSS approach. A second significant contribution of our work is the use of this nonlinear framework within a large-dimensional Dynamic Factor Model

(SDFM). What about the transmission mechanism of hysteresis? Following a negative demand shock, hysteresis appears to propagate in real activity through its effect on employment. Among the various transmission channels of hysteresis, long-term unemployment stands out as a prominent candidate. Contractionary shocks lead to job losses that are not recovered even when economic conditions improve. Once workers lose their jobs during recessions, it becomes more difficult for them to re-enter the job market. This may be due to skill erosion, the stigmatization of long-term unemployment, or structural changes in the labor market that make some jobs obsolete. All of these generate potential scarring effects, making a cyclical shock persistent or even permanent. Therefore, despite employing different assumptions and methodologies, our model yields results qualitatively consistent with those in Furlanetto et al. (2024). According to that paper, hysteresis primarily propagates through employment, while labor productivity is minimally affected. Although, our results reveal a negative and apparently persistent response of labor productivity to negative demand shocks, this is hardly statistically different from zero in the long run.

To conclude, negative demand shocks may indeed have more persistent and difficultto-reverse effects compared to positive ones. The findings highlight the critical need for macroeconomic policies to account for the directionality and lasting impact of demand shocks to mitigate long-term economic downturns.

FIGURES

FIGURES

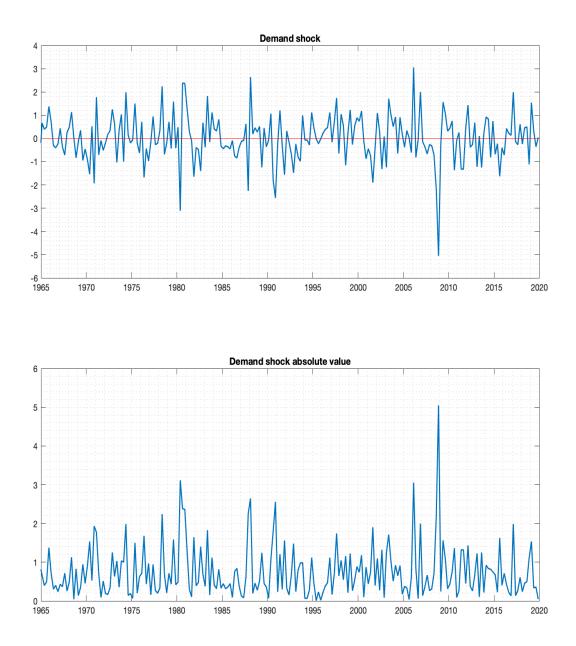


Figure A.3: Demand shock and its absolute value obtained from the first step.

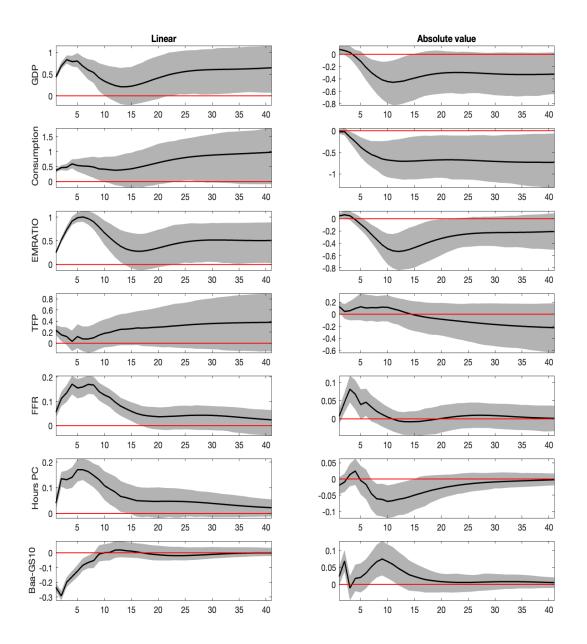


Figure A.4: Linear and nonlinear impulse responses to a demand shock u_t^d and to its absolute value $g(u_t^d) = |u_t^d|$. Solid lines represent point estimates, the gray areas are the 90% confidence bands. The cyan dotted lines are the impulse responses to a demand shock estimated in the linear model.

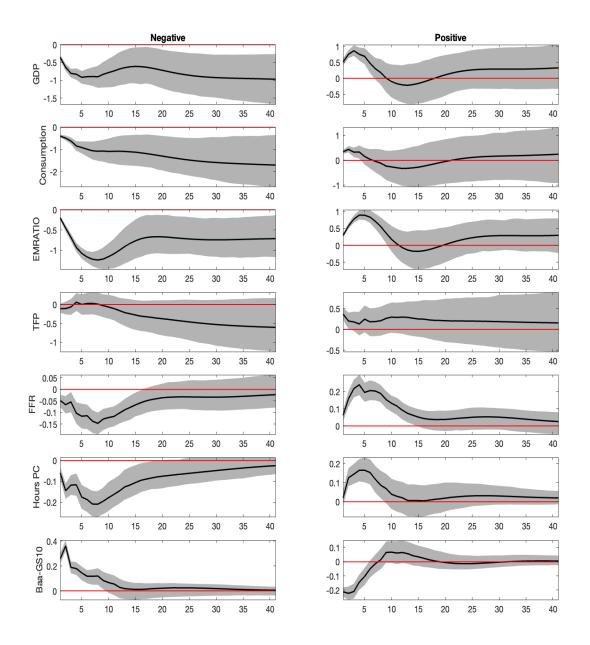


Figure A.5: Nonlinear impulse responses to a negative (first column) and a positive (second column) demand shock. Solid lines represent point estimates, the dark gray areas are the 90% confidence bands.

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Appendix

3.A. DATA DESCRIPTION AND DATA TREATMENT

The $N \times T$ dataset is made up of 114 US quarterly series, covering the period 1961-I to 2019-IV. Most series are from the FRED-QD database.⁸ TFP data series are from John Fernald's website (Fernald, 2012) while the Confidence data are available on the Michigan survey of consumer website.⁹ Following standard practice, consumption includes non-durables and services, while investment has been broadly defined to include consumer durables. Both measures are deflated. Monthly data, like the macroeconomic uncertainty measure estimated by Jurado et al. (2015), have been aggregated to get quarterly figures. Finally, it is worth noting that most series are expressed in per capita terms, dividing by population aged 16 years or more (civilian non-institutional population series) and stock market data have been deflated by the GDP deflator. We transform each series to reach stationarity. The complete list of variables and transformations is given in the table below.

For the description of each variable see McCracken and Ng (2020). For variables not in the FRED-QD dataset, refer to the Mnemonic and note. Treatment codes: 1 =no treatment; 2 = first difference, Δx_t ; $4 = \log(x_t)$; $5 = \log$ of the first difference, $\Delta \log(x_t)$.

ID	FRED-QD ID	Mnemonic	Treatment code	Note
1	1	GDPC1/CNP16OV	5	
2	2	PCECC96/CNP16OV	5	
3	3	PCDGx/CNP16OV	5	
4	4	PCESVx/CNP16OV	5	
5	5	PCNDx/CNP16OV	5	
6	6	GPDIC1/CNP16OV	5	
7	7	FPIx/CNP16OV	5	
8	8	Y033RC1Q027SBEAx/CNP16OV	5	
9	9	PNFIx/CNP16OV	5	
10	10	PRFIx/CNP16OV	5	
11	11	A014RE1Q156NBEA	1	
12	12	GCEC1/CNP16OV	5	
13	13	A823RL1Q225SBEA	1	
14	14	FGRECPTx/CNP16OV	5	
15	15	SLCEx/CNP16OV	5	
16	16	EXPGSC1/CNP16OV	5	
17	17	IMPGSC1/CNP16OV	5	
18	18	DPIC96/CNP16OV	5	
19	19	OUTNFB/CNP16OV	5	
20	20	OUTBS/CNP16OV	5	
21		(PCESVx+PCNDx)/CNP16OV	5	
22		(PCDGx+FPIx)/CNP16OV	5	
23	22	INDPRO/CNP16OV	5	
24	23	IPFINAL/CNP16OV	5	
25	24	IPCONGD/CNP16OV	5	
26	25	IPMAT/CNP16OV	5	
27	28	IPDCONGD/CNP16OV	5	
28	30	IPNCONGD/CNP16OV	5	
29	31	IPBUSEQ/CNP16OV	5	
30	35	PAYEMS/CNP16OV	2	
31	36	USPRIV/CNP16OV	2	
32	38	SRVPRD/CNP16OV	2	

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 $^{^{8}\}mathrm{The}$ FRED-QD is a large (248 series) quarterly macroeconomic database developed by McCracken and Ng (2020).

⁹http://www.sca.isr.umich.edu/

APPENDIX

ID	FRED-QD ID	MNEMONIC	Treatment Code	Note
33	39	USGOOD/CNP16OV	2	
34	51	USGOVT/CNP16OV	2	
35	57	CE16OV/CNP16OV (EMRATIO)	2	
36	58	CIVPART	2	
37	59	UNRATE	1	
38	60	UNRATESTx	1	
39	61	UNRATELTx	1	
40	62	LNS14000012	1	
41	$63 \\ 64$	LNS14000025	1	
$\frac{42}{43}$	04 74	LNS14000026 HOABS/CNP16OV	1 4	
44	76	HOANBS/CNP16OV	4	
45	77	AWHMAN	1	
46	79	AWOTMAN	1	
47	81	HOUST/CNP160V	5	
48	95	PCECTPI	5	
49	96	PCEPILFE	5	
50		GDPDEF	5	GDP: Implicit Price Deflator
51	97	GDPCTPI	5	
52	98	GPDICTPI	5	
53	120	CPIAUCSL	5	
54	121	CPILFESL	5	
55 56	122	WPSFD49207	5	
56 57	123	PPIACO WPSED40502	5	
$57 \\ 58$	124 126	WPSFD49502 PPIIDC	5 5	
58 59	120	WPU0561	5	
60	129	OILPRICEx	5	
61	135	COMPRNFB	5	
62	138	OPHNFB	5	
63	139	OPHPBS	5	
64	140	ULCBS	5	
65	142	ULCNFB	5	
66	143	UNLPNBS	5	
67		dtfp	1	Fernald's TFP growth
68		dtfp util	1	Fernald's TFP growth CU adjusted
69		dtfp I	1	Fernald's TFP growth - Inv
70		dtfp C	1	Fernald's TFP growth - Con
71		dtfp I util	1	Fernald's TFP growth CU - Inv
72		dtfp C util	1	Fernald's TFP growth CU - Con
73	144	FEDFUNDS	1	
74	145	TB3MS	1	
75	146	TB6MS	1	
76	147	GS1	1	
77 78	148 150	GS10 AAA	1	
79	151	BAA	1	
80	152	BAA10YM	1	
81	156	GS10TB3Mx	1	
82	100	BAA-AAA	1	
83		GS10-FEDFUNDS	1	
84		GS1-FEDFUNDS	1	
85		BAA-FEDFUNDS	1	
86	158	BOGMBASEREALx/CNP16OV	5	
87	160	M1REAL/CNP16OV	5	
88	161	M2REAL/CNP16OV	5	
89	163	BUSLOANSx/CNP16OV	5	
90	164	CONSUMERx/CNP16OV	5	
91	166	REALLNx/CNP16OV	5	
92 02	168	TOTALSLx/CNP16OV	5	
93 04	188	UMCSENTx Business Condition 12 Months	1	Mishiman Commune C
94 05		Business Condition 12 Months Business Condition 5 Years	1	Michigan Consumer Survey
$95 \\ 96$		Current Index	1	Michigan Consumer Survey Michigan Consumer Survey
90 97		Expected Index	1	Michigan Consumer Survey
98		News Index: Relative	1	Michigan Consumer Survey
99	197	UEMPMEAN	1	
100	201	GS5	1	
101	210	CUSR0000SAC	5	
102	211	CUSR0000SAD	5	
103	212	CUSR0000SAS	5	
104	213	CPIULFSL	5	
105	245	S&P 500	5	
106	246	S&P: indust	5	
107		S&P 500/GDPDEF	5	
108		S&P: indust/GDPDEF	5	
109		JLN Macro Unc 1-month	1	Jurado, Ludvigson and Ng Uncertaint
110		JLN Macro Unc 3-month	1	Jurado, Ludvigson and Ng Uncertaint

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APPENDIX

ID	FRED-QD ID	Mnemonic	Treatment Code	Note
112		DPCCRC1Q027SBEAx/CNP16OV	5	Real PCE Excluding food and energy
113		DFXARC1M027SBEAx/CNP16OV	5	Real PCE: Food
114		$\rm DNRGRC1Q027SBEAx/CNP16OV$	5	Real PCE: Energy goods and services

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