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Rating Systems and Procyclicality:
an Evaluation in a General
Equilibrium Framework

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Rating systems and procyclicality: an evaluation in a general equilibrium framework

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Abstract

The introduction of Basel II has raised concerns about the possible impact of risk-sensitive capital requirement on the business cycle. Several approaches have been proposed to deal with the procyclicality issue. In this paper we take a general equilibrium one, which is an appropriate framework for a comprehensive analysis of different proposals since it allows to account for banks' endogenous strategies in relation to the other agents' behaviour. The aim of the present paper is to set up a model which allows to evaluate different rating systems in relation to the procyclicality issue. Our set up extends previous models so as to allow the analysis of both the effects of different rating systems on banks' portfolios (as e.g. in Catarineu-Rabell *et al.* 2005) and the contagion effetcs relevant to financial stability (as e.g. in Goodhart *et al.* 2005). The paper presents a comparative statics analysis evluating a cycle-dependent and a neutral rating system with main focus on the banks' point of view.

Keywords: rating, procyclicality, Basel II, general equilibrium.
JEL: D52, E4, E5, G1,G2

1 Introduction

The introduction of Basel II has raised concerns about the possible impact of risk-sensitive capital requirement on the real economy: since it is widely recognised that credit risk factors are affected by economic conditions (see e.g. Bangia *et al.* 2002, and Nickell *et al.* 2000), risk sensitive capital requirements fluctuate over the business cycle, possibly causing an amplification of the same through the lending channel. Concerns about procyclicality have been expressed by several authors (e.g. Danielsson *et al.* 2001, Borio *et al.* 2001, Repullo and Suarez 2007). In particular, the concerns focus on the possible exacerbation of recessions due to credit shortage[1].

Most studies on financial stability and procyclicality in connection with Basel II have focused on the role of the rating system: the debate has basically developed around the comparison between *through the cycle* (ttc) versus *point in time* (pit) rating systems and several proposes have been done to deal with the procyclicality of the capital requirement (e.g. Kashyap and Stein (2004), Pederzoli and Torricelli (2005), Gordy and Howells (2006)). Gordy and Howells (2006) define three possible ways to deal with procyclicality under the IRB Basel framework. The forner two suggest either smoothing the input to the capital function (mainly adopting a *ttc* rating philosophy) or flattening the capital function itself (by reducing the sensitivity to the default probability). These two approaches have partly been followed in moving from the first to the last version of the document by the Basel Committee on Banking Supervision (see BCBS 2006). However they both lead to a loss in risk sensitivity, since they play on the trade-off between risk-sensitivity and prociclicality. In Gordy and Howells (2006) a the third approach is advocated, which consists in smoothing the output of the capital function for regulatory purpose and allows to avoid losses in terms of transparency. Pederzoli and Torricelli (2005) propose an alternative approach where a business cycle forecast is included into the estimation of default probabilities: this is consistent with the views expressed by Borio *et al.*(2001) that risk is built up during expansions and the high default rates observed during recessions are just a materialization of that risk. According to this view, a risk measure should be high in expansion before a recession and low at the bottom of the cycle in anticipation of an expansion. Hence a capital requirement based on such a risk measure would decline at the bottom of the cycle helping the economy out of the recession.

The trade-off between procyclicality and risk-sensitivity of capital

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[1]Recent papers stress the importance of setting up an appropriate framework for monitoring procyclicality, e.g. Masschelein (2007).

requirements, which emerges from the dependence of credit risk factors on the business cycle, needs to be analysed carefully. A few papers have provided an evaluation of the different rating proposals in terms of procyclicality. Gordy and Howells (2006) compare different rating philosophies by considering banks with different exogenous investment strategies and thus disregard the feedback impact of capital requirement on banks lending behaviour and hence on the economy. In order to overcome this limitation, a general equilibrium model is the appropriate framework for a comprehensive analysis of the procyclicality issue. Catarineu-Rabell *et al.* (2005) analyse the procyclicality issue by comparing different rating systems from the banks' profitability point of view: within a 2-periods, 2-states GE model with one bank, one corporate and one household, they find that banks would prefer *pit* rather than *ttc* rating system, with dangerous consequences in terms of procyclicality. However they do not account for the heterogeneity of agents, while we think this is an important feature in this context. The model used in Catarineu-Rabell *et al.* (2005) stems from the general theoretical model presented in Tsomocos (2003) and Goodhart *et al.*(2006). Goodhart *et al.*(2004) presents a smaller version of the original model which allows to obtain numerical solution while Goodhart *et al.* (2005) simplify the non-banking agents problems to reduced-form equations in order to perform a calibration agains real UK banking data. In sum, models proposed so far in the literature do not allow consideration of both the effects of different rating systems on banks'portfolios (as e.g. Catarineu-Rabell *et al.* 2005) and contagion effects relevant to financial stability (as e.g. Goodhart *et al.* 2005).

The purpose of this paper is to build on these models in order to set up a framework to evaluate different rating proposals in conjunction with the procyclicality issue.

The structure of the paper is as follows. In the following Section we present the agents' optimization problems. Section 3 presents the model solution (i.e. the initial equilibrium) and Section 4 is devoted to the evaluation of different rating systems by means of comparative statics analysis. Last Section concludes.

2   The model

The model we propose is a general equilibrium model of an exchange economy with money and bank. Within the structure of the general model in Tsomocos (2003) we simplify and conisder three sectors: the banking sector, the corporate sector and the household. The model includes corporate loans, interbank, and deposit markets; moreover, corporates and household trade perishable goods on the commodity market.

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While banks maximize a function of their profit, corporate and households aim at maximizing the intertemporal consumption.The model is characterized by heterogeneity of the agents, limited participation of banks in corporate and household and endogenous default. The structure of the model is synthetised in the following points:

1) Two periods t=0,1

2) Two states of the world in t=1: s=E (expansion) or R (recession)

3) Two banks: bank γ is a net borrower in the interbank market; bank δ is a net lender in the interbank market; both banks maximize expected utility of profits in t=1 subject to penalties on default and capital requirement constraints;

4) Two corporates α, β : each corporate borrows from a single bank (limited participation hypothesis) and maximizes expected utility of consumption minus a default penalty;

5) One household ϕ : the household deposits in a single bank and maximizes expected utility of consumption;

6) Central bank/regulator exogenously define: money supply M, minimum capital requirement in terms of capital ratio K (such as the 8% imposed by Basel II), default penalties λ.

The optimization problem for the single agents are presented in the following subsections.

2.1 Banks' optimization problem

There are two simultaneous optimization problems for the two heterogeneous banks. Both banks maximize the expected utility from profits and are subject to a capital constraint in the Basel II style. The difference between the two problems lies in the different role the banks play on the interbank and deposit market.

In order to formalize the problem for bank γ, let the relevant variables be defined as follows.

p_s= probability of state s;

m_α^γ = loan extension to corporate α;

μ^γ = money borrowed on the interbank market;

v_s^γ = repayment rate of bank γ in state s;

v_s^α = repayment rate of corporate α in state s;

c_0^γ = capital of bank γ in t=0;

λ^γ = default penalty for bank γ;

ρ = interbank interest rate;

r_m^γ = interest rate on corporate laons;

k^γ = bank γ' capital ratio;

w =Basel II credit risk weight;

π_s^γ =profit of bank γ in state s;

a^γ= bank γ coefficient of risk aversion;

$\Delta(x)$ =difference between right hand side and left hand side of inequality x.

The profit of bank γ in the generic state s is defined in equation 1 : it results as the difference between the amounts of money received and paid by bank γ in t=1 plus the money at hand from the firts period.

$$\pi_s^\gamma = v_s^\alpha \left(1 + r_m^\gamma\right) m_\alpha^\gamma - v_s^\gamma \mu^\gamma + \left(\frac{\mu^\gamma}{1+\rho} + c_0^\gamma - m_\alpha^\gamma\right) \tag{1}$$

The bank γ optimization problem is defined as follows:

$$\max_{m_\alpha^\gamma, \mu^\gamma, v_1^\gamma, v_2^\gamma} U^\gamma = \sum_s p_s \left[\pi_s^\gamma - a^\gamma \left(\pi_s^\gamma\right)^2 - \lambda \mu^\gamma \left(1 - v_s^\gamma\right)\right] \tag{2}$$

$$s.\ t.$$

$$m_\alpha^\gamma \leq \frac{\mu^\gamma}{1+\rho} + c_0^\gamma \tag{3}$$

$$v_s^\gamma \mu^\gamma \leq v_s^\alpha \left(1 + r_m^\gamma\right) m_\alpha^\gamma + \Delta(3) \qquad s = 1, 2 \tag{4}$$

$$k^\gamma \geq K \tag{5}$$

The bank maximizes its expected utility, as defined in equation 2: specifically, the utility in state s is defined by a quadratic function of state s profit less a penalty in case of default on the interbank debts. The bank chooses the amount of money to borrow from the interbank market (μ^γ), the amount of money to invest in loans to corporate (m_α^γ) and the its repayment rates (v_1^γ, v_2^γ) subject to budget and capital constraints. In t=0 investments are limited by money availability, as from the budget constraint in equation 3. Equation 4 states that revenues from investments in t=1 must cover the repayments to creditors. The capital constraint, defined in equation 5, mimics the Basel II requirement and applies in t=0. Since credit risk for banks arises from corporates defaultability, the capital ratio, defined in equation 6, accounts for investments in corporate loans.

$$k^\gamma = \frac{c_0^\gamma}{w \left(1 + r_m^\gamma\right) m_\alpha^\gamma} \tag{6}$$

The term w represents a credit risk measure for corporate loans and is modelled as an increasing function of the corporate α' probability of default. Specifically, equation 7 defines the risk weight w for exposure towards corporates as a function the corporate's probability of default

(PD) which mimics the Basel II formula[2].

$$w = 1.2PD - 1.9PD^2 \tag{7}$$

In equation 7 PD, which is the input to the Basel capital function, represents the expected default rate over the second period horizon. In order to model a forward-looking[3] pit rating system, the corporate α PD is defined in equation 8, where p_2 is the recession probability (and the expansion probability is $p_1 = 1 - p_2$)

$$PD = E(1 - v_s^\alpha) = p_1(1 - v_1^\alpha) + p_2(1 - v_2^\alpha) \tag{8}$$

With this modellization, the PD depends on the bisiness cycle forecast over the horizon considered through the recession probability. On the contrary a ttc rating system does not consider any dependence either on forecast or on current economic conditions

We now turn to the second bank optimization problem. Bank δ is a net lender on the interbank market and it collects deposits from household: this makes its optimization problem slighly different from the previous one. We assume that the bank cannot default on deposits[4]. Moreover, while corporate α is the counterpart of bank γ, corporate β is the counterpart of bank δ, as from the limited participation hypothesis.

In order to formalize the bank δ optimization problem, we report the relevant variables specific to bank δ:

m_β^δ = credit extension to corporate β;
d^δ = credit extension to bank γ;
μ_d^δ = money from deposits;
r_m^δ = interest rate on laons corporate;
r_d^δ = deposit interest rate;
v_s^β = repayment rate of corporate β to its creditor in state s;
c_0^δ = capital of bank δ in t=0;
k^δ = bank δ' capital ratio;
π_s^δ = profit of bank δ in state s;

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[2]The formula used for the risk weight is a proxy for the Basel II formula (M=1, LGD=45%): the introduction of the actual Basel formula would be much heavier from a computational point of view when solving the optimization problem. However we believe the distorsion is not relevant as the main features of the Basel function are preserved: the function we use is in fact convex and it reaches the maximum at the same point. The function is decreasing in the default probability from a certain level onward, like the unexpected loss in Basel II. However, the function is increasing for realistic values of the PD.

[3]Forward looking in that it accounts for a forecast of the business cycle rather than for current economic conditions.

[4]This hypothesis is equivalent to having a very high default penalty.

$a^\delta=$ bank δ coefficient of risk aversion.

As for bank γ, the profit of bank δ in state s (t=1) is defined by the net revenues in equation 9.

$$\pi_s^\delta = v_s^\beta \left(1 + r_m^\delta\right) m_\alpha^\delta + v_s^\gamma d^\delta \left(1 + \rho\right) - \mu_d^\delta + \left(\frac{\mu_d^\delta}{1 + r_d^\delta} + c_0^\delta - m_\beta^\delta - d^\delta\right) \tag{9}$$

The bank δ optimization problem is defined as follows:

$$\max_{m_\beta^\delta, \mu^\delta, \mu_d^\delta} U^\delta = \sum_s p_s \left[\pi_s^\delta - a^\delta \left(\pi_s^\delta\right)^2\right] \tag{10}$$

$$s. \ t.$$

$$m_\beta^\delta + d^\delta \leq \frac{\mu_d^\delta}{1 + r_d^\delta} + c_0^\delta \tag{11}$$

$$\mu_d^\delta \leq v_s^\delta \left(1 + r_m^\delta\right) m_\beta^\delta + v_s^\gamma d^\delta \left(1 + \rho\right) + \Delta(11) \quad s = 1, 2 \tag{12}$$

$$k^\delta \geq K \tag{13}$$

Bank δ maximizes its expected utility, which is a quadratic function of profits as it is shown in equation 10. Equation 11 represents the budjet constraint in t=0 and equation 12 states that revenues in t=1 plus money at hand form t=0 must cover debts. The capital constraint in equation 13 applies in t=0. The risk weighted assets for bank δ (equation 14) include both exposures towards corporate β and bank γ.

$$k^\delta = \frac{c_0^\delta}{w \left(1 + r_m^\delta\right) m_\alpha^\delta + \widetilde{w}(1 + \rho)d^\delta} \tag{14}$$

The risk weights w and $\widetilde{w}$ are increasing functions of the default probability of corporate β and bank γ respectively: the same considerations as from equations 7 and 8 hold.

2.2 Corporates' optimization problem

The economy is one without production and stochastic endowments play the role of production output. The corporates are poor agents in t=0 and they receive stochastic commodity and money endowments in t=1; therefore they borrow in t=0 in order to smooth intertemporal consumption. In order to present the optimization problems, in the following we define the additional relevant variables for both corporates, indicating with h either corporate α or β and with b either bank γ or δ respectively:

$e_s^h=$ commodity endowment in t=1;

m_s^h=monetary endowment in t=1;

b_0^h = expenditures for commodities in t=0;

q_s^h = quantity of commodity to sell in t=1;

μ^h = borrowings from the assigned bank;

v_s^h = repayment rate in t=1;

a^h=corporate h coefficient of risk aversion;

g_0, g_1, g_2=commodity prices in period t=0 and in the two states of period t=1 respectively.

x_0^h = corporate h commodity consumption in t=0;

x_s^h=corporate h commodity consumption in t=1.

The last two variables, representing consumption in the two periods, are defined respectively as $x_0^h = b_0^h/g_0$ (commodity expenditures over commodity price) and $x_s^h = e_s^h - q_s^h$ (commodity endowment minus commodity sale). The two corporates differ in endowments and risk aversion but their behaviours can be both represented with the following optimization problem:

$$\max_{b_0^h, q_s^h, \mu^h, v_1^h, v_2^h} U^h = x_0^h - a^h \left(x_0^h\right)^2 + \sum_s p_s \left[x_s^h - a^h \left(x_s^h\right)^2 - \lambda^h \mu_b^h \left(1 - v_s^h\right)\right]$$

(15)

s. t.

$$b_0^h \leq \frac{\mu^h}{1 + r_m^\delta}$$

(16)

$$v_s^h \mu^h \leq \Delta(16) + g_s q_s^h + m_s^h \quad s = 1, 2$$

(17)

Equation 15 states that the corporate chooses the amount of money borrowing to buy commodity in t=0, the commodities to sale in t=1 and the repayment rates to the bank in order to maximize its utility from consumption subject to a penalty for defaulted amounts. The constraints to the optimization problems state that the amount spent for commodity in t=0 is bounded by the amount of money borrowed from the bank (equation 16) and that the repayed amounts in t=1 are bounded by revenues from commodity sales plus money endowment (equation 17).

2.3 Households' optimization problem

The households (indicated by ϕ) are rich agents in period t=0 while they have no endowments in t=1: in the first period they sell part of their commodity and deposit money in order to smooth consumption over the second period. Based on the stylized fact that banks which are net lender on the interbank market collect most of the deposits, we simplify the model by assuming that the household deposits its whole

money endowment in bank δ. In the following the relevant variables and the optimization problem are presented:

$e_0^\phi =$ commodity endowment in t=0;

$m_0^\phi =$ monetary endowment in t=0;

$d_\delta^\phi =$ deposits in bank δ;

$b_s^\phi =$ expenditures for commodities in t=1;

$q_0^\phi =$ commodity sales in t=0;

$u =$ fixed money expenditures;

$a^\phi =$ household coefficient of risk aversion;

$x_0^\phi =$ household commodity consumption in t=0;

$x_s^\phi =$ household commodity consumption in t=1.

The last two variables, representing consumption in the two periods, are defined respectively as $x_0^\phi = e_0^\phi - q_0^\phi$ (commodity endowment minus commodity sale) and $x_s^\phi = b_s^\phi / g_s$ (commodity expenditures over commodity prices).

$$\max_{d_\delta^\phi, b_s^\phi, q_0^\phi} U^\phi = x_0^\phi - a^\phi \left(x_0^\phi \right)^2 + \sum_s p_s \left[x_s^\phi - a^\phi \left(x_s^\phi \right)^2 \right] \qquad (18)$$

$$s.\ t.$$

$$(a)\ d_b^\phi \leq m_0^\phi \quad (b)\ q_b^\phi \leq e_0^\phi \qquad (19)$$

$$b_s^\phi \leq \Delta(19-a) + g_0^\phi q_0^\phi + d^\phi \left(1 + r_d^\delta \right) - u \qquad s = 1, 2 \qquad (20)$$

Equation 18 states that households choose commodity trades and deposit in order to maximize the utility from consumption in both periods. The constraints in t=0 (equation 19) bound the deposit and the commodity sale to the money and commodity endowments respectively. The constraits in t=1 (equation 20) bound the expenditures for commodity to the revenues from sales and interests from deposits in t=0.

3 The initial equilibrium

The initial equilibrium is obtained by simultaneously solving the optimization problems for all the agents accounting for the market clearing conditions.

$$g_0 = \frac{b_0^\alpha + b_0^\beta}{q_0^\phi} \qquad g_s = \frac{b_s^\phi}{q_s^\alpha + q_s^\beta} \quad s = 1, 2 \qquad (21)$$

$$1 + r_m^\gamma = \frac{\mu^\alpha}{m^\gamma} \qquad 1 + r_m^\delta = \frac{\mu^\beta}{m^\delta} \qquad (22)$$

$$1 + r_d^\delta = \frac{\mu_d^\delta}{d^\phi} \qquad (23)$$

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$$1 + \rho = \frac{\mu^\gamma}{d^\delta + M} \qquad (24)$$

Equation 21 states commodity market clearing conditions for prices in the two periods. Equation 22 states the loan market clearing conditions for interest rates applied by bank γ and bank δ to corporate α and β respectively. Equation 23 states the deposit market clearing condition for the deposit interest rate and equation 24 states the interbank market clearing condition for the interbank interest rate.

An equilibrium occurs when all agents maximize their expected utilities and the market clearing conditions are satisfied. In order to formalize the definition of equilibrium, let $P = (g_0, g_1, g_2, r_m^\gamma, r_m^\delta, r_d^\delta, \rho)$, $D^\gamma(P) = \{(m_\alpha^\gamma, \mu^\gamma, v_1^\gamma, v_2^\gamma): (3) - (6) \text{ hold}\}$, $D^\delta(P) = \{(m_\beta^\delta, \mu^\delta, \mu_d^\delta): (11) - (14) \text{ hold}\}$, $D^h = \{(b_0^h, q_s^h, \mu^h, v_1^h, v_2^h): (16) - (17) \text{ hold}\}$, $D^\phi(P) = \{(d_\delta^\phi, b_s^\phi, q_0^\phi): (19) - (20) \text{ hold}\}$.

An equilibrium is defined as a set $(m_\alpha^\gamma, \mu^\gamma, v_1^\gamma, v_2^\gamma, m_\beta^\delta, \mu^\delta, \mu_d^\delta, b_0^\alpha, q_s^\alpha, \mu^\alpha, v_1^\alpha, v_2^\alpha, b_0^\beta, q_s^\beta, \mu^\beta, v_1^\beta, v_2^\beta, d_\delta^\phi, b_s^\phi, q_0^\phi; g_0, g_1, g_2, r_m^\gamma, r_m^\delta, r_d^\delta, \rho)$ such that:

1) $(m_\alpha^\gamma, \mu^\gamma, v_1^\gamma, v_2^\gamma) \in Arg \max_{(m_\alpha^\gamma, \mu^\gamma, v_1^\gamma, v_2^\gamma) \in D^\gamma(P)} U^\gamma$

2) $\left(m_\beta^\delta, \mu^\delta, \mu_d^\delta\right) \in Arg \max_{(m_\beta^\delta, \mu^\delta, \mu_d^\delta) \in D^\delta(P)} U^\delta$

3) $\left(b_0^h, q_s^h, \mu^h, v_1^h, v_2^h\right) \in Arg \max_{(b_0^h, q_s^h, \mu^h, v_1^h, v_2^h) \in D^h(P)} U^h \quad h = \alpha, \beta$

4) $\left(d_\delta^\phi, b_s^\phi, q_0^\phi\right) \in Arg \max_{(d_\delta^\phi, b_s^\phi, q_0^\phi) \in D^\phi(P)} U^\phi$

5) (21)-(24) hold.

As for the computation of the equilibrium[5], in line with the related literature we guess the first one by making hypotheses on interest rates and prices and numerically solving the five optimization problems[6].

In order to solve the model we need to attribute values to the exogenous variables: even if at this stage the model is not calibrated over real data, values are chosen to allow reasonable equilibrium values. The initial equilibrium is depicted in the Table 1 showing the equilibrium values of exogenous and endogenous variables.

The model is parametrized so that the capital for both banks is binding. As for the corporates, they have different risk features: corporate β is riskier than corporate α in that it is less risk averse (a^β lower than a^α) and its commodity and money endowments are more dispersed (m_s^h, e_s^h $h = \alpha, \beta$ in Table 1). As a consequence its repayment rate is lower and more dispersed in the initial equilibrium (v_s^h $h = \alpha, \beta$ in Table 1).

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[5]For the existence of the equilibrium we refer to the more general model in Tsomocos (2003).

[6]The problem is solved with the Matlab Optimization Toolbox.

Exogenous variables and parameters					
c_0^γ	c_0^δ	M			
0.13856	0.33412	0.38042			
e_0^ϕ	m_0^ϕ	u^ϕ			
6	2.0446	1.61			
e_1^α	e_2^α	e_1^β	e_2^β		
2.9	2.1	3	2		
m_1^α	m_2^α	m_1^β	m_2^β		
0.05	0.01	0.055	0.005		
Endogenous variables					
m^γ	μ^γ	ν_1^γ	ν_2^γ		
1.4374	1.4288	0.97087	0.94823		
m^δ	μ_ϕ^δ	d^δ			
1.4603	2.0855	0.9185			
b_0^α	μ^α	q_1^α	q_2^α	ν_1^α	ν_2^α
1.4374	1.8687	1.4216	1.3169	0.93965	0.92151
b_0^β	μ^β	q_1^β	q_2^β	ν_1^β	ν_2^β
1.4603	2.0152	1.3920	1.2749	0.85619	0.82495
q_0^ϕ	b_1^ϕ	b_2^ϕ	d^ϕ		
2.8978	3.3763	3.3694	2.0446		
Interest rates and prices					
r_m^γ	r_m^δ	r_d^δ	ρ		
0.3	0.38	0.02	0.1		
g_0	g_1	g_2			
1	1.2	1.3			

Table 1: Initial equilibrium values

According to the riskyness of corporates, the loan interest rate r_m^γ applied to corporate α is lower than the rate r_m^δ applied to corporate β in equilibrium. The model is solved assuming equal probability for expansion and recession for the second period: therefore the initial equilibrium represents a "neutral" situation in that no forecast on business cycle is considered. We assume that the pit and ttc rating systems give the same risk weight and we consider this situation as the starting point for comparative statics. Table 2 shows the conditional and unconditional utilities at the equilibrium point: the conditional utilities in expansion are obviously higher than the ones in recession for all agents.

These equilibrium values are the starting point for the comprative analysis presented in the following Section.

	bank γ	bank δ	corporate α	corporate β	household ϕ
$U\|_E$	0.3187	0.5899	1.1649	1.1819	2.497
$U\|_R$	0.2868	0.512	0.5908	0.4288	2.323
U	0.6055	1.1019	2.1105	2.0998	4.935

Table 2: Utilities at the initial equilibrium

4 Evaluation of rating systems

In this section we present the model outcomes as for the evaluation of different rating systems within economic scenarios defined in terms of state of the business cycle. In order to account for different rating philosophies the risk weight on risky assets in the capital ratio formula is modelled in equation 7 as an increasing function of the PD mimicking the Basel II formula. The final aim of this paper is to compare two rating systems: one business cycle neutral (i.e. resembling a ttc rating) and the other business cycle dependent (i.e. resembling a pit rating). Specifically, the comparison of the two rating systems is performed both from the banks' point of view and from a welfare perspective.

We define a pit rating system as the one where the PD in equation 7 and hence the risk weight changes according to the recession probability, while a ttc rating system as the one where the PD does not change. In order to reproduce these two rating systems we assume that banks first estimate their obligors'PD in a situation of equally likely expansion and recession as in the initial equilibrium: when the recession probability changes, the PD under pit system changes according to equation 8 while the PD under ttc remain constant.

In order to perform the comparison between the two rating systems, we consider a change in the recession probability and evaluate the utilities under the neutral rating system, where the change in the recession probabilty does not affect the capital requirement, and under the cycle-dependent one where the change in the recession probability alters the risk weights. We obtain new equilibria corresponding to different probabilities of recession by means of a comparative statics analysis and compare conditional and expected utilities.

Under the ttc rating system, the recession probability change does not affect the risk weights. Table 3 shows the directional changes due to an increase in recession probability.

In order to explain the various effects, we start from the household: the increase in the recession probability makes consumption relatively more convenient in t=0, therefore commodity selling in t=0 (q_0^ϕ) decreases in order to allow higher consumpion and expenditures for consumption in t=1 (b_s^ϕ) decrease. As a consequence commodity prices

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Banks decision variables											
m^γ	μ^γ	ν_1^γ	ν_2^γ		m^δ	μ_d^δ	d^δ				
↑	↑	↓	↓		↓	↓	↑				
Corporates decisin variables											
b_0^α	μ^α	q_1^α	q_2^α	ν_1^α	ν_2^α	b_0^β	μ^β	q_1^β	q_2^β	ν_1^β	ν_2^β
↑	↑ ≈	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
Household decision variables											
q_0^ϕ	b_1^ϕ	b_2^ϕ	d^ϕ								
↓	↓	↓	=								
Utilities											
U^γ	U^δ	U^α	U^β	U^ϕ							
↓	↓	↓	↓	↓							

Table 3: Directional changes due to an increase in recession probability under the ttc rating system

increase in t=0 (g_0 increases) while they decrease in both states in t=1 (g_1 and g_2 decrease). From the corporates' point of view, the increase in g_0 implies a reduction in counsumption (x_0^h) and the reduction in second period prices, together with the reduced expected endowment, produces a reduction in commodity selling (q_s^h): the total reduction in revenues leads to a reduction of repayment rates to banks in both states. Bank γ, which is net borrower on the interbank market, reacts by reducing repayments to bank δ (net lender) in order to keep its own profits; moreover, as it cannot diversify, it increases demand for interbank loans (μ^γ) in order to invest more in corporates: as a consequence the interbank interest rate increases while the loan interest rate decreases (this allows to keep compliance with the capital requirement). As for bank δ, both activities become less profitable due to lower repayment rates, but the higher interbank rate leads to increase the interbank deposits to the detriment of corporate loans. Recalling that bank δ collects deposits from household, the request of money from deposits μ_d^δ decreases with consequent reduction in deposit rate, which again reduces second period revenues for household and therefore emphasizes the chain effects described above. The loan rate for corporate β decreases despite the reduced supply since the demand decreases as well. In terms of utility, the increased recession probability determines a reduction in the utility of all the agents. The opposite effects hold for a reduction in recession probability.

We now turn to analyse the new equilibrium under the pit rating system. Table 4 shows the directional changes of the endogenous variables

following an increase of the recession probability.

Banks decision variables											
m^γ	μ^γ	ν^γ_1	ν^γ_2		m^δ	μ^δ_d	d^δ				
↓	↓	↑	↑		↑	↓	↓				
Corporates decisin variables											
b^α_0	μ^α	q^α_1	q^α_2	ν^α_1	ν^α_2	b^β_0	μ^β	q^β_1	q^β_2	ν^β_1	ν^β_2
↓	↓	↓	↓	↑	↑	↑	↓	↓	↓	↑	↑
Household decision variables											
q^ϕ_0	b^ϕ_1	b^ϕ_2	d^ϕ								
↓	↓	↓	=								
Utilities											
U^γ	U^δ	U^α	U^β	U^ϕ							
↑	↓	↓	↓	↓							

Table 4: Directional changes due to an increase in recession probability under the pit rating system

The main difference lies in the effect of recession probability on Basel II risk weight. Under the pit rating system, for both banks the increase in the recession probability means an increase in the risk weights inducing a direct effect on banks which is missing under ttc. Recalling the limited participation hypothesis, the corporate weight of bank δ is affected more strongly due to the higher dispersion of corporate β default rates. Bank γ, which cannot diversify, is forced to reduce its loans to corporate, and this reduction is associated to a decrease in interbank borrowings, which determines a reduction in the interbank interest rate. Moreover the increased expected penalty for default produces an increase in repayment rates, also emphasized by increased corporate repayment rates (as clarified later), which more than compensates the increase in risk weight of interbank loans for bank δ. The reduced interbank interest rate induces bank δ to switch from these to the corporate loans. Given the reduced profitability of activities, bank δ reduces the money from deposits determining a reduction in the deposit interest rate: this effect however is much weaker than under ttc. The reduced deposit interest rate means a reduced money availability in t=1 for the household, who reduces expenditures for commodities implying a reduction in their prices. Moreover, since the consumption in t=0 becomes relatively more convenient, the commodity sales in t=0 and expenditures for commodity in t=1 decrease, with similar effects as under ttc on prices. For corporates the reduction in second period commodity prices, together with decreased expected commodity endowment, determines a reduction in commodity selling and in money borrowing but the increased default

penalty more than compensate the effect on repayment which ultimately increase. The risk weights, however, are still higher than in the initial equilibrium. The total effect on interest rates for corporate loans is a reduction which is stronger for corporate β / bank δ. The increase in the recession probability turns out in an utility reduction for every agents, with the exception of bank γ. This apparently counterintuitive effect is due to two components: the increase in corporates repayment rates and the reduction in the interbank interest rate. The opposite effects hold for a reduction of the recession probability.

We now focus on the comparison between the neutral and the cycle-dependent rating system in terms of expected utility. We consider two different situations: one in which a recession is expected, i.e. an increase in the recession probability, and the other in which an expansion is expected (i.e. a reduction in the recession probability). When the recession becomes more likely, the pit system turns out to be more convenient for all the agents except for bank δ: this is mainly due to the fact that it suffers for the reduced interbank interest rate, which means reduced costs for bank γ and reduced profits for bank δ. Moreover, under pit system there are stronger reduction in corporates interest rates and weaker reduction in deposit rate with respect to ttc system which mean higher utility for corporates and househld respectively to the detriment of bank δ. The opposite effects prevail when an expansion is more likely. Therefore the preference for pit or ttc rating system depends on the point in the cycle, that is on the expectation relatively to the evolution of the business cycle over the time horizon considered for the second period.

Turning to the comparison of conditional utilities, we assume that the business cycle is correctly estimated, so that a recession probability higher then 0.5 turns out in an acual recession in t=1 (and viceversa for probability lower than 0.5) and consider the case of high recession probability in t=0 combined with realized recession in t=1. Since the recession was correctly expected, the conditional recession utility is higher than it would have been without forecast (or with wrong forecast). However, bank δ would be more satisfied (i.e. less damaged by recession) under ttc system since the risk features of corporate β determine a strong increase in risk weight (much stronger than for corporate α), the deposit rate decreases more strongly than under pit and the interbank rate increases. On the other side and partly for mirror reasons bank γ (as well as the other agents) has higher utility under pit. This result is driven by the role of bank δ on the interbank and deposit market and by the risk features of its debtors compared to bank γ. Similar considerations hold in case of expansion.

In order to make a thorough comparison relatively to the banks'

preferences, we analyse the size of the increases and reductions in utility moving from the pit to the ttc rating system in the two different phases of the cycle. This analysis gives less clear cut results. The increase in utility is higher than the decrease for bank γ: this means that bank γ would choose the pit rating system. The oucome is ambiguous for bank δ since the difference is very slight. The conclusions on the whole preferences are based on slight differences in the utilities and the robustness of these results to different parametrization of the model is under inspection.

5 Conclusions

In the light of the importance of the procyclicality issue related to the introduction of Basel II, we evelute different rating system within a general equilibrium framework. We set up a model with two heterogeneous banks, two corporates and an household and we solve numerically the model to find an initial equilibrium. Then, through a comparative statics analysis we evaluate a neutral and a cycle-dependent rating system. The main issue analysed is the comparison between the two ratings from the banks' point of view. The preference for pit or ttc rating systems depends on the point in the cycle and on the bank's features. Specifically, based on the comparison of conditional and expected utilities, it emerges that the net lender bank, which also extends loans to the riskier corporate, prefers the *ttc* rating, while the net borrower bank prefers the *pit* system when a recession is expected. These conclusions stem from the composition of different effects which the general equilibrium framework allows to highlight and quantify. The conclusions are on the whole (i.e. considering different states of the business cycle) less clear cut and deserve further analysis. In sum, our work shows that risk sensitivity of capital requirement can have different impact on the procyclicality issue depending on banks' features and business cycle conditions (expectations and realizations).

   The next step in our research agenda is twofold. On one hand, we plan to test the robustness of the results to different parametrizations of the model (e.g. different level of capitalization) and, as long as data availability allows it, to calibrate the model on rela data.

   On the other hand, in order to capture feedback effects of capital requirements on the business cycle, it would be useful to add a third period to the present model.

References

[1] Bangia A., Diebold F., Kronimus A., Schagen C., Schuermann T., 2002. Ratings migration and the business cycle, with application to credit portfolio stress testing. *Journal of Banking & Finance*, 26, 445-474.

[2] Basel Committee on Banking Supervision, (BCBS), 2006. International convergence of capital measurement and capital standards - A revised framework. June 2006.

[3] Borio C., Furfine C., Lowe P., 2001. Procyclicality of the financial system and Financial stability: issues and policy options. BIS Paper, 1.

[4] Catarineu-Rabell E., Jackson P., Tsomocos D., 2005. Procyclicality and the new Basel accord: banks'choice of loan rating system. *Economic Theory*, 26, 537-557.

[5] Daniellson J., Embrechts P., Goodhart C., Keating C., Muennich F., Renault O., Shin H. S., 2001. A academic response to Basel II. London School of Economics, Financial Markets Group, Special Paper 130.

[6] Goodhart C.A.E., Sunirand P., Tsomocos D., 2006. A model to analyse financial fragility. *Economic Theory*, 27, 107-142.

[7] Goodhart C.A.E., Sunirand P., Tsomocos D., 2005. A risk assessment model for banks. *Annals of Finance*, 1, 197-224.

[8] Goodhart C.A.E., Sunirand P., Tsomocos D., 2004. A model to analyse financial fragility: applications. *Journal of Financial Stability*, 1, 1-30.

[9] Gordy M., Howells B., 2006, Procyclicality in Basel II: can we treat the disease without killing the patient?. *Journal of Financial Intermediation,* 15, 395-417.

[10] Jokivuolle E., Vesala T., 2007. Portfolio effects and efficiency of lending under Basel II. Bank of Finland Research, Discussion Papers 13.

[11] Kashyap A., Stein J., 2004. Cyclical implications of the Basel II Capital Standards. *Federal Reserve Bank of Chicago Economic Perspective*, 1st Quarter, 18-31.

[12] Nickell P., Perraudin W., Varotto S., 2000. Stability of rating transitions. *Journal of Banking & Finance*, 24, 203–227.

[13] Masschelein N., 2007. Monitoring pro-cyclicality under the capital requirement directive: preliminary concepts for developing a framework. Natiolal Bank of Belgium, Working Paper Document 120.

[14] Pederzoli C., Torricelli C., 2005. Capital requirements and business cycle regimes: forward-looking modelling of default probabilities. *Journal of Banking & Finance*, 29, pp. 3121-3140.

[15] Repullo R., Suarez J., 2007. The procyclical effects of Basel II. Working paper.

[16] Tsomocos D., 2003. Equilibrium analysis, banking and financial instability. *Journal of Mathematical Economics*, 39, 619-655.