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On the Rationalizability of Observed Consumers' Choices when Preferences Depend on Budget Sets and (Potentially) on Anything Else*

Ennio Bilancini†

Abstract

We prove that defining consumers' preferences over budget sets is both necessary and sufficient to make every fully informative and finite set of observed consumption choices rationalizable by a collection of preferences which are transitive, complete, and monotone with respect to own consumption. Our finding has two important theoretical consequences. First, assuming that preferences depend on budget sets is illegitimate under the scientific commitments of revealed preference theory. Second, as long as consumers' preferences are not defined over budget sets, we can assume that preferences depend on observable objects other than own consumption without compromising the logical possibility to reject the model against observation. We however point out that, despite this logical possibility, in practice it can be almost impossible to reject a model where preferences are defined over objects that depend on budget sets. As an example of this we show that if preferences are defined over consumption choices of other individuals then rationalization fails only in cases of negligible practical interest.

JEL classification: B40, D11

Keywords: Revealed Preferences; Budget Sets; Rational Preferences; Rationalizability

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1 Introduction

The economic literature has repeatedly suggested that consumers' preferences can possibly depend on prices.[1] However, so far no systematic analysis has been conducted on the legitimacy of assuming price-dependent preferences under the scientific commitments of revealed preference theory. In other words, it is not yet fully clear if assuming that preferences depend on prices is compatible with the desideratum that observed consumption behaviors are not necessarily consistent with the hypothesis of rational choice.

In the present paper we investigate this issue by studying the rationalizability of consumers choices under the assumption that preferences depend on budget sets – the latter being the natural generalization of the assumption of price-dependent preferences to the case of non-linear budget sets (see e.g., Forges and Minelli, 2009). As in Afriat (1967) we suppose to observe a fully informative but finite set of consumer choices and we derive under what conditions observations are consistent with rational choice. Differently from Afriat's model, we do not constraint preferences to depend on consumption only, but we allow for the possibility that preferences depend on any observable object. The relevance of this generalization – which goes well beyond the consideration of price-dependent preferences – is discussed below.

Our main finding is that assuming preferences to depend on budget sets is a *necessary and sufficient* condition for making every fully informative and finite set of observed consumption choices rationalizable by a collection of preferences which are transitive, complete, and monotone with respect to own consumption. This result has two important consequences. First, under the assumption that preferences depend on budget sets the hypothesis of rational choice cannot be logically refuted by observed behavior. Therefore, assuming that individuals' preferences are price-dependent is methodologically illegitimate under the scientific commitments of revealed preference theory. Second, assuming that individuals do not care about budget sets is sufficient to obtain that observed behavior can possibly refute the hypothesis of rational choice. Since this result holds in a general framework where preferences can depend on any observable object, we can conclude that letting preferences depend on observable objects other than own consumption is legitimate provided that we maintain price-independence.

Of course, the *logical* possibility of refuting the hypothesis of rational choice does not imply the *practical* possibility to do so. Therefore, great

[1]For instance, economic agents may be interested in relative prices because they confer social status (Veblen, 1899) or because they are interpreted as an indication of quality (Scitovsky, 1944). The implications of price-dependent preferences for consumer demand theory and welfare theory have been investigated in a few famous contributions (Kalman, 1968; Ng, 1987; Pollak, 1977, 1978). More recently, the possibility of price-dependent preferences has been considered sufficiently important to justify the study of how it affects the equilibrium property of general equilibrium models (Balasko, 2003).

caution has to be had in assuming that preferences depend on objects other than own consumption. We substantiate this warning with an example where we show that if preferences are assumed to depend on the choices made by other individuals, then the hypothesis of rational choice can be refuted only in cases of negligible practical interest. In other words, our example shows that allowing preferences to depend on objects which, in turn, depend on budget sets may not be safe from a methodological standpoint. In our opinion this is a relevant issue since the hypothesis of "interdependent preferences" – i.e., dependent on others' choices – is gaining a certain consensus among economists (see e.g., Frank, 2005; Hopkins and Kornienko, 2004; Clark et al., 2008, and references therein).

A similar warning has been recently advanced by Carvajal (2009) who shows that the hypothesis of consumption externalities in competitive equilibria has testable implications, but admits that in practice carrying out a test could be an extremely hard task. We stress that, despite similar conclusions on practical testability, there is a substantial difference between our model and the one analyzed by Carvajal. In Carvajal (2009) consumption behavior is restricted by general equilibrium conditions which in our model are absent. Therefore, if we particularize our general finding about logical testability of rational behavior – i.e., price-independent preferences are sufficient for logical testability – to the case of consumption externalities, then we get a result which is stronger than Carvajal's one since it is obtained under fewer restrictions on behavior. At the same time, our warning on practical testability should be interpreted in a weaker sense since we do not consider restrictions on behavior which, if we observe competitive equilibria, could be available.

The present study is related to the stream of contributions regarding the rationalizability of observed choice (see e.g., Richter, 1966; Afriat, 1967; Matzkin, 1991; Varian, 1982, 2006; Forges and Minelli, 2009). One difference with respect to most papers belonging to this stream is that, in considering the issue of rationalizability by means of a preference ordering, we disregard the issue of representability of such an ordering by means of a utility function (see Chambers and Echenique, 2009, for an approach similar to ours). A more substantial difference is that we study rationalizability in the general case where individuals' preferences are defined over sets of non-choice objects and not only over the choice set of individuals. In this regard, our paper contributes to the literature by developing an original model where individuals may care about a countably infinite number of non-choice objects.

One further difference with respect to the traditional approach to rationalizability of consumer choices is that we focus on the behavior of *many* individuals. This is a natural generalization in our setting since preferences may be defined over the behavior of other individuals and, hence, fully informative observations have to contain information about everybody's choices. Other contributions on rationalizability have considered the behavior of more than one individual. Brown and Matzkin (1996) investigates the testable implications of competitive general equilibria in a Walrasian setting. Their model has been recently extended to a game theoretic setup by Deb (2009) and to the case of externalities and public goods by Carvajal (2009). As anticipated above, although we share with these contributions the focus on a population of individuals, differently from them we do not consider equilibrium restrictions on behavior. As a result, our positive findings about testable implications are stronger.

The seminal work of Chiappori (1988) on the collective model of household consumption has inspired a series of contributions focusing on the rational consumption behavior of group of individuals (see e.g., Cherchye et al., 2008, and references therein). In particular, the recent paper by Cherchye et al. (2010) studies the rationalizability of collective consumption choices and provides a collective version of the results found in Afriat (1967). The perspective on the problem of rationalizability, however, is quite different from ours as attention is given to the problem of missing information about how consumption is allocated inside the group considered – e.g., the household. Indeed, the main difference between models of rational collective consumption behavior and our model is that in the former observations are not fully informative. In particular, while quantities consumed by the group as a whole are observable, it is impossible to distinguish among quantities privately consumed by each group member and between private and public consumption inside the group. In other terms, the rationalizability studied in these models moves from the idea of limited information availability while here we consider the ideal case of full information.

The paper is organized as follows. Section 2 illustrates the mathematical preliminaries. Section 3 describes the model and adapts the standard idea of preference revelation to the case where individuals may also care about any number of non-choice objects. Section 4 defines rationalizability in our framework and provides the main results. Section 5 briefly concludes.

2 Preliminaries

Let R be a binary relation on a set W, where wRw' means that $(w, w') \in R$. The existence of a sequence $\{w_k\}_{k=1}^{K+1}$ in W such that $K > 0$, $w_1 = w$, $w_{K+1} = w'$ and $w_k R w_{k+1}$, for $k = 1, \dots, K$, is indicated by $wR^K w'$.

A binary relation R on W is *complete* if and only if for any $w, w' \in W$ either wRw' or $w'Rw$. A binary relation R on W is *reflexive* if and only if for any $w \in W$, wRw. A binary relation R on W is *transitive* if and only if for any $w, w', w'' \in W$, wRw' and $w'Rw''$ implies wRw''. The binary relation $\bar{R}$ on W is said to be the *transitive closure* of R on W and is defined as $\bar{R} \equiv \{(w, w') \in W^2 : \exists K > 0, wR^K w'\}$.

We denote with $\mathbb{N}$ the set of natural numbers and with $\mathbb{R}$ the set of real numbers. Let $X \subseteq \mathbb{R}^m$. The set $\partial X = \{x \in X : \forall y \in X \setminus \{x\}, \neg(y \geq x)\}$ is said to be the *upper frontier* of X.

Let $W \equiv \prod_{\lambda=1}^{\infty} A_\lambda$ whose generic element has the form $w = (a_1, \ldots, a_\lambda, \ldots)$. Let $\Lambda \equiv \{\lambda_1, \ldots, \lambda_k, \ldots\} \subseteq \mathbb{N} \setminus \{0\}$ be a set of indices where $\lambda_i \leq \lambda_j$ for all $1 \leq i \leq j \leq k$. We denote the Λ-projection of $w \in W$ with the map $\pi_\Lambda : W \to \prod_{\lambda \in \Lambda} A_\lambda$ such that $\pi_\Lambda(w) = (a_{\lambda_1}, \ldots, a_{\lambda_k}, \ldots)$ for all $w \in W$. Moreover, we denote with $\pi_{-\Lambda}$ the $(\mathbb{N} \setminus \Lambda)$-projection. To simplify the notation we set $w_\Lambda \equiv \pi_\Lambda(w)$ and $w_{-\Lambda} \equiv \pi_{-\Lambda}(w)$. Finally, whenever we write $w'' = (w_\Lambda, w'_{-\Lambda})$, with $w, w' \in W$, it is implicitly assumed that the components of both w_Λ and $w'_{-\Lambda}$ are reshuffled in such a way that $w'' \in W$.

3 The Model

3.1 Description of the Economy

We consider an economy populated by a finite number of individuals which we index according to the set $\{1, \ldots, n\} \subset \mathbb{N}$. The state space of the economy is $W \equiv \prod_{\lambda=1}^{\infty} A_\lambda$, where each $\lambda \in \mathbb{N}$ refers to a different characteristic of the economy while the elements of the set A_λ represent the possible specifications of the λ-th characteristic. Each state $w \in W$ is therefore an ordered tuple with countably infinite elements describing the actual characteristic of the economy (e.g., budget sets, choices of economic agents, weather conditions, time, etc).

Without loss of generality we further assume that characteristic 1 represents the commodity bundle purchased by individual 1, characteristic 2 represents the bundle purchased by individual 2, and so on and so forth up to characteristic n. Formally, we set $A_1 = \ldots = A_n = X \equiv \mathbb{R}_+^m$ where X is interpreted as the consumption set and m as the number of commodities for which there exists a market. Notice that some markets may be future markets. Moreover, and again without loss of generality, we assume that characteristic $(n+1)$ represents individual 1's budget set, characteristic $(n+2)$ represents individual 2's budget set, and so on and so forth up to characteristic $2n$. Formally, we set $A_{n+1} = \ldots = A_{2n} = \mathcal{B} \subset 2^{\mathbb{R}_+^m}$ where each $B \in \mathcal{B}$ is assumed to be such that if $x \in B$ then for any $x' \in X$, $0 \leq x' \leq x$, we have $x' \in B$.

Finally, for all $i \in \{1, \ldots, n\}$ and $w \in W$ we denote with $x_i(w) \equiv \pi_{\{i\}}(w)$ and $B_i(w) \equiv \pi_{\{n+i\}}(w)$, respectively, the bundle purchased by individual i and the budget set of individual i under the description of the economy w. Logically, we have that $x_i(w) \in B_i(w)$ for all $i \in \{1, \ldots, n\}$ and $w \in W$.[2]

[2]Notice that, as in Matzkin (1991) who generalized the model of Afriat (1967), budget sets are allowed to be non-linear and therefore prices do not explicitly appear.

3.2 Choices, Preferences and Preference Revelation

Individuals are assumed to be "pure consumers", in the sense that they can only choose what to consume. In other words, the choice set of individual i coincides with $B_i(w)$ for any $w \in W$.

Although individuals can only choose among alternatives in their budget set, they may care, in principle, about any characteristic of the economy. Let $\Lambda_i \subseteq \mathbb{N} \setminus \{0\}$ be the set containing the indices associated with the characteristics of the economy which individual i considers relevant – i.e., on which i' preferences are defined. We assume that $i \in \Lambda_i$, meaning that each individual at least cares about her own consumption. Then, i's preferences are defined over the set $\pi_{\Lambda_i}(W)$. We denote with $\succeq_i \subseteq \pi_{\Lambda_i}(W) \times \pi_{\Lambda_i}(W)$ the relation representing i's preferences. For any $w, w' \in W$, $w_{\Lambda_i} \succeq_i w'_{\Lambda_i}$ means, as usual, that w_{Λ_i} is at least as preferred as w'_{Λ_i}. (Again, w_{Λ_i} stands for the Λ_i-projection of w). When $w_{\Lambda_i} \succeq_i w'_{\Lambda_i}$ and not $w'_{\Lambda_i} \succeq_i w_{\Lambda_i}$ we say that i strictly prefers w_{Λ_i} over w'_{Λ_i} and we write $w_{\Lambda_i} \succ_i w'_{\Lambda_i}$. Furthermore, for any $w \in W$, we refer to $w_{-i} \equiv \pi_{\Lambda_i \setminus \{i\}}(w)$ as the "relevant" circumstances faced by individual i in the choice of $x_i(w)$ or, more briefly, as i's *circumstances of choice* in w. We stress that here the term "relevant" is a shorthand to indicate characteristics of the world over which preferences are defined. We say that the preference relation $\succeq_i$ is *locally non-satiated with respect to own consumption*, or A_i-locally non-satiated, if and only if for every $w \in W$ and $\epsilon > 0$ there exists $w' \in W$ such that $||x_i(w) - x_i(w')|| \leq \epsilon$, $w_{-i} = w'_{-i}$, $w'_{\Lambda_i} \succ_i w_{\Lambda_i}$. Finally, we say that the preference relation $\succeq_i$ is *monotone with respect to own consumption*, or A_i-monotone, if and only if, for every $w, w' \in W$, $x_i(w) > x_i(w')$ (meaning that $x_i(w)$ is greater than $x_i(w')$ in all its components) and $w_{-i} = w'_{-i}$ implies that $w_{\Lambda_i} \succ_i w'_{\Lambda_i}$. Clearly, A_i-monotonicity implies A_i-local non-satiation.

The fundamental presumption behind the mechanism of *direct preference revelation* is that when an individual is given the possibility to choose among different alternatives she reveals, by the very act of choosing, that what she has chosen is at least as preferred as all affordable alternatives. In other words, individuals' constrained choices are optimal. It must be noted, however, that direct preference revelation is limited to the relation between what the individual has chosen and all alternatives she could have actually chosen. This implies that, in the present framework, direct preference revelation can only rank alternatives in W that satisfy the *ceteris paribus* condition with respect to circumstances of choice since individuals cannot choose the latter. This will turn out to be the key to our results.

Our definition of an ideal set of observations regarding individuals' choices follows the basic idea just delineated. Let $D \subset W$ be a finite set whose elements are fully descriptive observations of the economy. We say that D is an *ideal dataset* if $D \subset W$ and for every $w \in D$ and $i \in \{1, \ldots, n\}$ we have that $x_i(w) \in \partial B_i(w)$. In other words, an ideal dataset contains all information about the state of the world and is such that individuals spend their entire budget. The latter assumption is standard. One typical interpretation is that D contains all possible uses of individuals' budgets.

Note that in this context we abstract from issues of strategic behavior and beliefs which are not central to this study. In this way we put ourselves in the most favorable situation in terms of observational information about individuals' preferences. This will help to highlight the purely logical nature of our findings.[3]

The binary relation $R_i \subseteq \pi_{\Lambda_i}(D) \times \pi_{\Lambda_i}(W)$ denotes i's directly revealed preferences. For every $w \in D$ and $w' \in W$, $w_{\Lambda_i} R_i w'_{\Lambda_i}$ if and only if $w_{-i} = w'_{-i}$ and $x_i(w') \in B_i(w)$. In other terms, under the hypothesis that individual i considers relevant the characteristics of the economy identified by Λ_i, the observation of $w \in D$ directly reveals that w_{Λ_i} is preferred to w'_{Λ_i} by individual i if and only if i's circumstances of choice in w and w' are the same and i could have chosen w' under the circumstances of choice faced in w. We emphasize that the requirement of equal circumstances of choice makes R_i crucially depend on the specification of Λ_i.

4 Rationalization of Observed Choices

Given an ideal dataset $D \subset W$, we say that the behavior of individual i is *rationalizable* under Λ_i if and only if there exists a preference relation $\succeq_i \subseteq \pi_{\Lambda_i}(W) \times \pi_{\Lambda_i}(W)$ satisfying

R1. transitiveness and completeness

R2. $w_{\Lambda_i} R_i w'_{\Lambda_i} \Rightarrow w_{\Lambda_i} \succeq_i w'_{\Lambda_i}$.

We say that an ideal dataset $D \subset W$ is rationalizable under $\{\Lambda_i\}_{i=1}^n$ if there exists a collection of preference relations $\{\succeq_i\}_{i=1}^n$ such that the behavior of each i can be rationalized.

Lemma 1 *Let $D \subset W$ be an ideal dataset and let $R_i \subseteq \pi_{\Lambda_i}(D) \times \pi_{\Lambda_i}(W)$ be the directly revealed preferences of individual i under Λ_i. Then, the following conditions are equivalent:*

i) the behavior of individual i can be rationalized by a preference relation which satisfies A_i-local non-satiation,

ii) $w_{\Lambda_i} \bar{R}_i w'_{\Lambda_i}$ and $w'_{\Lambda_i} R_i w_{\Lambda_i}$ imply that $x_i(w) \in \partial B_i(w')$,

iii) the behavior of individual i can be rationalized by a preference relation which satisfies A_i-monotonicity.

[3] For a discussion of the shortcomings of preference revelation in the presence of strategic behavior see Hausman (2000). Other insightful comments on related issues can be found in Varian (2006).

Proof. We will show that $i) \Rightarrow ii) \Rightarrow iii) \Rightarrow i)$.

We first show that $i) \Rightarrow ii)$. Suppose that the behavior of individual i is rationalized by the preference relation $\succeq_i \subseteq \pi_{\Lambda_i}(W) \times \pi_{\Lambda_i}(W)$ which satisfies A_i-local non-satiation. Consider $w, w' \in W$. We will show that jointly having that $w_{\Lambda_i} \bar{R}_i w'_{\Lambda_i}$, $w'_{\Lambda_i} R_i w_{\Lambda_i}$ and $x_i(w) \in B_i(w') \backslash \partial B_i(w')$ is impossible. Since $\succeq_i$ rationalizes i's behavior we have three cases: 1) $w_{\Lambda_i} \succ_i w'_{\Lambda_i}$, 2) $w'_{\Lambda_i} \succ_i w_{\Lambda_i}$, or 3) $w_{\Lambda_i} \succeq_i w'_{\Lambda_i}$ and $w'_{\Lambda_i} \succeq_i w_{\Lambda_i}$. If 1) holds then by R2 we have that not $w'_{\Lambda_i} \succeq_i w_{\Lambda_i}$ implies not $w'_{\Lambda_i} R_i w_{\Lambda_i}$. If 2) holds then by R1 (transitivity) we have that there exists no natural $K > 0$ such that $w_{\Lambda_i} \succeq_i^K w'_{\Lambda_i}$. In particular this implies that for any sequence $\{(w_k, w_{k+1})\}_{k=1}^K$ in W^2 with $w_1 = w$ and $w_{K+1} = w'$, there exists at least one natural k such that $((w_k)_{\Lambda_i}, (w_{k+1})_{\Lambda_i}) \notin \succeq_i$. Hence, by R2 and the fact that $D \subset W$ we have that for every sequence $\{(w_k, w_{k+1})\}_{k=1}^K$ in D^2 with $w_1 = w$ and $w_{K+1} = w'$, there exists at least one natural k such that $((w_k)_{\Lambda_i}, (w_{k+1})_{\Lambda_i}) \notin R_i$ which in turn implies that $w_{\Lambda_i} \bar{R}_i w'_{\Lambda_i}$ is impossible. Finally, suppose that 3) holds and that $w'_{\Lambda_i} R_i w_{\Lambda_i}$. By the definition of R_i we have that $x_i(w) \in B_i(w')$ and that $w'_{-i} = w_{-i}$. By A_i-local non-satiation we have that if $x_i(w) \in B_i(w') \setminus \partial B_i(w')$ then there exists w'' such that: $x_i(w'') \in B_i(w')$, $w''_{-i} = w_{-i}$, $w''_{\Lambda_i} \succ_i w_{\Lambda_i}$; by R1 (transitivity and completeness) this would imply that $w'_{\Lambda_i} \succ_i w_{\Lambda_i}$ which contradicts ii). We therefore conclude that $x_i(w) \in \partial B_i(w')$. Hence, if the behavior of i is rationalized by a preference relation which satisfies A_i-local non-satiation, then $w_{\Lambda_i} \bar{R}_i w'_{\Lambda_i}$ and $w'_{\Lambda_i} R_i w_{\Lambda_i}$ imply that $x_i(w) \in \partial B_i(w')$.

We show that $ii) \Rightarrow iii)$ in two steps. In the first step we show that, if $ii)$ holds, then we can construct a transitive and reflexive preference relation containing $\bar{R}_i$ which satisfies A_i-monotonicity. In step two we apply a standard result which guarantees the extendibility of the relation constructed in step one to a transitive and complete preference relation which preserves A_i-monotonicity. Suppose that $w_{\Lambda_i} \bar{R}_i w'_{\Lambda_i}$ and $w'_{\Lambda_i} R_i w_{\Lambda_i}$ imply that $x_i(w) \in \partial B_i(w')$. Define the binary relation $L_i \equiv \{(w_{\Lambda-i}, w'_{\Lambda-i}) \in \pi_{\Lambda_i}(W) \times \pi_{\Lambda_i}(W)((x_i(w) > x_i(w')) \wedge (w_{-i} = w'_{-i})) \vee ((w_{\Lambda-i} = w'_{\Lambda-i}) \wedge (w_{-i} = w'_{-i}))\}$. Let $H_i \equiv \overline{L_i \cup \bar{R}_i}$. By construction, H_i is both transitive and reflexive and is a subset of $\pi_{\Lambda_i}(D) \times \pi_{\Lambda_i}(W)$. Suppose that H_i does not satisfy A_i-monotonicity. Since, by construction, $x_i(w) > x_i(w')$ and $w_{-i} = w'_{-i}$ imply that $(w_{\Lambda_i}, w'_{\Lambda_i}) \in H_i$, we conclude that there exists $u, v \in W$ such that $x_i(u) > x_i(v)$, $u_{-i} = v_{-i}$ and $(v_{\Lambda_i}, u_{\Lambda_i}) \in H_i$. This means that there exists, for some natural $K > 0$, a sequence $\{((w_k)_{\Lambda_i}, (w_{k+1})_{\Lambda_i})\}_{k=1}^K \subseteq L_i \cup \bar{R}_i$ such that $w_1 = v$ and $w_{K+1} = u$. Since $((w)_{\Lambda_i}, (w')_{\Lambda_i}) \in L_i \cup \bar{R}_i$ implies that $w_{-i} = w'_{-i}$, we have that $(w_k)_{-i} = (w_{k+1})_{-i}$ for every $1 \leq k \leq K$; therefore, the whole sequence $\{((w_k)_{\Lambda_i}, (w_{k+1})_{\Lambda_i})\}_{k=1}^K$ cannot be contained in L_i otherwise we would have that $x_i(v) > x_i(u)$. In particular, since $x_i(u) > x_i(v)$, there must exist a subsequence $\{((w_k)_{\Lambda_i}, (w_{k+1})_{\Lambda_i})\}_{k=j}^l$, $1 \leq j \leq k \leq l \leq K$, which is a subset of $\bar{R}_i$, such that $(w_j)_{\Lambda_i} \bar{R}_i (w_{l+1})_{\Lambda_i}$ and $x_i(w_j) < x_i(w_{l+1})$.

Since $(w_j)_{\Lambda_i}\bar{R}_i(w_{l+1})_{\Lambda_i}$ implies that $(w_k)_{\Lambda_i}R_i(w_{k+1})_{\Lambda_i}$ for $l \leq k \leq j$, we have that $x_i(w_{k+1}) \in B_i(w_k)$ for $l \leq k \leq j$. From $x_i(w_{l+1}) \in B_i(w_l)$, $x_i(w_j) < x_i(w_{l+1})$ and the definition of budget sets follows that $x_i(w_j) \in B_i(w_l) \setminus \partial B_i(w_l)$ which in turn implies that $(w_l)_{\Lambda_i}R_i(w_j)_{\Lambda_i}$. This contradicts our initial assumption because we have that $(w_j)_{\Lambda_i}\bar{R}_i(w_l)_{\Lambda_i}$, $(w_l)_{\Lambda_i}R_i(w_j)_{\Lambda_i}$ and $x_i(w_j) \in B_i(w_l) \setminus \partial B_i(w_l)$. Hence, H_i satisfies A_i-monotonicity. Since H_i is both transitive and reflexive we can apply the result by Hansson (1968) (Lemma 3) which establishes the extendibility of H_i to a complete and transitive relation $H_i^E \supset H_i$ such that if $w_{\Lambda_i}H_iw'_{\Lambda_i}$ and not $w'_{\Lambda_i}H_iw_{\Lambda_i}$, then $w_{\Lambda_i}H_i^Ew'_{\Lambda_i}$ and not $w'_{\Lambda_i}H_i^Ew_{\Lambda_i}$.[4] The latter property implies that if H_i satisfies A_i-monotonicity then also H_i^E satisfies A_i-monotonicity. This proves that the behavior of individual i can be rationalized by a preference relation which satisfies A_i-monotonicity.

Finally, the fact that *iii)* $\Rightarrow$ *i)* trivially follows from the fact that A_i-monotonicity implies A_i-local non-satiation.■

Remark 1. Because of budget set non-linearity, we cannot extend the result of Lemma 1 to A_i-strong monotonicity – i.e., for any $w, w' \in W$ such that $w_{-i} = w'_{-i}$, $x_i(w) \geq x_i(w')$ and $x_i(w) \neq x_i(w')$ imply that $w'_{\Lambda_i} \succ_i w_{\Lambda_i}$. In fact, we could have $w, w' \in D$ such that $w_{-i} = w'_{-i}$, $x_i(w) \geq x_i(w')$ and $x_i(w) \neq x_i(w')$ implying that $w_{\Lambda_i}R_iw'_{\Lambda_i}$ and $w_{\Lambda_i}R_iw'_{\Lambda_i}$ which is incompatible with A_i-strong monotonicity (this is also evident in the light of the results of Richter (1966) and Matzkin (1991) which show that rationalizability by means of standard strong monotonic preferences is attainable only if the strong axiom of revealed preferences is satisfied).

We are now ready to study the rationalizability of an ideal dataset under the hypothesis that every individual considers the set of all possible budget sets a relevant characteristic of the economy.

Proposition 1 *Let $D \subset W$ be an ideal dataset. If $n + i \in \Lambda_i$ for every individual i then D is rationalizable by a collection of preferences such that the preferences of each individual i satisfy A_i-monotonicity.*

Proof. Consider individual $i \in \{1, \ldots, n\}$. If $w_{\Lambda_i}\bar{R}_iw'_{\Lambda_i}$ then there exists a sequence $\{((w_k)_{\Lambda_i}, (w_{k+1})_{\Lambda_i})\}_{k=1}^K$, where $w_1 = w$, $w_{K+1} = w'$ and $1 \leq k \leq K$, such that $(w_k)_{\Lambda_i}R_i(w_{k+1})_{\Lambda_i}$. The latter condition implies that $(w_k)_{-i} = (w_{k+1})_{-i}$ for every k and, in particular, $w_{-i} = w'_{-i}$. Since the $(n + i)$-th characteristic referes to i's budget set, from $n + i \in \Lambda_i$ and $w_{-i} = w'_{-i}$ we get that $B_i(w) = B_i(w')$. Hence, by budget exhaustion we have that $x_i(w), x_i(w') \in \partial B_i(w) = \partial B_i(w')$. This implies that having both $w_{\Lambda_i}\bar{R}_iw'_{\Lambda_i}$ and $w'_{\Lambda_i}R_iw_{\Lambda_i}$ implies $x_i(w') \in \partial B_i(w)$. By Lemma 1 we

[4] See also Suzumura (1976, 1983) (theorem 3, theorem A(5)), Donaldson and Weymark (1998) (theorem), Duggan (1999), (application C).

get rationalizability of i's behavior by means of A_i-monotone preferences. Applying this reasoning to every individual gives the result.■

Proposition 1 states that the hypothesis that individuals consider their budget sets relevant characteristics of the economy is a sufficient condition to make any finite set of ideal observations of individuals' choices rationalizable by means of a collection of transitive and complete preferences which are monotone (or locally non-satiated) in own consumption. Next Proposition states that such a hypothesis is also a necessary condition.

Proposition 2 *If $n+i \notin \Lambda_i$ for some individual i then there exists an ideal dataset $D \subset W$ which is not rationalizable by a collection of preferences such that the preferences of individual i satisfy A_i-local non-satiation.*

Proof. We prove the Proposition by showing that if $n+i \notin \Lambda_i$ for some individual $i \in \{1,\ldots,n\}$ then there exists a finite ideal dataset $\tilde{D}$ such that i's revealed preferences do not satisfy statement ii) of Lemma 1 and, hence, i's behavior cannot be rationalized by a preference relation satisfying A_i-local non-satiation. Suppose that $n+i \notin \Lambda_i$. Consider the behavior of individual i in the dataset $\tilde{D} \equiv \{w, w'\} \subset W$ such that $w_{-i} = w'_{-i}$. Notice that, under the hypothesis that $n+i \notin \Lambda_i$, such a dataset is feasible independently of what are the other elements of Λ_i besides i; moreover, provided that $x_i(w) \in \partial B_i(w)$ and $x_i(w') \in \partial B_i(w')$, the budget sets $B_i(w)$ and $B_i(w')$ can be set freely. Suppose that $x_i(w') \in B_i(w') \cap (B_i(w) \setminus \partial B_i(w))$. Hence, by the definition of preference revelation, we get that that $w_{\Lambda_i} R_i w'_{\Lambda_i}$, $w'_{\Lambda_i} R_i w_{\Lambda_i}$ and $x_i(w') \in B_i(w) \setminus \partial B_i(w)$ which in turn implies that statement ii) of Lemma 1 is false.■

Remark 2. Together Lemma 1, Proposition 1 and Proposition 2 imply that rationalizability by means of either A_i-local non-satiated or A_i-monotone preferences obtains for every finite ideal dataset if and only if the preferences of every individual depend on her own budget set.

Remark 3. In Proposition 2, the set Λ_i may contain infinitely many characteristics of the economy and some of these may depend on actual budget sets. Hence, we have also proved that assuming that individuals' preferences depend on characteristics of the economy which in turn depend on budget sets does not lead to the rationalizability of every finite ideal dataset.

As stated in Remark 3, the fact that individuals' preferences depend on some characteristic of the economy that depend on actual budget sets does not logically prevent rationalizability by means of A_i-non satiated (or A_i-monotone) preferences. However, one may wonder about the difficultly of obtaining the failure of rationalizability under such a hypothesis. In order to shed some light on this issue we find useful to investigate an instance of this assumption. More precisely, we consider the case where individuals only care about their own and others' consumption choices, budget sets are linear and all individuals face the same price vector. Clearly, we can apply Proposition 2 because $(n+i) \notin \Lambda_i$ for every i. Moreover, since others' consumption choices depend on actual prices, this case satisfies the hypothesis that preferences are defined over characteristics of the economy (other than own consumption) which depend on actual budgets sets. It turns out that in such a case rationalizability can fail only if a rather strong condition is met.

Let us denote with $\hat{W}$ the state space of an economy where the following assumptions hold. First, $A_{2n+1} = \ldots = A_{3n+1} = \mathbb{R}^m_+$. Second, for every $i \in \{1, \ldots, n\}$ and $w \in \hat{W}$, $\partial B_i(w) = \{x \in X : 0 \leq p(w)x = p(w)e_i(w)\}$ where $e_i(w) \equiv \pi_{\{2n+i\}}(w)$ and $p(w) \equiv \pi_{\{3n+i\}}(w)$ represent, respectively, the vector of endowments of individual i and the vector of ruling prices.

Proposition 3 *Suppose that $\Lambda_j = \{1, \ldots, n\}$ for every individual j. Then, a finite ideal dataset $D \subset \hat{W}$ is not rationalizable by a collection of preferences, such that the preferences of each individual j satisfy A_j-monotonicity, only if there exists $i \in \{1, \ldots, n\}$ and $w, w' \in D$ such that $p(w) \neq p(w')$, $p(w)x_i(w) > p(w)x_i(w')$ and $x_j(w) = x_j(w')$ for all $j \neq i$.*

Proof. Suppose that the claimed necessary condition is not satisfied, i.e., that for every $w, w' \in W$ and $i \in \{1, \ldots, n\}$, either

(a) $p(w) = p(w')$ or

(b) $p(w)x_i(w) \leq p(w)x_i(w')$ or

(c) $x_j(w) \neq x_j(w')$ for some $j \neq i$.

We will show that statement *ii*) of Lemma 1 is satisfied for every $i \in \{1, \ldots, n\}$. Suppose that $w'_{\Lambda_i} \bar{R}_i w_{\Lambda_i}$. If (b) holds then we obtain that $w_{\Lambda_i} R_i w'_{\Lambda_i}$ only if $p(w)x_i(w) = p(w)x_i(w')$ which implies that $x_i(w') \in \partial B_i(w)$, consistently with statement *ii*) of Lemma 1. If (c) holds then $w_{-i} \neq w'_{-i}$ and, hence, we have not $w_{\Lambda_i} R_i w'_{\Lambda_i}$ which, again, is consistent with statement *ii*) of Lemma 1. Now consider the case where (a) holds. If $p(w)x_i(w) \leq p(w')x_i(w') = p(w)x_i(w')$ then we are in case (b). Suppose, instead, that $p(w)x_i(w) > p(w)x_i(w')$. We will show that this implies that there exist two elements of $\hat{W}$ which satisfy neither (a), (b), nor (c). Since $w'_{\Lambda_i} \bar{R}_i w_{\Lambda_i}$, there exists a sequence $\{((w_k)_{\Lambda_i}, (w_{k+1})_{\Lambda_i})\}_{k=1}^K$ in $\hat{W} \times \hat{W}$, with $w_1 = w'$, $w_{K+1} = w$ and $1 \leq k \leq K$, such that $(w_k)_{\Lambda_i} R_i (w_{k+1})_{\Lambda_i}$ and $w^k_{-j} = w^{k+1}_{-j}$ for every $j \neq i$. This implies that, for every k, w^k and w^{k+1} do not satisfy condition (c). Moreover, $p(w^k)x_i(w^k) \geq p(w^k)x_i(w^{k+1})$. In particular, $p(w')x_i(w') \geq p(w')x_i(w^2)$. Since $p(w) = p(w')$ and $p(w)x_i(w) >$

$p(w)x_i(w')$, we obtain that $p(w)x_i(w) > p(w)x_i(w^2)$ which implies that condition (b) is not satisfied by w and w^2. If w and w^2 do not satisfy (a) we are done. If they do, then $p(w) = p(w^2)$ and we can apply the same reasoning applied so far to get that $p(w)x_i(w) > p(w)x_i(w^3)$ and so on and so forth. Since K is finite and $p(w)x_i(w) > p(w)x_i(w)$ is impossible, we necessarily get that there exists some k such that w^k and w^{k+1} satisfy neither (a), (b), nor (c).■

Remark 4. From the necessary condition stated in Proposition 3 and budget exhaustion follows that if $e_j(w) = e_j(w')$ then $e_j(w) = x_j(w) = x_j(w')$. Therefore, an ideal finite dataset whose elements are constants over individuals' endowments can fail to be rationalized only if everybody but one find optimal to consume their endowments under two different price systems.

5 Conclusions

In this paper we have proved that the dependency of preferences on budget sets is both *necessary and sufficient* for making every fully informative and finite set of observed consumption choices rationalizable by a collection of preferences which are transitive, complete, and monotone with respect to own consumption.

We believe that our result is important in two main respects, one related to *sufficiency* and the other to *necessity*. As regards sufficiency, our finding shows that economic models where preferences are assumed to be price-dependent are methodologically problematic under the scientific commitments of revealed preference theory. Indeed, our result indicates that, if we want to have a model with price-dependent preferences which could be possibly rejected by (fully informative) observation, then we have to add restrictions on behavior which do not derive from rational choice theory alone. For what concerns necessity, our result shows that price-independent preferences ensure that the hypothesis of rational behavior always produces implications which can be rejected by (fully informative) observation, no matter how extravagant we allow preferences to be. This conclusion suggests that modelers may consider the extension of the domain of preferences beyond consumption more benevolently since, as long as we maintain price-independence, letting preferences depend on other observable objects is not illegitimate under the scientific commitments of revealed preference theory.

We stress that these considerations are fundamentally logical, abstracting from practical issues of testability. As we have shown in the paper, a model which provides testable implications from a logical standpoint may well be untestable in practice. We therefore call for great caution in the interpretation of our results.

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