WORKING PAPER SERIES

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Working Paper 147

December 2020

www.recent.unimore.it

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Common Components Structural VARs

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December 1, 2020

Abstract

Small scale VAR models are subject to two major issues: first, the information set might be too narrow; second, many macroeconomic variables are measured with error. The two features produce distorted estimates of the impulse response functions. We propose a new procedure, called Common Components Structural VARs (CC-SVAR), which solves both problems. It consists in (a) treating the variables, prior to estimation, in order to extract their common components; this eliminates measurement errors; (b) estimating a VAR with $m > q$ common components, that is a singular VAR, where q is the number of shocks driving the economy; this solves the fundamentalness problem. SVARs and CC-SVARs are compared in the empirical analysis of monetary policy and technology shocks. The results obtained by SVARs are not robust, in that they strongly depend on the choice and the treatment of the variables considered. On the contrary, using CC-SVARs (i) contractionary monetary shocks produce a decrease of prices independently of the variables included in the model, (ii) irrespective of whether hours worked enter the model in log-levels or growth rates, technology improvements produce an increase in hours worked.

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†Luca Gambetti acknowledges the financial support of the Spanish Ministry of Economy and Competitiveness through grant ECO2015-67602-P and through the Severo Ochoa Programme for Centres of Excellence in R&D (SEV-2015-0563), and the Barcelona Graduate School Research Network. Contact: Office B3.1130 Departament d'Economia i Historia Economica, Edifici B, Universitat Autonoma de Barcelona, Bellaterra 08193, Barcelona, Spain. Tel. +34 935814569; e-mail: luca.gambetti@uab.cat

JEL classification: C32, E32.
Keywords: Structural VAR models, structural factor models, nonfundamentalness, measurement errors.

1 Introduction

Since the seminal paper by Sims (1980), Structural Vector Autoregressive (SVAR) models have become the main tool for applied macroeconomic analysis. In the SVAR approach, the macroeconomic variables in the vector X_t are driven by the q-dimensional vector of structural shocks u_t, and react to these shocks according to linear impulse-response functions (IRF). The structural shocks are obtained as linear combinations of the VAR residuals by imposing suitable identifying restrictions derived from economic theory.

However, it is well known by now that the vector of structural shocks not always can be represented as a linear transformation of the vector of innovations. When this is the case, u_t is *non-fundamental* for X_t, and SVAR analysis fails. Non-fundamentalness occurs, in the usual interpretation, when the information set available to the agents is larger than the linear space spanned by current and past values of X_t.[1] An obvious example is that in which the number of variables is smaller than the number of shocks. But even when the number of shocks and variables coincide, which is the standard assumption in SVAR analysis, the information contained in the history of X_t can be deficient. Early papers containing examples of non-fundamental economic models are Hansen and Sargent (1991) and Lippi and Reichlin (1993). More recent works are Fernández-Villaverde et al. (2007), Alessi et al. (2011), Sims (2012), Leeper et al. (2013), Forni and Gambetti (2014), Forni et al. (2019).[2]

A second problem with SVAR analysis is represented by the presence of measurement errors in macroeconomic variables. The fact that many macroeconomic aggregates are affected by measurement error is indisputable. Still, the problem has been largely neglected in the literature. There is an implicit widespread belief that the consequences on SVAR analysis are not serious. However, Giannone et al. (2006) and Lippi (2020) show that this view is wrong (see also the simulations in the present paper). Indeed, even small measurement errors can generate substantial distortions in the estimates of the impulse response functions, yielding misleading results.[3]

The difficulties caused by non-fundamentalness and measurement errors might be used to recommend not to use SVAR models for macroeconomic analysis, an additional argument for authors who argue that Dynamic Stochastic General Equilibrium (DSGE) models should become the standard tool in empirical macroeconomics, see in particular Chari et al. (2008).

The opposite view is upheld in the present paper. We show that the two problems can be satisfactorily solved within the SVAR approach and that the advantages of VAR techniques, simplicity and data-driven estimation of the dynamic relationships in particular, can be carried out provided that the data X_t is replaced by a vector of common components estimated by means of High-Dimensional Dynamic Factor techniques.

[1]An interesting case of non-fundamentalness not arising from superior information of economic agents is that of 'noise' shocks (Forni et al. (2017a,b)). We do not deal with this case in the present paper.

[2]Whether the problem of non-fundamentalness is empirically important or not has been a matter of debate in recent years. Some authors claim that non-fundamentalness has not necessarily dramatic consequences (Sims and Zha (2006), Sims (2012), Beaudry et al. (2019)). On the other hand, Forni et al. (2014) find that narrow information sets distort the estimated effects of news shocks, while several papers, see in particular Bernanke et al. (2005), Forni and Gambetti (2010) and Miranda-Agrippino and Ricco (forthcoming), insist on the importance of large information sets for the estimation of monetary policy shocks.

[3]Lippi (2020) shows that if the variables contain meaurement errors the shocks obtained by standard identifying restrictions contain dynamic mixtures of the structural shocks and measurement errors. These dynamic contamination effects of measurement errors are special cases of the dynamic contamination effects of aggregation, as analyzed in Forni and Lippi (1997) and Forni and Lippi (1999).

The starting point of our analysis is the assumption that X_t is part of an n-dimensional dataset x_t, with n large, which follows the Structural Dynamic Factor Model (SDFM) introduced in Stock and Watson (2005) and Forni et al. (2009).[4] In this model each variable in the dataset is the sum of an *idiosyncratic* and a *common component*.[5] The idiosyncratic components are either non-correlated or weakly correlated to each other and have a natural interpretation as causes of variation of the x's that are specific to one or just a few variables, like regional or sectoral shocks and measurement errors. In particular, for the big aggregates like income, consumption, investment, or general stock price indexes, in which all local or sectoral shocks should have been already averaged out, the idiosyncratic component can be interpreted as only containing measurement errors or variable-specific high-frequency fluctuations of no interest for macroeconomic analysis. The common components on the contrary account for the commonality among the variables of the dataset. This is because they are different linear combinations of the same r common factors, with r small as compared to n. The dynamics of the model is given by the fact that the common factors are a (possibly infinite) Moving Average of q common shocks. An important feature of the model is that the vector of the factors is *singular*, i.e. its dimension r is larger than the number of driving shocks q. As the common shocks affect pervasively the common components and the variables in x_t, they can be interpreted as *structural macroeconomic shocks*.

The SDFM can be easily related to a DSGE model linearized around the steady state and written in state-space form. The variables in the DSGE correspond to a selection of the common components and the states correspond to the factors (up to suitable transformations). Notice that in the DSGE we typically have singularity, in that the number of states r is larger than the number of shocks q, as in the SDFM.

The main contribution of the present paper is a new general method for estimation and identification of structural macroeconomic shocks and the corresponding IRFs; we call it Common Component Structural VAR (CC-SVAR). Its main features are:
(a) High-Dimensional Dynamic Factor Model techniques are employed to estimate the common components.
(b) The SVAR analysis is applied to an m-dimensional vector of common components, call it Y_t, which includes the variables of interest, with $m > q$. Thus we use a *singular* stochastic vector for our analysis.

With respect to the SVAR, advantages of the CC-SVAR are:
(i) The dynamic contamination produced by measurement errors disappears or becomes negligible using the common components.
(ii) By a key result in the theory of singular stochastic vectors, the q-dimensional vector of structural shocks is generically fundamental for the m-dimensional vector Y_t. That is, the fundamentalness problem is solved once X_t has been replaced by Y_t.
(iii) The number of series to include in the CC-SVAR varies from the minimum $m = q + 1$, sufficient to obtain singularity and therefore fundamentalness, and the maximum $m = r$. We argue that $m = r$ is the best choice. By contrast, in standard SVAR analysis the choice of the number of variables is left to the judgment of the researcher, with the risk of mis-specification.
(iv) With the CC-SVAR, the results are much less sensitive to the choice of the variables to include in the model.

[4] See also Stock and Watson (2016).

[5] The representation of the x's as common plus idiosyncratic components is the basic feature of the High-Dimensional Dynamic Factor Model introduced in Forni et al. (2000), Stock and Watson (2002b,a), Bai and Ng (2002).

Point (iv) needs clarification. We show that, when using the CC-SVAR, different sets of common components yield similar results, since the models are asymptotically equivalent. Moreover, with $m = r$, the results are identical even for finite samples. This is in sharp contrast with standard SVARs, where changing a single series may change dramatically the size and even the sign of the estimated impulse-response functions, as documented in Section 5. The intuition is that if the structural shocks are fundamental for the m-dimensional vectors of common components, different vectors yield analogue information and results. By contrast, when information is deficient (because of non-fundamentalness) and, in addition, is contaminated by measurement errors, changing a single variable may produce large effects, depending on the quality of the information conveyed by this variable about the shocks of interest.

While the theoretical assumptions underlying the CC-SVAR and the SDFM are the same—a large dataset, common and idiosyncratic components, common factors driven by the structural shocks—the estimation procedures are different. Estimation of SDFMs, as proposed in the previous literature[6] consists of two steps: (1) estimating a singular VAR for the r factors, (2) inserting that VAR in the relationship between the variables of interest and the factors. We refer to that method as the Standard Procedure. With respect to it, advantages of the CC-SVAR are:

(I) Our method estimates *directly* a SVAR, i.e. the structural shocks and IRFs, for a vector including the relevant variables jointly with any other variable the researcher may be interested in, a decisive gain in clarity with respect to the Standard Procedure.

(II) We propose a procedure to set the value of r which is more robust to possible mis-specification.

(III) The CC-SVAR, unlike the Standard Procedure, does not require estimation of q. We show by simulations that this does not affect estimation accuracy, while avoiding large errors implied by a possible mis-specification.

The CC-SVAR is also related to the FAVAR model introduced in Bernanke and Boivin (2003), Bernanke et al. (2005). The basic advantage of CC-SVARs is that, unlike FAVARs, they only include common components, not the variables, which are as a rule contaminated by measurement errors.

In the empirical part of the paper, we apply the CC-SVAR method to the study of two highly debated problems in macroeconometrics: (1) the effects of monetary policy shocks on the main macroeconomic variables, (2) the effect of technology shocks on hours worked. In both cases we show that the results of SVAR analysis are not robust. In particular, the sign of the effect of a contractionary policy shock on prices is sensitive to the variables included in the model, and the sign of the effect of a positive technology shock on hours worked depends on whether hours are taken in differences or in log-levels. These puzzling phenomena disappear with CC-SVARs. Contractionary monetary policy shocks reduce prices independently of the variables included. Positive technology shocks produce an increase in hours worked, regardless of whether the specification in log-levels or in differences is used, solving the old-standing controversy about the effects of technology shocks on hours worked.[7]

The paper is organized as follows. In Section 2 the effects of non-fundamentalness and measurement errors on SVAR analysis are illustrated by means of the simple model studied in Leeper et al. (2013). In Section 3 we provide a short presentation of the SDFM, discuss fundamentalness in the light of singular vector time series theory and explain in detail the

[6] See again Stock and Watson (2005), Forni et al. (2009), Stock and Watson (2016).

[7] See Galí (1999a) and Christiano et al. (2003b).

CC-SVAR procedure. In Section 4 we present some simulation exercises comparing the CC-SVAR with the SVAR, the Standard Procedure SDFM and the FAVAR. Section 5 presents the empirical applications mentioned above. Section 6 concludes.

2 Non-Fundamentalness and Measurement Errors in a Simple Model

The model discussed in Leeper et al. (2013) is employed here as a laboratory to discuss the consequences of narrow information sets (non-fundamentalness) and measurement errors. The model is a simple Real Business Cycle (RBC) model with log preferences, inelastic labor supply and two shocks: $u_{a,t}$, a technology shock, and $u_{\tau,t}$, a tax shock. A nonstandard feature of the model is the fact that the tax shock has a delayed effect on taxes, the so-called fiscal foresight. The equilibrium capital accumulation is

$$k_t = \alpha k_{t-1} + a_t - \kappa \sum_{i=0}^{\infty} \theta^i E_t \tau_{t+i+1} \tag{1}$$

where $0 < \alpha < 1$, $|\theta| < 1$, $\kappa = (1-\theta)\tau/(1-\tau)$, τ being the steady state tax rate, and a_t, k_t and τ_t are the log deviations from the steady state of technology, capital and the tax rate, respectively. Technology and taxes are assumed, for simplicity, to be *i.i.d* processes,

$$\begin{aligned} a_t &= u_{a,t} \\ \tau_t &= u_{\tau,t-2}, \end{aligned}$$

where $u_{\tau,t}$ and $u_{a,t}$ are i.i.d. shocks that economic agents can observe. The second equation implies a delay of two periods. Solving for k_t we obtain the following equilibrium MA representation:

$$\begin{pmatrix} a_t \\ k_t \\ \tau_t \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \dfrac{-\kappa(L+\theta)}{1-\alpha L} & \dfrac{1}{1-\alpha L} \\ L^2 & 0 \end{pmatrix} \begin{pmatrix} u_{\tau,t} \\ u_{a,t} \end{pmatrix} = B(L)u_t. \tag{2}$$

2.1 Full versus Narrow Information Sets

In the standard approach to the estimation of the impulse-response functions, as the variables are driven by two shocks we should estimate a SVAR including two of the three variables in the system. However, the vector $u_t = (u_{\tau,t}\ u_{a,t})'$ is non-fundamental for all pairs of variables. Indeed, considering the square subsystem given by the first two variables (technology and capital), the determinant $\kappa(z+\theta)/(1-\alpha z)$ vanishes for $z = -\theta$, which is less than unity in modulus. The determinant of the submatrix corresponding to technology and capital vanishes at $z = 0$, and so does the one corresponding to capital and taxes. This implies that standard SVAR techniques are unlikely to correctly estimate the dynamic effect of the fiscal shock.

A quantitative assessment of the distortion caused by non-fundamentalness in the two-dimensional SVARs within system (2) is obtained here by a simulation exercise. We generate 1000 different dataset with 200 time observations from the model (2) using the parameterization in Leeper et al. (2013): $\alpha = 0.36$, $\theta = 0.2673$ and $\tau = 0.25$ and $u_t \sim N(0, I)$. For each of the datasets we estimate a VAR(4) including taxes and capital and we identify the tax shock by imposing that it is the only one driving cumulated taxes in the long run, a restriction that is clearly satisfied in the model. Panel (a) of Figure 1 plots the estimated impulse-response functions for a tax shock. The red dashed lines are the theoretical impulse response functions. The solid lines represent the mean (across datasets) of the point estimates. The grey areas represent the 16th and 84th percentiles of the point estimate distribution. As the red lines lie outside the bands, the true effects are very badly estimated. The responses obtained by the SVAR neatly anticipate the peak response in the true impulse response functions. Both taxes and capital react immediately and then the effects vanish.

Thus when only part of the information is used, current and past values of taxes and capital, the estimates of the impulse-response functions can be substantially distorted.

However, the information contained in current and past values of technology, capital and taxes is sufficient to recover the vector u_t. Indeed, the matrix $B(L)$ in (2) can be inverted, in the sense that there exists a stable 3×3 finite polynomial matrix $D(L)$ such that $D(L)B(L) = B(0)$, so that we have the VAR(3) representation with 3 lags

$$\begin{pmatrix} 1 & 0 & 0 \\ \dfrac{(\theta-L)L}{\theta^2} & \dfrac{(1-\alpha L)(\theta^2-\theta L+L^2)}{\theta^2} & \dfrac{\kappa L}{\theta^2} \\ \dfrac{-L^2}{\kappa\theta} & \dfrac{(1-\alpha L)L^2}{\kappa\theta} & 1-\dfrac{L}{\theta} \end{pmatrix} \begin{pmatrix} a_t \\ k_t \\ \tau_t \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\kappa\theta & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} u_{\tau,t} \\ u_{a,t} \end{pmatrix}. \tag{3}$$

To verify stability, observe that the determinant of $D(L)$ is $(1-\alpha L)/\theta$ and recall that $|\alpha| < 1$.

Note that singularity of the vector $(a_t\ k_t\ \tau_t)'$ implies that its 3×3 spectral density is singular at all frequencies. However, the covariance matrix which is necessary to estimate a VAR is not singular. Therefore, using the same data as in the previous exercise, we estimate a VAR(3) for the full vector $(a_t\ k_t\ \tau_t)'$.[8]

We identify the tax shock by assuming that it is the only one affecting cumulated taxes in the long run, thus a Blanchard and Quah (1989) identification scheme withe the tax shock ordered first. The results are displayed in Panel (b) of Figure 1. Using the full information set, the impulse response functions are estimated extremely well, the red dashed and solid black lines perfectly overlapping.

It is important to note that this result, that a correct estimation of the impulse-response functions is obtained by enlarging the information available to the econometrician, crucially depends in our simple model on the fact that an additional variable is added without increasing the sources of uncertainty, that is *without adding additional shocks*, which is tantamount to the fact that the enlarged vector of available variables is singular.

2.2 Measurement Errors

Typically, many of the macroeconomic variables used in SVAR models are affected by measurement error. To understand the implications of this we use another simulation exercise. We augment the three variables in model (2) with *i.i.d* measurement errors:

$$\begin{pmatrix} a_t \\ k_t \\ \tau_t \end{pmatrix} = B(L)u_t + \begin{pmatrix} \xi_t^a \\ \xi_t^k \\ \xi_t^\tau \end{pmatrix},$$

[8] Almost identical results are obtained with a VAR with four lags (not shown here).

with the assumption that the ξ's are orthogonal to the shocks $u_{a,t}$ and $u_{\tau,t}$ at all leads and lags. The data are generated using the same parameterization of the previous section, with $\xi_t^\tau = \xi_t^k = 0$ and ξ_t^a accounting for 5% of the variance of the series a_t. Using the full vector we estimate again a VAR(3) with the same identification scheme.[9] Panel (c) of Figure 1 reports the estimated impulse-response functions. Surprisingly, with a measurement error as small as that used in the generation of the data, and affecting only one of the variables, the effects of the tax shock are very badly estimated. Thus, even when information seems sufficient to correctly recover the impulse-response functions, a small measurement error may cause substantial distortion in the estimates. The reason for this will be made clear in Section 3.4.

3 Common Factors, Common Components, CC-SVAR

First, we give a short, formal presentation of the SDFM. Then we discuss the fundamentalness issue in the light of singular vector time series theory. Next we introduce and discuss in detail the CC-SVAR estimation procedure. Finally, we compare the CC-SVAR with the estimation procedure proposed in previous literature, see Stock and Watson (2005), Forni et al. (2009) in particular, and the Factor Augmented VARs, see Bernanke et al. (2005).

3.1 Structural Dynamic Factor Models

Let x_t be a dataset of n macroeconomic variables. We assume that x_t is weakly stationary, possibly after detrending. A rigorous definition of the SDFM requires that the vector x_t is part of an infinite-dimensional vector, so that we can make assumptions by letting n tend to infinity, see Forni et al. (2000), Stock and Watson (2002b,a), Bai and Ng (2002).

We assume that the variables x_{it} can be represented as

$$x_{it} = \chi_{it} + \xi_{it}, \quad i = 1, \dots, n, \tag{4}$$

where the following conditions are fulfilled.

(SDFM1) The variables ξ_{it}, called *idiosyncratic components*, are *weakly correlated* across different i's. The formal condition is asymptotic: the eigenvalues of the variance-covariance matrix of the ξ's are bounded as n tends to infinity. This entails, for example, that the mean of the ξ's tends to zero as n tends to infinity:

$$\lim_{n\to\infty} \mathrm{E}\left[\frac{1}{n}\sum_{i=1}^{n}\xi_{it}\right]^2 = 0,$$

which is obviously true if the ξ's are mutually orthogonal with an upper bound for the variance of ξ_{it}, but is also true if some "local" non-zero covariance among the ξ's is allowed.

(SDFM2) The variables χ_{it} are called the *common components*. Given t, the χ's, for $i \in \mathbb{N}$, span a finite-dimensional space, call r such dimension. This implies that there exists an r-dimensional vector F_t, weakly stationary, such that

$$\chi_{it} = A_{i1}F_{1t} + \cdots + A_{ir}F_{rt} = A_iF_t \quad \text{or} \quad \chi_t = AF_t, \tag{5}$$

[9] Augmenting the number of lags does not improve estimates (not shown here).

where χ_t is the n-dimensional vector of the χ's and A, the *factor loading* matrix, is $n \times r$. The coordinates of F_t are called the *static factors* and (5) the *static representation* of the common components. Moreover, the factors F_t are *pervasive*, in that all of them affect, with a few possible exceptions, all the variables x_{it}. And they are *non-redundant*. For example, if $\chi_{it} = A_i F_{1t} + A_i F_{2t}$, then we would have only the factor $F_{1t} + F_{2t}$. Again, the formal condition is asymptotic: as n tends to infinity, the r-th eigenvalue of the variance-covariance matrix of the χ's diverges.

(SDFM3) The idiosyncratic components are orthogonal to the factors at all leads and lags. Thus ξ_{it} is orthogonal to χ_{js} for all i, j, t and s.

(SDFM4) The r-dimensional vector F_t has the VARMA representation

$$H(L)F_t = K(L)u_t, \tag{6}$$

where (i) u_t is the q-dimensional orthonormal vector of the *structural common shocks*, (ii) $H(L)$ is a stable $r \times r$ matrix polynomial with $H(0) = I$, (iii) $K(L)$ is an $r \times q$ polynomial matrix.

A crucial observation is that usually the static representation (5) is a convenient transformation of a deeper dynamic set of relationships. Consider the following simple example:

$$\chi_{it} = a_{i0}u_t + a_{i1}u_{t-1}, \tag{7}$$

where u_t is a scalar white noise. In this case, to obtain (5) we define $F_{1t} = u_t$, $F_{2t} = u_{t-1}$, so that $r = 2$ and $\chi_{it} = A_{i1}F_{1t} + A_{i2}F_{2t}$, with $A_{i1} = a_{i0}$, $A_{i2} = a_{i1}$. In this case the VARMA for F_t is

$$\begin{pmatrix} F_{1t} \\ F_{2t} \end{pmatrix} = \begin{pmatrix} 1 \\ L \end{pmatrix} u_t \quad \text{or, equivalently,} \quad \begin{pmatrix} 1 & 0 \\ -L & 1 \end{pmatrix} \begin{pmatrix} F_{1t} \\ F_{2t} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} u_t. \tag{8}$$

We see that F_t is *singular*, i.e. it has dimension $r = 2$ but is driven by the 1-dimensional white noise u_t. This example and its obvious generalization, $r > q$, are consistent with empirical evidence, see, e.g., Giannone et al. (2005), Amengual and Watson (2007), Forni and Gambetti (2010), Luciani (2015) for US macroeconomic databases, Barigozzi et al. (2014) for the euro area. Therefore we assume throughout

(SDFM5) $r > q$, i.e. the stochastic vector F_t, the VARMA (6), is singular.

Using (6) and the resulting MA representation

$$F_t = \left[H(L)^{-1}K(L)\right] u_t = B_F(L)u_t, \tag{9}$$

we obtain an MA representation for the n-dimensional vector χ_t:

$$\chi_t = \left[AB_F(L)\right] u_t = B_\chi(L)u_t. \tag{10}$$

As $n > r > q$ the vector χ_t is singular. Lastly, define

$$\chi_t^\psi = \psi \begin{pmatrix} \chi_t \\ F_t \end{pmatrix},$$

where ψ is a $m \times (n + r)$ matrix of ones and zeros selecting m variables among the χ's and the factors. We have

$$\chi_t^\psi = \psi \begin{pmatrix} \chi_t \\ F_t \end{pmatrix} = \psi \begin{pmatrix} B_\chi(L) \\ B_F(L) \end{pmatrix} u_t = B_\psi(L)u_t. \tag{11}$$

Notice that, by equations (9) and (10), $B_\psi(L)$ is a matrix of rational functions in L, i.e. χ_t^ψ has a VARMA representation. If $m > q$, χ_t^ψ is a singular stochastic vector. Moreover, as the coordinates of the vector χ_t are linear combination of the r coordinates of F_t, if $m > r$, χ_t^ψ has a singular variance-covariance matrix. We assume:

(SDFM6) If $m \leq r$ the variance-covariance matrix of χ_t^ψ is non singular for all choice vectors ψ.

3.2 Singularity and the Fundamentalness of the Structural Shocks

At the end of the present section we make one further assumption on the SDFM, namely that if $m > q$ the structural shocks u_t are fundamental for χ_t^ψ. We start with an illustration of our point by means of a simple example. Consider the 2-dimensional vector $\chi_t^\psi = (\chi_{1t}\ \chi_{2t})'$, where

$$\begin{aligned} \chi_{1t} &= u_t + b_1 u_{t-1} \\ \chi_{2t} &= u_t + b_2 u_{t-1}, \end{aligned} \tag{12}$$

u_t being a scalar white noise. The vector χ_t^ψ is dynamically singular, since it has two entries ($m = 2$) driven by just one shock ($q = 1$).

If $b_1 \neq b_2$ we have

$$u_t = \frac{b_2\chi_{1t} - b_1\chi_{2t}}{b_2 - b_1}.$$

This can be used to replace u_{t-1} in (12), obtaining

$$\begin{aligned} \chi_{1t} &= \frac{b_1}{b_2 - b_1}(b_2\chi_{1,t-1} - b_1\chi_{2,t-1}) + u_t \\ \chi_{2t} &= \frac{b_2}{b_2 - b_1}(b_2\chi_{1,t-1} - b_1\chi_{2,t-1}) + u_t, \end{aligned}$$

which is a *finite* autoregressive representation for the singular MA(1) vector χ_t^ψ. Thus u_t belongs to the space spanned by current and past values of χ_t^ψ. We conclude that the white noise u_t in (12) is fundamental and that χ_t^ψ has a finite autoregressive representation *for all values of the parameters* b_1 *and* b_2*, with the exception of the line* $b_1 = b_2$, thus *generically* if $(b_1\ b_2)$ belongs to an open subset of $\mathbb{R}^2$.

Note that u_t is fundamental for χ_t^ψ even if it is not fundamental for χ_{1t} nor for χ_{2t}. For instance, if $b_1 = 2$ and $b_2 = 3$, u_t cannot be recovered by any univariate autoregression, even if it were possible to use an infinite number of lags; despite this, it can be recovered by a bivariate singular VAR with just one lag.

Model (12) provides an elementary example of a general result proved in Anderson and Deistler (2008a,b), referred to as AD in the sequel. Let $z_t = G(L)w_t$, where z_t is r-dimensional and w_t is q-dimensional with $r > q$. Moreover assume that the entries of $G(L)$ are rational functions of L:

$$\frac{g_{ij,0} + \cdots + g_{ij,s_1}L^{s_1}}{1 - \tilde{g}_{ij,1}L - \cdots - \tilde{g}_{ij,s_2}L^{s_2}}. \tag{13}$$

AD prove the following:

(AD1) If the matrix $G(L)$ is zeroless, i.e. $G(z)$ has rank q for all complex numbers z, then there exists a *finite* $r \times r$ stable matrix polynomial $C(L)$ such that $C(L)G(L) = G(0)$ (we say that $C(L)$ is a left inverse of $G(L)$), so that z_t has the *finite* VAR representation $C(L)z_t = G(0)w_t$.

As $G(0)$ has maximum rank (because $G(L)$ is zeroless), w_t lies in the space spanned by current and past values of z_t, i.e. w_t is fundamental for z_t.
(AD2) Suppose that the vector of the $d = (s_1 + 1)s_2 rq$ coefficients of the entries (13) varies in an open set $\mathcal{G} \subseteq \mathbb{R}^d$. Then the subset of $\mathcal{G}$ where $B(L)$ is not zeroless has dimension lower than d. We say that *generically the matrix* $G(L)$ *is zeroless*. In other words, if the d coefficients $g_{ij,k}$, $\tilde{g}_{ij,h}$ are free to vary independently of one another ($\mathcal{G}$ is an open set), then the non-zeroless matrices $G(L)$ lie in a lower dimensional, negligible set.

Going back to our vector χ_t^ψ, if $B_\psi(L)$ is zeroless, by AD1 we have

$$D_\psi(L)\chi_t^\psi = v_t^\psi = B_\psi(0)u_t \tag{14}$$

where $D_\psi(L)$ is a finite matrix polynomial and u_t is fundamental for χ_t^ψ.

However, it easy to see that when the matrix polynomials $B_\psi(L)$ are obtained within a theory-based macroeconomic model, we cannot apply straightforwardly AD2 to conclude that generically $B_\psi(L)$ is zeroless, so that by AD1 (14) holds. Indeed, the coefficients of the entries of $B_\psi(L)$ depend on a vector of 'deep' parameters, like in the example of equation (2), so that they do not vary independently of one another. In the Appendix, Section A, we discuss this difficulty and its solution in detail. We conclude that, although the independence assumption in AD2 does not hold, it is reasonable to assume that $B_\psi(L)$ is generically zeroless:

(SDFM7) If $r \geq m > q$, then generically $B_\psi(L)$ is zeroless.

Thus, if $m > q$, then generically:
(i) u_t is fundamental for all m-dimensional vectors χ_t^ψ.
(ii) χ_t^ψ has the finite VAR representation (14).
(iii) Moreover, suppose that F_t has another MA representation with an orthonormal fundamental white noise w_t. In view of the orthonormality assumpion in (SDFM4), a standard result is that $w_t = \rho u_t$, where ρ is a $q \times q$ orthogonal matrix (see Rozanov (1967), pp. 56-7; see also Section 3.2 in Forni et al. (2009)).

3.3 CC-SVAR Analysis

The procedure for the estimation of a SDFM in Stock and Watson (2005), Forni et al. (2009), the Standard Procedure, is the following:
(i) Estimation of the number of factors r and commmon shocks q. This is done by means of existing information criteria, see Section 3.6 for details.
(ii) Estimation of the factors F_t, the factor loadings A and the common components $\chi_t = AF_t$ in equation (5). The factors are estimated by the ordinary principal components of the large data set. The factor loadings are estimated by the eigenvectors corresponding to the largest r eigenvalues of the variance-covariance matrix of the x's.
(iii) Estimation of a standard VAR for the estimated factors $\hat{F}_t$ to get an estimate of the Wold representation

$$\hat{F}_t = \hat{B}(L)\hat{v}_t. \tag{15}$$

(iv) Rank reduction. The estimated factors $\hat{F}_t$ contain a residual of the idiosyncratic components, which completely disappears only asymptotically and are therefore not exactly singular. As a consequence, the vector $\hat{v}_t$ has rank r, not q, although the last $r - q$ eigenvalues of its variance-covariance matrix are close to zero. In the Standard Procedure singularity is forced on $\hat{v}_t$ by means of rank-reduction techniques described in detail in Stock and Watson (2005), Forni

et al. (2009). For example, in Forni et al. (2009) the vector $\hat{v}_t$ is replaced by the $\hat{q}$-dimensional vector $C\hat{v}_t$, where C is the $q \times r$ matrix whose rows are the eigenvectors corresponding to the first $\hat{q}$ eigenvalues of the variance-covariance matrix of $\hat{v}_t$.
(v) Identification of the structural shocks and impulse-response functions by SVAR techniques, to get an estimate of the structural shocks u_t and the corresponding impulse response matrices $B_F(L)$ in (9). The impulse response functions of the χ's are obtained according to (10) as $\hat{B}_\chi(L) = \hat{A}\hat{B}_F(L)$.

The CC-SVAR is the alternative estimation procedure summarily described below and discussed in detail in the next subsections.
(I) We estimate r, F_t and χ_t as above.
(II) We select an m dimensional vector χ_t^ψ containing the common components of X_t, the variables of interest, plus other common components or factors, so that with $r \geq m > q$. Step (iii) of the Standard Procedure is replaced by the estimation of a VAR for χ_t^ψ. We do not apply any rank-reduction technique, see Section 3.7.
(III) The identification restrictions are applied directly to the VAR estimated in (II).

3.4 CC-SVAR, the choice of m

As observed in the previous subsection the estimated factors and vectors $\hat{\chi}_t^\psi$ are not exactly singular. In particular, the singular VAR in (14) is estimated by means of a non-singular VAR. In Appendix B we provide a short discussion about consistency of such VAR estimates.

The fact that $\hat{\chi}_t^\psi$ is not exactly singular may produce serious consequences: if $B_\psi(L)$ is close to the non-zeroless region, it is possible that u_t can be recovered using χ_t^ψ, but not using $\hat{\chi}_t^\psi$. To see this, consider the following example:

$$\begin{aligned} \chi_{1t} &= u_{t-1} \\ \chi_{2t} &= a_2 u_t + u_{t-1}. \end{aligned} \tag{16}$$

Here $B_\psi(L)$ is zeroless unless $a_2 = 0$. If $a_2 \neq 0$,

$$\frac{1}{a_2}(\chi_{2t} - \chi_{1t}) = u_t,$$

so that u_t lies in the econometrician's information set. Now suppose that $\hat{\chi}_{2t} = \chi_{2t} + \epsilon_t$, ϵ_t being a small residual idiosyncratic term. For simplicity, assume that $\hat{\chi}_{1t}$ is estimated without error, i.e. $\hat{\chi}_{1t} = \chi_{1t}$. The above expression becomes

$$\frac{1}{a_2}(\hat{\chi}_{2t} - \hat{\chi}_{1t}) = u_t + \frac{1}{a_2}\epsilon_t.$$

Now if $|a_2|$ is large, we can still get u_t with a good approximation; but as $|a_2|$ approaches 0 (i.e. the non-zeroless region), the error grows without bound. For instance, if u_t is unit variance and ϵ_t has standard deviation 0.01, with $a_2 = 1$ the error is negligible, but with $a_2 = 0.01$ the error has the same size as u_t.

The above example and discussion sheds some light on the fact, observed in Section 2.2, that a small measurement error may have effects as large as those shown in Figure 1, Panel (c). Our simulation exercises in Section 4 suggest that, with $m = q + 1$, cases like the one of the example above may occur.

Clearly, the larger is m, the more unlikely they are. For instance, in the above example, if we have a third common component $\chi_{3t} = a_3 u_t + u_{t-1}$, the non-zeroless region is defined by $a_2 = a_3 = 0$, so that we only have problems when both $|a_2|$ and $|a_3|$ are close to 0. In our simulations, problematic cases no longer occur when m is larger than $q + 1$.

3.5 CC-SVAR, again on m and the choice of ψ

The previous subsection provides a first important motivation for setting m larger than $q+1$. Indeed, we recommend setting m equal to its largest possible value, i.e. $m = r$. Additional arguments in favor of this choice are the following.

Firstly, in empirical situations, q is unknown and has to be determined by existing information criteria. Such criteria, albeit consistent, may deliver wrong results in small samples, as shown by the fact that often different criteria give different results. Therefore if we set $m = \hat{q}+j$, with, say, $j = 1$ or 2, we cannot be sure that our m is greater than the true q. Thus setting $m = r$ is the safest choice. As we choose the maximum value $\hat{r}$ for m and because we do not apply rank-reduction techniques, see Section 3.6, estimation of q in a CC-SVAR is not strictly necessary. On the other hand, checking that $\hat{r}$ is actually greater than $\hat{q}$ is recommended.

A second motivation in favor of $m = \hat{r}$ is that, if $m = \hat{r}$, the estimated shocks of interest and the corresponding estimated IRFs are the same, irrespective of the choice of ψ —an equality that holds only asymptotically in the case $\hat{q} < m < \hat{r}$. The intuition is simple: since the entries of $\hat{\chi}_t^{\psi}$ are linear combinations of the estimated factors in $\hat{F}_t$ (i.e. the first m principal components of our large data set), when $\hat{\chi}_t^{\psi}$ is m-dimensional it spans the same linear space as $\hat{F}_t$, for any ψ. A formal proof is provided in Appendix C.

This fact has two important consequences. The first is that selecting the variables to be included in the CC-SVAR is not an issue. The natural choice is the set of variables which are needed for identification and, if required to complete the information set, factors, or the common components of other variables of interest. The second is that, if we are interested in the IRFs of some variables which have not been included in the CC-SVAR, we can simply estimate another CC-SVAR including these variables. This practice, which is common in empirical work, is questionable within the standard SVAR framework, since, as confirmed by the simulations and the empirical applications in Sections 4 and 5, changing the variables may change dramatically the information set and therefore the estimated shock of interest. By contrast, it is perfectly justified within the CC-SVAR approach, when setting $m = r$.

Our third motivation is parsimony. Intuition suggests that, as m increases, the number of lags which are needed in the singular VAR decreases, implying a representation with a smaller number of parameters to estimate. In Appendix D we provide an illustrative example. Our intuition is also confirmed by our first simulation exercise in Section 4.

Lastly, the observations above about estimating q obviously apply to r as well, with many consistent criteria providing different estimates in small samples. In Section 4 we see that if $\hat{r}$ increases from values below r, the true value, to values above r, the IRFs change from bad to good estimates, as $\hat{r}$ reaches r, and get stable for values greater than r, see Section 4.2.

This finding is used in empirical applications, where r is not known, see step (E5) of the estimation procedure in Section 3.6 and Section 5. We use the estimate $\hat{r}$ as the baseline specification and estimate the IRFs. Then we assess the robustness of the results by using a range of values for $\hat{r}$ around the baseline. We choose the final value of $\hat{r}$ as that at which stabilization of the IRFs occurs.

3.6 CC-SVAR, the estimation procedure in detail

The estimation procedure outlined at the end of Section 3.3, with the CC-SVAR replacing the VAR for the factors, is now pursued.

(E0) Select a large data set with n series and T observations for each series. Transform the series to get stationarity and standardize them.
(E1) Estimate r. Out of the vast literature, beginning with Bai and Ng (2002), proposing consistent estimates $\hat{r}$, in the empirical applications in Section 5 we use Alessi et al. (2010). To check that $\hat{r} > \hat{q}$, our choice for the estimation of q is Hallin and Liška (2007).
(E2) Given $\hat{r}$, estimate the factors F_t, the factor loadings A and the common components $\chi_t = AF_t$ in equation (5). The estimate of the factors, $\hat{F}_t$, is given by the first $\hat{r}$ ordinary principal components of our data set. The factor loading estimator $\hat{A}$ is given by the eigenvectors corresponding to the largest $\hat{r}$ eigenvalues of the variance-covariance matrix of the x's. The estimated common component vector $\hat{\chi}_t$ is given by $\hat{A}\hat{F}_t$. Finally de-standardize the common component by multiplying each one of them by the standard deviation of the corresponding series.
(E3) Choose an $r \times n$ matrix ψ selecting the variables or the factors which are necessary for identification, along with other variables of interest. Estimate a VAR for $\hat{\chi}_t^{\psi} = \psi \left(\hat{\chi}_t' \;\; \hat{F}_t' \right)'$, to get an estimate of the matrix $D_{\psi}(L)$ and the VAR innovations v_t^{ψ} (see equation (14). Forni et al. (2009) show, under suitable assumptions, that the resulting estimates of the raw impulse-response functions $\hat{D}_{\psi}^{-1}(L)$ converge in probability to their population counterpart with rate $\max\left(1/\sqrt{n}, 1/\sqrt{T}\right)$.
(E4) Identify the structural shocks and the IRFs by SVAR techniques applied to $\hat{D}_{\psi}^{-1}(L)$ and v_t^{ψ}. Let us recall that we do not impose any rank reduction.
(E5) The steps (E2), (E3) and E4) are repeated for several values of r in an interval $[r_1, \; r_2]$ around the baseline value $\hat{r}$ estimated in (E1). Starting with r_1, we choose the final value of r as that for which the IRFs become stable.
(E6, optional) Estimate the whole panel of impulse response functions as $\hat{A}\hat{A}_{\psi}^{-1}\hat{B}_{\psi}(L)$, where $\hat{A}_{\psi} = \psi\hat{A}$. Alternatively, repeat steps (E3) and (E4) with a different ψ, selecting the additional variables of interest.

The estimation procedure above refers to datasets that are either stationary or are made stationary by the common-practice replacement of trending series by the first differences of the logs. On the other hand VAR in differences are misspecified if the variables are cointegrated. Here we adopt the simplest solution to this difficulty, namely the specification of non-stationary variables in log levels rather than growth rates. This requires the estimation of the common components of the levels (see Barigozzi et al. (forthcoming) for a thorough discussion of non-stationary dynamic factor models). The following procedure is recommended here:

(E0′) Starting from the original data set, not treated to reach stationarity, take the first difference (or the first difference of the logs) of all series (included the ones that are already stationary). Standardize the resulting series.
(E1′) Same as (E1).
(E2′) Follow (E2). Then take the cumulated sum of all common component to get the common components of the variables in levels.
(E3′) Same as (E3). If we want to get the same results irrespective of ψ we have to include a linear trend in the VAR. This is because different sets of cumulated common component

series span linear spaces differing by a deterministic linear trend.
(E4′ and E5′) Same as (E4) and (E5) respectively.

3.7 CC-SVARs, SVARs, the Standard-Procedure SDFMs and FAVARs

The advantages of CC-SVARs over SVARs, regarding fundamentalness and measurement errors, have been discussed in detail in the Introduction and Section 2, see also Sections 4 and 5.

Compared to the Standard Procedure SDFM, an important advantage of the CC-SVAR is that the common components of the variables of interest appear directly in the estimation step. Thus a natural and transparent procedure.

Secondly, uncertainty in the estimate $\hat{r}$ is explicitly taken into account. We start the estimation procedure with $\hat{r}$ determined by one of the available criteria, but then, see step (E5) in the estimation procedure, repeat the steps (E2), (E3) and (E4) for increasing values of r in an interval $[r_1,\ r_2]$ around $\hat{r}$, until stability of the IRFs is achieved. This "correction" of r is sensible and produces better IRFs, as compared to the Standard Procedure, in which only the IRFs corresponding to the baseline $\hat{r}$ are estimated, see Section 4.

Thirdly, in the Standard Procedure SDFM the identification techniques are applied to the residuals of the VAR estimated for F_t *after rank reduction.* Thus the residuals of the VAR are linearly combined two times, firstly for rank reduction, secondly for identification. On the other hand, in the CC-SVAR the identification techniques are applied to the (non-exactly-singular) residuals of the estimated VAR without reducing the rank, see step (E4). The simulations in Section 4 show that when $m = r$ in the CC-SVAR and $\hat{q}$ is not less than the true q in the Standard Procedure, the IRFs estimated by the two procedures are very similar. However, if $\hat{q}$ underestimates q, the Standard Procedure performs badly, this strongly suggesting that imposing the identification restrictions also takes care of the rank reduction. In conclusion, the estimation of q and the rank reduction are not necessary and a potential source of error.

FAVAR models, as introduced and studied in Bernanke and Boivin (2003) and Bernanke et al. (2005), estimate a SVAR for a vector including the observable variables of interest and the principal components of a large macroeconomic data set (the estimated factors). They are close to CC-SVARs in that they include the factors, which provide information free of measurement errors. On the other hand, with a few exceptions, the observable variables contain measurement errors, with negative consequences on the estimates. We see this in a simulation exercise in Section 4.3, where the CC-SVAR and the FAVAR performances are compared. Empirical situations in which the inclusion of some observable variables in a CC-SVAR is recommended are discussed in Appendix E.

4 Simulations

The procedure described in Section 3.6 is now applied to simulated data sets based on the model discussed in Section 2. Firstly we rewrite model (2) in static-factor form. Let

$$F_t = (k_t\ u_{a,t}\ u_{\tau,t}\ u_{\tau,t-1}\ u_{\tau,t-2})'.$$

The 5-dimensional vector F_t has the following singular VAR(1) representation:

$$\begin{pmatrix} k_t \\ u_{a,t} \\ u_{\tau,t} \\ u_{\tau,t-1} \\ u_{\tau,t-2} \end{pmatrix} = \begin{pmatrix} \alpha & 0 & -\kappa & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} k_{t-1} \\ u_{a,t-1} \\ u_{\tau,t-1} \\ u_{\tau,t-2} \\ u_{\tau,t-3} \end{pmatrix} + \begin{pmatrix} 1 & -\kappa\theta \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} u_{a,t} \\ u_{\tau,t} \end{pmatrix}. \tag{17}$$

Defining $Z_t = (a_t \ k_t \ \tau_t)'$, we have

$$Z_t = A^Z F_t + \xi_t^Z \tag{18}$$

where

$$A^Z = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

we generate a vector z_t including 100 additional time series ($T = 200$) as

$$z_t = A^z F_t + \xi_t^z \tag{19}$$

where A^z is the 100×5 matrix matrix of the loadings. The entries of A^z are generated independently from a standardized normal distribution. We define $x_t = (Z_t' \ z_t')'$ and $\xi_t = (\xi_t^{Z\prime} \ \xi_t^{z\prime})'$. We generate the measurement errors ξ assuming that $\xi_{it} \sim N(0, \sigma_i^\xi)$ where σ_i^ξ is uniformly distributed in the interval $(0, 0.5)$, so that different variables have measurement errors of different size (on average, the idiosyncratic components account for about 11% of total variance).

4.1 Changing m and ψ

In our first exercise, Simulation 1, we assess the performance of the CC-SVAR for different values of m. We estimate the common components using the true number of factors, i.e. $r = 5$. We run: (a) a VAR(4) with the common components of capital and taxes and the first principal component ($m = 3$); (b) a VAR(1) with the common components of capital and taxes and the first two principal components ($m = 4$); (c) a VAR(2) with the same variables (again $m = 4$); (d) a VAR(1) with the common components of capital and taxes and the first three principal components ($m = 5$). As above, we identify the tax shock by imposing that it is the only one affecting cumulated taxes in the long run. We repeat the exercise for 1000 data sets.

Figure 2 reports the results. The red dashed lines are the theoretical impulse response functions. The solid lines are the mean point estimates (mean over the different datasets) and the grey areas represent the 16th and 84th percentile of the point-estimate distribution. The results for specification (a) are reported in Panel (a). We see that there is a sizable bias and a large variability of the results, especially for taxes. This disappointing result is discussed below. Here we only observe that the number of lags included in the VAR is not responsible for it. Indeed, a similar result (not shown) is obtained with 8 lags instead of 4.

Panel (b) and (c) show results for specifications (b) and (c), respectively. The difference is the number of lags included: just one lag in Panel (b) and two lags in Panel (c). Comparing the two panels, it is seen that when $m = 4$ we need two lags in the VAR to get good estimates of the impulse response functions. Panel (d) confirms that, with $m = 5$, just one lag is enough, consistently with equation (17). In both Panels (c) and (d), the dynamics are estimated extremely well, with the mean impulse response functions almost overlapping with the theoretical ones. Notice that, with the more parsimonious model in (d), the variability of the estimates is somewhat smaller at large lags. In the present case the advantage of specification (d) is modest, since T is relatively large and the number of parameters to estimate is small even for specification (c). But for shorter data sets or data sets requiring a larger number of parameters, like the ones of the empirical applications in Section 5, the advantage of a more parsimonious specification is somewhat greater.

To shed some light on the disappointing result obtained with $m = 3$, we run a second simulation exercise, Simulation 2, analyzing what happens when changing ψ, i.e. the variables included in the CC-SVAR, for different values of m. For this exercise, we generate just one data set. As above, we use five principal components to estimate the common components.

To begin, we set $m = 3$. Then we estimate one hundred of different CC-SVAR(4) specifications, including the common components of capital and taxes, plus the common component of the $3 + i$-th variable, $i = 1, ..., 100$.

The result is reported in Figure 3, Panel (a). The red lines are the 100 estimated impulse response functions, the black lines are the true impulse response functions. We see that there are several specifications which produce bad estimates, despite the fact that we have $m = q+1$. We repeat the exercise by using the true common components in place of the estimated ones. The result is reported in Panel (b). With the true common components the results are good, consistently with the zeroless assumption (SDFM7). Hence the bad results of Panel (a) are due to the fact that the estimated common components are close to singular, though not exactly singular. When ψ is such that $B_\psi(L)$ is close to the non-zeroless region, the small idiosyncratic residual, which is still present in the estimated common components, produces large estimation errors.

Panels (c) and (d) show results for $m = 4$ and $m = 5$, respectively. We use four lags as before. In Panel (c) we include the same (estimated) common components of Panel (a), plus the first principal component as the fourth variable, equal for all specifications. We see that in this case the problem arising with $m = 3$ is solved. This is because matrices $B_\psi(L)$ very close to the non-zeroless region are much more unlikely, and actually never occur for this data set.[10]

Finally, in Panel (d) we have $m = 5$: the common components of capital and taxes, the third common component, changing across specifications, plus the first two principal components, which are kept fixed for all ψ. Consistently with the analysis in Section 3.5, all specifications produce exactly the same result, so that they produce a single line.

4.2 Changing r

In Simulation 3 we suppose that r is not known and use the criterion (E5), see Section 3.6, to determine the final value of $\hat{r}$. We try some values of $\hat{r}$ between 2 and 7. In all cases we set $m = \hat{r}$. For $m = \hat{r} = 2$ we estimate a CC-SVAR(2) including the common components of capital and taxes. For $m = \hat{r} = 3$ we estimate a CC-SVAR(2) including the common components of capital and taxes and the first principal component. For $m = \hat{r} = 7$ we estimate a CC-SVAR(2) including the common components of capital and taxes and the first five principal components. As usual, we repeat the exercise for 1000 data sets.

[10]Indeed, we did not find bad specifications for $m = 4$ even for several other data sets, not shown here.

Figure 4 shows the results. In panels (a) and (b), corresponding to $m = \hat{r} = 2$ and $m = \hat{r} = 3$ respectively, the impulse response functions are badly estimated, whereas for $m = \hat{r} = 7$ the results are pretty good, and very similar to those already obtained for $m = \hat{r} = 5$. Thus, with our simulated data, the criterion (E5) to determine the final value of $\hat{r}$ produces the correct result.

4.3 Standard-Procedure SDFM, CC-SVAR and FAVAR

In Simulation 4 we compare the CC-SVAR with the Standard Procedure for the SDFM and the FAVAR. Firstly, we estimate: (a) a SDFM with a too small number of common shocks, i.e. $\hat{q} = 1$, and (b) a SDFM with the correct number of shocks, i.e $\hat{q} = 2$. In both cases $\hat{r}$ is, correctly, equal to 5. Secondly, we estimate (c) a CC-SVAR with $m = \hat{r} = 5$. Finally, we estimate (d) a FAVAR including capital, taxes, technology and the first two principal components. In all cases we use two lags in the estimation. Again, we perform 1000 replications.

The results are reported in Figure 5. Panel (a) shows the results for the mis-specified SDFM (a). Not surprisingly, with this data generating process, where $q = 2$, setting $\hat{q} = 1$ has dramatic consequences on the estimates of the impulse response functions. With a different DGP and a larger q we can expect a smaller bias. However, the point is that, in real data applications, q can be underestimated, leading to sizable estimation errors.

Panels (b) and (c) refer to the correctly specified SDFM (b) and the CC-VAR (c), respectively. It is hard to see any difference between the two figures. This suggests that the rank reduction step typical of the factor model can be ignored with no consequences on the quality of the estimates. Moreover, as argued above, with the CC-SVAR (with $m = r$) we do not need an estimate of q, which is safer, in view of the results of Panel (a).

Finally, panel (d) reports the results for the FAVAR model. Owing to measurement errors, the estimates are clearly worse than those in panels (b) and (c).

Simulation 5 deals again with the choice of the specification of the variables included in the model. Like in Simulation 2, we use just one data set, but, unlike in Simulation 2, here we compare the SVAR, the FAVAR and the CC-SVAR. Regarding the SVAR model, we estimate one hundred of three-variable VAR(2) specifications, including capital, taxes, and the $3 + i$-th variable, $i = 1, ..., 100$. The results are reported in Figure 6, Panel (a). The figure shows that the choice of the third variable produces huge differences in the estimated impulse response functions, both because of the information delivered by the common component of the third variable and the extent of the contamination induced by the measurement error. Panel (b) refers to FAVAR models including capital, taxes, the $3 + i$-th variable, $i = 1 : 100$, plus the first two principal components. Again we use two lags. Here the estimated IRFs are much closer to each other, since information is not deficient. However, there is still some variability due to the size of the measurement error included in the third variable. Panel (c) refers to the CC-VAR, where, as already seen above, all lines are identical.

5 Applications

We illustrate the advantages of CC-SVAR analysis, as an alternative to SVAR, by means of two applications, the first on monetary policy shocks, the second on technology shocks. Our main results are: (I) as a consequence of non-fundamentalness and measurement errors the results of the SVAR analysis are quite unstable, depending on which variables are included in the vector. Thus the conclusions on the effects of structural shocks on macroeconomic variables are not robust, (II) some improvement in robustness is obtained with FAVAR models, although the effects of measurement errors are still evident, (III) with CC-SVAR instability completely disappears and robust conclusions can be drawn. Independently of variables used, contractionary monetary policy shocks reduce prices, and positive technology shocks increase hours worked.

5.1 Number of factors in McCraken and Ng dataset

In our applications we use both the monthly and quarterly dataset of McCracken and Ng (2016).[11] We exclude a few variables to obtain balanced panels and we end up with a monthly and quarterly dataset with 122 and 215 variables respectively. We transform each series to reach stationarity. We apply the criterion proposed by Alessi et al. (2010) and find a number of static factors $\hat{r} = 8$ for the monthly dataset and $\hat{r} = 10$ for the quarterly dataset. Thus we use, as baseline specification, $\hat{r} = 8$ and $\hat{r} = 10$ in the monthly and quarterly dataset respectively.

5.2 Monetary policy shocks

In the first application we use the monthly dataset to study the effects of monetary policy shocks. We start with the following exercise. We consider 118 different VAR specifications characterized by different vectors X_t^j, $j = 1, ..., 118$. Each of them includes five variables. Four of them are common to all vectors: the unemployment rate, industrial production growth, inflation and a policy rate. Each model includes an additional variable of the panel which differs across models.

For each of the 118 specifications, we identify the shock using three different identification schemes. Firstly, a Cholesky scheme. In this case the time span is 1966:M1-2008:M12, since we exclude the ZLB period. The ordering of the five variables is the following: the unemployment rate, industrial production growth, inflation, the federal funds rate and the fifth additional variable. The monetary policy shock is the fourth one.

The second and the third schemes are based on the proxy SVAR method (Mertens and Ravn (2013) and Stock and Watson (2018). In the first we use the Gertler and Karadi (2015) instrument (GK henceforth). In the second the Miranda-Agrippino and Ricco (forthcoming) instrument (MAR henceforth). For the last two identification schemes the span is 1991:M2-2008:M12. The common component are still computed using the sample 1966:M1-2008:M12. The policy rate in this case is the 1-year bond, to be consistent with the specifications used in both the above mentioned papers.

The first column of Figures 7-9 reports the estimated IRFs. Each red solid line represents the impulse response function of a particular specification, so that each box contains 118 different lines. The most striking result is the high degree of heterogeneity in the estimated responses, despite the fact that specifications differ only for the fifth variable. The result resembles the one of the simulation exercise of Figure 6. With all identifications schemes, there are specifications for which prices increase and specifications for which they decrease. The effects on unemployment and industrial production estimated using the external instrument approach are highly heterogeneous in term of magnitude. When using the GK instrument the effects of a contractionary shock appear to be expansionary for real economic activity variables for most of the specifications. All in all, the results suggest that drawing robust conclusions

[11] The data set is available at https://research.stlouisfed.org/econ/mccracken/fred-databases/.

about the propagation mechanisms of monetary policy shocks is very hard. Indeed, the effects differ substantially across specifications both qualitatively and quantitatively.

To understand the effects of enlarging the information set, we augment each 5-variable specification with the 8 principal components. We then run a FAVAR with 13 variables and apply the three identifications schemes. In this case information is complete but still the model can suffer the problem arising from the presence of measurement error.

The results are reported in the second column of Figures 7-9. Completing information seems to have important consequences for all three identification schemes. Firstly, the price puzzle is greatly reduced with the Cholesky identification scheme, as observed in Bernanke et al. (2005), and disappears with the two proxy SVAR schemes. Secondly, the effects are quite consistent across specifications in terms of sign. Thirdly, the effects are much more similar in terms of magnitude.

To understand the implications of measurement errors we repeat the same exercise as before but replacing the variables with their common components. So, we estimate 118 different CC-VAR specifications which include the common components of the interest rate, industrial production growth, inflation and unemployment, plus a fifth common component which changes for each specification and three principal components. These models should be free of measurement error and include all the relevant information. We use the first 3 principal components (but using three common component of any triple of variables would yield the same results). The results are reported in the third column of Figures 7-9. The impulse response functions overlap for all of the 118 different specifications and the effects are in line with common wisdom about the effects of a contractionary monetary policy shock. The price puzzle does not occur, even using the standard Cholesky scheme.

In the specifications above the I(1) are taken in log-differences. We also perform the SVAR, the FAVAR and the CC-SVAR using the levels of all variables. For the CC-SVAR we follow the procedure (E0$'$)-(E5$'$) at the end of Section 3.6. Figure 10 displays the results for the MAR proxy SVAR identification scheme. The first three columns correspond to the three columns of the previous figures. The fourth column is the CC-SVAR for the common components augmented with a deterministic linear trend, see (E3$'$) in Section 3.6.

Overall, the results obtained with the CC-SVAR for the three identification schemes are qualitatively very similar. In particular, the sign of the responses are the same: a contractionary monetary policy shock leads to a short-run reduction in real activity and a reduction in prices. The effects of the two proxy CC-SVARs are also quantitatively similar, while this is not the case, in general, when using a SVAR with five variables. Relative to those estimated with external instruments, the effects on real economic activity and prices estimated with a Cholesky scheme are slightly smaller.

To assess the robustness of the results, we repeat the CC-SVAR analysis using $m = \hat{r} = 6, 8, 10$ common components. When $\hat{r} = 10$ we include in the VAR the five common components plus the first five principal components. When $\hat{r} = 6$ we include in the VAR the five common components plus the first principal components. The results are displayed in Figure 11. The result with $\hat{r} = 10$ are very similar to those obtained with $\hat{r} = 8$. On the contrary when $\hat{r} = 6$ a few differences emerge, which is likely the consequence of incomplete information, $\hat{r}$ being too small.

In conclusion, using the SVAR analysis, the results obtained with the three identification schemes, including the external instrument approach, crucially depend on the specification of the model, more precisely on the choice of the variables. For some specifications the IRFs are plausible, others produce puzzling results.

On the other hand, the IRFs of the CC-SVAR analysis, with the three identification procedures, apart from minor quantitative differences, are very similar, a result that runs counter the growing consensus that high frequency identification with external instruments is a better approach to identify monetary policy shocks, in comparison to the Choleski scheme. Moreover, the IRFs are in line with the standard view of the transmission mechanism of monetary policy shocks, in which contractionary policy shocks reduce prices and slow down economic activity in the short run.

5.3 Technology shocks and hours worked

In the second application we study the effects of technology shocks on hours worked. The empirical result has relevant implications for economic theory.

According to the existing SVAR literature, the effect of technology shocks depends crucially on the treatment applied to the time series of hours worked, see in particular Galí (1999b) and Christiano et al. (2003a)). In a bivariate SVAR with labor productivity and hours, the sign of the response of hours depends on whether hours are entered in log-levels or growth rates. In the first case hours increase, while in the second hours fall.

Here we show that, when information is properly taken into account, hours increase, independently of the data treatment chosen.

We consider the quarterly dataset, which includes 225 variables. We estimate 223 three-variable SVAR specifications. All specifications include the growth rate of labor productivity and the growth rate of per-capita hours. The third variable differs across specifications (we use all of the remaining 223 series). We identify the technology shock following Galí (1999b), assuming that it is the only shock affecting labor productivity in the long run. Panel (a) of Figure 12 reports the estimated impulse response functions. The response of hours is negative in most specifications. However, in a few of them hours increase in a hump-shaped manner. So a decrease in hours worked after a positive technology shock is not a fully robust finding.

Column 1 of Panel (b) in Figure 12 reports the response of labor productivity and hours when hours enter in log-levels. Again there are 223 specification differing in the third variable included in the model. In this case the response of hours is positive in most of the specifications, but not in all. In a few specification indeed hours decrease, either on impact or in the long-run. As previously, an increase in hours after a positive technology shocks is not a fully robust finding.

Column 2 of panels (a) and (b) report the response of labor productivity and hours worked estimated in a FAVAR including labor productivity and hours (in growth rates panel (a), in levels panel (b)), a third variable again, and 10 principal components, the number of static factors determined in Section 5.1. In all these specifications information is complete. A very important result is that the responses of hours corresponding to the two treatments (panels (a) and (b) respectively) are qualitatively similar.

The third column reports the responses obtained when instead of the three variables one uses their common components together with the first seven principal components in a CC-SVAR. As expected, for each treatment of the hours, the responses are identical across specifications. Moreover, enhancing the result obtained with the FAVAR, the responses are very similar across treatments: hours are nearly zero on impact and then increase, reaching their maximum after around two years.

As in the previous application in Section 5.2, we repeat the CC-SVAR analysis using $m = \hat{r} = 8, 10, 12$ common components. The results are displayed in Figure 11. The result with $\hat{r} = 12$ are very similar to those obtained with $\hat{r} = 10$. On the contrary when $\hat{r} = 8$ a few differences emerge, which is likely the consequence of incomplete information, $\hat{r}$ being too small.

The most important result of the CC-SVAR analysis is that, as we have seen, the response of hours does not depend on whether hours worked are taken in first differences or in levels. Apart from a minor difference in the sign of the first impact, in both cases a small impact effect is followed by a significant positive hump-shaped increase. Thus we contribute to solving, or at least substantially smoothing, the old-standing controversy about the effects of technology shocks on hours worked, see Galí (1999a) and Christiano et al. (2003b): Hours, in line with Christiano et al. (2003b), do increase following a positive technology shock.

Another remarkable difference of CC-SVAR with respect to SVAR results is that labor productivity, and thus output, increases very slowly in the CC-SVAR with both treatments of hours, which is not the case with the SVAR using first differences. This, together with a slow increase of hours, is consistent with the view of technology as a news shock. With this interpretation of technology, the response of hours estimated in the CC-SVAR is fully in line with both New-Keynesian models with nominal wage and price rigidities, see e.g. Barsky and Sims (2009), Christiano et al. (2010), Barsky et al. (2015)), and with RBC models featuring frictions like habit formation in consumption, adjustment costs of investment, or with the assumption on preferences in Jaimovich and Rebelo (2009) (see also Schmitt-Grohé and Uribe (2012)).

6 Conclusion

CC-SVARs apply SVAR techniques to singular vectors including the common components of the variables of interest. We claim that CC-SVARs provide a solution to the difficulties arising with possible non-fundamentalness of the structural shocks and measurement errors in macroeconomic variables. In two applications to relevant empirical problems the CC-SVAR produces results that, unlike those obtained with SVAR analysis, are both sensible and robust with respect to changes in specification.

Although we have introduced and discussed the CC-SVAR technique with reference to the DFM model described in Section 3.1, a similar method applies in the General Dynamic Factor Model, that is when the assumption of a finite number of static factors does not necessarily hold and χ_t^{ψ} is estimated by frequency-domain methods, see Forni et al. (2015, 2017). This however is left to future research.

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Figures

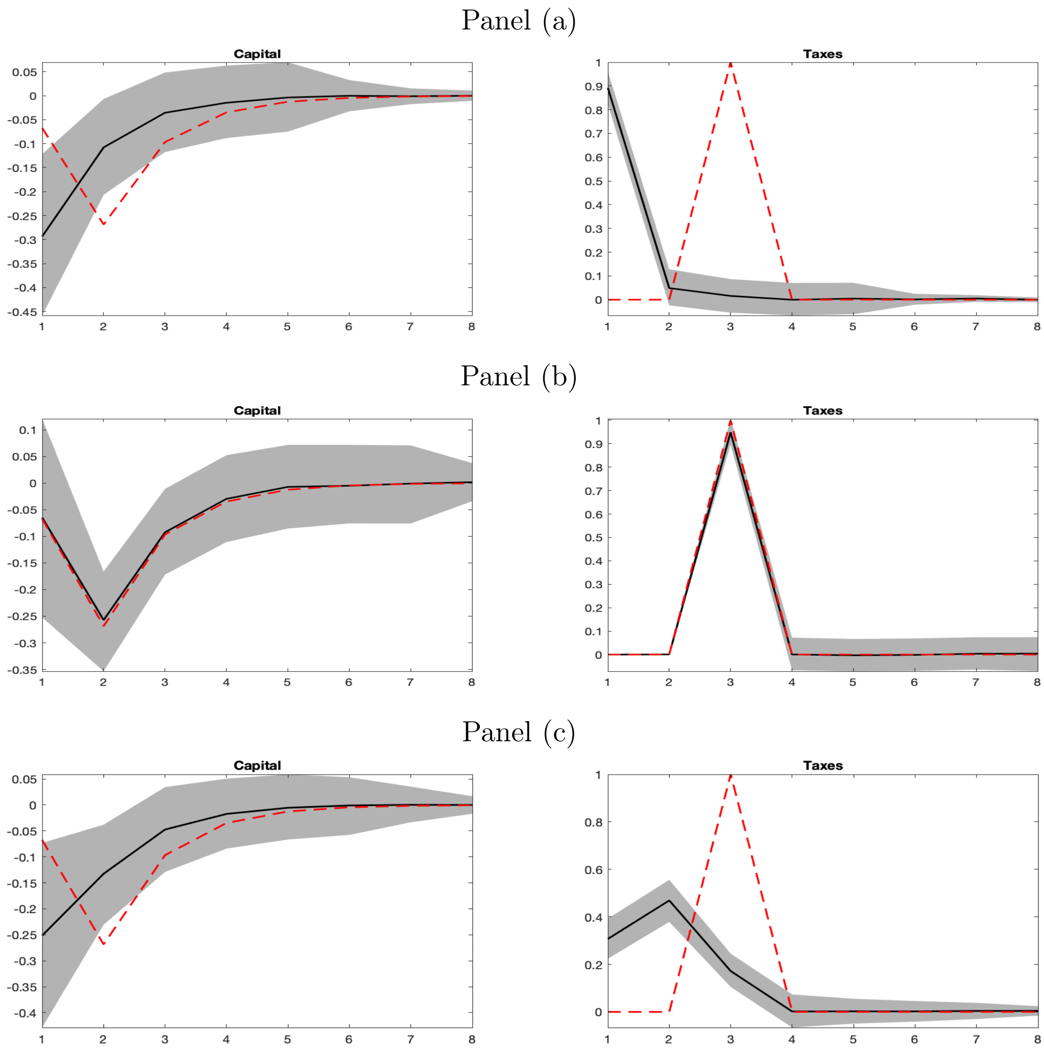


Figure 1: Simulation: Non-fundamentalness and measurement errors. Estimated IRFs for the tax shock. The red dashed lines are the theoretical IRFs. The solid lines represent the mean (across 1000 simulated datasets) of the point estimates. The grey areas represent the 16th and 84th percentiles of the point estimate distribution. Panel (a): SVAR(4) with Capital and Taxes. Panel (b): SVAR(3) with Capital, Taxes and Technology. Panel (c): SVAR(3) with Capital, Taxes and Technology when Technology is measured with a 5% error.

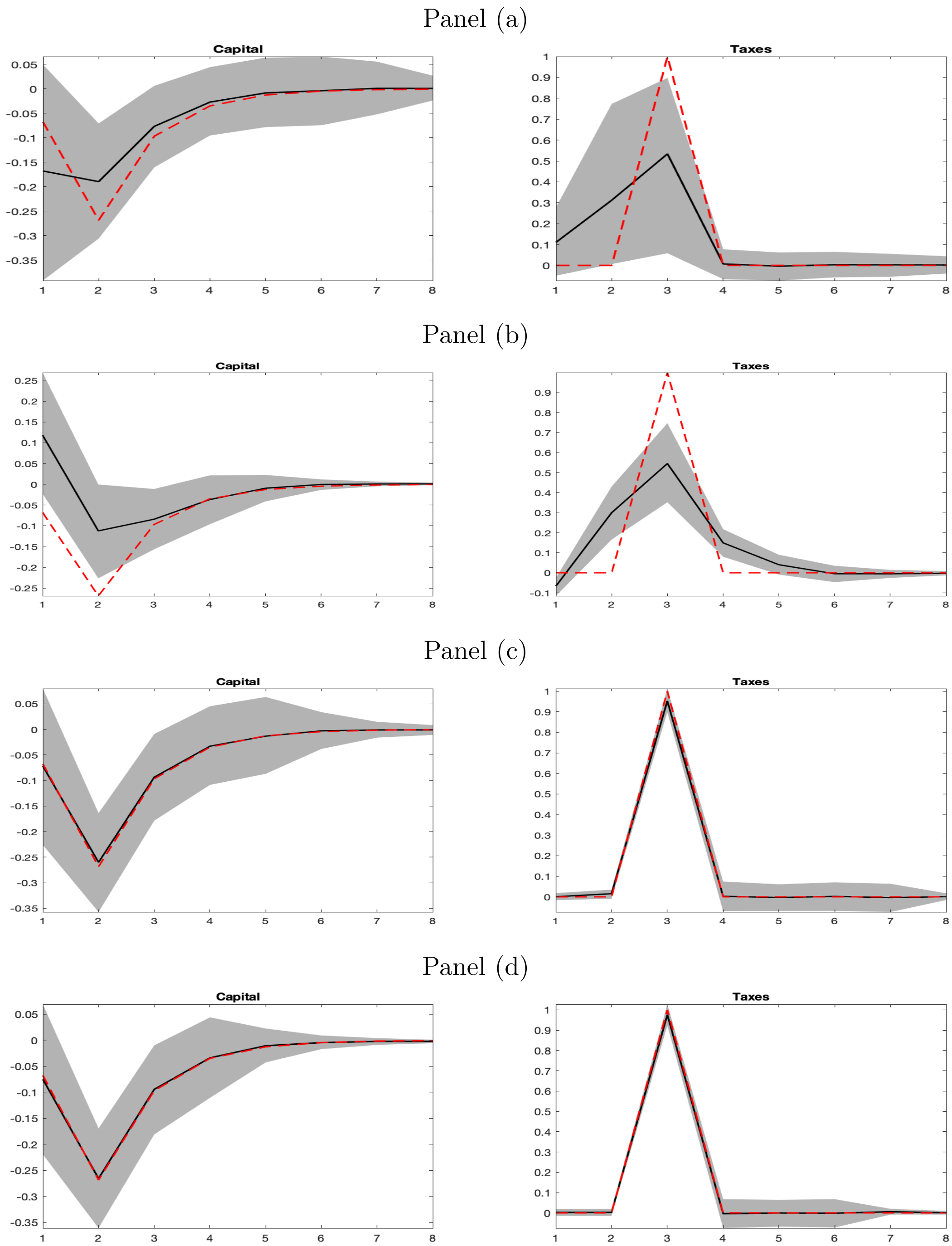


Figure 2: Simulation 1: The choice of m.
Estimated IRFs for the tax shock. The red dashed lines are the theoretical IRFs. The solid lines represent the mean (across 1000 simulated datasets) of the point estimates. The grey areas represent the 16th and 84th percentiles of the point estimate distribution. Panel (a): CC-SVAR(4) with Capital, Taxes and the first principal component ($m = 3$). Panel (b): CC-SVAR(1) with Capital, Taxes and the first 2 principal components ($m = 4$). Panel (c): CC-SVAR(2) with Capital, Taxes and the first 2 principal components ($m = 4$). Panel (d): CC-SVAR(1) with Capital, Taxes and the first 3 principal components ($m = 5$).

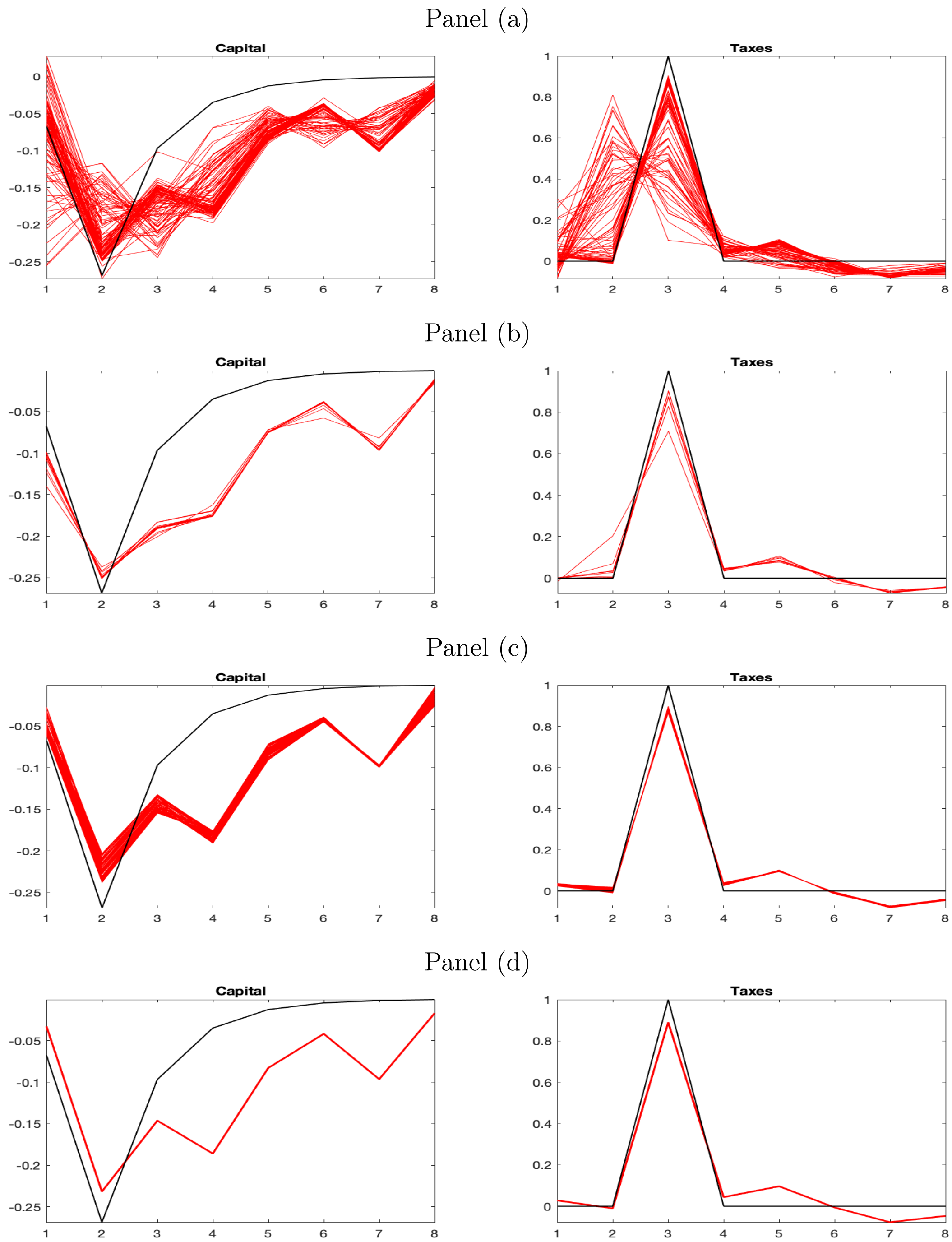


Figure 3: Simulation 2: the choice of ψ with $m < r$ and $m = r$.
Estimated IRFs for the tax shock, for a single simulated data set. The black lines are the theoretical IRFs. The red lines are the CC-SVAR estimates obtained with different variable specifications. Panel (a): CC-SVAR(4) with Capital, Taxes and a third variable, changing across specifications ($m = 3$). Panel (b): same as Panel (a) with the true common components in place of the estimated ones. Panel (c): CC-SVAR(4) with Capital, Taxes the changing variable and the first principal component ($m = 4$). Panel (d): CC-SVAR(4) with Capital, Taxes, the changing variable and the first 2 principal components ($m = 5$).

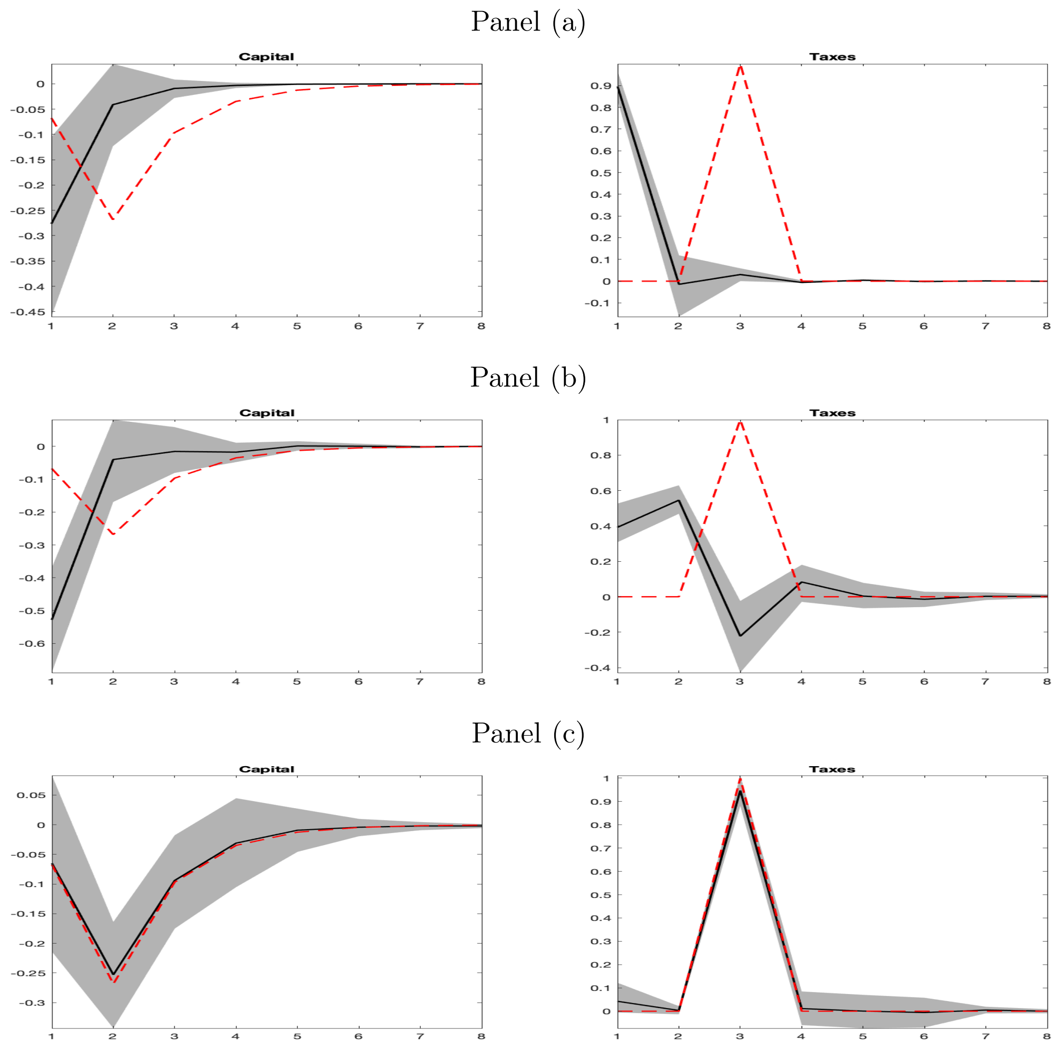


Figure 4: Simulation 3: The choice of $\hat{r}$. Results for $m = \hat{r} < r$ and $m = \hat{r} > r$. Estimated IRFs for the tax shock. The red dashed lines are the theoretical IRFs. The solid lines represent the mean (across 1000 simulated datasets) of the point estimates. The grey areas represent the 16th and 84th percentiles of the point estimate distribution. Panel (a): CC-SVAR(2) with $\hat{r} = m = 2$ (Capital and Taxes). Panel (b): CC-SVAR(2) with $\hat{r} = m = 3$ (Capital, Taxes and the first principal component). Panel (c): CC-SVAR(2) with $\hat{r} = m = 7$ (Capital, Taxes and the first 5 principal components).

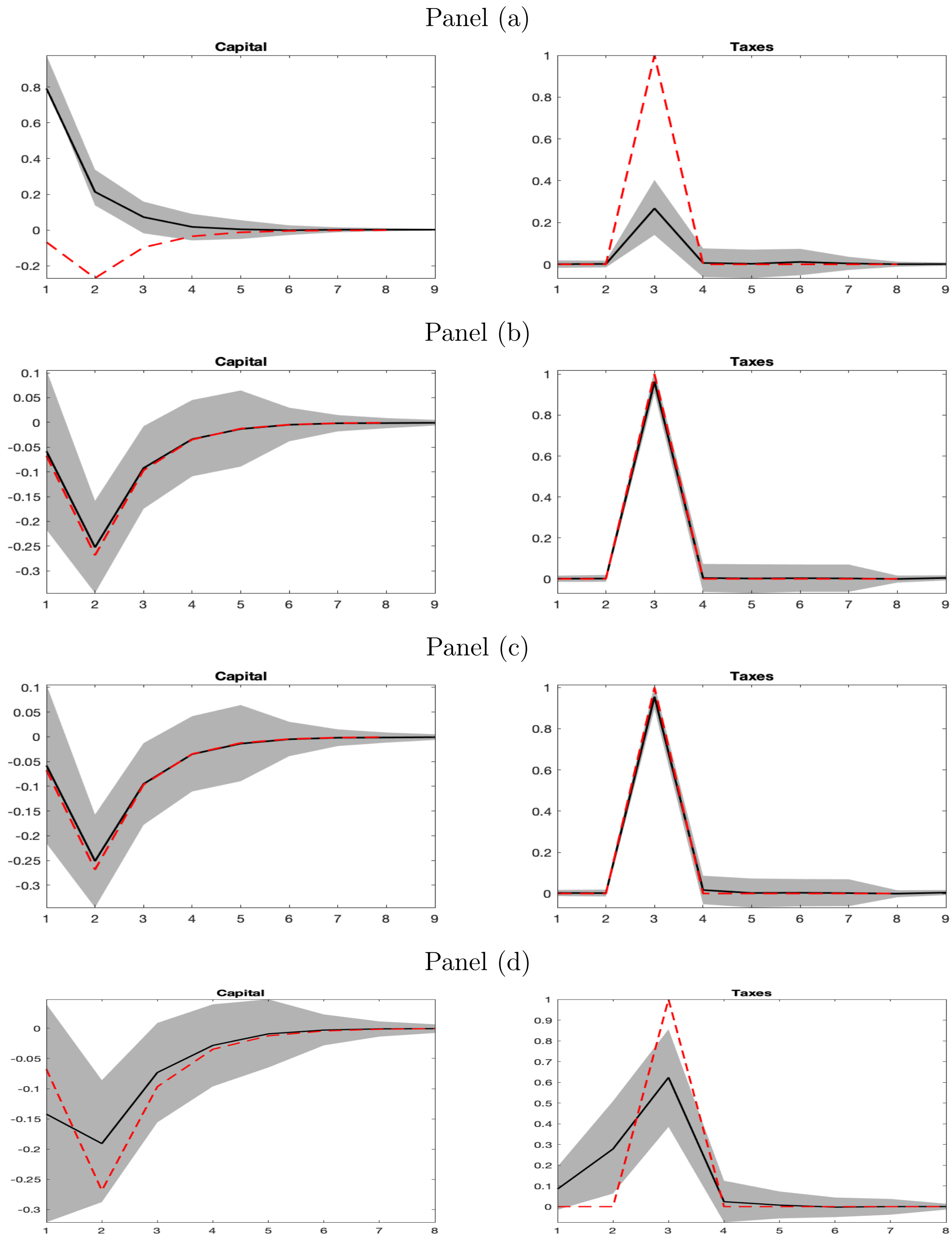


Figure 5: Simulation 4: Standard DFM, CC-SVAR, FAVAR
Estimated IRFs for the tax shock. The red dashed lines are the theoretical IRFs. The solid lines represent the mean (across 1000 simulated datasets) of the point estimates. The grey areas represent the 16th and 84th percentiles of the point estimate distribution. Panel (a): DFM(2) with $\hat{q} = 1 < q$ $(\hat{r} = r = 5)$. Panel (b): DFM(2) with $\hat{q} = q = 2$ $(\hat{r} = r = 5)$. Panel (c): CC-SVAR(2) with Capital, Taxes and the first 3 principal components $(m = \hat{r} = 5)$. Panel (d): FAVAR(2) with Capital, Taxes and the first 3 principal components.

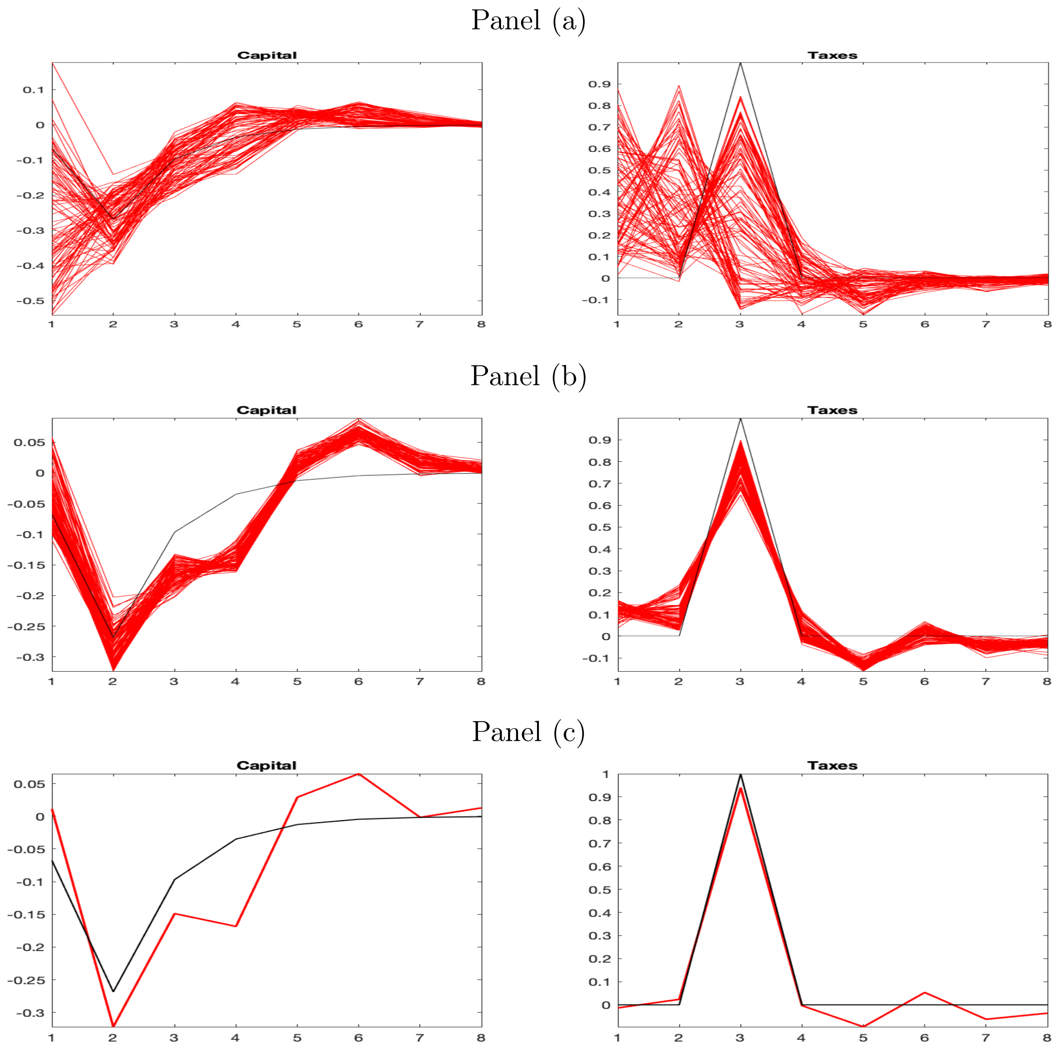


Figure 6: Simulation 5: Different variable specifications for a deficient VAR, the FAVAR and the CC-SVAR.
Estimated IRFs for the tax shock, for a single simulated data set. The black lines are the theoretical IRFs. The red lines are the CC-SVAR estimates obtained with different variable specifications. Panel (a): SVAR(2) with Capital, Taxes and a third variable, changing across specifications. Panel (b): FAVAR(2) with Capital, Taxes, a third variable, changing across specifications, and the first two principal components. Panel (c): CC-SVAR(2) with Capital, Taxes, a third variable, changing across specifications, and the first two principal components.

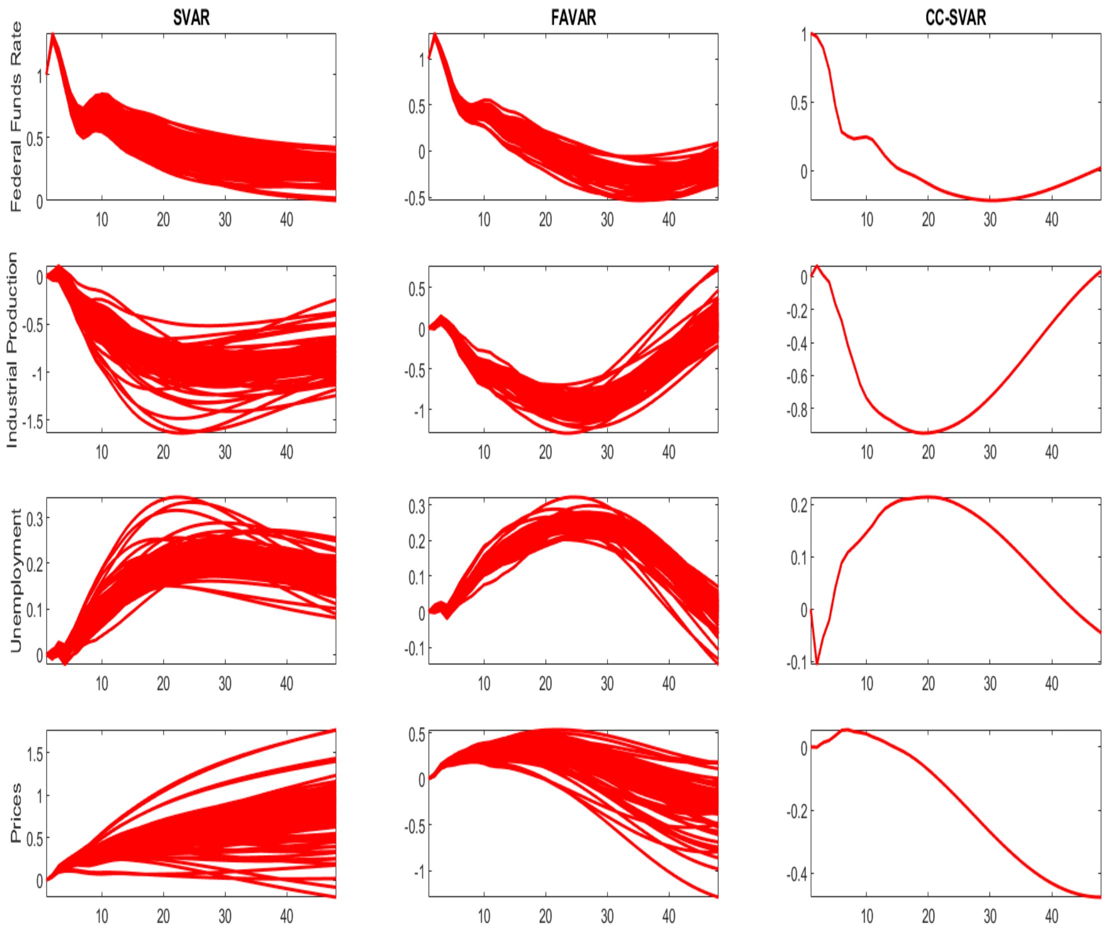


Figure 7: US MONTHLY DATA. THE IRFS OF A MONETARY POLICY SHOCK. CHOLESKY IDENTIFICATION.

The red lines are the CC-SVAR estimates obtained with different variable specifications. First column: SVAR(6) for 118 five-variable specifications, differing for the fifth variable. Second column: FAVAR(6) the variables in the first column are augmented with the first 8 principal components. Third column: CC-SVAR(6): the variables in the first column are replaced with their common components; in addition, we include the first 3 principal components ($\hat{r} = 8$).

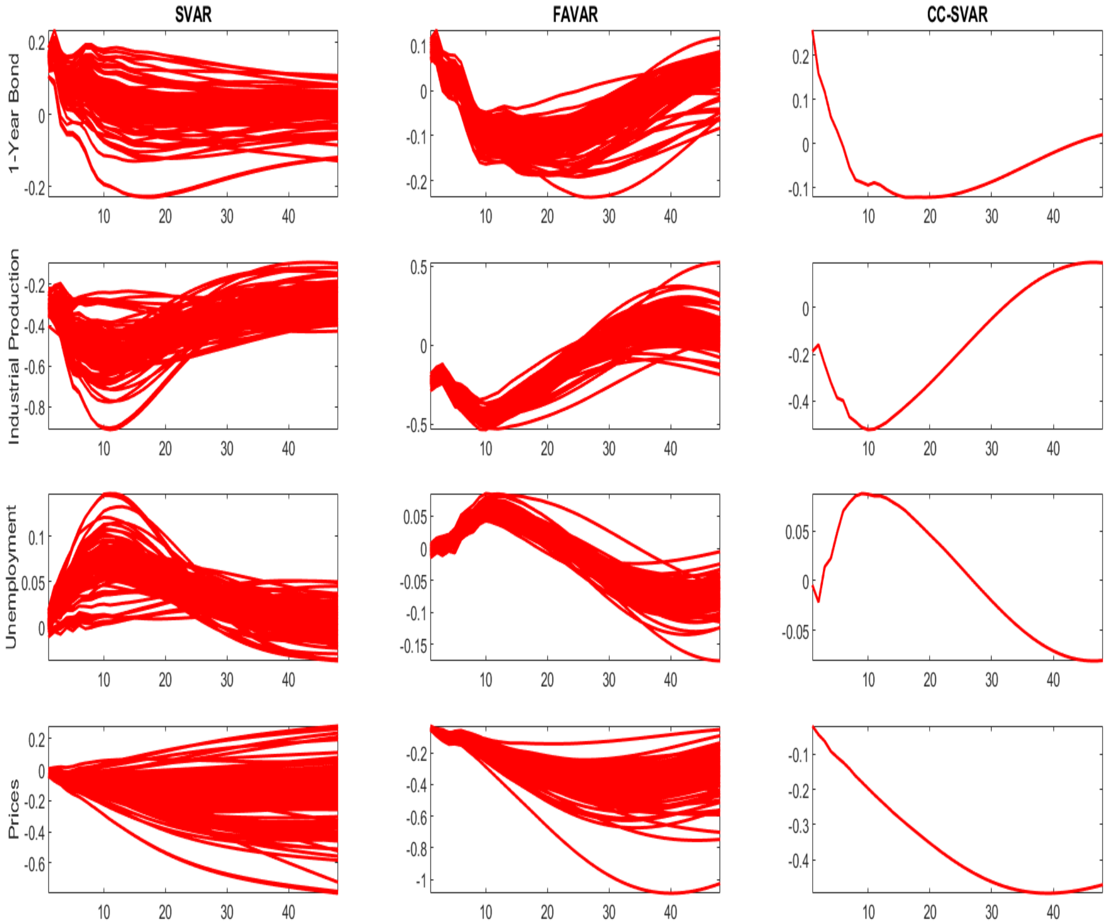


Figure 8: US MONTHLY DATA. THE IRFS OF A MONETARY POLICY SHOCK. PROXY MAR IDENTIFICATION.
The red lines are the CC-SVAR estimates obtained with different variable specifications. First column: SVAR(6) for 118 five-variable specifications, differing for the fifth variable. Second column: FAVAR(6) the variables in the first column are augmented with the first 8 principal components. Third column: CC-SVAR(6): the variables in the first column are replaced with their common components; in addition, we include the first 3 principal components ($\hat{r} = 8$).

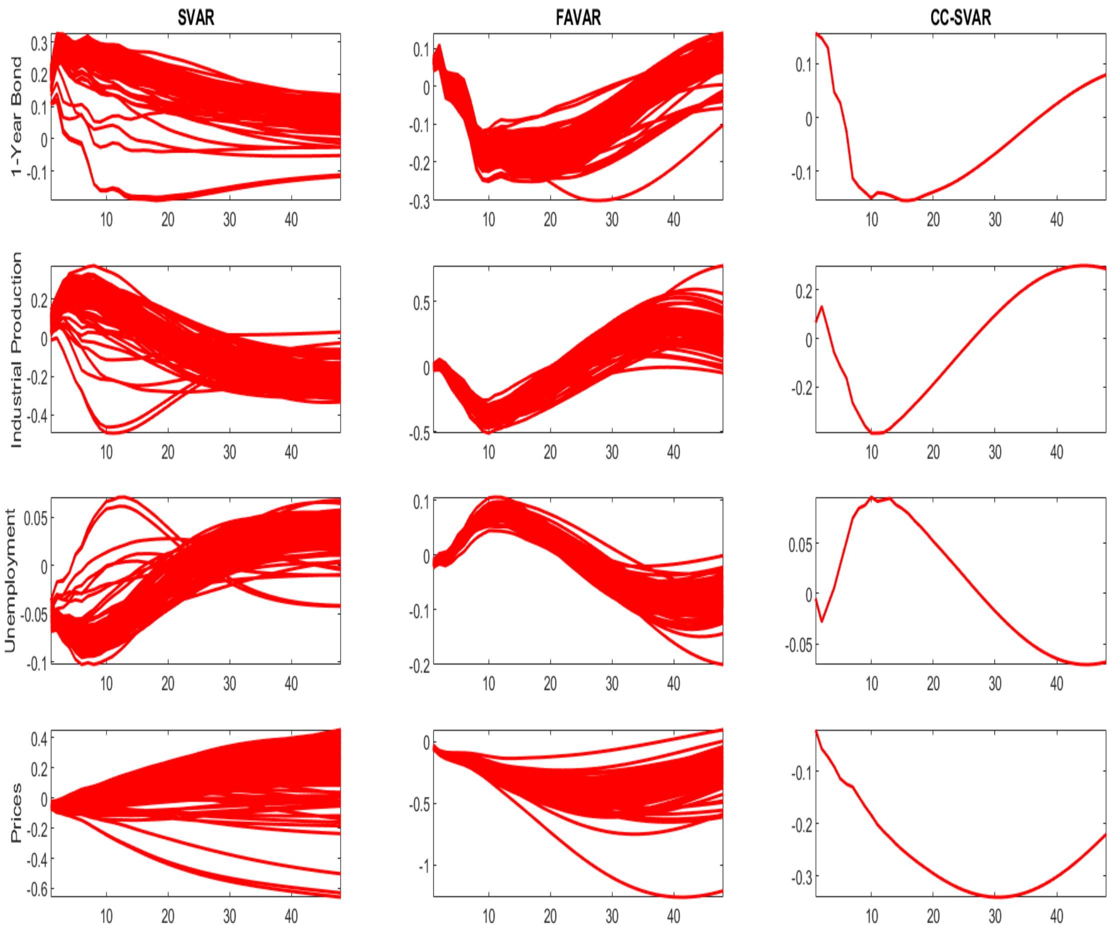


Figure 9: US MONTHLY DATA. THE IRFS OF A MONETARY POLICY SHOCK. PROXY GK IDENTIFICATION.

The red lines are the CC-SVAR estimates obtained with different variable specifications. First column: SVAR(6) for 118 five-variable specifications, differing for the fifth variable. Second column: FAVAR(6) the variables in the first column are augmented with the first 8 principal components. Third column: CC-SVAR(6): the variables in the first column are replaced with their common components; in addition, we include the first 3 principal components ($\hat{r} = 8$).

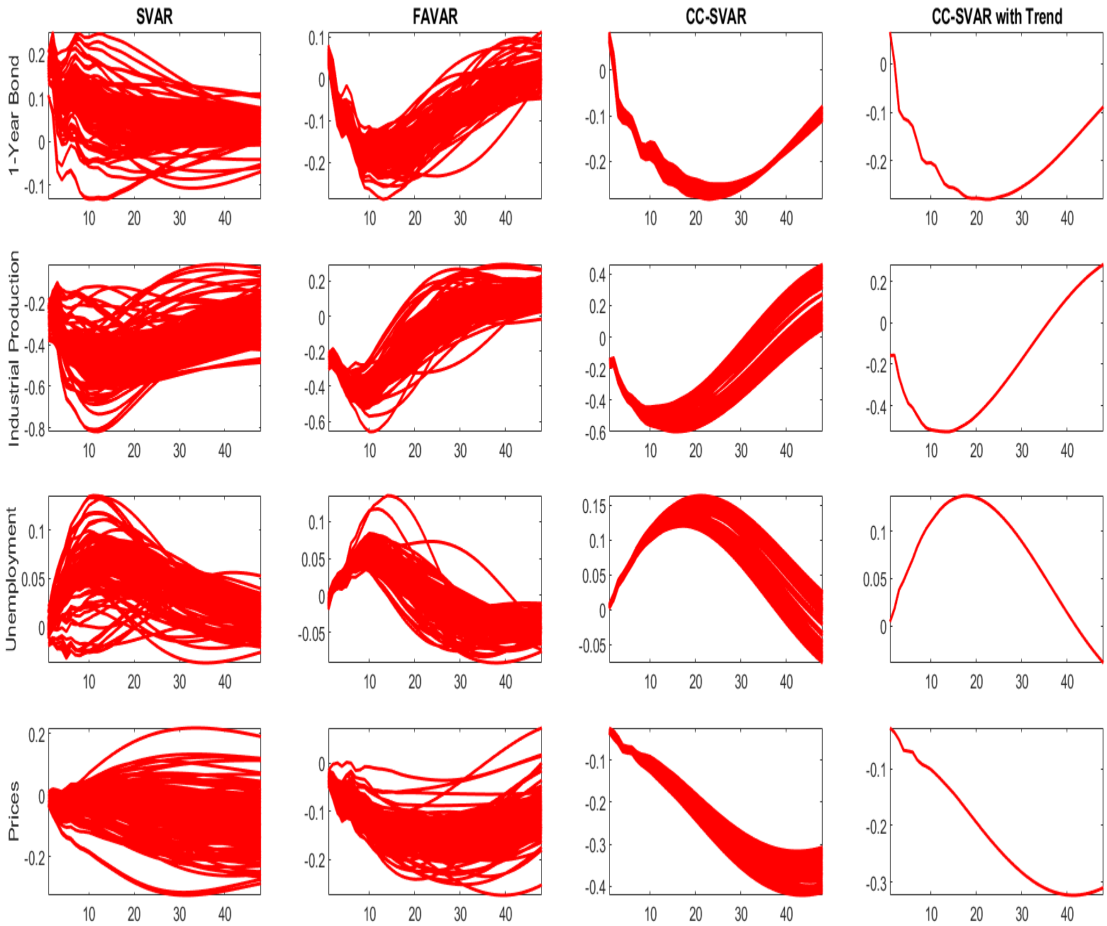


Figure 10: US monthly data in levels. The IRFs of a monetary policy shock. Proxy MAR identification.
The red lines are the CC-SVAR estimates obtained with different variable specifications. First column: SVAR(6) for 118 five-variable specifications, differing for the fifth variable. Second column: FAVAR(6). The variables in the first column are augmented with the first 8 principal components. Third column: CC-SVAR(6). The variables in the first column are replaced with their common components; in addition, the first 3 principal components are included ($\hat{r} = 8$). Fourth column: CC-SVAR(6), linear trend. As in the third column, but with a linear trend.

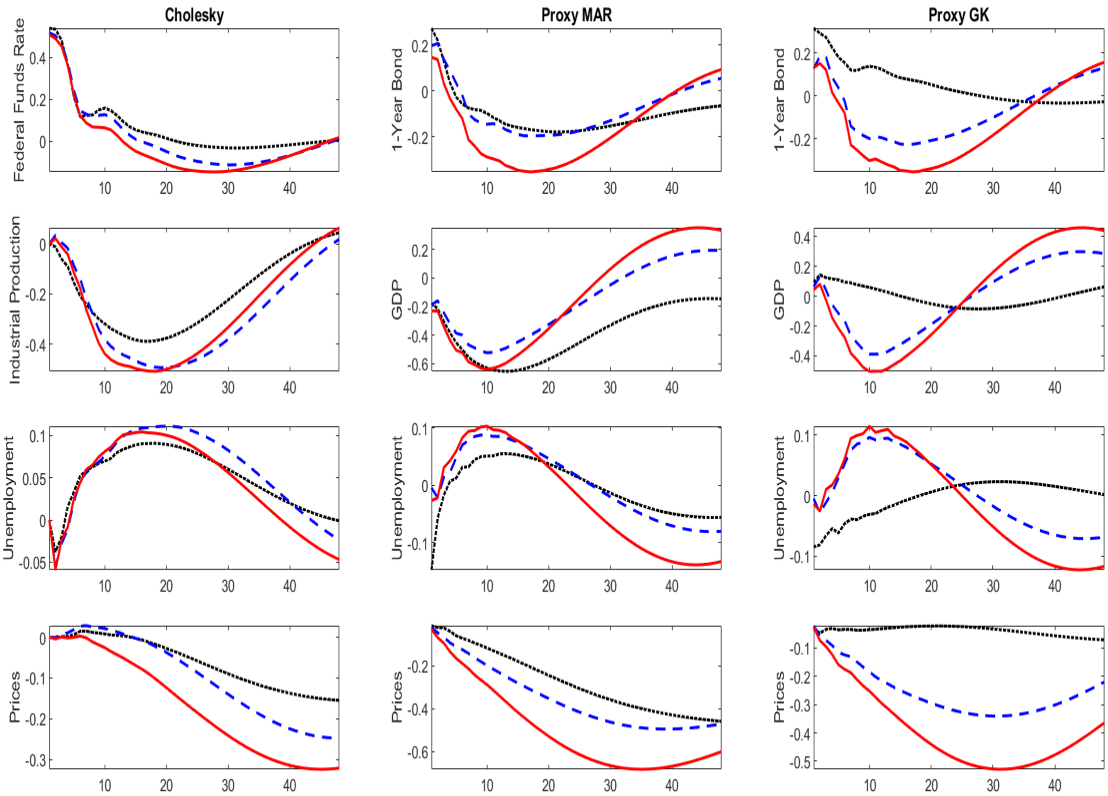
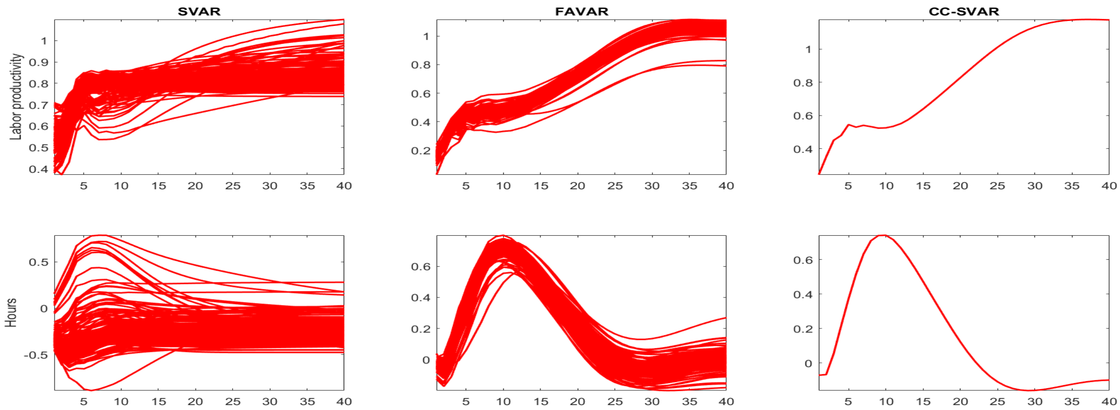


Figure 11: US MONTHLY DATA. THE IRFS OF A MONETARY POLICY SHOCK.
CC-SVAR(6) with $m = r$, using different values of r. Black dotted line: $r = 6$. Blue dashed line: $r = 8$. Red solid line: $r = 10$.

Panel (a)



Panel (b)

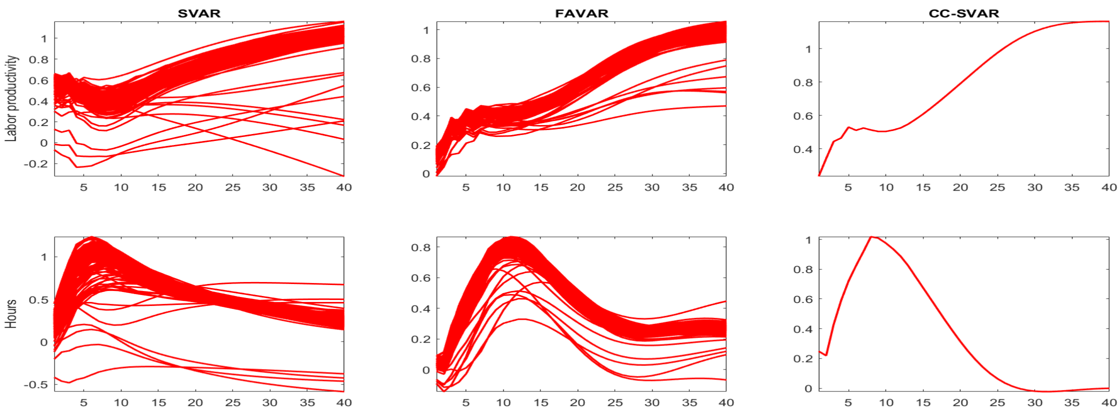


Figure 12: US QUARTERLY DATA. THE IRFS OF A PRODUCTIVITY SHOCK. GALÍ (1999B) LONG-RUN IDENTIFICATION.
The red lines are the CC-SVAR estimates obtained with different variable specifications. Panel (a): Hours in first differences. Panel (b): Hours in levels. First column: SVAR(6) for 118 five-variable specifications, differing for the fifth variable. Second column: FAVAR(6) the variables in the first column are augmented with the first seven principal components. Third column: CC-SVAR(6): the variables in the first column are replaced with their common components; in addition, we include the first seven principal components ($\hat{r} = 8$).

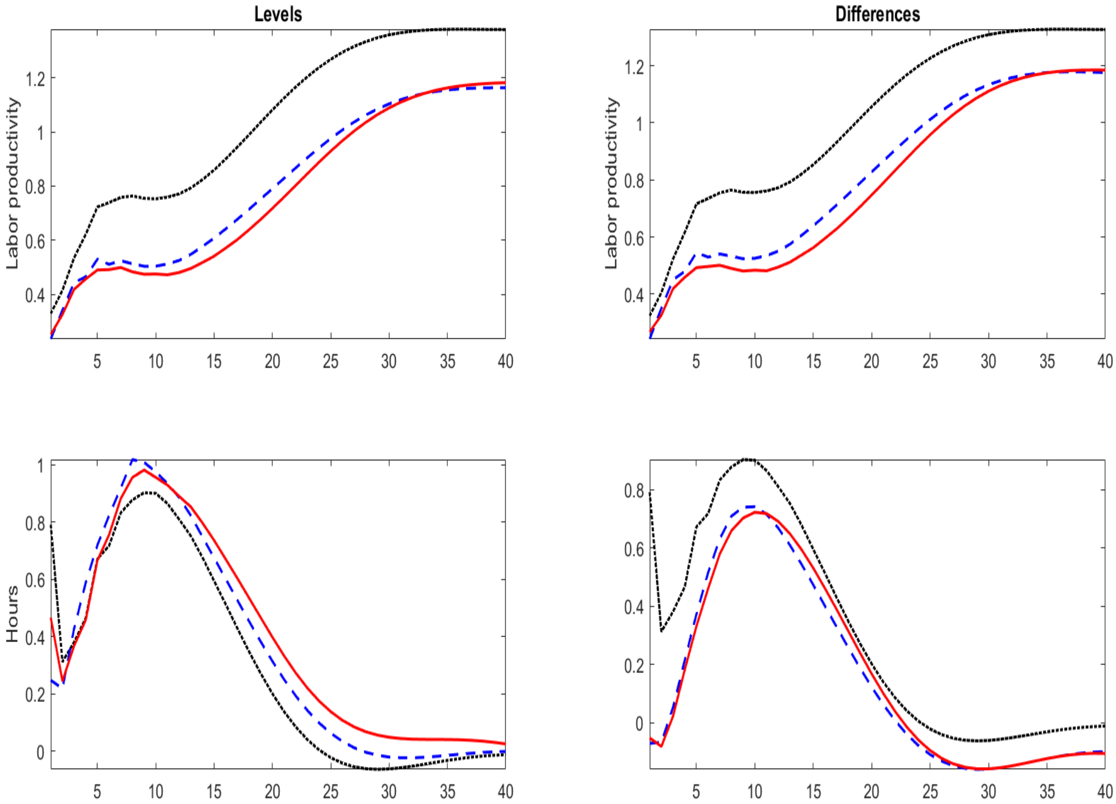


Figure 13: US MONTHLY DATA. THE IRFS OF A TECHNOLOGY SHOCK.
CC-SVAR(6) with $m = \hat{r}$, using different values of $\hat{r}$. Black dotted line: $\hat{r} = 8$. Blue dashed line: $\hat{r} = 10$. Red solid line: $\hat{r} = 12$.

Appendix

A 'Deep' Parameters and the Alternative Principle

Consider firstly the model studied in Section 2, which is reported here for convenience:

$$\begin{pmatrix} a_t \\ k_t \\ \tau_t \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \frac{-\kappa(L+\theta)}{1-\alpha L} & \frac{1}{1-\alpha L} \\ L^2 & 0 \end{pmatrix} \begin{pmatrix} u_{\tau,t} \\ u_{a,t} \end{pmatrix} = B(L)u_t. \tag{2}$$

The matrix $B(L)$ depends on the 3-dimensional parameter vector $\Pi = (\kappa\ \theta\ \alpha)$. It is zeroless unless $\kappa = 0$ or $\theta = 0$, thus generically zeroless if Π varies in an open set $\mathcal{C} \subseteq \mathbb{R}^3$. We see here that the assumption of AD2, in this case that the coefficients of the entries of the matrix $B_\psi(L)$ vary independently of one another, is sufficient but not necessary for generic zerolessness of $B_\psi(L)$.

Secondly, consider a generalization of model (12). Let

$$\begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = \begin{pmatrix} a_1 + b_1 L \\ a_2 + b_2 L \end{pmatrix} u_t = B(L)u_t, \tag{20}$$

We see that $B(L)$ is zeroless unless

$$a_1 b_2 - a_2 b_1 = 0. \tag{21}$$

Now suppose that all the 4 coefficients a_i and b_i are structural polynomial functions of the scalar parameter $\Pi \in \mathcal{C}$, where $\mathcal{C}$ is an open subset of $\mathbb{R}$. Equation (21) becomes

$$a_1(\Pi)b_2(\Pi) - a_2(\Pi)b_1(\Pi) = p(\Pi) = 0. \tag{22}$$

As $p(\Pi)$ is a polynomial function then the following *alternative principle* holds: Either $p(\Pi)$ vanishes for a finite number of points, a lower-dimensional set, or it vanishes identically. Equivalently, either generically $B(L)$ is zeroless or it is not zeroless for all $\Pi \in \mathcal{C}$. Now, the functions a_i and b_i are structurally determined in a way that bears no relationship to equation (21). It is therefore reasonable to assume that they do not fulfill the restriction (22) for all $\Pi \in \mathcal{C}$, so that, by the alternative principle, $B(L)$ is generically zeroless, like in model (2).

The same argument can be applied in general. Let the entries of $B_\psi(L)$ be

$$\frac{b_{ij,0} + \cdots + b_{ij,s_1} L^{s_1}}{1 - \tilde{b}_{ij,1} L - \cdots - \tilde{b}_{ij,s_2} L^{s_2}}.$$

and assume that the coefficients $b_{ij,k}$ and $\tilde{b}_{ij,h}$ are polynomial functions of a vector of structural parameters Π which varies in an open set $\mathcal{C} \subseteq \mathbb{R}^\nu$. The condition for non-zerolessness of $B_\phi(L)$ is a set of polynomial equations, among the coefficients $b_{ij,k}$ and $\alpha_{ij,h}$ of the entries of $B_\psi(L)$:

$$p_1(\mathbf{b}) = 0, \quad \ldots, \quad p_K(\mathbf{b}) = 0, \tag{21$'$}$$

where $\mathbf{b}$ is the vector gathering all the b's and $\tilde{b}$'s. Using the polynomial dependence of $\mathbf{b}$ on Π we generalize (22):

$$p_1(\mathbf{b}(\Pi)) = 0, \quad \ldots, \quad p_K(\mathbf{b}(\Pi)) = 0. \tag{22$'$}$$

Then either ($22'$) holds for a lower dimensional subset of $\mathcal{C}$ or for all $\mathcal{C}$, this is the alternative principle in its general form. Again, the function $\mathbf{b}$ bears no relationship to equations ($21'$). Thus we believe that it is reasonable to assume that for at least one value of $\Pi \in \mathcal{C}$ the matrix $B_\psi(L)$ is zeroless, so that, by the general version of the alternative principle, $B_\psi(L)$ is generically zeroless.

B A Singular VAR Approximated by a Non-Singular VAR

Firstly note that non-singularity of $\hat{\chi}_t^\psi$ is only caused by the finiteness of n. Thus, for simplicity, assume that T has already reached infinity. Our estimate, given n, is

$$\hat{\chi}_t^\psi = \chi_t^\psi + \zeta_t,$$

where ζ_t, the idiosyncratic residual, tends to 0 as n tends to infinity. Again, for simplicity assume that ζ_t is a non-singular m-dimensional white noise orthogonal to u_t at any lead and lag. Using $\chi_t^\psi = D_\psi(L)^{-1}B_\psi(0)u_t$, we obtain

$$D_\psi(L)\hat{\chi}_t^\psi = B_\psi(0)u_t + D_\psi(L)\zeta_t = B_{\psi,\zeta}(L)w_t^{\psi,\zeta},$$

so that $\hat{\chi}_t^\psi$ has a non-singular VARMA structure (a consequence of the non-singularity of ζ_t). Suppose that we estimate a VAR

$$\hat{D}_{\psi,\zeta}(L)\hat{\chi}_t^\psi = \hat{w}_t^{\psi,\zeta}, \tag{23}$$

where the length of $\hat{D}_{\psi,\zeta}(L)$ is greater or equal to the length of $D_\psi(L)$ (as should be suggested by the BIC criterion for example). As n tends to infinity, $\hat{\chi}_t^\psi$ and its autocovariance function converge to χ_t^ψ and its autocovariance function respectively. It is easy to show that: (i) $\hat{D}_{\psi,\zeta}(L)$ converges to $D_\psi(L)$, (ii) $\hat{w}_t^{\psi,\zeta}$, the non-singular residual of the (finite) orthogonal projection in (23), collapses in the limit into the singular residual of the (finite) orthogonal projection in (14), that is $B_\psi(0)u_t$ (note that both the residuals $\hat{w}_t^{\psi,\zeta}$ and $B_\psi(0)u_t$ are uniquely determined, whereas u_t and $B_\psi(L)$ are not).

C Equivalence of a VAR with $\hat{F}_t$ and a VAR with $\hat{\chi}_t^\psi$ when $m = r$

Consider the population VAR(p) of equation (14), along with its sample OLS counterpart

$$\hat{\chi}_t^\psi = \hat{\mu}_\psi - \hat{D}_{\psi,1}\hat{\chi}_{t-1}^\psi - \cdots - \hat{D}_{\psi,p}\hat{\chi}_{t-p}^\psi + \hat{v}_t^\psi, \quad t = p+1, \dots, T, \tag{24}$$

Moreover, consider the corresponding OLS equation for $\hat{F}_t$, i.e.

$$\hat{F}_t = \hat{\mu}_F - \hat{D}_{F,1}\hat{F}_{t-1} - \cdots - \hat{D}_{F,p}\hat{F}_{t-p} + \hat{v}_t^F, \quad t = p+1, \dots, T. \tag{25}$$

Let $Y_k = \begin{pmatrix}\hat{\chi}_{p+1-k}^\psi & \hat{\chi}_{p+2-k}^\psi & \cdots & \hat{\chi}_{T-k}^\psi\end{pmatrix}$, $Z_k = \begin{pmatrix}\hat{F}_{p+1-k} & \hat{F}_{p+2-k} & \dots & \hat{F}_{T-k}\end{pmatrix}$, $\iota = \begin{pmatrix}1 & 1 & \dots & 1\end{pmatrix}$, $V_\psi = \begin{pmatrix}v_{p+1}^\psi & v_{p+2}^\psi & \dots & v_T^\psi\end{pmatrix}$, $V_F = \begin{pmatrix}v_{p+1}^F & v_{p+2}^F & \dots & v_T^F\end{pmatrix}$. With this notation, equations (24) can be re-written as

$$Y_0 = \hat{\mu}_\psi\iota - \hat{D}_{\psi,1}Y_1 - \cdots - Y_p\hat{D}_{\psi,p} + V_\psi,$$

where V_ψ fulfills the OLS orthogonality properties $V_\psi \iota' = V_\psi Y_k' = 0$, $k = 1, \ldots, p$. Now we have $\hat{\chi}_t^\psi = \hat{A}_\psi \hat{F}_t$, where $\hat{A}_\psi$ is $r \times r$. If the entries of $\hat{\chi}_t^\psi$ are not collinear (in which case the VAR cannot be estimated), then $\hat{A}_\psi$ is invertible and $\hat{A}_\psi^{-1} Y_k = Z_k$. Pre-multiplying the above equation by $\hat{A}_\psi^{-1}$, and replacing Y_k with $\hat{A}_\psi Z_k$, we get

$$Z_0 = \hat{A}_\psi^{-1} \hat{\mu}_\psi \iota - \hat{A}_\psi^{-1} \hat{D}_{\psi,1} \hat{A}_\psi Z_k - \cdots - \hat{A}_\psi^{-1} \hat{D}_{\psi,p} \hat{A}_\psi Z_k + \hat{A}_\psi^{-1} V_\psi.$$

Now, the orthogonality properties of V_ψ imply $\hat{A}_\psi^{-1} V_\psi \iota = 0$ and $\hat{A}_\psi^{-1} V_\psi Z_k' = \hat{A}_\psi^{-1} V_\psi Y_k \hat{A}_\psi^{-1} = 0$. By uniqueness of the orthogonal projection, we then have $\hat{A}_\psi^{-1} V_\psi = V_F$. Hence the $r-$dimensional subspace of $\mathbb{R}^{T-p}$ spanned by V_F is the same as the one spanned by V_ψ, for any ψ, so that, by imposing the same identification conditions, we get the same estimated structural shocks.

As for the impulse response functions, we have $\hat{A}_\psi^{-1} \hat{\mu}_\psi = \hat{\mu}_F$ and $\hat{A}_\psi^{-1} \hat{D}_{\psi,p} \hat{A}_\psi = \hat{D}_{F,k}$, so that $\hat{D}_F(L) = \hat{A}_\psi^{-1} \hat{D}_\psi(L) \hat{A}_\psi$. By inverting the VAR filter for $\hat{F}_t$ we get the estimated Wold IRFs $\hat{D}_F^{-1}(L) = \hat{A}_\psi^{-1} \hat{D}_\psi^{-1}(L) \hat{A}_\psi$. The corresponding IRFs for $\hat{\chi}_t^\psi = \hat{A}_\psi \hat{F}_t$ are $\hat{A}_\psi \hat{D}_F^{-1}(L) = \hat{D}_\psi^{-1}(L) \hat{A}_\psi$, i.e linear combinations of the IRFs $\hat{D}_\psi^{-1}(L)$ obtained with $\hat{\chi}_t^\psi$. Again, identification conditions take care of producing identical results, for any ψ.

D Lags in Singular VARs: An Example

The simple example below illustrates the point about parsimony made at the end of Section 3.3, point (A). Let

$$S_t = \begin{pmatrix} f_t \\ g_t \\ h_t \end{pmatrix} = P_0 \eta_t + P_1 \eta_{t-1} + P_2 \eta_{t-2} = P_0 \eta_t + \begin{pmatrix} P_1 & P_2 \end{pmatrix} \begin{pmatrix} \eta_{t-1} \\ \eta_{t-2} \end{pmatrix}, \tag{26}$$

where η_t is scalar white noise and the matrices P_j are 3×1, so that S_t has dimension 3 and dynamic rank 1. Suppose that $P_2 \neq 0$ and that Q_1 and Q_2 are two linearly independent 3×1 vectors orthogonal to P_2. We have:

$$\begin{aligned} Q_1' S_{t-1} &= Q_1' P_0 \eta_{t-1} + Q_1' P_1 \eta_{t-2} \\ Q_2' S_{t-1} &= Q_2' P_0 \eta_{t-1} + Q_2' P_1 \eta_{t-2} \end{aligned}$$

so that

$$\begin{pmatrix} \eta_{t-1} \\ \eta_{t-2} \end{pmatrix} = \begin{pmatrix} Q_1' P_0 & Q_1' P_1 \\ Q_2' P_0 & Q_1' P_1 \end{pmatrix}^{-1} \begin{pmatrix} Q_1' \\ Q_2' \end{pmatrix} S_{t-1} = R S_{t-1}.$$

Replacing in (26) we obtain

$$S_t = \left[\begin{pmatrix} P_1 & P_2 \end{pmatrix} R \right] S_{t-1} + P_0 \eta_t = U_1 S_{t-1} + P_0 \eta_t. \tag{27}$$

Thus the MA(2) in (26) has a VAR(1) representation.

Consider now the 2–dimensional vector

$$\tilde{S}_t = \begin{pmatrix} f_t \\ g_t \end{pmatrix} = \tilde{P}_0 u_t + \tilde{P}_1 \eta_{t-1} + \tilde{P}_2 \eta_{t-2}. \tag{28}$$

Let $\tilde{Q}$ be a 2×1 vector orthogonal to $\tilde{P}_2$. We have:

$$\tilde{Q}'\tilde{S}_{t-1} = \tilde{Q}'\tilde{P}_0\eta_{t-1} + \tilde{Q}'\tilde{P}_1\eta_{t-2},$$

so that

$$\eta_{t-2} = \frac{1}{\tilde{Q}'\tilde{P}_1}\left[\tilde{Q}'\tilde{S}_{t-1} - \tilde{Q}'\tilde{P}_0\eta_{t-1}\right].$$

Replacing in (27) we obtain

$$\tilde{S}_t - \frac{1}{\tilde{Q}'\tilde{P}_1}\tilde{P}_2\tilde{Q}'\tilde{S}_{t-1} = \tilde{P}_0\eta_t + \left[\tilde{P}_1 - \frac{1}{\tilde{Q}'\tilde{P}_1}\tilde{P}_2\tilde{Q}'\tilde{P}_0\right]\eta_{t-1} = \tilde{P}_0\eta_t + P^*\eta_{t-1}.$$

Repeating the same procedure we remove η_{t-1} and obtain the VAR(2) representation

$$\tilde{S}_t = \tilde{U}_1\tilde{S}_{t-1} + \tilde{U}_2\tilde{S}_{t-2} + \tilde{A}_0\eta_t. \quad (29)$$

Each equation of the 3-dimensional VAR(1) in (27) has 4 parameters (3 for S_{t-1} plus 1 for the coordinate of P_0), an advantage with respect to (29), where the parameters in each equation are 5.

E Variables which should be included in the CC-SVAR without treatment

While macroeconomic variables often contain large measurement errors, it is also true that the estimated common components inevitably contain an estimation error. Therefore, using the estimated common components instead of the variables themselves has advantages and disadvantages. We argue in the main text that, as a general rule, using the estimated common components is the better choice. But there are two relevant exceptions. The first is when a variable has no measurement error. The second when a variable has a common component which is very poorly estimated. Let us briefly discuss these two cases in turn.

Firstly, there are variables with a small measurement error. For example, it can be argued that many financial variables are observed without error or with negligible error. Such variables coincide with their common components. If we have well-founded *a priori* reasons to believe that a variable is free of measurement error we should include it in the CC-SVAR without the treatment in (E2).

Second, it should be observed that, for a given observable variable, a good estimation of its common component crucially depends on the composition of the dataset. Large macroeconomic datasets usually include many prices and many interest rates but few stock market indexes, few public-spending or tax-revenue variables, few variables related to total factor productivity. If a particular aspect of the macroeconomic system is poorly represented in the dataset, the common components of the related series might be poorly estimated. We can get an indication of how serious is this issue for a particular series by computing the percentage of the variance explained by the estimated common component. If it is around 90%, the residual is likely a measurement error. If it is, say, 70%, the residual is likely to contain relevant information. In this case, by replacing the variable with the estimated common component, the researcher risks throwing the baby out with the bath water. Building an ad hoc dataset, which best represents the phenomena to be studied, is the main way to solve the problem. However, this is not always possible, as the necessary information may not be available. Hence, despite the measurement error, the lesser evil can be to include the variable in the CC-SVAR without the treatment in (E2).

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