On a Unified Framework for the Consistent Comparison of Experimental Tests in Isotropic Hyperelasticity

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Keywords: hyperelastic materials, material characterization, deformation invariants

This work proposes a unified framework for the interpretation and comparison of material response functions obtained from different homogeneous deformation states in isotropic hyperelasticity. The analysis encompasses a broad spectrum of experimental configurations, including uniaxial tension and compression, biaxial loading, constrained biaxial deformation, simple shear, angular shear, and unequal biaxial tests. Within this perspective, homogeneous deformations are interpreted as paths in the invariant plane, leading to a novel classification based on trajectories along the boundary of the admissible invariant domain, its bisector, or its interior. Such a classification provides new insight into the role of deformation modes in probing constitutive behavior.

Starting from equilibrium considerations, the derivatives of the strain-energy density function are systematically derived for a broad class of homogeneous deformations. These quantities constitute the fundamental material response functions and provide a consistent basis for constitutive characterization. A central concept introduced within this framework is that of energetic exhaustiveness, according to which homogeneous experiments directly identify the derivatives of the stored energy function and therefore provide exhaustive constitutive information. Within the quasi-static regime, the theory demonstrates that experiments performed under different homogeneous deformation modes must yield identical energy derivatives when conducted consistently. This result is established analytically and validated through the classical data of Treloar, where uniaxial and biaxial tests produce overlapping response functions when interpreted within the proposed framework. The methodology is further extended beyond quasi-static conditions through a novel criterion that enables the comparison of experiments performed at different deformation rates and loading configurations while preserving consistency in the identification of energy derivatives. This development reduces the dependence on time-consuming quasi-static testing protocols and provides practical guidelines for experimental design and data interpretation.

The formulation is expressed in terms of Criscione–Lode invariants associated with the natural (Hencky) strain measure, highlighting several advantages over traditional approaches based on Cauchy–Green invariants. Experimental results obtained from uniaxial compression and biaxial tension tests further support the theoretical framework, demonstrating a remarkable consistency of the derived response functions across different loading conditions.

References

[1] Falope, F. O., Lanzoni, L., Tarantino, A. M., "Energetic Exhaustiveness for the Direct Characterization of Energy Forms of Hyperelastic Isotropic Materials", *J. Mech. and Phys. of Solids*, 193, 105885, 2024.

[2] Falope, F. O., "The Full-Field Coverage Characterization over the Energy Domain of Hyperelastic Isotropic Materials", *Mechanics of Materials*, 217, 105656, 2026.

[3] Treloar, L. R. G., *The Physics of Rubber Elasticity*, Oxford University Press, New York, 1975.

[4] Falope, F. O., Vjerdha, J., Lanzoni, L., Tarantino, A. M., "Unified Response Functions in Isotropic Hyperelasticity: Toward a Consistent Comparison of Experiments Based on Energy Derivatives", submitted.