GUB Covers and Power-Indexed formulations for Wireless Network Design*

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We propose a pure 0-1 formulation for the wireless network design problem, i.e. the problem of configuring a set of transmitters to provide service coverage to a set of receivers. In contrast with classical mixed integer formulations, where power emissions are represented by continuous variables, we consider only a finite set of powers values. This has two major advantages: it better fits the usual practice and eliminates the sources of numerical problems which heavily affect continuous models. A crucial ingredient of our approach is an effective basic formulation for the single knapsack problem representing the coverage condition of a receiver. This formulation is based on the GUB cover inequalities introduced by Wolsey (1990) and its core is an extension of the exact formulation of the GUB knapsack polytope with two GUB constraints. This special case corresponds to the very common practical situation where only one major interferer is present. We assess the effectiveness of our formulation by comprehensive computational results over realistic instances of two typical technologies, namely WiMAX and DVB-T.

Key words: Wireless Network Design; Power Discretization; 0-1 Linear Programming; GUB Cover Inequalities; Strong Formulation.

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1. Introduction

Wireless communication systems constitute one of the most pervasive phenomena of every-day life. Television and radio programs are distributed through broadcasting networks (both terrestrial and satellite), mobile communication is ensured by cellular networks, internet service is provided through broadband access networks. Moreover, a number of

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security services are provided by ad-hoc wireless networks. All these networks have grown very rapidly during the last decades, generating dramatic congestion of radio resources such as frequency channels. Wireless networks provide different services and rely on different technologies and standards. Still, they share a common feature: they all need to reach users scattered over an area with a radio signal that must be strong enough to prevail against other unwanted interfering signals.

The perceived quality of service thus depends on several signals, wanted and unwanted, generated from a large number of transmitting devices. Due to the increasing size of the new generation networks, co-existing in an extremely congested radio spectrum and subject to local and international constraints, establishing suitable power emissions for all the transmitters has become a very difficult task, which calls for sophisticated optimization techniques.

Since the early 1980s several optimization models have been developed to design wireless networks. It is claimed that the use of automatic and optimization-oriented planning techniques may lead to cost reduction of up to 30% (Dehghan 2005). Concretely, recent experiences have clearly shown that the adoption of optimization techniques results in sensible increases in the quality of coverage plans and in a more effective and efficient use of the limited resources that a network administrator has at disposal - see the case of *atesio* for UMTS networks in Germany (ATESIO 2000) and the case of the *CORG* for DVB-T networks in Italy (CORG 1998).

Two fundamental issues must be faced when designing a wireless network: localizing the transmitters and dimensioning their power emissions. In most models, power emissions are represented as continuous decision variables. This choice typically yields ill-conditioned constraint matrices and requires the introduction of very large coefficients to model disjunctive constraints. The corresponding relaxations are very weak and state-of-the-art Mixed-Integer Linear Programming solvers are often affected by numerical instability. The use of continuous decision variables also contrasts with the telecommunications practice. In fact, the actual design specifications of real life antennas are always expressed as rational numbers with bounded precision and, consequently, assume a finite number of values.

Motivated by the above remarks we propose a pure 0-1 formulation for the problem that is obtained by considering only a finite set of power values. This formulation has two basic advantages: first, the ensuing model better fits the usual practice and, second, the numerical problems produced by the continuous variables are sensibly reduced. Indeed, the new approach allows us to find better solutions to large practical instances with less computational effort. In addition, the model fits the common network planning practice of considering a small number of power values and it directly models power restrictions that are often imposed by the technology (e.g., Mallinson et al. 2007). The situation where only two power values (on, off) are allowed is not rare (Ridolfi 2010). Finally, the new approach easily allows for generalizations of the model, such as power consumption minimization or antenna diagram optimization.

For our purposes, a wireless network can be described as a set of transmitters B distributing a telecommunication service to a set of receivers T. A receiver is said to be *covered* (or *served*) by the network if it receives the service within a minimum level of quality. The set B actually contains all *candidate* transmitters: in general, only a subset of B will be activated to cover the set T. Transmitters and receivers are characterized by a number of locations and radio-electrical parameters (e.g., geographical coordinates, power emission, transmission frequency). The *Wireless Network Design Problem* (WND) consists of establishing suitable values for such parameters with the goal of maximizing the coverage (or a revenue associated with the coverage).

Each transmitter $b \in B$ emits a radio signal with power $p_b \in [0, P_{\max}]$. We remark that a transmitter b such that $p_b = 0$ is actually not activated and thus not deployed in the network. The power $p(t)$ received by receiver t from transmitter b is proportional to the emitted power p_b by a factor $\tilde{a}_{tb} \in [0, 1]$, i.e. $p(t) = \tilde{a}_{tb} \cdot p_b$. The factor $\tilde{a}_{tb}$ is called *fading coefficient* and summarizes the reduction in power that a signal experiences while propagating from b to t. The value of a fading coefficient depends on many factors (e.g., distance between the communicating devices, presence of obstacles, antenna patterns) and is commonly computed through a suitable propagation model. For a detailed presentation of all technical aspects, we refer the reader to Rappaport (2001).

To simplify the discussion, we assume here that all the transmitters of the network operate at the same frequency. This assumption is dropped in Section 5 where we introduce the real-life application which motivated our developments. Among the signals received from transmitters in B, receiver t can select a *reference signal* (or *server*), which is the one carrying the service. All the other signals are interfering.

A receiver t is regarded as served by the network, specifically by server $\beta \in B$, if the ratio of the serving power to the sum of the interfering powers (*signal-to-interference ratio* or *SIR*) is above a threshold δ' (Rappaport 2001), the *SIR threshold*, whose value depends on the technology and the desired quality of service:

$$\frac{\tilde{a}_{t\beta} \cdot p_\beta}{\mu + \sum_{b \in B \setminus \{\beta\}} \tilde{a}_{tb} \cdot p_b} \geq \delta'. \tag{1}$$

Note the presence of the system noise $\mu > 0$ among the interfering signals. Since each transmitter in B is associated with a unique received signal, in what follows we will also refer to B as the set of signals received by t. By letting $\delta = -\mu \cdot \delta' < 0$ and letting:

$$a_{tb} = \begin{cases} \tilde{a}_{tb} & \text{if } b = \beta \\ \delta' \cdot \tilde{a}_{tb} & \text{otherwise} \end{cases}$$

for every $b \in B$, inequality (1) can be transformed into the so-called *SIR inequality* by simple algebra operations:

$$\sum_{b \in B \setminus \{\beta\}} a_{tb} \cdot p_b - a_{t\beta} \cdot p_\beta \leq \delta. \tag{2}$$

For every $t \in T$, we have one inequality of type (2) for each potential server $\beta \in B$. Receiver t is served if at least one of these inequalities is satisfied or, equivalently, if the following disjunctive constraint is satisfied:

$$\bigvee_{\beta \in B} \left(\sum_{b \in B \setminus \{\beta\}} a_{tb} \cdot p_b - a_{t\beta} \cdot p_\beta \leq \delta \right). \tag{3}$$

The above disjunction can be represented by a family of linear constraints in the p variables by introducing, for each $t \in T$ and each $b \in B$, a binary variable x_{tb} that is equal to 1 if t is served by b and to 0 otherwise. For each $\beta \in B$, the following constraint is then introduced:

$$\sum_{b \in B \setminus \{\beta\}} a_{tb} \cdot p_b - a_{t\beta} \cdot p_\beta - M \cdot (1 - x_{t\beta}) \leq \delta \tag{4}$$

where M is a large positive constant (big-M). When $x_{t\beta} = 1$ then (4) reduces to (2); when instead $x_{t\beta} = 0$ and M is sufficiently large (for example, we can set $M = -\delta + \sum_{b \in B \setminus \{\beta\}} a_{tb} \cdot P_{max}$), (4) is satisfied for any feasible power vector and becomes redundant. Constraints of type (4) appear in the Mixed-Integer Linear Programs (MILP) for the

WND presented in several papers in different application contexts, such as radio and video broadcasting (e.g., Mannino et al. 2006, 2009), GSM (e.g., Mathar and Schmeinck 2005), UMTS (e.g., Amaldi et al. 2006a, Eisenblätter and Geerdes 2008, Kalvenes et al. 2006, Naoum-Sawaya and Elhedhli 2010), WiMAX (Zhang 2009). Such MILPs are informally called *big-M formulations*. For a comprehensive description of the main elements that constitutes such models we refer to the recent book by Kennington et al. (2010) and to Amaldi et al. (2006b). For a more detailed discussion about how modeling an UMTS network, we refer the reader to Eisenblätter et al. (2002), and additionally to Siomina et al. (2006), where focus is on dimensioning pilot channel powers rather than the overall power emissions, considered as fixed.

The MILPs have been also tailored to cope with uncertainty affecting parameters of the model: in (Rosenberger and Olinick 2007) and (Olinick and Rosenberger 2008), two stochastic optimization approaches are presented to establish a robust location plan of the transmitters to tackle fluctuations in the traffic demand; in (Heikkinen and Prekopa 2004) and (Bienstock and D'Andreagiovanni 2009), Stochastic and Robust Optimization are respectively adopted to tackle the uncertainty affecting the fading coefficients.

WND instances of practical interest typically correspond to very large MILPs. In principle, such programs can be solved by standard Branch-and-Cut and by means of effective commercial solvers such as IBM ILOG Cplex (2010). However, it is well-known that the presence of a great number of constraints of type (4) results in ill-conditioned instances, due to the large variability of the fading coefficients, and weak bounds, due to the presence of the big-M coefficients. Furthermore, the resulting coverage plans are often unreliable (e.g., Kalvenes et al. 2006, Kennington et al. 2010, Mannino et al. 2009). In some cases, feasible WND instances can be even considered as unfeasible. In practice, only small-sized WND instances can actually be solved to optimality.

It is interesting to note that though these problems are known, only a limited number of papers of the wide literature about the WND has tried to overcome them. Kalvenes et al. (2006) proposed to execute a post-processing procedure that tries to repair coverage errors by eventually dropping service of a number of receivers. Naoum-Sawaya and Elhedhli (2010) focused on networks based on *Code Division Multiple Access (CDMA)* and adopted Benders' decomposition to obtain a new problem where the big-M coefficients are eliminated. However, the fading coefficients are still present, thus maintaining a relevant source of numerical problem. Inspired by practical observations about DVB networks, Mannino et al. (2009) considered a relaxation of the WND problem, obtained by including a single interfering transmitter in each SIR constraint and solved by a heuristic approach. Finally, Eisenblätter and Geerdes (2008) proposed a new approach for reducing interference in an UMTS network to increase the overall capacity of the network, under the assumption of perfect power control.

All the previously cited work are based on modeling the power emission of a transmitter as a continuous variable. In this paper, we follow instead a different path: we discretize the continuous power variables and consider only a finite number of feasible values. We stress that discretization is a classical tool in combinatorial optimization (e.g., Dyer and Wolsey 1990) and in telecommunication modeling (e.g., Castorini et al. 2008, Fridman et al. 2008, Mallinson et al. 2007), but, to our best knowledge, no effort has been made to go beyond the simple use of discretized SIR inequalities and replace them by more combinatorial inequalities. By using discretization, we are instead able to completely eliminate the two main sources of numerical issues, namely the fading and the big-M coefficients. We accomplish this by introducing a set of (strong) valid inequalities for the resulting 0-1 problem that radically improve the quality of obtained solutions. Additionally, solutions do not contain errors.

In the next section, we introduce our new contribution to the WND, the Power-Indexed formulation. In Section 3, we prove that for a special case that is very relevant in practice (single server interfered by a single transmitter), we can characterize the convex hull of the knapsack polytope associated with discrete power levels. In Section 4, we describe our solution approach to the WND. Finally, extensive computational results on realistic instances of WiMAX and DVB-T networks are presented in Section 5, showing that the new approach outperforms the one based on the big-M formulation.

2. A Power-Indexed formulation for the WND

As discussed in the previous section, a classical and much exploited model for the WND belongs to the class of the so-called big-M formulations and writes as:

$$\max \quad \sum_{t \in T} \sum_{b \in B} r_t \cdot x_{tb} \tag{BM}$$

$$\text{s.t.} \quad \sum_{b \in B \setminus \{\beta\}} a_{tb} \cdot p_b - a_{t\beta} \cdot p_\beta - M \cdot (1 - x_{t\beta}) \leq \delta \qquad t \in T,\ \beta \in B \tag{5}$$

$$\sum_{b \in B} x_{tb} \leq 1 \qquad t \in T \tag{6}$$

$$0 \leq p_b \leq P_{max} \qquad b \in B$$

$$x_{tb} \in \{0,1\} \qquad t \in T,\ b \in B$$

where r_t is the revenue (e.g. population, number of customers, expected traffic demand) associated with receiver $t \in T$ and the objective function is to maximize the total revenue. Constraint (5) is the SIR inequality (4) introduced in Section 1 and constraint (6) ensures that each receiver is served at most once.

Technology-dependent versions of (BM) can be obtained from the basic formulation by including suitable constraints or even new variables. For example, in the case of WiMAX networks, a knapsack constraint involving the service variables x_{tb} is added to (BM) to model the bandwidth capacity of each transmitter $b \in B$ (Zhang 2009). In the case of antenna diagram design, the number of power variables associated with each transmitter b is multiplied by 36 to represent the power emissions along the 36 directions which approximate the horizontal radiation pattern, and new constraints are included to represent physical relations between different directions (Mannino et al. 2009).

As observed in the introduction, the model (BM) has serious drawbacks both in terms of dimension of the solvable instances and of numerical instability. We tackle these issues by restricting the variables p_b to assume value in the finite set $\mathcal{P} = \{P_1, \ldots, P_{|\mathcal{P}|}\}$ of feasible power values, with $P_1 = 0$ (*switched-off value*), $P_{|\mathcal{P}|} = P_{max}$ and $P_i > P_{i-1}$, for $i = 2, \ldots, |\mathcal{P}|$. To this end, we introduce a binary variable z_{bl}, which is 1 iff b emits at power P_l. Since b is either switched-off or emitting at a positive value in $\mathcal{P}$, we have:

$$\sum_{l \in L} z_{bl} = 1 \qquad b \in B$$

where $L = \{1, \ldots, |\mathcal{P}|\}$ is the set of power value indices or simply *power levels*. Then we can write:

$$p_b = \sum_{l \in L} P_l \cdot z_{bl} \qquad b \in B. \tag{7}$$

By substituting (7) in (5), we obtain the following SIR constraint that only involves 0-1 variables:

$$\sum_{b \in B \setminus \{\beta\}} a_{tb} \sum_{l \in L} P_l \cdot z_{bl} - a_{t\beta} \sum_{l \in L} P_l \cdot z_{\beta l} - M \cdot (1 - x_{t\beta}) \leq \delta$$

The following *discrete big-M formulation* (DM) for the WND with a finite number of power values directly derives from (BM):

$$\max \quad \sum_{t \in T} \sum_{b \in B} r_t \cdot x_{tb} \tag{DM}$$

$$\text{s.t.} \quad \sum_{b \in B \setminus \{\beta\}} a_{tb} \sum_{l \in L} P_l \cdot z_{bl} - a_{t\beta} \sum_{l \in L} P_l \cdot z_{\beta l} + M \cdot x_{t\beta} \leq \delta + M \qquad t \in T,\ \beta \in B \tag{8}$$

$$\sum_{b \in B} x_{tb} \leq 1 \qquad t \in T$$

$$\sum_{l \in L} z_{bl} = 1 \qquad b \in B \tag{9}$$

$$x_{tb} \in \{0,1\} \qquad t \in T,\ b \in B$$

$$z_{bl} \in \{0,1\} \qquad b \in B,\ l \in L.$$

Note that, due to (7), every p_b also satisfies $0 \leq p_b \leq P_{max}$. As a consequence, the box constraints on p_b and thus variable p_b are dropped from the formulation.

The Power-Indexed formulation is obtained from (DM) by substituting each *knapsack SIR constraint* (8) with a set of *GUB cover inequalities* (Wolsey 1990).

In the following, we denote a GUB cover inequality by the acronym *GCI*. The GCIs constitute a stronger version of simple cover inequalities of a knapsack constraint, and are defined by exploiting the presence of the additional constraints (9), that are called *generalized upper bound* (GUB) constraints.

Before introducing the GCIs, we recall some related definitions and concepts introduced in (Wolsey 1990). We consider the set of binary points $Y = P \cap B^n$, where $P \subseteq R^n_+$ is the polytope defined by:

$$(i) \sum_{j \in N_1} a_j \cdot y_j - \sum_{j \in N_2} a_j \cdot y_j \leq a_0$$

$$(ii) \sum_{j \in S_i} y_j \leq 1 \text{ for } i \in I_1 \cup I_2 \tag{10}$$

$$y \in R^n_+,$$

where $N = N_1 \cup N_2,\ N_1 \cap N_2 = \emptyset,\ a_j > 0$ for $j \in N$, $\bigcup_{i \in I_1} S_i = N_1$, $\bigcup_{i \in I_2} S_i = N_2$ and, finally $S_i \cap S_l = \emptyset$ if $i, l \in I_k$ with $i \neq l$ for $k = 1, 2$. In other words, the variables of the knapsack

(10.i) are partitioned into a number of subsets, and at most one variable can be set to 1 for each subset. Each of these subsets thus defines a GUB constraint (10.ii). Furthermore, by definition each subset is entirely contained either in N_1 or N_2 and thus the coefficients of the corresponding 0-1 variables have the same sign in the knapsack constraint (10.i).
A set $C = C_1 \cup C_2$ is a *GUB cover* for Y if:

$$(i)\ C_k \subseteq N_k \qquad \text{for } k = 1, 2$$

$$(ii)\ |C_k \cap S_i| \leq 1 \quad \text{for } i \in I_k \text{ and } k = 1, 2$$

$$(iii)\ \sum_{j \in C_1} a_j - \sum_{j \in C_2} a_j > a_0.$$

On the basis of the GUB cover C, it is easy to build a standard cover inequality which is valid for the set Y. Such constraint can be lifted by including new variables, by exploiting the GUB inequalities (10.ii). In particular, with the GUB cover C we associate the following sets:

$$I_k^+ = \{i \in I_k : C_k \cap S_i \neq \emptyset\} \qquad \text{for } k = 1, 2$$

$$S_i^+ = \{j \in S_i : a_j \geq a_l \text{ for } l \in C_1 \cap S_i\} \text{ for } i \in I_1^+$$

$$S_i^+ = \{j \in S_i : a_j \leq a_l \text{ for } l \in C_2 \cap S_i\} \text{for } i \in I_2^+.$$

For each set S_i with one element in the cover, S_i^+ represents the set of elements which may be added to the cover in order to lift the corresponding inequality. In particular, if the elements of S_i correspond to non-negative coefficients a_j of the knapsack, then we can add all the elements that correspond to coefficients that are larger than a_l (i.e., the coefficient of the element of S_i in the GUB cover). We instead include all the elements with smaller coefficient in the case of negative coefficients.

In (Wolsey 1990), Wolsey proves that if $C = C_1 \cup C_2$ is a *GUB cover*, the following GUB cover inequality (GCI) is valid for Y:

$$\sum_{i \in I_1^+} \sum_{j \in S_i^+} y_j \leq |C_1| - 1 + \sum_{i \in I_2^+} \sum_{j \notin S_i^+} y_j + \sum_{i \in I_2 \setminus I_2^+} \sum_{j \in S_i} y_j. \tag{11}$$

When $I_2^+ = I_2$ and $|I_2| = 1$, such valid inequality reduces to:

$$\sum_{i \in I_1^+} \sum_{j \in S_i^+} y_j + \sum_{i \in I_2^+} \sum_{j \in S_i^+} y_j \leq |C_1|. \tag{12}$$

Now, let us focus on a single knapsack constraint (8) of (DM) associated with testpoint $t \in T$ and server $\beta \in B$, along with constraints (9) for $b \in B$ and the valid inequality $x_{t\beta} \leq 1$. We can cast this into the GUB framework introduced by Wolsey by making the following associations:

$$N_1 = \{(b,l) : \, b \in B \backslash \{\beta\}, \, l \in L\} \, \cup \, \{(t,\beta)\}$$
$$N_2 = \{(\beta,l) : \, l \in L.\}$$

Observe that, with a slight abuse of notation, in the definition of N_1 we are also including index (t,β) corresponding to variable $x_{t\beta}$. Similarly, we let:

$$I_1 = \{b : \, b \in B \backslash \{\beta\}\} \, \cup \, \{(t,\beta)\}$$
$$I_2 = \{\beta\}.$$

Indeed, for each $b \in B$ at most one variable z_{bl} can be equal 1, for $l \in L$, and we have $S_b = \{(b,l) : l \in L\}$ for all $b \in B$. Also, we let $S_{t,\beta} = \{(t,\beta)\}$ be the singleton corresponding to variable $x_{t\beta}$. Observe that we have $N_1 = S_{t,\beta} \cup (\bigcup_{b \in B \backslash \{\beta\}} S_b)$ and $N_2 = S_\beta$.

Before translating conditions (i), (ii) and (iii) into our setting, we provide an intuitive explanation of how a CGI is build for formulation (DM). For a fixed couple of receiver and server and a fixed subset of interferers, a GUB cover corresponds to one serving power level and a combination of interfering power levels that jointly deny the coverage of the receiver by the server. Thereafter, the lifting is done by considering lower serving power levels and higher interfering power levels. We now proceed to define formally the GCI. To this purpose, consider first the coverage condition (2) corresponding to receiver $t \in T$ with server $\beta \in B$. Suppose that the server β is emitting at power value $p_\beta = P_\lambda$, for some $\lambda \in L$. Let $\Gamma = \{b_1, \ldots, b_{|\Gamma|}\} \subseteq B \backslash \{\beta\}$ be a set of interferers (for t when β is its server) and let $q_1, \ldots, q_{|\Gamma|}$ be power levels for each interferer in Γ such that:

$$a_{tb_1} \cdot P_{q_1} + \cdots + a_{tb_{|\Gamma|}} \cdot P_{q_{|\Gamma|}} - a_{t\beta} \cdot P_\lambda > \delta. \tag{13}$$

In other words, receiver t is not served when t is assigned to server β emitting at power value P_λ, and the interferers $b_1, \ldots, b_{|\Gamma|}$ are emitting at power values $p_{b_1} = P_{q_1}, \, \ldots, \, p_{b_{|\Gamma|}} = P_{q_{|\Gamma|}}$, respectively.

By letting $C_1 = \{(b_i, q_i) : i = 1, \ldots, |\Gamma|\} \cup \{(t,\beta)\}$ and $C_2 = \{(\beta,\lambda)\}$, it follows that $C = C_1 \cup C_2$ is a cover of (8). Also, it is not difficult to see that C is a GUB cover, since $C_1 \subseteq N_1$,

$C_2 \subseteq N_2$, $|C_1 \cap S_b| \leq 1$, for all $b \in I_1$ and $|C_2 \cap S_\beta| = 1$. We also have $I_1^+ = \Gamma \cup \{(t,\beta)\}$ and $I_2^+ = \{\beta\}$.

Since $a_{tb} \cdot P_l < a_{tb} \cdot P_{l+1}$ for all $b \in B$ and $l = 1, \ldots, |L|-1$, we have that $S_{b_i}^+ = \{(b_i, q_i), (b_i, q_{i+1}), \ldots, (b_i, q_{|L|})\}$ for $b_i \in B \backslash \{\beta\}$, $S_{t,\beta}^+ = \{(t,\beta)\}$ and $S_\beta^+ = \{(\beta,1), \ldots, (\beta,\lambda)\}$.

It follows from (12) that, for $t \in T$, $\beta \in B$, the inequality

$$x_{t\beta} + \sum_{l=1}^{\lambda} z_{\beta l} + \sum_{i=1}^{|\Gamma|} \sum_{j=q_i}^{|L|} z_{b_i j} \leq |\Gamma| + 1 \tag{14}$$

is valid for the set of binary vectors satisfying (8) and (9).

Now, for all the subsets of interferers $\Gamma \subseteq B \backslash \{\beta\}$, denote by $L^I(t,\beta,\lambda,\Gamma)$ the set of $|\Gamma|$-tuples $q \in L^{|\Gamma|}$ satisfying (13). The following proposition follows immediately by the validity of (14):

Proposition 1. *Given $t \in T$, $\beta \in B$, the family of inequalities:*

$$x_{t\beta} + \sum_{l=1}^{\lambda} z_{\beta l} + \sum_{i=1}^{|\Gamma|} \sum_{j=q_i}^{|L|} z_{b_i j} \leq |\Gamma| + 1 \tag{15}$$

defined for $\Gamma \subseteq B \backslash \{\beta\}$, $\lambda \in L$, $q \in L^I(t,\beta,\lambda,\Gamma)$, is satisfied by all the binary solutions of (8) and (9).

It can be formally shown that the reverse is also true, namely all binary solutions to (15) and (9) also satisfy (8). It follows that the following formulation, that we call Power-Indexed (PI), is valid for the WND (with finite set of power values):

$$\max \quad \sum_{t \in T} \sum_{b \in B} r_t \cdot x_{tb} \tag{PI}$$

$$\text{s.t.} \quad x_{t\beta} + \sum_{l=1}^{\lambda} z_{\beta l} + \sum_{i=1}^{|\Gamma|} \sum_{j=q_i}^{|L|} z_{b_i j} \leq |\Gamma| + 1 \qquad t \in T,\ \beta \in B,\ \Gamma \subseteq B \backslash \{\beta\}, \ \lambda \in L,\ q \in L^I(t,\beta,\lambda,\Gamma) \tag{16}$$

$$\sum_{b \in B} x_{tb} \leq 1 \qquad t \in T \tag{17}$$

$$\sum_{l \in L} z_{bl} = 1 \qquad b \in B \tag{18}$$

$$x_{tb} \in \{0,1\} \qquad t \in T,\ b \in B \tag{19}$$

$$z_{bl} \in \{0,1\} \qquad b \in B,\ l \in L. \tag{20}$$

The above formulation contains a very large number of GCIs (potentially exponential in $|B|$ for all $t \in T$). To cope with this we proceed in a standard fashion by initially considering a subset of all inequalities and subsequently generating new inequalities when needed. In Section 4, we give the details of our column and row generation approach to solve the WND along with a heuristic routine for separating violated GCIs (16). The overall behaviour of the row generation approach is strongly affected by the quality of the initial relaxation. In the context of WND, a particularly well-suited choice consists of including only the GCIs (16) corresponding to interferer sets Γ with $|\Gamma| = 1$; we denote such initial relaxation by (PI^0). This choice has several major advantages.

First, the number of constraints in (PI^0) is small and can be generated efficiently. In the next section, we actually show that, for each $t \in T, \beta \in B$ and $b \in B \backslash \{\beta\}$, the number of non-dominated GCIs (16) is at most $|L|$.

Second, as the Power-Indexed formulation (PI) is derived from the discretized SIR formulation (DM), so (PI^0) can be thought as derived from a relaxation (DM^0) of (DM). Namely, the relaxation (DM^0) is obtained from (DM) by replacing, for each $t \in T$ and each $\beta \in B$, the SIR inequality (8) with the family of inequalities (one for each interferer):

$$a_{tb} \sum_{l \in L} P_l \cdot z_{bl} - a_{t\beta} \sum_{l \in L} P_l \cdot z_{\beta l} + M \cdot x_{t\beta} \leq \delta + M \qquad b \in B \backslash \{\beta\}. \tag{21}$$

Clearly, each inequality of type (21) is dominated by the original inequality (8) from which it derives, and the 0-1 solutions to (DM^0) may not be feasible for (DM). Nevertheless, in many applicative contexts (DM^0) appears to be a very good approximation of (DM). Indeed, this type of relaxation has been introduced in (Mannino et al. 2009) to cope with DVB network design problems, and successfully applied to the design of the Italian national reference DVB network. Similarly, our experiments reported in Section 5 show that (PI^0) is a good approximation of (PI). Indeed, the number of inequalities not in (PI^0) generated by our Branch-and-Cut is always very small. This can be well explained by the practical observation that, for a given receiver, there exists most of the time one particular interferer whose signal is much stronger than the others (see Section 5 for a more detailed discussion).

A third and most crucial feature of (PI^0) relates to the strength of its GCIs. In the next section we show that, for each $t \in T$, $\beta \in B$ and $b \in B \backslash \{\beta\}$, the family of GCIs associated with (21) along with the trivial facets define the corresponding *GUB knapsack polytope*, i.e. the convex hull of the 0-1 solutions to the knapsack SIR constraint (21) and its corresponding GUB constraints (9). This is a very desired property which explains why the LP-relaxations of (PI^0) provide much tighter bounds than those provided by (DM^0), thus in turn implying more effective searches and the capability to solve larger instances.

Summarizing, (PI^0) can be easily generated, is a good approximation of the original problem and provides strong LP-relaxations.

3. The GUB knapsack polytope for the single-interferer SIR inequality

For a receiver $t \in T$, server $\beta \in B$ and a single interferer $b \in B \setminus \{\beta\}$, let us consider the family of GCIs associated with the constraint (21):

$$x_{t\beta} + \sum_{l=1}^{\lambda} z_{\beta l} + \sum_{j=q}^{|L|} z_{bj} \leq 2 \qquad \lambda \in L,\ q \in L^I(t, \beta, \lambda, \{b\}). \tag{22}$$

Since $P_l > P_{l-1}$ for $q = 2, \ldots, |L|$, the set $L^I(t, \beta, \lambda, \{b\})$ of interfering levels of b for a server power level λ can be written as $\{q(\lambda), q(\lambda)+1, \ldots, |L|\}$, where $q(\lambda) = \min\{l \in L : a_{tb} \cdot P_l - a_{t\beta} \cdot P_\lambda > \delta\}$. It follows that the subfamily of inequalities (22) associated with λ is dominated by the single inequality corresponding to $q(\lambda)$. Finally, observe that $q(\lambda') \geq q(\lambda)$ for $\lambda' \geq \lambda$.

In order to simplify the notation, we now let $u = x_{t\beta}$, $v_l = z_{\beta l}$ for $l \in L$ and $w_l = z_{bl}$ for $l \in L$. After removing the dominated GCIs, the remaining family can be rewritten as:

$$u + \sum_{l=1}^{\lambda} v_l + \sum_{l=q(\lambda)}^{|L|} w_l \leq 2 \qquad \lambda = 1, \ldots, |L|. \tag{23}$$

The following theorem extends a result presented in (Wolsey 1990) (Proposition 3.1), also providing an alternative and simpler proof for it.

Proposition 2. *The polytope P defined as the set of points $(u, \mathbf{v}, \mathbf{w}) \in \mathbb{R}^{1+2|L|}$ satisfying (23) and the constraints $0 \leq u \leq 1$, $\mathbf{0} \leq \mathbf{v} \leq \mathbf{1}$ and $\mathbf{0} \leq \mathbf{w} \leq \mathbf{1}$ is the convex hull of the 0-1 solutions to (21).*

Proof of Proposition 2. Let A be the 0-1 coefficient matrix associated with the set of constraints (23). We first show that A is an *interval matrix*, i.e. in each column the 1's appear consecutively (Nehmauser and Wolsey 1988).

We start by noticing that $A = (U|V|W)$ where U is the column associated with the variable u; $V \in \{0,1\}^{|L| \times |L|}$ is the square matrix associated with the variables $v_1, \ldots, v_{|L|}$; and $W \in \{0,1\}^{|L| \times |L|}$ is the square matrix associated with the variables $w_1, \ldots, w_{|L|}$.

The vector U has all the elements equal to 1 as u is included in every constraint (23). The matrix $V = [n_{ij}]$ with $i,j = 1,\ldots,|L|$ is lower triangular and such that $n_{ij} = 1$ for $i \geq j$. Indeed, the constraint (23) corresponding with $\lambda \in L$ includes exactly the v variables $v_1,\ldots,v_\lambda$.

Finally, consider the matrix $W = [m_{ij}]$ with $i,j = 1,\ldots,|L|$. First, observe that for all $\lambda, j \in L$, we have:

$$m_{\lambda j} = 1 \iff j \geq q(\lambda).$$

Recalling that for every $\lambda', \lambda \in L$ with $\lambda' \geq \lambda$, we have $q(\lambda') \geq q(\lambda)$, it follows that, for all $\lambda \leq \lambda'$, $m_{\lambda' j} = 1 \Longrightarrow j \geq q(\lambda') \Longrightarrow j \geq q(\lambda) \Longrightarrow m_{\lambda j} = 1$. W is thus an interval matrix and as U and V are interval matrices as well, it follows that A is an interval matrix and thus totally unimodular.

Finally, if we denote by $\bar{A}$ the matrix associated with the constraints (23) and the box constraints on variables $u, \mathbf{v}, \mathbf{w}$, then $\bar{A}$ is obtained by extending A with I and $-I$, where I is the identity matrix of size $1 + 2|L|$. Thus $\bar{A}$ is a totally unimodular matrix (Nehmauser and Wolsey 1988) and, since the right hand sides of the constraints are integral, the vertices of P are also integral, completing the proof. $\square$

4. Solution Algorithm

The solution algorithm is based on the (PI) formulation for the WND and consists of two basic steps: (i) a set $\mathcal{P}$ of feasible power values is established; (ii) the associated formulation is solved by row generation and Branch-and-Cut. We start by describing step (ii) and we come back to step (i) later in this section.

In the following, for a fixed power set $\mathcal{P}$, we denote the solution algorithm for the associated (PI) formulation as SOLVE-PI($\mathcal{P}$). Since the (PI) formulation has in general an exponential number of constraints of type (16), we apply row generation. Namely, we start by considering only a suitable subset of constraints and we solve the associated relaxation. We then check if any of the neglected rows is violated by the current fractional solution. If so, we add the violated row to the formulation and solve again, otherwise we proceed with standard Branch-and-Cut (as implemented by the commercial solver Cplex). The separation of violated constraints is repeated in each branching node.

At node 0, the initial formulation (PI^0) includes only a subset of constraints (16), namely those including one interferer (i.e., $|\Gamma| = 1$). In Section 2 and Section 3 we discussed why this is a good choice for (PI^0). Indeed, in our case studies, only a low number of additional constraints is added by separation during the iterations of the algorithm.

4.1. Separation.

We now proceed to show how violated constraints are separated. Let (x^*, z^*) be the current fractional solution. In Section 2 we have showed that constraints (16) are GUB cover inequalities of (8). In order to separate a violated GCI of type (16), we make use of the exact oracle introduced by Wolsey (1990) and heuristically solve it by extending the standard (heuristic) approach to the separation of cover inequalities described in (Nehmauser and Wolsey 1988).

To this end, let us first select a receiver $t \in T$ and one of its servers, say $\beta \in B$. We want to find a GCI of type (16) that is associated with t and β, and is violated by the current solution (x^*, z^*). In other words, we want to identify a power level $\lambda \in L$ for β, a set of interferers $\Gamma = \{b_1, \ldots, b_{|\Gamma|}\} \subseteq B \backslash \{\beta\}$ and an interfering $|\Gamma|$-tuple of power levels $q = (q_1, \ldots, q_{|\Gamma|}) \in L^I(t, \beta, \lambda, \Gamma)$, such that:

$$x^*_{t\beta} + \sum_{l=1}^{\lambda} z^*_{\beta l} + \sum_{i=1}^{|\Gamma|} \sum_{j=q_i}^{|L|} z^*_{b_i j} > |\Gamma| + 1. \tag{24}$$

Recall that $q \in L^I(t, \beta, \lambda, \Gamma)$ if

$$\sum_{i=1}^{|\Gamma|} a_{tb_i} \cdot P_{q_i} - a_{t\beta} \cdot P_\lambda > \delta. \tag{25}$$

We solve the separation problem by defining a suitable 0-1 Linear Program. In particular, in order to identify a suitable pair (β, λ) we introduce, for every $l \in L$, a binary variable $u_{\beta l}$, which is 1 iff $l = \lambda$. Similarly, we introduce binary variables u_{bl} for all $b \in B \backslash \{\beta\}$ and $l \in L$, with $u_{bl} = 1$ iff $(b, l) = (b_i, q_i)$, where $b_i \in \Gamma$ and q_i is the corresponding interfering power level. Then $u \in \{0,1\}^{|B(t)| \times |L|}$ satisfies the following system of linear inequalities:

$$\sum_{b \in B \backslash \{\beta\}} a_{tb} \sum_{l \in L} P_l \cdot u_{bl} - a_{t\beta} \sum_{l \in L} P_l \cdot u_{\beta l} > \delta \tag{26}$$

$$\sum_{l \in L} u_{bl} = 1 \qquad b \in B. \tag{27}$$

Constraint (26) ensures that u is the incidence vector of a cover of (8), whereas constraint (27) states that u satisfies the GUB constraints.

Observe now that $|\Gamma| = \sum_{b \in B \setminus \{\beta\}} \sum_{l \in L} u_{bl}$. So, if u identifies a violated GCI (24), we must have:

$$\sum_{l \in L} u_{\beta l} \sum_{k=1}^{l} z^*_{\beta k} + \sum_{b \in B \setminus \{\beta\}} \sum_{l \in L} u_{bl} \sum_{k=l}^{|L|} z^*_{bk} \; > \sum_{b \in B \setminus \{\beta\}} \sum_{l \in L} u_{bl} + 1 - x^*_{t\beta}. \tag{28}$$

In order to (heuristically) search for a violated inequality, we proceed in a way which resembles the classical approach for standard cover inequalities (Nehmauser and Wolsey 1988), by considering the following linear program (SEP), introduced by Wolsey (1990):

$$Z = \max \quad \sum_{l \in L} u_{\beta l} \sum_{k=1}^{l} z^*_{\beta k} + \sum_{b \in B \setminus \{\beta\}} \sum_{l \in L} u_{bl} \cdot \left(\sum_{k=l}^{|L|} z^*_{bk} - 1 \right) \tag{SEP}$$

$$\text{s.t.} \quad \sum_{b \in B \setminus \{\beta\}} a_{tb} \sum_{l \in L} P_l \cdot u_{bl} - a_{t\beta} \sum_{l \in L} P_l \cdot u_{\beta l} \geq \delta \tag{29}$$

$$\sum_{l \in L} u_{bl} = 1 \qquad b \in B$$

$$u_{bl} \geq 0 \qquad b \in B,\, l \in L.$$

It is easy to notice that the feasible region of (SEP) contains all binary vectors satisfying (26) and (27). Let Z be the optimum value to (SEP). If $Z \leq 1 - x^*_{t\beta}$ then no binary vector u satisfies (28) and consequently no violated constraint exists. If $Z > 1 - x^*_{t\beta}$ then a violated constraint may exist, and we resort to a heuristic approach to find it. In particular, observe first that Z can be computed by relaxing the knapsack constraint (29) in a Lagrangian fashion and then by solving the resulting Lagrangian dual, namely:

$$Z = \min_{\eta \geq 0} \quad Z(\eta)$$

where $\eta \in \mathbb{R}^+$ is the Lagrangian multiplier and:

$$\begin{aligned} Z(\eta) = \max_{u \geq 0} \quad & \sum_{l \in L} u_{\beta l} \sum_{k=1}^{l} z^*_{\beta k} + \sum_{b \in B \setminus \{\beta\}} \sum_{l \in L} u_{bl} \cdot \left(\sum_{k=l}^{|L|} z^*_{bk} - 1 \right) \\ & + \; \eta \cdot \left(\sum_{b \in B \setminus \{\beta\}} a_{tb} \sum_{l \in L} P_l \cdot u_{bl} - a_{t\beta} \sum_{l \in L} P_l \cdot u_{\beta l} - \delta \right) \\ \text{s.t.} \quad & \sum_{l \in L} u_{bl} = 1 \qquad b \in B. \end{aligned}$$

For fixed $\eta \geq 0$, the objective $Z(\eta)$ can be easily computed by inspection. To simplify the notation we rewrite the objective function of the above linear program as:

$$-\delta \cdot \eta \ + \ \max_{u \geq 0} \sum_{b \in B} \sum_{l \in L} c_{bl}(\eta) \cdot u_{bl} \tag{30}$$

where, for every $b \in B, l \in L$, we let:

$$c_{bl}(\eta) = \begin{cases} \sum_{k=1}^{l} z^*_{\beta k} - \eta \cdot a_{t\beta} \cdot P_l & \text{if } b = \beta \\ \\ \sum_{k=l}^{|L|} z^*_{bk} - 1 + \eta \cdot a_{tb} \cdot P_l & \text{if } b \in B \backslash \{\beta\}. \end{cases}$$

For fixed $\eta \geq 0$, an optimal solution $u(\eta)$ to the inner maximization problem can be found by inspection as follows. For each $b \in B$, identify a power level $l_b \in L$ which maximizes the coefficient in (30), namely $c_{bl_b}(\eta) = \max_{l \in L} c_{bl}(\eta)$; then, for each $b \in B$ and each $l \in L$, let:

$$u_{bl}(\eta) = \begin{cases} 1 & \text{if } l = l_b \\ 0 & \text{otherwise.} \end{cases}$$

It is straightforward to see that, for all $\eta \geq 0$, $u(\eta) \geq 0$ satisfies all constraints (27) and maximizes (30). For $\eta \geq 0$, the function $Z(\eta)$ is convex and unimodal and the optimum solution η^* can be found efficiently by applying the *Golden Section Search Method* (Gerald and Wheatley 2004). Suppose now that $Z(\eta^*) > 1 - x^*_{t\beta}$ (otherwise no violated constraints exist). If, in addition, $u(\eta^*)$ also satisfies (26), then the positive components of the binary solution $u(\eta^*)$ are in one-to-one correspondence to the variables of a violated constraint. Otherwise the algorithm returns no violated cover.

Finally, when the current solution (x^*, z^*) is purely 0-1, we perform an exact separation by directly checking the satisfaction of each of the constraints (16).

4.2. The Algorithm

We come back now to the first step in our algorithm, namely the choice of the set of admissible power values $\mathcal{P}$. Large sets are in principle more likely to produce better quality solutions. However, the ability of the solution algorithm to find optimal or simply good-quality solutions is strongly affected by $|\mathcal{P}|$, as we will show in more details in the computational results section. Thus, the size and the elements of $\mathcal{P}$ should represent a suitable compromise between these two opposite behaviors. Moreover, the effectiveness of the Branch-and-Cut is typically affected by the availability of a good initial feasible solution. Thus, we decided to iteratively apply SOLVE-PI($\mathcal{P}$) to a sequence of power sets $\mathcal{P}_0 \subset \mathcal{P}_1 \subset \cdots \subset \mathcal{P}_r$. Each invocation inherits all the generated cuts, the best solution found so far and the corresponding lower bound from the previous invocation. More precisely, if we denote by -99 the switched-off state (in dBm), and $P_{\min}^{dBm}$, $P_{\max}^{dBm}$ are the (integer) minimum and maximum power values (in dBm), then we have $\mathcal{P}_0 = \{-99, P_{\max}^{dBm}\}$, $\mathcal{P}_1 = \{-99, P_{\min}^{dBm}, \left\lfloor \frac{P_{\max}^{dBm} - P_{\min}^{dBm}}{2} \right\rfloor, P_{\max}^{dBm}\}$ and $\mathcal{P}_r = \{-99, P_{\min}^{dBm}, P_{\min}^{dBm} + 1, \ldots, P_{\max}^{dBm}\}$. The structure of the intermediate power sets will be described in Section 5. Observe that the actual power values are only used in the separation oracle where the dB values are converted into the original non-dB values.

The overall approach, denominated *WPLAN*, is summarized in Algorithm 1, where i denotes the current iteration, along with the associated best solution found x_i, the corresponding value LB_i, and the set of feasible powers $\mathcal{P}_i$. If SOLVE-PI($\mathcal{P}_i$) is executed in less than the iteration time limit TL_i then the residual time τ_i is used to increase the time limit of the following iteration (i.e., $\mathrm{TL}_{i+1} := \mathrm{TL}_{i+1} + \tau_i$). The initial incumbent solution x_{-1} corresponds to all transmitters switched off and no receiver served ($LB_{-1} = 0$).

Algorithm 1 WPLAN

Input: the power sets $\mathcal{P}_0, \mathcal{P}_1, \ldots, \mathcal{P}_r$, the iteration time limit TL_i for $i = 0, \ldots, r$

Output: the best solution x_r

$LB_{-1} := 0$

for $i = 0$ to r **do**

1. Invoke SOLVE-PI($\mathcal{P}_i$) with lower bound LB_{i-1}, incumbent x_{i-1} and TL_i

2. Get x_i, LB_i and τ_i

3. $TL_{i+1} := TL_{i+1} + \tau_i$

end for

Return x_r

5. Computational Results

The model that we have considered so far has a very simple and basic structure and applies to the main wireless technologies. More precisely, it can be effectively used if the service coverage condition of a receiver is expressed by means of a SIR constraint (1). As pointed out in Section 2, each technology generally requires its own peculiar parameter values and additional constraints and/or variables to model its own specific features.

In this section, we present computational results concerning realistic instances of two important wireless technologies: the *IEEE Standard 802.16* (WiMAX 2004) and the Terrestrial Digital Video Broadcasting (DVB-T, ETSI 2006).

The target of these tests is manyfold. First, we compare the new (PI) formulation to the two big-M formulations (BM) and (DM) and show that (PI) outperforms (BM) and (DM) both in terms of quality of bounds and quality of solutions. Then, we illustrate specific features of the solution algorithm WPLAN and we motivate the iterative approach with increasing power sets. Finally, we assess the ability of WPLAN to tackle realistic network design instances. The tests were performed under Windows XP 5.1 operating system, with 1.80 GHz Intel Core 2 Duo processor and 2×1024 MB DDR2-SD RAM. The algorithm is implemented in C++ (under Microsoft Visual Studio 2005 8.0), whereas the commercial MILP solver ILOG Cplex 10.1 is invoked by ILOG Concert Technology 2.3.

In the following two subsections, we provide a concise description of the main specific features of the two technologies and we highlight their impact on the basic model that we presented in Section 2. Furthermore, we describe the characteristics of the realistic instances that we consider for each technology.

5.0.1. WiMAX Network Design The first set of instances refers to a *WiMAX network* and were developed with the *Technical Strategy and Innovations Unit* of *British Telecom Italia* (BT). WiMAX is the common name used to indicate the *IEEE Standard 802.16* (WiMAX 2004). Specifically, we consider the design of a *Fixed WiMAX Network* that provides broadband internet access.

The major amendments concern the introduction of different frequency channels, channel capacity and traffic demand. To model the additional features of a WiMAX network, the formulations (BM) and (PI) must include additional variables to take into account multiple frequencies (denoted by set F) and multiple transmission schemes (denoted by set H). Furthermore, we need to introduce additional constraints to model the capacity of each frequency to accommodate traffic generated by users. For a detailed description of these additional features, both from technological and modeling perspective, we refer the reader to (Zhang 2009).

All the instances correspond to an urban area of the city of Rome (Italy), selected in agreement with the engineers at BT, who considered it as a representative residential traffic scenario. Each activated transmitter can emit by using integer power levels in the range [20,40] dBm. We define three types of instances, denoted by SX, where X is the instance identifier ranging in $\{1,\ldots,7\}$, RX with $X = \{1,\ldots,4\}$ and QX with $X = \{1,\ldots,4\}$. For the SX instances, the traffic is uniformly distributed among the TPs and we assign unitary revenue to each TP (i.e. $r_t = 1$). Finding an optimal coverage plan thus corresponds to define the plan with the maximum number of covered TPs. Only one frequency and one burst profile are allowed. For the RX instances, we consider a traffic distribution based on the actual distribution of the buildings. We also introduce multiple frequencies and burst profiles. In this case, the revenue of each testpoint is proportional to the traffic generated. Finally, the QX instances include an increasing number of candidate sites and focus on a single frequency network with multiple burst profiles. The dimension of each instance is resumed in Table 1.

Table 1 Description of the WiMAX test-bed instances

ID	S1	S2	S3	S4	S5	S6	S7	R1	R2	R3	R4	Q1	Q2	Q3	Q4
$\|T\|$	100	169	196	225	289	361	400	400	441	484	529	400	441	484	529
$\|B\|$	12	12	12	12	12	12	18	18	18	27	27	36	36	36	36
$\|F\|$	1	1	1	1	1	1	1	3	3	3	3	1	1	1	1
$\|H\|$	1	1	1	1	1	1	1	4	4	4	4	4	4	4	4

5.0.2. DVB-T Network Design The second set of instances refers to networks based on the *Terrestrial Digital Video Broadcasting* technology (DVB-T, ETSI 2006). Indeed, our algorithm has been used to design the *reference networks of the Italian DVB-T plan*, comprising 25 national and hundreds of regional single-frequency networks. Unfortunately, due to non-disclosure agreements, we cannot reproduce and distribute the details of the real life instances. Nevertheless, we have synthesized 9 instances using the same digital terrain and propagation model, the same population database and, finally, the same technical assumptions defined by the *Italian Authority for Telecommunications* (Agcom). As a consequence, our instances and solutions constitute a valid proxy of the real networks planned by the Authority and currently under deployment by the italian broadcasters.

Each instance corresponds to a regional area of Italy, with an extent ranging from about 3.500 to about 30.000 km^2. The network represented in an instance is constituted by a set of transmitters B that synchronously broadcast the same telecommunication service on the same frequency over a target area. Each transmitter can emit by using a subset of power levels in the range [-40,26] dBkW. Service coverage is evaluated in a set of testpoints T and the revenue obtained by covering a testpoint is equal to the population living in the corresponding elementary portion of territory. The coverage is assessed through an adapted version of the SIR inequality (2): the rules of distinction between serving and interfering signals and summation of signals comes from the adoption of *Orthogonal Frequency Division Multiplexing* (OFDM) in the DVB-T technology. For a detailed description of how the SIR inequality is built, we refer the reader to Mannino et al. (2006). The dimension of each instance is shown in Table 2.

We stress that the coefficient matrices associated to the DVB-T instances are in general more ill-conditioned than those associated to the WiMAX instances. This can be intuitively explained by considering that DVB-T networks involve transmitters that are much more powerful than those used by a WiMAX network. Such transmitters are able to broadcast signals at very long distance. As a consequence, weak signals can be picked up also far away from the target area, creating interference that may be very small when compared to the powerful signal of closer serving transmitters (for example, Italian transmitters in Sardegna may interfere transmissions in Tunisia and Southern France). The ratio between the largest and the smallest fading coefficient of a DVB-T SIR inequality is thus in general much larger than that of a WiMAX SIR inequality. Numerical instability phenomena become therefore more marked.

Table 2 Description of the DVB-T test-bed instances

ID	DVB1	DVB2	DVB3	DVB4	DVB5	DVB6	DVB7	DVB8	DVB9
|T|	2003	1741	5618	4466	2704	4421	197	3400	2003
|B|	127	188	411	202	113	215	109	183	127

5.1. Numerical Results and Comparisons

We have pointed out in Section 1 that the solutions to (BM) and (DM) returned by state-of-the-art MILP solvers such as Cplex can be affected by numerical inaccuracy, i.e. the SIR inequalities of testpoints recognized as covered are actually unsatisfied (similar problems were also reported in Kalvenes et al. (2006), Kennington et al. (2010) and Mannino et al. (2009)). We detect such coverage errors by evaluating the solutions *off-line*: after the optimization process, we verify that the SIR inequality corresponding to each nominally covered testpoint is really satisfied by the power vector of the returned solution. This is not the only issue, as, in the case of some instances, (BM) and (DM) can be even wrongly evaluated as infeasible.

In our experience, tuning the parameters of Cplex is crucial to reduce coverage errors and to contain the effects of numerical instability. Furthermore, in the case of (DM), tuning is essential to ensure that the problem is correctly recognized as feasible. After a series of tests, we established that, in the case of (BM) and (DM), an effective setting consists of turning off the *presolve* and on the *numerical emphasis*. Moreover, we turn off the generation of the *mixed-integer rounding cuts* and of the *Gomory fractional cuts* as we observed no advantages in the quality of the bounds and a sensible increase in running times.

Assessing the strength of the Power-Indexed formulation. The first group of experiments is designed to assess the strength of (PI) comparing it with (BM) and (DM). To this end, we focus on a single instance of our test-bed (instance S4 presented in Table 1) and detail the behaviour of WPLAN for each invocation of SOLVE-PI($\mathcal{P}$). The sets of power values in the first three invocations of SOLVE-PI($\mathcal{P}$) are (in dBm) $\mathcal{P}_1 = \{-99, 40\}, \mathcal{P}_2 = \{-99, 20, 30, 40\}$ and $\mathcal{P}_3 = \{-99, 20, 25, 30, 35, 40\}$, respectively. Then, in each of the following invocations, $\mathcal{P}$ is expanded by including two more values (suitably spaced). To analyse the behaviour of the single iterations and establish an effective sequence of power sets, we set a time limit of 1 hour for each invocation of the solution algorithm for (PI) and (DM).

In order to evaluate the quality of (PI) w.r.t. (DM), we apply WPLAN to (DM) (note that in this case the solution procedure SOLVE-PI is replaced by the simple solution of (DM) by Cplex). In Table 3, for each iteration of WPLAN, we report the number |L| of considered power levels, the number of GCIs included in the initial formulation (PI^0) and the number of GCIs separated during the current iteration. Additionally, for both (PI) and (DM), we report the upper bound at node 0 (UB), the value |T*| of the final solution (number of covered testpoints) and the final gap. When the solution contains coverage errors, two values are presented in the |T*| column, namely the nominal value of the best solution returned by Cplex (in brackets) and its actual value computed by re-evaluating the solution *off-line*.

The last line of the table shows the results obtained for (BM) by setting a time limit of 3 hours. Note that in this case, the second column reports the number of SIR (big-M)

constraints (5) included in (BM). This number is by definition also the number of SIR (big-M) constraints (8) included in (DM).

Table 3 Behaviour of WPLAN for instance S4

	GCIs		(PI)			(DM)		
|L|	init	added	UB	|T*|	gap%	UB	|T*|	gap%
2	5743	17	199.2193	106	0.00	218.3465	91	125.65
4	9035	7	204.2500	111	0.00	219.0015	97 (98)	102.68
6	14312	13	206.6261	111	59.03	219.3488	100 (101)	115.70
8	17142	45	209.4200	111	67.51	219.7349	100 (101)	122.98
10	24638	6	210.0000	111	79.99	220.2788	100 (101)	123.14
12	27799	1	211.7000	111	82.05	219.9144	100 (101)	124.01
14	35944	0	212.0000	111	83.46	220.1307	100 (101)	123.58
16	38496	10	214.5930	111	85.48	220.3000	100 (101)	125.00
18	45425	2	215.8000	111	86.44	220.1091	100 (101)	124.83
20	48918	2	218.0000	111	89.99	220.0560	100 (101)	125.00
22	57753	3	218.0000	111	90.83	220.3720	100 (101)	125.00
(BM)	1170	-	221.3925	93	97.18	-	-	-

The figures in Table 3 are representative of the typical behaviour of WPLAN on all instances of our test-bed. They allow us to make some relevant observations. First, the size of (PI) grows quickly with the number of power levels, and is typically much larger than that of (BM) and (DM). This is counterbalanced by the quality of the upper bounds, which are consistently better for (PI) and, most important, the quality of the solutions found. Interestingly, the best solution is found quite early in the iterative procedure, namely for $|\mathcal{P}| \leq 6$. A similar behaviour is observed for the other WiMAX instances reported in Table 5 and the DVB-T instances in Table 7 as well. This motivated our choice of the sequence of feasible power values in the final version of WPLAN for WiMAX: most of the computational effort is concentrated on small cardinality power sets, and only one large set. More precisely, there will be only 4 iterations, corresponding to 2, 4, 6 and 22 power levels, respectively.

Finally, we note that the number of generated GCIs is small. Also, in most cases the GCIs include only two interferers, and in any case never more than three. In other words, even though many interferers can reach a given testpoint, only very few of them (in most cases only one) give a significant contribution to the overall interference.

The performance of the Power-Indexed approach over the test-bed. In this subsection, we comment the results over our WiMAX and DVB-T benchmark instances. For an exhaustive report of the results through tables, we refer the reader to the Appendix of this paper. In all experiments, we set a time limit of 3 hours for the solution of (BM) and (DM) and for WPLAN applied to (PI). As in subsection 5.1, we solve (DM) by an adapted version of WPLAN (we recall that in this case the solution procedure SOLVE-PI is replaced by the simple solution of (DM) by Cplex).

Besides the results obtained by solving the "pure" models (BM) and (DM), we report also the results obtained by trying to stabilize (BM) and (DM) through Cplex *indicator constraints* and by strengthening (DM) through a suitable subset of our GCIs. The indicator constraints constitute a way to express relationships between variables and may reduce the flaws of big-M formulations. In our work, we check if declaring the big-M coverage constraints of (BM) and (DM) by means of Cplex indicator constraints (Cplex 2010) can improve the quality of solutions. We denote the resulting formulations by adding the symbol "+" to the acronym (e.g., BM+). Furthermore, we investigate if it is convenient to strengthen (DM) by simply including the GCIs corresponding with the single-interferer condition (i.e., $|\Gamma| = 1$, see Section 4). We denote the resulting formulation by *(DM & GCI1)*. This investigation is motivated by the fact that such subset of GCIs seems to be very effective to discover high quality solutions fast.

The results show that WPLAN applied to (PI) outperforms (BM) and (DM) in terms of quality of the solutions found and, in most cases, running times to obtain them (the running times obviously include also the time spent by the separation oracle). Coverage errors, in particular, are completely eliminated. Even if in principle the reduced and quite small number of power values considered by WPLAN could result in poorer coverage w.r.t. (BM), the results clearly show that this is not the case. On one hand, this happens as a small number of well-spaced power values suffices in practice to obtain good coverage; indeed, it is common practice in WiMAX network planning to neglect intermediate values, i.e. a device is either switched-off or activated at its maximum power (Ridolfi 2010). On the other hand, the size of the (BM) formulation and the ill-conditioned constraint matrix, along with the presence of the big-M coefficients, makes the solution process unstable, the solutions found unreliable and the branching tree extremely large. Indeed, due to rounding errors and numerical instability, several solutions to (BM) turn out to be infeasible when verified off-line. The effects of numerical instability become more marked in the case of the DVB-T test bed: the feasible solutions to (BM) of all but one of the instances contain coverage errors that entail the loss of up to 20.000 users. Furthermore, in contrast to the good performance of WPLAN, several instances seem to be very difficult for (BM) and no feasible solution is retrieved by the time limit.

WPLAN applied to (PI) also outperforms (DM) solved by the adapted WPLAN algorithm. The results show that in general the simple discretization of the power range does not suffice to get better solutions than those obtained by (BM). Indeed, in many cases, the performance of (DM) is worse than that of (BM) and coverage errors are still strongly present.

Finally, after having pointed out the advantages of a pure GCI-based approach, we assess if stabilizing (DM) by indicator constraints or strengthening (DM) by GCIs can lead to remarkable advantages. The results reported show that stabilizing by indicator constraints does not allow to reach the quality of the solutions obtainable by the pure GCI formulation (PI). This behavior can be explained by the simple observations that (PI) allows us to get rid of the major sources of instability in (BM) and (DM), namely the bad-conditioned coefficients of the constraint matrix (not affected by the use of Cplex indicator constraints) and the big-M coefficients. Though in a significative number of cases the value of the best solution is higher than that of solutions obtained by pure (BM) and (DM), coverage errors are still (heavily) present. Moreover, the value of the solutions is anyway lower than those obtained by (PI). Strengthening (DM) by GCIs seems to be more effective than stabilizing by indicator constraints: in many cases (DM & GCI1) reaches better solutions w.r.t. (DM) and (DM+) However, also in this case, there are still a few solutions that contain errors and the final value is anyway lower than that obtained through (PI). Finally, also in the case of (DM+ & GCI1), the stabilization by indicator constraints seems to decrease the value of solutions obtained within the time limit, without being able to completely avoid coverage errors.

Comparisons between warm and cold start for (PI). Finally, in Table 4 we show the impact of the iterative approach WPLAN on the quality of the solutions found for (PI) in the case of the WiMAX instances. A similar behaviour is observed also in the case of the DVB-T instances. In particular we compare *cold starts*, which correspond to invoking SOLVE-PI($\mathcal{P}$) without benefiting from cuts and lower bounds obtained at former invocations, with *warm starts* which, in contrast, make use of such information. The value of the best solutions found during successive invocations of SOLVE-PI both under warm and cold starts are shown in the columns identified by $|L| = n$, where n denotes the number of corresponding power levels. The value of the best solution found at the first invocation is in column $|L| = 2$, while the value of the best solution and the number of levels used to find it are shown in column $|T^*|$ and $|L^*|$, respectively.

Table 4 **Comparisons between warm and cold starts**

ID	|T*|	|L*|	|L|=2	WARM START |L|=4	WARM START |L|=6	COLD START |L|=4	COLD START |L|=6
S1	74	6	69	72	74	71	58
S2	107	4	72	107	107	80	63
S3	113	4	83	113	113	108	101
S4	111	4	75	111	111	100	97
S5	86	6	76	84	86	83	81
S6	170	4	127	170	170	110	127
S7	341	4	296	341	341	314	196
R1	400	2	400	-	-	399	304
R2	441	4	416	441	-	394	355
R3	427	2	427	427	427	414	Out
R4	529	2	529	-	-	512	Out
Q1	67	2	67	67	67	*	*
Q2	211	4	196	211	211	156	Out
Q3	463	2	463	463	463	Out	Out
Q4	491	2	491	491	491	Out	Out

For all S-instances the best solution can be found only due to warm start. Note that SOLVE-PI encounters increasing difficulties in finding good solutions as the number of power levels increases (in the case of the apparently hard instance Q1, for 3 and 5 power levels, no feasible solution is found within the time limit when cold start is adopted). This is mainly due to the large size of the corresponding instances, that, in some cases denoted by *Out*, makes Cplex run out of memory while building the model. However, a good initial solution provided to SOLVE-PI can be improved in most cases. We have already observed that for a larger number of levels (i.e. > 6), no improved solutions can be found for all the WiMAX instances in our test-bed. Finally, for R1 and R4 a solution covering the entire target area is found already with $|L| = 2$, while for R2 such a solution is found with $|L| = 4$ (and warm-start).

6. Conclusions.

The coverage condition in wireless network design problems is typically modeled by linearizing the signal-to-interference ratio and by including the notorious big-M coefficients. The resulting Mixed-Integer Programs are very weak and ill-conditioned, hence unable to solve large instances of real networks. In this paper, we show how power discretization, a common modeling approach among professionals, can constitute the first step to define formulations that are noticeably stronger than the classical ones. These pure 0-1 formulations are based on GUB cover inequalities, that completely eliminates the source of numerical instability. This new Power-Indexed approach outperforms the classical big-M models, both in terms of quality of solutions found and of strength of the bound, as showed by an extensive computational study on real WiMAX and DVB-T instances.

Appendix. Tables of comparison.

In this appendix, we present tables that exhaustively report the computational results about comparisons between (BM), (DM) and (PI), that we have commented in Section 5.1. The results are reported in Tables 5 and 6 for the WiMAX instances and in Tables 7 and 8 for the DVB-T instances. All results are obtained by setting a time limit of 3 hours. We recall that (BM+), (DM+) respectively denote the versions of (BM),(DM) stabilized through Cplex *indicator constraints*, whereas (DM & GCI1) denotes the version of (DM) strengthened through the GCIs corresponding with the single-interferer condition (i.e., $|\Gamma| = 1$).

The value of the best solutions found within the time limit is shown in column |T*| for WiMAX and column *COV%* for DVB-T (*COV%* is the percentage of population covered with service). The *gap%* columns report the *nominal* (i.e., before checking solution correctness) percentage gap between the upper and lower bound at termination, the *time* column specifies when the best solution is found (in *seconds*), whereas the last column |L*| is the number of power levels used in the iteration in which WPLAN obtains the best solution. In the columns |T*| and *COV%*, the expression "Out" indicates that Cplex runs out of memory while building the model. Finally, to denote some specials situations, we adopt the following conventions: i) the expression *"None by TL"* indicates that the solver is not able to find a feasible solution within the time limit; ii) the expression *"Infeasible*"* indicates that the solver wrongly considers the problem as being infeasible.

We briefly resume the three main observations that can be made on the basis of the results (discussed in more detail in Section 5.1): 1) WPLAN applied to (PI) outperforms (BM) and (DM) for all the WiMAX and DVB-T instances; 2) in most cases, the use of indicator constraint leads to finding solutions of lower value than those provided by pure (DM) and this reduction in value is not compensated by a complete elimination of coverage errors; 3) strengthening (DM) by GCIs in general enhances the solving performance, but solutions containing errors are still generated.

The higher performance of our approach based on the Power-Indexed formulation (PI) is especially apparent for all of the DVB instances and the WiMAX R-instances. In particular, several instances seem to be quite easy for WPLAN but very difficult for (BM) and (DM). Indeed, when no time limit is imposed to the solution of (BM), Cplex runs out of memory after about ten hours of computation without getting sensible improvements in the bounds. On the contrary, in the case of WiMAX instances like R1, R2 and R4, SOLVE-PI($\mathcal{P}$) finds the optimum solution (when $|\mathcal{P}| = 2$) in less than 1 hour. The higher performance is also highlighted in the case of instance Q1 that turns out to be hard: both (BM) and (DM) with 2 power levels cannot find any feasible solution with non-zero value within the time limit, while, in contrast, (PI) finds a solution with value 67.

Table 5 Comparisons between (BM), (DM), (DM & GCI1) and WPLAN (WiMAX instances)

ID	|T|	(BM)			(DM)			(DM & GCI1)			WPLAN		
		|T*|	gap% (nom)	time	|T*|	time	|L*|	|T*|	time	|L*|	|T*|	time	|L*|
S1	100	63 (78)	13.72	10698	65 (70)	8705	6	67	8275	6	74	10565	6
S2	169	99 (100)	56.18	10705	86	6830	4	101	6405	4	107	5591	4
S3	196	108	79.54	4010	61 (100)	4811	4	95	6908	4	113	5732	4
S4	225	93	103.43	10761	100 (101)	6507	6	100	6891	4	111	7935	4
S5	289	77	202.24	10002	73 (76)	4602	4	72 (82)	7088	4	86	10329	6
S6	361	154	130.76	8110	121 (138)	5310	4	149	5724	4	170	8723	4
S7	400	259 (266)	49.67	8860	120 (121)	4100	4	239	6003	4	341	7154	4
R1	400	370	7.57	10626	284 (328)	1066	2	304	3424	2	400	1579	2
R2	441	302 (303)	45.03	3595	393 (394)	4713	4	375 (384)	4371	4	441	1244	4
R3	484	99	385.86	10757	188	2891	2	306	3440	2	427	3472	2
R4	529	283 (286)	84.96	10765	307	3026	2	399	3152	2	529	2984	2
Q1	400	0	-	-	0	-	-	37	3108	2	67	2756	2
Q2	441	191	130.89	9124	158 (179)	6282	4	156	6932	4	211	7132	4
Q3	484	226	112.83	3392	290 (292)	2307	2	316	3091	2	463	3323	2
Q4	529	145 (147)	264.83	6623	273 (280)	1409	2	343	2248	2	491	3053	2

Table 6 Comparisons between (BM+), (DM+), (DM+ & GCI1) and WPLAN (WiMAX instances)

ID	|T|	(BM+) |T*|	(BM+) gap% (nom)	(BM+) time	(DM+) |T*|	(DM+) time	(DM+) |L*|	(DM+ & GCI1) |T*|	(DM+ & GCI1) time	(DM+ & GCI1) |L*|	WPLAN |T*|	WPLAN time	WPLAN |L*|
S1	100	63 (71)	24.48	7556	53 (56)	7551	6	68 (70)	8650	6	74	10565	6
S2	169	99 (100)	65.92	10322	73	7036	4	67	8816	4	107	5591	4
S3	196	101 (103)	85.43	10241	Infeasible*	-	-	72 (75)	7101	4	113	5732	4
S4	225	71	179.2	8213	97 (102)	6820	4	93	6566	4	111	7935	4
S5	289	69	262.55	6561	75	6695	6	75	6710	4	86	10329	6
S6	361	81 (107)	235.51	5630	98 (116)	5672	4	108	6102	4	170	8723	4
S7	400	238	67.23	7141	157 (241)	6008	4	158	7059	4	341	7154	4
R1	400	309 (340)	17.05	4320	292	2355	2	188	3004	2	400	1579	2
R2	441	329	30.06	7809	296	4682	4	371	4798	4	441	1244	4
R3	484	0	-	-	185 (201)	3150	2	374	3256	2	427	3472	2
R4	529	249 (253)	112.44	9203	278	2933	2	329	2871	2	529	2984	2
Q1	400	0	-	-	0	-	-	53	3400	2	67	2756	2
Q2	441	115	283.47	6135	137	5024	4	128	7082	4	211	7132	4
Q3	484	238 (263)	82.88	5162	258	2644	2	291	2808	2	463	3323	2
Q4	529	252	109.92	7212	378 (416)	2118	2	341	2391	2	491	3053	2

Table 7 Comparisons between (BM), (DM), (DM & GCI1) and WPLAN (DVB-T instances)

ID	(BM) COV%	(BM) gap%	(BM) time	(DM) COV%	(DM) time	(DM) |L*|	(DM & GCI1) COV%	(DM & GCI1) time	(DM & GCI1) |L*|	WPLAN COV%	WPLAN time	WPLAN |L*|
DVB1	95.10 (95.49)	2.67	8707	96.76	10020	6	94.80	7817	6	97.26	7193	6
DVB2	96.15 (96.51)	1.35	9510	89.75 (96.98)	8639	6	96.03	9194	6	97.14	9305	6
DVB3	None by TL	-	-	70.43	6155	4	70.80	6891	4	71.08	6544	4
DVB4	None by TL	-	-	77.55 (83.50)	5650	4	80.19 (84.85)	7123	4	88.98	725	2
DVB5	94.94 (96.18)	0.68	9804	92.25 (94.30)	7701	6	95.93 (96.11)	8826	8	96.25	9677	8
DVB6	None by TL	-	-	71.73	5922	4	74.09	6421	4	74.55	5840	4
DVB7	94.82 (100.00)	0.00	65	80.49 (100.00)	293	8	96.84 (99.83)	311	10	99.47	182	9
DVB8	78.63	22.24	6621	84.35	6212	4	83.54	6049	4	84.35	6293	4
DVB9	None by TL	-	-	95.26	5003	4	95.57	5447	4	96.60	1538	2

Table 8 Comparisons between (BM+), (DM+), (DM+ & GCI1) and WPLAN (DVB-T instances)

ID	(BM+) COV%	(BM+) gap%	(BM+) time	(DM+) COV%	(DM+) time	(DM+) |L*|	(DM+ & GCI1) COV%	(DM+ & GCI1) time	(DM+ & GCI1) |L*|	WPLAN COV%	WPLAN time	WPLAN |L*|
DVB1	95.38 (95.50)	2.85	9203	93.45 (96.62)	9642	6	94.27	8559	6	97.26	7193	6
DVB2	96.95	0.90	9940	96.22 (96.77)	10077	6	94.59	9803	6	97.14	9305	6
DVB3	None by TL	-	-	69.94	6774	4	70.21	7105	4	71.08	6544	4
DVB4	65.53 (65.65)	49.18	9120	83.07 (86.67)	6910	4	77.57 (86.02)	7008	4	88.98	725	2
DVB5	95.14 (95.41)	1.61	6194	94.03	9180	6	95.02	10092	8	96.25	9677	8
DVB6	None by TL	-	-	70.56 (70.96)	6701	4	70.30	6338	4	74.55	5840	4
DVB7	96.91 (100.00)	0.00	244	92.87 (99.81)	573	10	98.10	414	10	99.47	182	9
DVB8	58.51	64.24	10086	80.09 (80.12)	7050	4	83.80	7122	4	84.35	6293	4
DVB9	None by TL	-	-	95.85	6108	4	94.44	6966	4	96.60	1538	2

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